A Standard Task-Based Solow Model Delivers AI-Driven Double-Digit Growth*

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Abstract

This supplement to our blog post shows that it is easy to write down a theoretical model that delivers AI-driven explosive growth in the short term. We embed a task-based production function into an otherwise standard Solow model. The share of automated tasks α equals the capital share of a Cobb-Douglas production function. With a standard calibration (initial $\alpha = 1/3$, saving rate 20%, depreciation 5%) and technology growth chosen to deliver 2% GDP growth, we ask what happens as α increases over time. Full automation $\alpha \to 1$ eliminates diminishing returns to capital: the economy becomes an "AK" economy, and continued technology growth then produces a growth explosion — GDP grows double-exponentially, with a growth rate that itself grows exponentially. Our main quantitative exercise is a gradual transition to full automation over 2025–2045: GDP growth accelerates from 2% to about 12% by 2035 and, after a brief dip, rises without bound thereafter, while the labor share converges to zero. Letting the saving rate respond to the exploding return to capital accelerates the take-off substantially: with a modest increase in the saving rate, growth reaches 15% per year already in 2034 and 20% by 2045.

1 Model Setup

The model combines two standard building blocks. The accumulation side is the Solow (1956) growth model: a fixed — or, in a variant, interest-sensitive — fraction of output is

*Most of this document and the code producing the figures and tables are AI-generated. A replication package is available here.

invested in capital. The production side is a task-based model in which output requires the completion of tasks that can be performed by workers or, when automated, by machines — the particular formulation follows Moll (2025) and Aghion, Jones and Jones (2019). This section describes the two blocks in turn and then our calibration.

1.1 Capital accumulation

Capital accumulation follows the standard Solow model: a constant fraction s of output Y is invested, capital K depreciates at rate δ, and labor supply is constant (normalized to $L = 1$). Aggregate investment I and consumption C are

$$I = sY, \quad C = (1 - s)Y$$

and capital accumulation is $\dot{K} = I - \delta K$ so that

$$\dot{K} = sY - \delta K. \tag{1}$$

The fixed saving rate is a strong assumption in the present context: automation will turn out to generate an enormous rise in the return to capital, and a fixed s lets none of it feed back into accumulation. In our quantitative experiments we therefore also consider a constant-elasticity generalization in which saving responds to the capital return R,

$$s(t) = 1 - (1 - s_0)\left(\frac{R(t)}{R_0}\right)^{-\sigma}, \tag{2}$$

so that the consumption rate C/Y falls with elasticity σ as the rental rate rises above its initial value R_0. Before automation begins, R is constant at R_0, so $s = s_0$ and the model is exactly the Solow model above; $\sigma = 0$ recovers it at all dates. The parameter σ plays the role of the intertemporal elasticity of substitution in a Ramsey model.[1]

[1] Under full automation GDP is asymptotically pure capital income, $Y = RK$, so (2) implies consumption per unit of wealth $C/K = (1 - s_0)R_0^{\sigma} R^{1-\sigma}$ — the continuous-time analogue of the textbook result that a CRRA household with IES σ and discount factor β facing a linear return R saves the fraction $(\beta R)^{\sigma}/R$ of its wealth. At $\sigma = 1$ the marginal propensity to consume out of wealth is independent of the return, the signature of logarithmic utility: a log-utility Ramsey household with discount rate disciplined by the pre-automation BGP ($r_0 - g_Y^* = 6.7\% - 2\% = 4.7\%$) would choose $s = 1 - 0.047/R$ under full automation, of the same form as (2) with $\sigma = 1$, which gives $s = 1 - 0.093/R$.

1.2 Production: a task-based model

Production requires completing a list of *tasks* $i = 1, \dots, N$. Output of the final good is a symmetric Cobb-Douglas aggregate of "task services" X_i:

$$Y = AN\, X_1^{1/N} X_2^{1/N} \cdots X_N^{1/N}, \tag{3}$$

where A is a technology parameter which grows exogenously at rate g_A. The normalization N ensures that A is measured in aggregate rather than per-task units: when total inputs are spread evenly across the N tasks, each task receives $1/N$ of them, and the factor N exactly undoes this division.

