Lecture 10: Unemployment

Macroeconomics EC2B1
Benjamin Moll

1 Measuring the Labor Market

Also see Kurlat chapter 7.1. There are three groups of people:

(1) Employed, E.

(2) Unemployed, U: not employed, but actively searched for work at some time during the last four week.

(3) Not in labor force, NL: neither employed nor unemployed.

Definitions: unemployment rate, u, participation rate, p, employment-population ratio, n:

$$u = \frac{U}{E+U}$$

$$p = \frac{E+U}{E+U+NL}$$

$$n = \frac{E}{E+U+NL}$$

2 Brief Recap of Standard Market of Labor Market

Recall the model from lectures 1 and 2. Apologies for the change of notation (from $u(c) - v(h)$ to $U(c) - V(h)$) which is due to the fact that we will need lower-case u and v to denote unemployment and vacancies below.

Households:

$$\max_{c,h} \; U(c) - V(h) \quad \text{s.t.} \quad c = wh + \Pi \quad \Rightarrow \quad \frac{V'(h)}{U'(c)} = w$$

Firms:

$$\Pi = \max_n f(n) - wn \quad \Rightarrow \quad f'(n) = w$$

Equilibrium: $n = h$ and $c = f(n)$ and hence

$$\frac{V'(n)}{U'(c)} = f'(n) \tag{1}$$

No involuntary unemployment! Worker's always set $MRS = w$, hence the only reason for decrease in n at given w are changing preferences, e.g. increase in disutility of work. Depending on preferences and the implied income and substitution effects, a fall in employment can also be due to a fall in firm labor demand (e.g. due to a decline in firm productivity) which depresses the wage w and hence labor supply. But also in this case any decline in employment is a voluntary optimal response to a lower wage w as opposed to involuntary unemployment.

Parametric example: This example will be useful below. Assume $U(c) - V(h) = \log c - \gamma h$ and $f(n) = An$. Then $V'(h)/U'(c) = \gamma c$ and hence (1) becomes

$$\gamma c = A \text{ with } c = An$$

so that equilibrium employment is $n^* = 1/\gamma$. As discussed in lecture 2, in this example income and substitution effects cancel and equilibrium employment is independent of productivity A.

3 Diamond-Mortensen-Pissarides Model (Static Version)

3.1 Matching Function

Assumption of standard model: recruiting is costless and firms can find as many workers as they want. Change this assumption. Assume that in order to increase employment, firms have to post a vacancy at cost κ which captures firms having a human resources (HR) division and so on. But posting a vacancy does not guarantee recruiting a worker. Instead we make the following assumption: when the unemployment rate is u and firms in the economy post a total of v vacancies, a total of $m = M(u, v)$ new jobs are created (equivalently, $m = M(u, v)$ "matches" are formed). We also assume that the size of the total labor force is fixed at one (i.e. there is no labor force participation decision) and unemployment equals $u = 1 - n$.

Suppose initial employment is n_0, i.e. n_0 workers have already been matched.[1] Then

[1] We here take n_0 as exogenously given. The standard version of the DMP model is a dynamic (infinite-horizon) model that endogenizes where this n_0 comes from, namely from the search and matching process in the previous period. This is more satisfactory in terms of the economics but also much more complicated in terms of the math than the simple one-period model presented here.

employment is

$$n = n_0 + M(u, v).$$

The function $M(u, v)$ is the so-called "matching function". Think of it as a production function that takes unemployed and vacancies as inputs and produces matches. Do we think that's an accurate description of search and matching in reality? Of course not, but noone complains about aggregate production functions $F(k, n)$ either.

It is common to assume that the matching function takes a Cobb-Douglas functional form (in the same way production functions are often assumed to be Cobb-Douglas, $F(k, n) = zk^\alpha n^{1-\alpha}$):

$$m = M(u, v) = \bar{\mu} v^\eta u^{1-\eta}, \quad 0 < \eta < 1$$

and we will make this assumption going forward. The number of matches can be usefully written as

$$m = \bar{\mu} v^{\eta-1} u^{1-\eta} v = \bar{\mu} \theta^{\eta-1} v$$

where

$$\theta = \frac{v}{u} = \frac{v}{1-n} \tag{2}$$

is the "vacancy-unemployment ratio" or "labor market tightness" (see explanation below).

Now consider the problem of an individual firm that understands that this is how the labor market works and considers posting some vacancies. This firm calculates the "vacancy filling rate", i.e. the probability of filling a given vacancy as follows. When all firms in the economy post a total of v vacancies, then each vacancy generates $m/v = \bar{\mu}\theta^{\eta-1}$ matches; because each firm has the same probability of being matched, the vacancy filling rate from the point of an individual firm is

$$\mu(\theta) = \bar{\mu}\theta^{\eta-1}.$$

Continuing this logic, the law of motion of employment for an individual firm is

$$n = n_0 + \mu(\theta) v. \tag{3}$$

Note that $\mu'(\theta) = \bar{\mu}(\eta - 1)\theta^{\eta-2} < 0$ so the higher is θ the harder is it to recruit a worker. Therefore θ is also referred to as "labor market tightness". Importantly, individual firms take tightness, θ as given. The right way to think about it is that θ is the ratio of *aggregate* vacancies to unemployment. Each individual firm takes into account how many extra workers it can recruit by increasing v, but not how that increasing v increases the *aggregate* number of vacancies and therefore the tightness of the labor market.[2]

3.2 Firm and Household Behavior

Firms: Profits of a firm with existing workers n_0 are

$$\Pi = \max_v \; An - wn - \kappa v, \quad n = \mu(\theta)v + n_0,$$

i.e. profits are production An minus wage payments wn minus the cost of posting vacancies κv. The cost of posting vacancies is paid to households (i.e. it is a transfer to households which will show up in the household budget constraint).[3] This captures the idea of HR services being provided to the firm and the corresponding payments ending up with households.

