Lectures 7 & 8: New Keynesian Model (Mankiw-Weinzierl version)

Macroeconomics EC2B1
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These notes discuss one version of the New Keynesian model, namely the one outlined by Mankiw and Weinzierl (2011). We will first cover this New Keynesian model because – as you will see – it ties in quite nicely with our microfounded approach to macroeconomics. I will then give a "history lesson" on the "old" Keynesian model that some of you have seen before and which does not have any microfoundations (and relies on a graphical analysis). The reason for proceeding in this order, is that I expect you will find the New Keynesian model easier to understand.

1 Real Model: see Lecture Notes 5

The Mankiw-Weinzierl version of the New Keynesian model builds on the general equilibrium model of investment and capital accumulation covered in lecture notes 5 (see section 2 of the model writeup). As a reminder, the economy has the following primitives:

- Preferences: households have utility function

$$U(C_1) + \beta U(C_2) \quad \text{with} \quad U(C) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \tag{1}$$

- Technology: firms have production function

$$Y_t = A_t K_t, \quad t = 1, 2$$

  and capital accumulates according to $K_2 = I_1 + (1-d)K_1$ with $d = 1$, i.e.

$$K_2 = I_1$$

- Resource constraints (feasibility):

$$\begin{aligned} &\text{goods in period 1:} \quad C_1 + I_1 = Y_1 \\ &\text{goods in period 2:} \quad C_2 = Y_2 \end{aligned}$$

In the competitive equilibrium, households maximize utility (1) subject to the budget constraint

$$C_1 + \frac{C_2}{1+r_1} = W \tag{2}$$

and firms maximize profits

$$W = \max_{K_2} \left\{ A_1 K_1 - K_2 + \frac{A_2 K_2}{1+r_1} \right\} \tag{3}$$

As we have shown in lecture 5, consumption satisfies an Euler equation

$$C_1 = \frac{\left(\frac{1}{\beta(1+r_1)}\right)^{\sigma}(1+r_1)}{1+\left(\frac{1}{\beta(1+r_1)}\right)^{\sigma}(1+r_1)} W, \quad C_2 = \frac{1+r_1}{1+\left(\frac{1}{\beta(1+r_1)}\right)^{\sigma}(1+r_1)} W \tag{4}$$

and the competitive equilibrium allocation is:

$$\begin{aligned}
C_1 &= \frac{\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1 \\
C_2 &= \frac{A_2}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1 \\
I_1 &= \frac{1}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1 \\
Y_1 &= A_1 K_1 \\
Y_2 &= \frac{A_2}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1
\end{aligned} \tag{5}$$

This model is a baby two-period version of Real Business Cycle (RBC) model. As we have discussed, this model has the feature that the welfare theorems hold and so there is no room for macroeconomic stabilization policy (monetary and fiscal policy).

The New Keynesian model adds "nominal rigidities" in the form of sticky prices to this model. As you will see, adding nominal rigidities changes the model's behavior and policy implications quite dramatically.

2 Introducing Money and Inflation

2.1 Price Level and Inflation

So far we have ignored nominal factors and the whole model was in real terms. In particular, we simply set the price of final goods in both period $P_1 = P_2 = 1$. This was without loss of generality because we simply expressed all prices (in the model above the interest rate, r_1) in units of the consumption good (i.e. "saving one unit of apples (more generally consumption goods) today pays back $(1 + r_1)$ units of apples tomorrow"). In order to talk meaningfully about inflation, we reintroduce nominal prices. For instance, we write the household budget constraint in nominal terms (dollars) as

$$P_1 C_1 + \frac{P_2 C_2}{1 + i_1} = P_1 \Pi_1 + \frac{P_2 \Pi_2}{1 + i_1} \tag{6}$$

where i_1 is the *nominal* interest rate (interest rate in terms of dollars rather than apples). Similarly, the firm's problem becomes

$$\Omega = \max_{K_2} \; P_1(Y_1 - K_2) + \frac{P_2}{1 + i_1} A_2 K_2. \tag{7}$$

For reasons that will become clear momentarily, we also introduce "period $t = 0$" (a pre-period) in which a reference price P_0 is determined. The idea is that this reference price P_0 is determined some time before period 1, before households and firms know economic conditions like A_1, A_2 etc.