Each task can be produced with labor; in addition, a subset of tasks is *automatable*, meaning it can also be produced with capital:

$$X_i = \begin{cases} L_i & \text{if task } i \text{ is not automatable,} \\ L_i + \varphi K_i & \text{if task } i \text{ is automatable,} \end{cases} \tag{4}$$

where K_i, L_i denote capital and labor used in task i, and φ is *machine productivity*: one unit of capital does the work of φ workers. The key idea is that *machines substitute for tasks, not jobs* (see Zeira, 1998; Acemoglu and Restrepo, 2018). Let n denote the number of automatable tasks and

$$\alpha = \frac{n}{N} = \text{share of automatable tasks.} \tag{5}$$

Automation is an increase in α. The economy's resource constraints are $\sum_i K_i = K$ and $\sum_i L_i = L$, where K is the capital stock accumulated according to (1).

Factor markets are competitive: in equilibrium, the wage w and the rental rate of capital R equal the marginal products of the two factors given the optimal allocation of inputs across tasks,

$$w = \frac{\partial Y}{\partial L}, \qquad R = \frac{\partial Y}{\partial K}. \tag{6}$$

At each date, competition among firms allocates the aggregate stocks K and L efficiently across tasks — only the intertemporal allocation is pinned down by the Solow saving rule (1) rather than by optimization — and this within-period efficient allocation yields a simple reduced form.

Proposition 1 (Reduced-form production function)**.** *Efficiently allocating capital and labor across tasks in* (3)–(4) *yields*

$$Y = \begin{cases} Z\,K^{\alpha}L^{1-\alpha}, \quad Z = \dfrac{\varphi^{\alpha}A}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} & \textit{if } \dfrac{L}{\varphi K} < \dfrac{1-\alpha}{\alpha} \quad (\textit{“capital cheaper”}), \\ A\,(\varphi K + L) & \textit{if } \dfrac{L}{\varphi K} \geq \dfrac{1-\alpha}{\alpha} \quad (\textit{“cost parity”}). \end{cases} \tag{7}$$

In the capital-cheaper regime machines do tasks strictly more cheaply than workers, $R/\varphi < w$, and all automatable tasks are fully automated; in the cost-parity regime labor also works in automatable tasks and machines and workers do tasks at the same cost, $w = A$ and $R = \varphi A$, so $w = R/\varphi$.

Proof. By symmetry, the optimal allocation equalizes X_i within the automated group and within the non-automated group. Let $L_a \in [0, L]$ denote labor allocated to automatable tasks; then $X_i = (\varphi K + L_a)/n$ for automatable tasks and $X_i = (L - L_a)/(N - n)$ for the others, and maximizing (3) amounts to maximizing

$$\alpha \log \frac{\varphi K + L_a}{n} + (1-\alpha) \log \frac{L - L_a}{N - n}$$

over L_a. The objective is strictly concave with derivative $\frac{\alpha}{\varphi K + L_a} - \frac{1-\alpha}{L - L_a}$. If $L/(\varphi K) < (1-\alpha)/\alpha$, the derivative is negative at $L_a = 0$, so the corner $L_a = 0$ is optimal: capital is spread evenly over automatable tasks ($K_i = K/n$) and labor over the rest ($L_i = L/(N-n)$). The Cobb-Douglas marginal products are then $w = (1-\alpha)Y/L$ and $R = \alpha Y/K$, so $R/\varphi < w$ is equivalent to $\alpha L < (1-\alpha)\varphi K$, the capital-cheaper condition. Substituting into (3),

$$Y = AN \left(\frac{\varphi K}{n}\right)^{n/N} \left(\frac{L}{N-n}\right)^{(N-n)/N} = A \left(\frac{\varphi K}{\alpha}\right)^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha} = ZK^{\alpha}L^{1-\alpha}.$$

If instead $L/(\varphi K) \geq (1-\alpha)/\alpha$, the first-order condition holds at an interior L_a that equalizes task services across *all* tasks, $X_i = (\varphi K + L)/N$, which is feasible precisely under this condition. Then $Y = A(\varphi K + L)$, with marginal products $w = A$ and $R = \varphi A$. □

In the capital-cheaper regime all automatable tasks are automated, and the task model delivers a Cobb-Douglas production function whose capital share *equals the share of automated tasks* α. The cost-parity branch is usually ignored, but it becomes relevant below: as $\alpha \to 1$ the threshold $(1-\alpha)/\alpha \to 0$, so the economy necessarily ends up in that regime.

