Lecture 9: Speculative Asset Bubbles

Macroeconomics EC2B1
Benjamin Moll

These notes are about speculative bubbles. A bubble is a situation in which the price of an asset does not reflect its "fundamental value" (we will be more precise below). The idea will be that speculators buy the asset only because they expect its price to rise in the future. This can give rise to self-fulfiling expectations: "prices rise because they are expected to rise."

An asset pays dividends $\{y_t\}_{t=0}^{\infty} = (y_0, y_1, ...)$. Individuals who discount the future with discount factor $\beta = 1/(1+r)$ trade this asset with each other. How will this asset be priced?

Example: Housing. In this case, y_t is the per-period benefit received from owning the house. This could be either the rent an individual obtains from renting it out or the benefit she gets from living in it herself. In light of this example and for sake of concreteness, we will sometimes say "house" instead of "asset" and "rent" instead of dividend.

The following arbitrage condition must hold for the price, p_t:

$$p_t = y_t + \beta p_{t+1} \tag{1}$$

This says that, if the price of the house at date $t+1$ is p_{t+1}, then the price today, p_t, has to equal this price discounted by β plus the dividend income. If this equation did not hold, there would be an arbitrage opportunity. For instance, if $p_t < y_t + \beta p_{t+1}$, an individual could buy the house today, rent it out for one time period, sell it tomorrow and make a windfall. This situation is ruled out.

Equation (1) is a difference equation (you may have encountered difference equations when studying the Solow model). What does it imply for the equilibrium price?

Claim: A solution to the difference equation (1) is

$$p_t = \sum_{j=0}^{\infty} \beta^j y_{t+j} = y_t + \beta y_{t+1} + \beta^2 y_{t+2} + \beta^3 y_{t+3} ... \tag{2}$$

Verify: write (2) as

$$p_t = y_t + \beta \underbrace{[y_{t+1} + \beta y_{t+2} + \beta^2 y_{t+3} + ...]}_{p_{t+1}} = y_t + \beta p_{t+1}$$

Hence we have verified that (2) satisfies the difference equation (1).

Equation (2) says that the price of the asset equals the present discounted value (PDV) of future dividends. For example, consider the case of a constant dividend $y_t = \bar{y}$ for all t. In that case

$$p_t = \frac{\bar{y}}{1-\beta}.$$

Would be call the situation where the price of the asset equals the PDV of dividends (2) a bubble? Certainly not! In fact, the PDV of future dividends *is* the correct notion of the fundamental value of an asset. So (2) says that the price of the asset equals its fundamental value which, by definition, is no bubble.

How then can bubbles arise? The key is to realize that (2) is not the *unique* solution to the pricing equation (1). In addition to the no-bubble-solution (2), (1) has many more solutions that all correspond to bubbles. To see this consider the extreme case where the asset pays no dividend, $y_t = 0$ for all t so that (1) becomes

$$p_t = \beta p_{t+1}. \tag{3}$$

One obvious solution is $p_t = 0$ (this is the no-bubble solution, i.e. expression (2) for the special case $y_t = 0$). But, and this is the crucial insight, another solution to (3) is

$$p_t = c\left(\frac{1}{\beta}\right)^t = c\beta^{-t} \tag{4}$$

for *any* constant c.

Verify: write (4) as

$$p_t = c\beta^{-t} = \beta \underbrace{\left[c\beta^{-(t+1)}\right]}_{p_{t+1}} = \beta p_{t+1}$$

Hence we have verified that (4) satisfies the difference equation (3).

What does the time path of (4) look like? if $c > 0$, because $\beta < 1$, the price explodes exponentially! [MAKE A GRAPH] Of course, the fundamental value of the asset is zero, hence this is a *pure bubble*. This bubble arises purely due to speculation. That is, individuals buy a worthless asset solely because they expect to sell it at a higher price tomorrow. The bubble therefore has the feature that "prices rise because they are expected to rise."

Now, let's put things together and consider the general case of any sequence of dividends.

Claim: The general solution to (1) is

$$p_t = \underbrace{\sum_{j=0}^{\infty} \beta^j y_{t+j}}_{\text{fundamental value}} + \underbrace{c\beta^{-t}}_{\text{bubble component}} \tag{5}$$

for any constant c.

Verify: write (5) as

$$p_t = y_t + \beta \underbrace{\left[y_{t+1} + \beta y_{t+2} + \beta^2 y_{t+3} + ... + c\beta^{-(t+1)}\right]}_{p_{t+1}} = y_t + \beta p_{t+1}$$

Hence we have verified that (5) satisfies the difference equation (1).

The special case $c = 0$ corresponds to the no-bubble solution (2). But this solution is *not unique* and there are many other solutions corresponding to different values of c. All of these have the feature that the price of the asset does not equal its fundamental value (the PDV of future dividends), which is to say there is a bubble. In fact, and as indicated in (5), the term $c\beta^{-t}$ *is* the bubble.

All these bubble solutions arise due to speculation, i.e. the hope of selling the asset at an elevated price tomorrow, and have the feature that "prices rise because they are expected to rise."