Definition: The *inflation rate* in this economy is

$$\pi_2 = \frac{P_2 - P_1}{P_1}, \qquad \pi_1 = \frac{P_1 - P_0}{P_0}$$

Note that the main role of "pre-period" 0 and the reference price P_0 is to enable us to define short-run inflation π_1 (the other role is to motivate price stickiness in Section 4 below).

Definition: The *real interest rate* is the nominal interest rate adjusted for inflation:

$$1 + r_1 = \frac{1 + i_1}{P_2/P_1} = \frac{1 + i_1}{1 + \pi_2} \tag{8}$$

The relation between real and nominal interest rates can be written in a simpler fashion using the approximation that $r_1 \pi_2$ is negligible if both r_1 and π_2 are small (this is a very commonly used approximation you may have come across in other courses):

$$1 + i_1 = (1 + r_1)(1 + \pi_2) \quad \Rightarrow \quad i_1 \approx r_1 + \pi_2 \tag{9}$$

This equation is known as the *Fisher equation*, after economist Irving Fisher.

2.2 Money Supply and Money Demand

A satisfactory treatment of nominal variables (variables in terms of dollars) also requires an analysis of money demand and money supply (demand and supply of dollars). We simply assume that the government/central bank sets the money supply. More on this in section 6 below. To discuss briefly the issue of money demand, we introduce the concept of *velocity* which you have already encountered in EC1B1.

Definition: *Velocity*, which we denote by V_t, is the average number of times a piece of money turns over in a year.

So consider an economy with nominal GDP, i.e. the total amount of purchases in terms of dollars, equal to P_tY_t. Velocity answers the question: How large is the required stock of money in the economy? Suppose money can be used only once, i.e. velocity is $V_t = 1$. Then clearly, the required stock of money is $M_t = P_tY_t$. Suppose money can be used twice, $V_t = 2$. Then $M_t = P_tY_t/2$. More generally, if money can be used V_t times

$$M_tV_t = P_tY_t \tag{10}$$

This is equation is known as the *quantity equation* and you have already seen it in EC1B1.[1]

There are different theories of velocity or, equivalently, money demand:

- Quantity theory (see EC1B1): $V_t = V$ fixed.
- Other theories of money demand covered in EC1B1, for example the one Lecture 8 "Monetary Policy in Modern Economies" in which velocity is given by $\log V_t = \phi i_t + v_t$ where ϕ is a parameter, i_t is the nominal interest rate, and v_t is a money demand shock.
- Baumol-Tobin model:[2] Consider a consumer who spends PC dollars on consumption each year. Denote by T the amount of time (in fractions of year) between trips to the bank.

[1] Arguably a more useful way of thinking about this equation is just as an accounting identity that defines V_t. That is, suppose you observe an economy with nominal GDP P_tY_t and stock of money, M_t. Then you conclude that velocity is $V_t = P_tY_t/M_t$.

[2] The description here is purposely brief and only sketches the model. If you want to read more on this (you don't have to), good references are chapter 10.4 of Kurlat and Chapter 4 of Doepke-Lehnert-Sellgren available here `https://faculty.wcas.northwestern.edu/mdo738/book.htm`.

Then $1/T$ is the number of trips per year. If the consumer spaces out his consumption expenditure equally over the year, then he spends PCT dollars between trips to the bank. If we assume that his money holdings decline linearly, $m(t) = Pc(T-t)$ at a point $0 \leq t \leq T$, then his average money holdings over this period are $M = (PCT^2/2)/T = PCT/2$.[3] In this theory, the velocity is $V_t = 2/T$. This follows from setting $Y_t = C$ (market clearing without capital), $M_t = M$, and the quantity equation (10). In the Baumol-Tobin model, T is then a choice variable that depends on the costs of holding money such as the interest rate.

- Cash-in-advance (CIA) models. This model simply assumes that purchasing goods requires some cash-on-hand which has to be put aside in advance. The CIA constraint is $P_t C_t \leq M_t$. Hence, with $Y_t = C_t$ velocity is assumed to be $V_t = 1$.