1.3 Calibration

Table 1 summarizes our calibration, which is as standard as possible. The initial share of automatable tasks is $\alpha = 1/3$, the usual capital share. The saving rate is $s = 0.2$ and depreciation $\delta = 0.05$. We choose g_A so that GDP growth on the pre-automation balanced growth path is 2% per year; Section 2 shows this requires $g_A = (1-\alpha) \times 2\% = 1.33\%$, which equals measured TFP growth in the model and is in line with postwar US data. We normalize $Y = L = 1$ at the initial (detrended) steady state, implying $K(0) = K/Y = 2.86$, a rental rate $R = \alpha Y/K = 11.7\%$, and a wage $w = 1 - \alpha = 2/3$.

Machine productivity φ requires more care, for two reasons. First, it is *not identified by pre-automation aggregates*: by (7), φ enters output only through the product $\varphi^\alpha A$ in the TFP term Z, so it is absorbed into measured TFP. Second, the model has a single φ: the machines that automated the first third of tasks and the AI systems that will automate the rest are treated identically. We calibrate φ from an outside moment at the start of automation: the cost of a machine performing a task relative to a worker. Replacing one worker at a task requires $1/\varphi$ units of capital at annual user cost R/φ (the rental rate equals the net return plus depreciation, $R = r + \delta$), so this cost ratio is

$$\rho \equiv \frac{R/\varphi}{w} \qquad \Longrightarrow \qquad \varphi = \frac{R}{\rho\, w}. \tag{8}$$

We set $\rho = 0.10$ — machines perform the tasks they can do at one-tenth the cost of the workers they replace, loosely motivated by the compute cost of current AI systems relative to wages for comparable work. With $R = 11.7\%$ and $w = 2/3$ this yields $\varphi = 1.75$; the implied technology level is $A(0) = 0.31$. Note that ρ is an equilibrium object rather than a parameter: along the pre-automation BGP the wage grows while R is constant, so ρ drifts down over time, and $\rho = 0.10$ disciplines its level at the moment automation begins. Since post-automation growth scales with $\varphi^{1-\alpha}$ (Section 3), this moment does real quantitative work.

2 Pre-Automation Balanced Growth Path

Before automation begins, $\alpha = 1/3$ is constant and A grows at rate g_A. The capital-cheaper condition in Proposition 1 holds ($L/(\varphi K) = 0.20 < (1-\alpha)/\alpha = 2$, and it continues to hold as K grows), so

$$Y = Z K^\alpha L^{1-\alpha}:$$

Parameter	Value	Target / source
α (initial share of automatable tasks)	1/3	capital share
s (saving rate)	0.20	investment/GDP
δ (depreciation rate)	0.05	standard
g_A (technology growth)	0.0133	2% GDP growth pre-automation
φ (machine productivity)	1.75	machine/worker task cost $\rho = 0.10$
L (labor)	1	normalization
Implied: K/Y	2.86	(data: ≈ 3)
Implied: TFP growth	1.33%	(data: $\approx 1.3\%$)
Implied: $r = \alpha Y/K - \delta$	6.7%	
Implied: $A(0)$	0.31	

Table 1: Calibration.

a standard Cobb-Douglas economy whose measured TFP Z grows at rate g_A.

Balanced growth path. Let $x \equiv K/Y$ denote the capital-output ratio. Taking logs and time derivatives of the production function, $g_Y = g_A + \alpha g_K$, and from (1), $g_K = s/x - \delta$. Hence

$$\frac{d \log x}{dt} = g_K - g_Y = (1-\alpha)\left(\frac{s}{x} - \delta\right) - g_A, \tag{9}$$

whose right-hand side is strictly decreasing in x: the capital-output ratio converges monotonically to a unique balanced growth path (BGP). On the BGP, $g_K = g_Y \equiv g_Y^*$, so

$$g_Y^* = \frac{g_A}{1-\alpha} \qquad \text{and} \qquad x^* = \frac{s}{g_Y^* + \delta}. \tag{10}$$

Quantitative results. With $\alpha = 1/3$, equation (10) gives $g_Y^* = \frac{3}{2} g_A$, so our 2% growth target requires TFP growth of 1.33% per year — close to its postwar US average. The capital-output ratio is $x^* = 0.20/0.07 = 2.86$, close to the US value of about 3. The net return to capital is $r = \alpha Y/K - \delta = 6.7\%$; the wage grows at 2% per year; the labor share is constant at $1 - \alpha = 67\%$. Note the standard Solow logic underlying (10): because of diminishing returns to capital ($\alpha < 1$), all long-run growth comes from $g_A > 0$, amplified by the factor $1/(1-\alpha) = 1.5$ through induced capital accumulation; if $g_A = 0$, growth peters out.

3 Full Automation Limit

Now consider the limit $\alpha \to 1$: all tasks can be performed by machines. Since the threshold $(1-\alpha)/\alpha \to 0$, the cost-parity branch of (7) applies:

$$Y = A\,(\varphi K + L), \tag{11}$$

with $w = A$ and $R = \varphi A$: labor simply works alongside machines at task-cost parity. Since K grows without bound while L is fixed, $Y \approx \varphi AK$ asymptotically — production is linear in capital. The task-based Solow model becomes an *AK model*: diminishing returns to capital vanish because there are no longer any tasks that machines cannot do.

3.1 From AK to double-exponential growth

To see what the AK structure implies, consider first a constant technology level A. Combining (11) with (1),

$$\frac{\dot K}{K} = s\varphi A\left(1 + \frac{L}{\varphi K}\right) - \delta \;\longrightarrow\; g = s\varphi A - \delta,$$

the standard AK result: capital accumulation alone sustains perpetual exponential growth — at a *constant* rate that is increasing in the level of technology φA. This is the key stepping stone: under full automation, the level of technology determines the *growth rate* of the economy, not just its level.

It follows that when A continues to grow at g_A, so that $A(t) = A_0 e^{g_A t}$, the growth rate itself must grow. Ignoring the (asymptotically negligible) L term, capital accumulation becomes

$$\frac{\dot K}{K} = s\varphi A_0 e^{g_A t} - \delta,$$

which integrates to

$$\log K(t) = \log K(0) + \frac{s\varphi A_0}{g_A}\left(e^{g_A t} - 1\right) - \delta t. \tag{12}$$

GDP grows *double-exponentially*: $\log Y$ itself grows exponentially. Equivalently, the growth rate is no longer constant but grows exponentially at rate g_A,

$$g_Y(t) \approx s\varphi A_0 e^{g_A t} - \delta + g_A. \tag{13}$$

The interaction is key: technology growth that previously produced a constant 2% now produces an ever-accelerating growth rate, because each improvement in A raises the return to capital accumulation permanently rather than being absorbed by diminishing returns. This is the sense in which "AK plus growing A" generates a growth explosion (cf. Aghion, Jones and Jones, 2019).

Two structural properties of (13). First, the *pace* of the explosion is asymptotically governed by g_A: the growth rate (net of the constant $-\delta + g_A$) doubles every $\log(2)/g_A \approx 52$ years, so at the historical pace of technology growth the explosion unfolds over decades rather than years; a faster g_A — e.g. if AI also accelerates the growth of A itself — would compress this timeline. Second, machine productivity shifts the *timing*: since $\varphi A_0 \propto \varphi^{1-\alpha}$ (recall φ^α is absorbed into pre-automation TFP), raising φ translates the entire explosion path forward in time by $(1-\alpha)\log\varphi/g_A$ — about 28 years for our $\varphi = 1.75$ relative to $\varphi = 1$. This full-automation economy is a building block rather than an exercise of independent interest: it is the long-run limit of the gradual transition to which we now turn.