Mankiw-Weinzierl restrict themselves to the simplest such theory, the quantity theory with constant velocity. They use slightly different notation. In particular they write the quantity equation (top of p.7) as

$$\mathbb{M}_t V = P_t C_t, \quad V = \frac{1}{\phi}$$

In their theory, $\mathbb{M}_t$ is only the currency used for buying consumption goods, C_t, (as opposed to all output, $Y_t = C_t + I_t$). Since V is constant they simply redefine $M_t = \mathbb{M}_t V = \mathbb{M}_t/\phi$ and write

$$M_t = P_t C_t \tag{11}$$

(they also argue that the cost of holding money is negligible which implies that (a) this can be ignored when writing budget constraints – the more conventional way would be to build this in as in a CIA model – and (b) that velocity $V = 1/\phi$ is large).

3 Flexible Prices: Monetary Neutrality

We are interested in examining the equilibrium of the economy in particular the role played by money and inflation. We first consider the case with flexible final goods prices. Later, we will examine the case with sticky prices which is the defining assumption of a (New) Keynesian Model.

[3] More formally, using that $m(t) = Pc(T-t)$, average money holdings are

$$M \equiv \frac{1}{T}\int_0^T m(t)dt = \frac{1}{T}PC\left[T^2 - T^2/2\right] = PCT/2$$

Definition: *Neutrality of Money* means that a change in monetary variables like nominal interest rates or the stock of money affects only nominal variables in the economy such as prices and nominal wages but has no effect on real (inflation-adjusted) variables, like employment, real GDP, and real consumption.

The purpose of this section is to argue that *with flexible prices*, monetary neutrality holds in the above economy. More concretely this will simply mean that real variables, $(C_1, C_2, I_1, Y_1, Y_2)$ do not depend on policy tools of the central bank, that is the money supply M_2 or the nominal interest rate, i_1. Only the nominal variables, in particular, P_1 and P_2 do. In contrast, when we examine sticky prices below, monetary neutrality will not hold anymore.

As already noted, the problem of a firm is (7). The problem of a household is to maximize utility (1) subject to (6). The key to showing monetary neutrality is to realize that we can simply rewrite the problems of firms and households in real terms. As a result, everything will be exactly as in the real model in section 1. To this end, consider for instance the budget constraint of the household (6). Dividing through by the price level in period one, P_1, we have

$$C_1 + \frac{P_2 C_2}{(1+i_1)P_1} = \Pi_1 + \frac{P_2 \Pi_2}{(1+i_1)P_1}$$

Using the definition of the real interest rate (8), we can immediately see that this constraint is simply the budget constraint of the household in real terms (2). Similarly, defining the firm's value in real terms as $W = \Omega/P_1$, the firm's problem can be written as a maximization over real variables (3).

With this insight, all real variables are found in the exact same way as above. In particular, the equilibrium real variables are as in (5). It is easy to see that monetary neutrality holds in this economy, that is monetary policy instruments (money supply, interest rates) do not affect these real variables. Instead the nominal variables are given by

Claim: The nominal variables in this economy are

$$\begin{aligned}
P_1 &= \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{A_1 K_1} \frac{M_2}{1+i_1} \\
P_2 &= \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{A_2 A_1 K_1} M_2 \\
M_1 &= \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{1+i_1}
\end{aligned} \tag{12}$$

Derivation: The Euler equation in terms of nominal variables is

$$\frac{U'(C_1)}{\beta U'(C_2)} = \frac{1+i_1}{P_2/P_1} \quad \Rightarrow \quad \frac{C_2}{C_1} = [\beta(1+i_1)]^{\sigma}\left(\frac{P_1}{P_2}\right)^{\sigma} \tag{13}$$

Using (11), we can eliminate prices and obtain

$$\left(\frac{C_2}{C_1}\right)^{1-\sigma} = [\beta(1+i_1)]^{\sigma}\left(\frac{M_1}{M_2}\right)^{\sigma}$$

$$M_1 = \frac{1}{\beta}\left(\frac{C_2}{C_1}\right)^{\frac{1}{\sigma}-1}\frac{M_2}{1+i_1} = \frac{1}{\beta}(\beta A_2)^{1-\sigma}\frac{M_2}{1+i_1}$$

where the second equality follows because the Euler equation (13) can still be written in terms of real variables as $C_2/C_1 = [\beta(1+r_1)]^{\sigma} = (\beta A_2)^{\sigma}$. Rearranging, this is the last equation in (12). The expression for P_1 then follows from $P_1 = M_1/C_1$ and the expression for C_1 in (5). Similarly, the expression for P_2 follows from $P_2 = M_2/C_2$ and the expression for C_2 in (5).