4 The Transition to Full Automation

Finally, consider a gradual transition. The economy starts on its pre-automation BGP; automation begins in 2025 and the share of automatable tasks rises smoothly from 1/3 to 1 over 20 years:

$$\alpha(t) = \frac{1}{3} + \frac{2}{3}\left(3\tau(t)^2 - 2\tau(t)^3\right), \qquad \tau(t) = \min\{t/20,\, 1\}, \tag{14}$$

where t denotes years since 2025 — an S-shaped path with $\dot{\alpha} = 0$ at both ends, shown in Figure 1. Technology A grows at $g_A = 1.33\%$ throughout. We solve (1) with the two-regime production function (7) numerically, both for the baseline fixed saving rate and for the endogenous saving rate (2) with $\sigma = 0.1$ (code in `codes/task_solow.py`). Table 2 and Figures 2–3 report the results.

GDP and growth. Growth accelerates from 2% to a peak of about 12.5% around 2035 and rises without bound after a brief dip:[2] 15% by 2066, 20% by 2085, 37% by 2125.

[2]The dip — from 12.5% around 2035 to 10.1% in 2041 — reflects the end of the automation dividend. In the capital-cheaper regime, $g_Y = g_A + \alpha g_K + \dot{\alpha}\log\left(\frac{\varphi w}{R}\right)$: ongoing automation contributes $\dot{\alpha}$ times the log cost gap between workers and machines, which starts at $\log(1/\rho) = \log 10$ and shrinks to zero at the regime switch, where cost parity $w = R/\varphi$ holds and output no longer depends on α. Growth then falls back to what technology and capital deepening deliver until the accelerating AK dynamics take over.

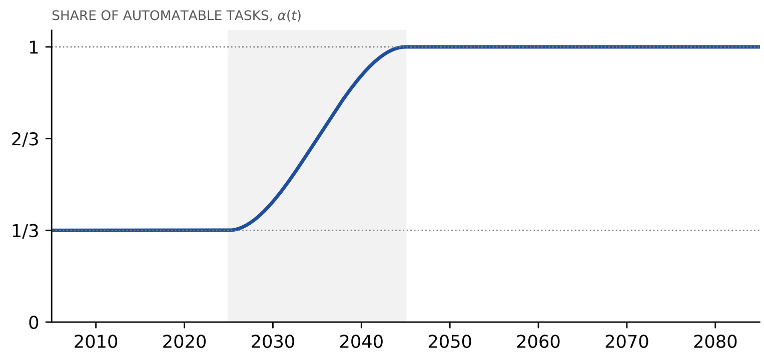

Figure 1: The automation path: the share of automatable tasks $\alpha(t)$ rises smoothly from 1/3 to 1 between 2025 and 2045, equation (14). Here and in subsequent figures, the shaded band is the 2025–2045 automation phase.

Year	2030	2035	2040	2045	2065	2085	2125
α	0.44	0.67	0.90	1	1	1	1
GDP growth (%/yr)	10.4	12.5	10.7	10.7	14.8	20.4	37.4
GDP ($Y(2025) = 1$)	1.4	2.5	4.6	7.8	97	3,225	2.5×10^8
GDP / no-automation GDP	1.3	2.1	3.4	5.2	44	971	3.4×10^7
GDP growth, endog. saving (%/yr)	11.1	16.3	19.4	19.9	28.6	40.5	78.3

Table 2: The transition to full automation: α rises from 1/3 to 1 over 2025–2045.

There is no post-transition BGP. The level effects compound quickly: by 2045 GDP is 7.8 times its 2025 level and 5.2 times the no-automation counterfactual; by 2125 it exceeds the counterfactual by a factor of 3×10^7.

The regime switch. In 2041, when $\alpha \approx 0.92$ and $K \approx 6.9$, the capital-cheaper condition $L/(\varphi K) < (1 - \alpha)/\alpha$ fails and the economy switches to the cost-parity regime: so many tasks have been automated that the displaced labor's wage has been pushed down to the machines' cost of performing tasks, $w = R/\varphi$, and machines lose their strict cost advantage. From then on production is effectively $Y = A(\varphi K + L)$ and the economy behaves as an AK economy *before* automation is even complete.