Note from (12), that the nominal price levels P_1 and P_2 are proportional to the money supply M_2, which means that an increase in the money supply of 10% will increase price levels by 10%. A decrease in the interest rate i_1 increases the price level P_1 while not affecting P_2 which means lower long-run inflation $1+\pi_2 = P_2/P_1 = (1+i_1)/A_2$ or deflation. In summary, printing money is inflationary and interest rates are deflationary in the long-run.

There is a simple and direct intuition for why interest rate cuts are deflationary that uses the Fisher equation (9), i.e. $i_1 = r_1 + \pi_2$. Monetary neutrality is equivalent to the real interest rate not changing in response to monetary policy: in the model above the real interest rate is always just $1+r_1 = A_2$). In that case the Fisher equation, can simply be written as $\pi_2 = i_1 - r_1$. If the central bank lowers nominal interest rates, i_1, it must be that inflation, π_2, falls so as to keep the real interest rate constant. In the blogosphere, this idea is sometimes discussed under the name "Fisherism" or "Neo-Fisherism."

As we just discussed this model has the feature that real variables in (5) are determined completely independently of the nominal variables in (12). Therefore real and nominal variables could be analyzed separately. This property is called *classical dichotomy*.

4 Sticky Prices

We will now change one single assumption in the above model, namely that prices are perfectly flexible in the short run (period one). We will argue that this changes results dramatically. But first, let's discuss why we think that prices may be sticky.

4.1 Stickiness in Reality and Economic Modeling

I like the following quote: “While I regard the evidence for such stickiness as overwhelming, the assumption of at least temporarily rigid nominal prices is one of those things that works beautifully in practice but very badly in theory.” (Paul Krugman, see `https://web.mit.edu/krugman/www/islm.html`).

Possible reasons for stickiness in reality:

- Menu costs: the assumption is that a firm has to pay a fixed cost whenever it wants to change prices (e.g. a restaurant has to print a new menu). Hence the firm only changes its price when the payoff from so doing is large enough to justify paying the fixed cost.

- Rational inattention: the assumption is that acquiring information is costly and that therefore firms optimally choose to not pay attention to what's going on all the time, in particular not to what's happening to monetary policy.

Both of these stories have similarly realistic implications but are quite hard to model. In particular, the models are too complicated to embed in simple representative agent dynamic models like the one here. In practice therefore, sticky prices are often modeled with a technique called "Calvo-pricing". The assumption is that with an exogenous probability, say α , a firm may change its price, but with probability $1 - \alpha$ it is forced to keep its previous price. This assumption is obviously not satisfactory and it is often ridiculed (the “Calvo-fairy” that tips you on the shoulder and says “now you're allowed to change your price”). The Calvo model is an example of what's called "time-dependent sticky prices". This is in contrast to “state-dependent models” such as the menu cost and inattention models, explained above. Time-dependent models are considered to be less satisfactory by most economists.

With this discussion in mind, we will make an even more simplistic and extreme assumption than Calvo pricing: the price level in period one, P_1 is completely fixed and cannot be changed. It equals the reference price from pre-period $t = 0$, i.e. $P_1 = P_0$. The key friction is that the price at $t = 1$ is set before households and firms know the economic conditions at $t = 1$ and hence it cannot respond to any changes to these economic conditions (as it would with flexible prices). This assumption will break monetary neutrality. In contrast, the price level in period two P_2 is flexible.

4.2 Equilibrium with Sticky Prices

First note that a model with a completely fixed price level is necessarily a model in which market clearing doesn't hold for one or more markets. In our case, it is the market for final goods in period one that is not necessarily in equilibrium, and in particular demand for goods which we denote by Y_1^D can be smaller than the supply of goods $Y_1^S = A_1K_1$. To see this, suppose we start out in a case where the final goods market in period one is in equilibrium, $Y_1^D = Y_1^S$. Then a negative shock hits that decreases demand Y_1^D (for instance, we will show below that a negative shock to future A_2 has that feature). We will show below that (mot surprisingly) demand, Y_1^D, is a downward-sloping function of P_1. If P_1 were flexible, the price would adjust downwards so as to equate demand and supply again. But if P_1 is sticky – or as here completely fixed – this is not possible and there will have to be an excess supply of final goods in period one, $Y_1^D < Y_1^S$. We therefore replace the market clearing condition with the inequality $Y_1 \leq A_1K_1$.[4] In fact, the case where the inequality is strict, will be the "interesting" case in which there is a role for government policy. In this sense, this version of the New Keynesian model is a model of *disequilibrium*.

Claim: The equilibrium with sticky prices is given by (cf. Mankiw-Weinzierl equations (31)-(36) with $g_2 = 0$).

$$\begin{aligned}
C_1 &= \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)P_1} \\
C_2 &= A_2 \frac{M_2}{(1+i_1)P_1} \\
I_1 &= \frac{M_2}{(1+i_1)P_1} \\
Y_1 &= \left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right] \frac{M_2}{(1+i_1)P_1} \\
Y_2 &= A_2 \frac{M_2}{(1+i_1)P_1} \\
1 + r_1 &= A_2 \\
P_2 &= \frac{1+i_1}{A_2} P_1
\end{aligned} \tag{14}$$

Derivation: Consumption still satisfies (4). However, now we cannot use the market clearing condition $Y_1 = A_1K_1$ anymore because it may not hold. This implies that the real value of the firm is $W = Y_1 \leq A_1K_1$. In the derivation of (5) in Lecture notes 5, we substituted $W = A_1K_1$ into (4). This step now breaks down because now $W = Y_1$ but we do not know the value of Y_1.

Instead we proceed as follows. First note that from the firm's first order condition, the

[4] We can never have the case $Y_1 > A_1K_1$ because equilibrium quantities are always determined by the short side of the market, that is $Y_1 = \min\{Y_1^D, Y_1^S\}$, i.e. $Y_1 = Y_1^D$ if $Y_1^D < Y_1^S$ and $Y_1 = Y_1^S$ if $Y_1^S < Y_1^D$.

real interest rate is still given by $1 + r_1 = A_2$ (see the comment in the next paragraph on this somewhat surprising result). Hence, because i_1 is chosen by the monetary policy authority and P_1 is fixed, P_2 is determined from the definition of the real interest rate (8). Rearranging, gives the last equation in (14). Consumption in period two, C_2, is then immediately pinned down from equilibrium in the money market, $P_2 C_2 = M_2$. Rearranging gives the expression for C_2. Next, use the equation for C_2 and $C_2 = A_2 K_2$ to get the expression for $I_1 = K_2$. Finally, we can find the expression for C_1 from the Euler equation

$$\frac{C_2}{C_1} = [\beta(1 + r_1)]^\sigma = (\beta A_2)^\sigma \quad \Rightarrow C_1 = \left(\frac{1}{\beta A_2}\right)^\sigma C_2$$

and the expression for C_2. Therefore, we have expressions for C_1 and I_1 and hence

$$Y_1 = C_1 + I_1 = \left[\left(\frac{1}{\beta A_2}\right)^\sigma A_2 + 1\right] \frac{M_2}{(1 + i_1)P_1}.$$

Comment on equilibrium real interest rate and investment: As can be seen from the last equation in (14) (the equation for P_2), the equilibrium real interest rate $1+r_1 = (1+i_1)P_1/P_2$ satisfies $1 + r_1 = A_2$. This looks like monetary policy does not affect real interest rates just like in the model with flexible prices (in which this was a result of monetary neutrality). This result is somewhat surprising given that we made a big deal out of monetary neutrality *not* holding with sticky prices. What is going on?

In a nutshell, this result is a bit of an artefact of a particular assumption we made in order to get nice analytical formulae, namely that production is linear and features constant returns to capital $Y_2 = A_2 K_2$. Intuitively, this assumption makes investment demand perfectly elastic as a function of the real interest rate and results in investment responding very strongly which pushes up period 2 output $Y_2 = C_2$ and pushes down the price level P_2 until the real interest rate equals $1 + r_2 = A_2$. With the more realistic (but analytically less tractable) assumption of diminishing returns to capital, monetary policy instead *does* affect the real interest rate r_1 as you may have expected. The result that the equilibrium real interest rate is unaffected by monetary policy is therefore due to a special assumption made purely for tractability and should not be taken too seriously.[5]

To make the point that this result is special to the constant returns assumption, change