Endogenous saving. The green lines in Figure 2 replace the fixed saving rate with the interest-sensitive rule (2) and $\sigma = 0.1$. The motivation is that the transition generates an enormous rise in the return to capital which the fixed s entirely ignores: the rental rate rises from 11.7% in 2025 to 67% at the regime switch and keeps growing, while under

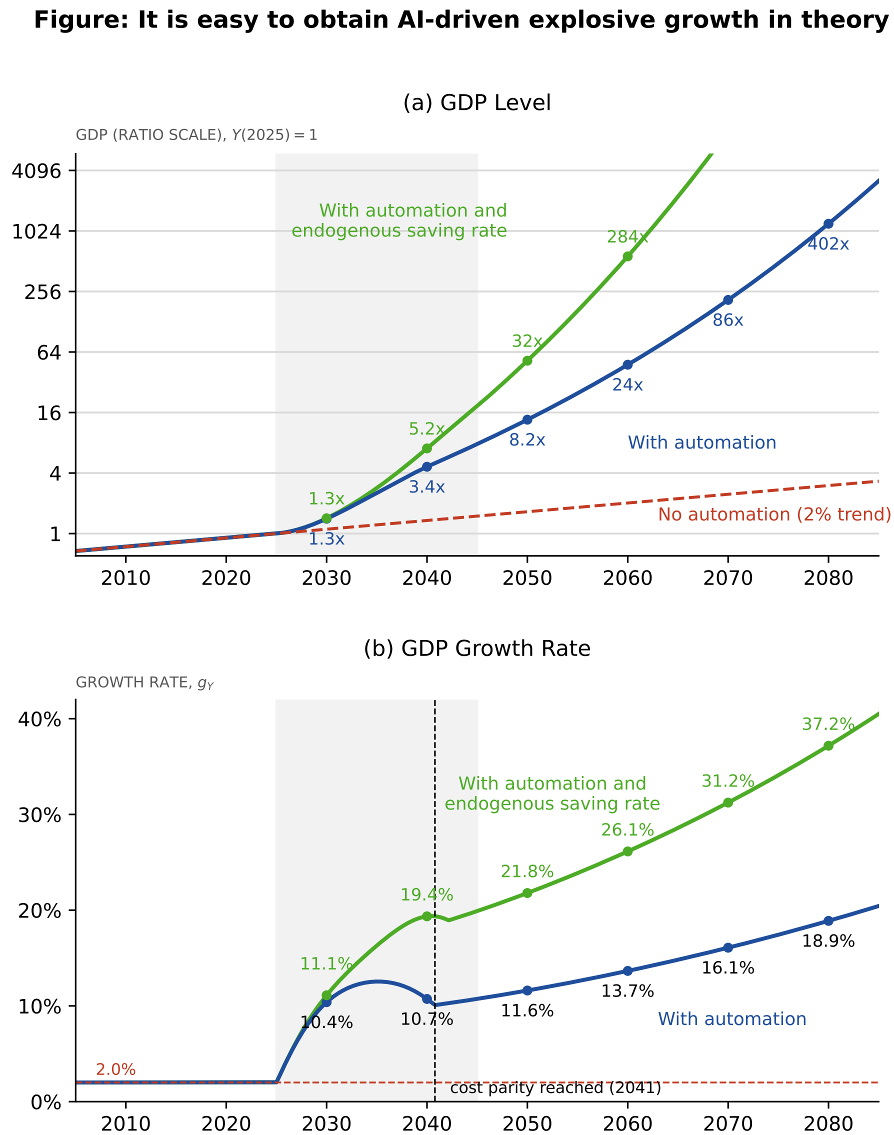

Figure 2: The transition to full automation. Panel (a): GDP (ratio scale) with and without automation; the labels on the dots report GDP relative to the no-automation trend. Panel (b): the GDP growth rate, with values at each decade. Blue: baseline fixed saving rate; green: endogenous saving rate (2) with $\sigma = 0.1$. The shaded band is the 2025–2045 automation phase; the dashed vertical line in panel (b) marks the baseline's switch to the cost-parity regime in 2041.

full automation the growth rate $s\varphi A - \delta$ depends directly on the fraction s of output reinvested. Even the modest elasticity $\sigma = 0.1$ therefore has dramatic effects: the saving rate rises only from 20% to 33% by 2045 — and stays below 40% even by 2125 — yet growth reaches 15% per year already in 2034 (versus 2066 in the baseline), 20% by 2045,