[5] Also note that the seeming zero pass-through from i_1 to r_1 is an equilibrium outcome. The right way to think about it is that there definitely is passthrough from the former to the latter and that monetary policy is very much non-neutral. In fact, the whole issue is that it is "too non-neutral" in the sense that investment demand is infinitely elastic and responds extremely strongly to changes in monetary policy and this is what pushes the equilibrium real rate to adjust to equal $1 + r_1 = A_2$.

the assumption that production in the second period is $Y_2 = F(K_2) = A_2 K_2^\alpha$ with $0 < \alpha \leq 1$. Fortunately one does not have to resolve the whole model to make this point – instead one can separately analyze one block of the model, namely the determining the real interest rate r_1, investment I_1, and second-period price level P_2. We will show below that these become

$$1 + r_1 = \alpha^\alpha A_2 \left(\frac{(1+i_1)P_1}{M_2} \right)^{1-\alpha} \tag{15}$$

$$K_2 = I_1 = \alpha \frac{M_2}{(1+i_1)P_1} \tag{16}$$

$$P_2 = \frac{1}{\alpha^\alpha A_2} [(1+i_1)P_1]^\alpha M_2^{1-\alpha} \tag{17}$$

As expected, when $\alpha = 1$, the real interest rate equals $1 + r_1 = A_2$ again and investment I_1 and the price level P_2 boil down to the expressions in (14). More importantly, whenever $\alpha < 1$ (i.e. there are diminishing returns to capital), the real interest rate r_1 in (15) now depends on monetary policy i_1 and M_2. In particular, a nominal rate cut (a decrease in i_1) results in a decrease in r_1. The same is true when the central bank prints more money, i.e. M_2 increases. Note that with flexible prices, the same would *not* be true, i.e. monetary neutrality still holds even when $\alpha < 1$.

Let's derive expressions (15) to (17). To pin down $r_1, I_1 = K_2$ and P_2 it is sufficient to work with the following three equations: (i) the firm's optimality condition for investment,[6] (ii) the quantity equation in period 2, and (iii) the Fisher equation:

$$\alpha A_2 K_2^{\alpha-1} = 1 + r_1,$$

$$M_2 = P_2 C_2 = P_2 A_2 K_2^\alpha,$$

$$1 + r_1 = \frac{1+i_1}{P_2/P_1}.$$

Combining the first and third equations, we have $\alpha A_2 K_2^{\alpha-1} = (1+i_1)P_1/P_2$. Substituting for P_2 from the second equation, we have an equation for K_2

$$\alpha A_2 K_2^{\alpha-1} = \frac{(1+i_1)P_1}{M_2} A_2 K_2^\alpha$$

[6]The derivation of this equation is as above and in Lecture 5: firms choose K_2 to maximize profits

$$\Omega = \max_{K_2} \; P_1(Y_1 - K_2) + \frac{P_2}{1+i_1} A_2 K_2^\alpha$$

which is the same problem as (7) but with the new production function AK_2^α in place of AK_2. The corresponding optimality condition condition is $\alpha A_2 K_2^{\alpha-1} = 1 + r_1$ with $1 + r_1 = \frac{1+i_1}{P_2/P_1}$.

Rearranging gives (16). Substituting back into the first equation gives (15). Finally, using the third equation yields (17).

5 Policy in the New Keynesian Model

Sticky prices introduce a friction that implies that the welfare theorems break down. Therefore, unlike in the RBC model of section 1, there is a role for policy. Policy can broadly be split into two categories:

(1) Monetary policy: central bank sets interest rate or money supply.

(2) Fiscal policy: government spending, tax cuts, stimulus checks, “cash for clunkers” (trade in old car for new more fuel-efficient car and get cash).

Below, we will consider a recession and what policy is called for. We will show that in “normal times” monetary policy is sufficient to correct the friction implied by sticky prices. However, in “abnormal times”, fiscal policy may be useful (stimulus package).

6 Monetary Policy

6.1 What Is the Central Bank’s Policy Instrument – The Money Supply or the Interest Rate?

Before we continue, a brief detour seems appropriate. In the above model, we have assumed that the policy maker (central bank), sets the nominal interest rate in period 1, i_1, but the money supply in period 2, M_2 (look at equations (14) and you will see the ratio of the two enters). When the media report on changes in central bank policy, they often just say that the central bank has raised or lowered interest rates. So why don’t we follow this strategy here in both periods? The short answer is that because this is a two period model, there is only one interest rate, namely the one between periods one and two, i_1. So we cannot choose i_2 as a policy instrument. Standard New Keynesian models are modeled as infinite horizon economies which implies that this problem doesn’t occur (there’s always a tomorrow) and the policy instrument is simply the sequence $\{i_t\}_{t=0}^{\infty}$. More generally, though it should be noted that the two are really equivalent. Showing the equivalence of (i_1, M_2) and (M_1, M_2) as policy instruments in the present framework will be part of the next problem set. You should also read:

- the blog post by Mankiw on the question "What Is the Fed's Policy Instrument – The Money Supply or the Interest Rate?" here `http://gregmankiw.blogspot.com/2006/05/is-lm-model.html`
- Williamson Ch.12 on how monetary policy is conducted in practice. In particular, you should know the concepts of "open market operations" and "helicopter drops".

6.2 A Recession and the Response of Monetary Policy

We will now consider the effect of a recession, as above triggered by a decrease in future productivity A_2. First, compare the effects of this drop in the flexible price and sticky price models. Assume that the IES is $\sigma < 1$ (Mankiw and Weinzierl's canonical case). We have:

- Flexible prices: $C_1 \downarrow, C_2 \downarrow, I_1 \uparrow, \bar{Y}_1, Y_2 \downarrow$
- Sticky prices: $C_1 \downarrow, C_2 \downarrow$, $\bar{I}_1$, $Y_1 \downarrow, Y_2 \downarrow$ (the last by more than in the flexible price case).

So the recession is worse with sticky prices and therefore there is a role for policy. The optimal monetary policy is simply the one that undoes all the distortions induced by price stickiness, thereby equating the equilibrium allocations under sticky prices (14) to that under flexible prices (5). Most directly, this policy is the one that delivers "full employment", $Y_1 = A_1 K_1$, in (14) or

$$\frac{M_2}{(1+i_1)P_1} = \frac{1}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1 \tag{18}$$

Under the assumption $\sigma < 1$, it can be seen that the optimal policy response to a drop in A_2 is to decrease short-term interest rates, i_1, or to increase future money supply, M_2. Again, this policy undoes all the distortions due to sticky prices. Therefore, there is no need for fiscal policy (which we haven't even introduced into the model yet).

Note that this policy only works *because* prices, P_1 are sticky. If instead prices were flexible, the increase in aggregate demand due to an increase in M_2 or decrease in i_1 would be immediately offset by an increase in P_1. You can verify this from the first equation in (12).

6.3 Feasibility of Monetary Policy: Zero Lower Bound

One important question is whether the optimal policy above is always feasible. One constraint on policy is the so-called zero lower bound

Definition: The *zero lower bound* (ZLB) is the requirement that nominal interest rates cannot be negative, $i_1 \geq 0$.

The reason for this constraint is that money is an alternative asset to bonds. Money always pays an interest rate of zero. Therefore, if the interest rate on bonds would go negative, no one would lend. If they did, they would get less money back than they lent, and they would be better off putting their money in their mattress. As we will see, if interest rates are at the zero lower bound, monetary policy will be impotent. Because of this feature the zero lower bound is sometimes referred to as a "liquidity trap" (see for instance the paper by Krugman, 1998, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap"). However, the term zero lower bound is more descriptive so we will use that one.

In order to analyze the effect the ZLB has on the economy, some more notation is useful. In particular we denote *pre*-recession variables with a "ˆ". For instance productivity drops from $\hat{A}_2$ to $A_2 < \hat{A}_2$. We further assume that the pre-recession economy features full employment that is $\hat{Y}_1 = A_1K_1$ (no hat needed on K_1 because it is fixed). For now, also that the money supply in period two is fixed $M_2 = \hat{M}_2$. We can therefore write the relative frop in output as

$$\frac{Y_1}{\hat{Y}_1} = \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1 + \left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2} \frac{1 + \hat{i}_1}{1 + i_1}$$

where we use that prices are fixed in the short-run, $P_1 = \hat{P}_1$ and $M_2 = \hat{M}_2$. Since $\hat{Y}_1 = A_1K_1$

$$Y_1 = \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1 + \left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2} \frac{1 + \hat{i}_1}{1 + i_1} A_1K_1$$