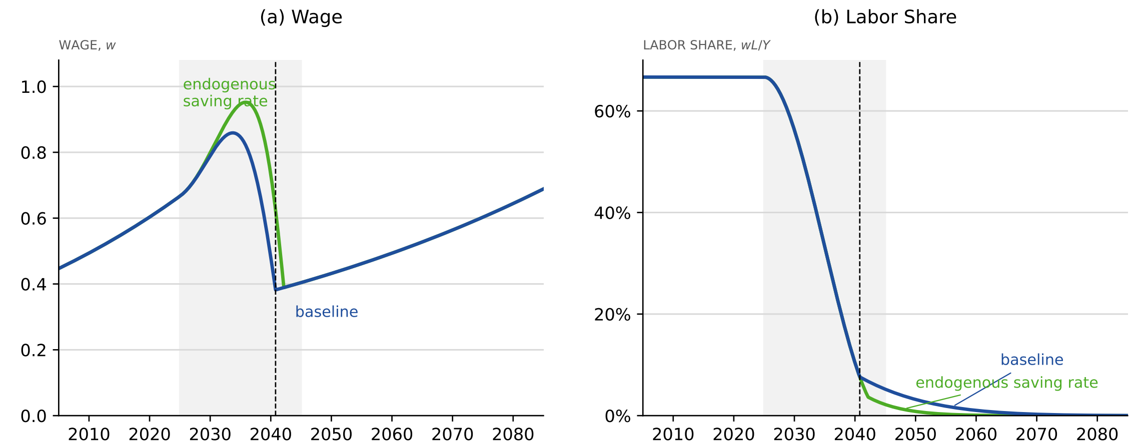

Figure 3: Factor prices during the transition. Panel (a): the wage w (2025 GDP = 1). Panel (b): the labor share. The shaded band is the 2025–2045 automation phase. Blue: baseline; green: endogenous saving rate (2) with $\sigma = 0.1$. The dashed vertical line marks the baseline's switch to the cost-parity regime ($w = R/\varphi = A$) in 2041.

and 40% by 2085.

Wages and the labor share. Figure 3 shows the distributional side, which is far less benign than the GDP side. In the capital-cheaper regime the wage is $w = (1 - \alpha)Y/L$: automation has an offsetting *displacement effect* ($1 - \alpha$ falls) and *productivity effect* (Y rises). The productivity effect wins early and the displacement effect wins late: the wage rises 29%, from 0.67 to a peak of 0.86 around 2034, then falls 56% to 0.38 at the regime switch — 43% *below* its pre-automation level. After the switch, workers earn the machine cost of doing their tasks: $w = A(t)$ grows only at the technology growth rate $g_A = 1.33\%$ — while GDP growth explodes past 10% and 20% — and the wage does not regain its 2034 peak until around 2102. The labor share falls from 67% to $1 - \alpha(t^*) \approx 8\%$ at the switch and then to $L/(\varphi K + L) \to 0$: essentially all of the exploding GDP accrues to capital owners. The endogenous saving rate (green) raises wages during the boom through faster capital deepening, but pushes the labor share toward zero even faster after the switch. Who owns the capital thus becomes the central distributional question in a fully automated economy.

5 Concluding Remarks

The engine of the task-based Solow model is an interaction: full automation converts the economy to an AK technology, so that the level of technology determines the growth rate rather than the level of output — and continued technology growth therefore turns ordinary 1.33% technology growth into a double-exponential GDP path. Along the transition, aggregate and distributional outcomes decouple: GDP growth explodes while wages boom, bust, and then stagnate relative to the economy.

The model is deliberately minimal. The saving rate responds to the exploding return to capital only through the reduced form (2), not through explicit intertemporal optimization. Tasks are symmetric, so there are no "bottleneck" tasks whose weak-link character could tame the explosion (Aghion, Jones and Jones, 2019). The automation path $\alpha(t)$ and machine productivity φ are exogenous, and capital is produced one-for-one from output with no adjustment costs. Each of these margins matters quantitatively and is worth exploring.

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