This can be written in terms of the nominal interest rate as

$$1 + i_1 = \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1 + \left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2} (1 + \hat{i}_1) \frac{A_1K_1}{Y_1}$$

The interest rate that delivers $Y_1 = A_1K_1$ is then

$$1 + i_1 = \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1 + \left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2} (1 + \hat{i}_1)$$

We need $1 + i_1 \geq 0$. Therefore it can be seen that the ZLB is more likely to bind if the pre-recession interest rate, $\hat{i}_1$, is already close to zero. Furthermore, the RHS of this equation is an increasing function of A_2. Hence there is a cutoff $A_2|_{conventional}$ such that $i_1 \geq 0$ if and only if

$A_2 \geq A_2|_{conventional}$. That is, productivity cannot drop too far, otherwise the ZLB binds. For instance, consider the case where the pre-recession interest rate is already at zero, $\hat{i}_1 = 0$. In that case, a productivity drop $A_2 < \hat{A}_2$ always implies that the interest rate that delivers "full employment", $Y_1 = A_1 K_1$, is negative. Therefore monetary policy has no power.

6.4 Fiscal vs. Monetary Policy

We have shown that if the ZLB binds, monetary policy looses its power. For now keep the assumption that M_2 is fixed and ask: can fiscal policy undo the distortions due to sticky prices? We introduce fiscal policy in the exact same way as Mankiw-Weinzierl, that is we denote government spending by G_t, lump-sum taxes by T_t and assume that the government has a present value budget constraint

$$P_1 G_1 + \frac{P_2 G_2}{1+i_1} = P_1 T_1 + \frac{P_2 T_2}{1+i_1}$$

The resource constraints are now

$$C_1 + I_1 + G_1 = Y_1, \quad C_2 + G_2 = Y_2 \tag{19}$$

and the household budget constraint is

$$P_1 C_1 + \frac{P_2 C_2}{1+i_1} = P_1(\Pi_1 - T_1) + \frac{P_2(\Pi_2 - T_2)}{1+i_1}$$

We will also denote by $g_t = G_t/(A_t K_t)$ the share of government purchases in full employment.

Claim: the equilibrium with sticky prices satisfies (these are equations (31) to (36) in the paper)

$$
\begin{aligned}
C_1 &= \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)P_1} \\
C_2 &= A_2 \frac{M_2}{(1+i_1)P_1} \\
I_1 &= \frac{1}{1-g_2} \frac{M_2}{(1+i_1)P_1} \\
Y_1 &= \frac{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2(1-g_2)}{1-g_2} \frac{M_2}{(1+i_1)P_1} + G_1 \\
Y_2 &= A_2 \frac{1}{1-g_2} \frac{M_2}{(1+i_1)P_1} \\
P_2 &= \frac{1+i_1}{A_2} P_1
\end{aligned}
\tag{20}
$$

Derivation: The expression for C_1 and C_2 are derived in the same was as in the derivation of (14). Output in period 2, Y_2, can then be found from (19) which becomes $C_2 = Y_2(1-g_2)$ or $Y_2 = C_2/(1-g_2)$. Similarly the expression for investment follows because $I_1 = K_2 = Y_2/A_2$. Finally output in period 1 is found from substituting the expressions for I_1 and C_1 into (19).

The equilibrium has the feature that consumption, C_1 and C_2 are *not affected* by changes in government spending. As Mankiw-Weinzierl put it: "the government-spending multiplier here is precisely one. Here, as in that model, an increase in government spending puts idle resources to work and raises income. Consumers, meanwhile, see their income rise but recognize that their taxes will rise by the same amount to finance that new, higher level of government spending. As a result, consumption and investment are unchanged and the increase in income precisely equals the increase in government spending."

Therefore there *is* room for government intervention. By increasing G_1 the government can achieve full employment $Y_1 = A_1 K_1$. However, note that the shortfall in private consumption ($C_1 + I_1 < A_1 K_1$) is made up by public consumption, i.e. G_1 fills the gap $C_1 + I_1 + G_1 = A_1 K_1$. That is, fiscal policy restores the first-best level of GDP Y_1, but *not* consumption C_1. Therefore this is a second-best policy.

Summarizing, in "normal" times when the ZLB doesn't bind, monetary policy is sufficient to restore the flexible price equilibrium. In "abnormal" times, when the ZLB binds there is a role for fiscal policy.