The Lagrangean for the firm profit maximization problem is

$$\mathcal{L} = An - wn - \kappa v + \lambda[\mu(\theta)v + n_0 - n]$$

The FOCs for n and v are

$$A = w + \lambda, \quad \kappa = \lambda\mu(\theta)$$

Combining

$$A = w + \frac{\kappa}{\mu(\theta)} \tag{4}$$

It can be seen that the standard model with a frictionless labor market is the special case where the cost of posting a vacancy $\kappa = 0$ so that $A = w$.

Households: There is a representative household with a large number of members. A fraction n of household members are employed and a fraction $u = 1 - n$ are unemployed. Importantly, households *do not* get to choose how much they work, i.e. they cannot choose how many household members are employed. They just take whatever work they can find. Household utility is simply (note the absence of $\max_n$)

$$E(n) = U(c) - V(n) \quad \text{with} \quad c = wn + \kappa v + \Pi.$$

Note the household budget constraint which includes κv as income, reflecting that the cost of posting vacancies is paid to households as we discussed above.

[2] There is therefore a congestion effect: if all firms increase their vacancies, recruiting is less efficient.

[3] Alternative assumptions are possible as well, for example that the cost of posting vacancies is paid in productive resources, in particular it could use up consumption goods or hours of work.

The marginal value to the household of having an extra member who is employed is

$$E'(n) = U'(c)w - V'(n).$$

Household members will be happy to be employed whenever $E'(n) \geq 0$ or equivalently

$$w \geq \frac{V'(n)}{U'(c)} \tag{5}$$

i.e. whenever the wage exceeds their disutility of working (converted into dollar terms by dividing by marginal utility $U'(c)$). Equivalently, we can say that $V'(n)/U'(c)$ is the household's value of leisure and household members will be happy to be employed whenever the wage exceeds this value of leisure.

Wage Determination:[4] If a worker and a firm manage to match, then they need to decide what wage the worker will be paid. We are going to assume that they bargain over the wage in a specific way, known as Nash bargaining (after economist John Nash). The outcome will be that the equilibrium wage satisfies

$$w = \phi A + (1 - \phi)\frac{V'(n)}{U'(c)} \tag{6}$$

where $0 \leq \phi \leq 1$ is the worker's bargaining power. Here is how this works. The worker and firm first look at what's going to happen if they *cannot* reach an agreement:

- the firm will have an unfilled vacancy that will produce no output and no profits
- the worker will be unemployed and get the value of leisure $B = V'(n)/U'(c)$ (see the discussion in the preceding paragraph)

After doing that, the worker and firm say to each other: "if we do reach an agreement, we'll get a total of A instead of B, so that's $A - B$ better. Let's find a deal so that we split the $A - B$ of surplus that is generated by this job." They then set the wage w so that the worker gets a fraction ϕ of the surplus and the firm gets $1 - \phi$. This is the sense in which the parameter ϕ measures the bargaining power of the worker. This means that the wage will be

$$w = B + \phi(A - B) = \phi A + (1 - \phi)B \quad \text{with} \quad B = \frac{V'(n)}{U'(c)}$$

[4] This paragraph borrows heavily from chapter 7.5 of Kurlat's book, including much of the exact wording.

which is (6). Another way of saying this is that A and B are the firm's and worker's "threat points" in the wage negotiation process, i.e. the firm will never pay a wage higher than A and the worker will never accept a wage lower than B. So the bargained wage will lie somewhere between A and B. For completeness the firm's profits will be $A - w = (1-\phi)(A-B)$ and similarly the wage exceeds the outside option of workers by $B - w = \phi(A - B)$.

For the remainder of the analysis, specialize the utility function

$$U(c) = \log c, \quad V(n) = \gamma n \quad \Rightarrow \quad \frac{V'(n)}{U'(c)} = \gamma c$$

Hence

$$w = \phi A + (1-\phi)\gamma c. \tag{7}$$

Think of A and $B = \gamma c$ as the "threat points" of firms and workers. The households will never accept wage lower than that and the firms will never pay a wage higher than that.

3.3 Equilibrium

There are two markets in this economy: the goods market and the labor market. The goods market clearing condition is[5]

$$c = An \tag{8}$$

As already discussed, the labor market is such that households take whatever work they can find subject to the wage exceeding their value of leisure – see (5).

An equilibrium is characterized by three equations: the firm's first order condition (4), the wage determination equation (7), and the law of motion for n (3). By doing some algebra these can be written as

$$A - w = \frac{\kappa}{\mu(\theta)} \tag{9}$$

$$w = \phi A + (1-\phi)\gamma A n \tag{10}$$

$$n = \mu(\theta)\theta(1-n) + n_0 \tag{11}$$

where (11) follows from combining (2) and (3) and (10) follows from substituting the goods

[5]You can check that this is the correct goods market clearing condition by using Walras' law: substituting the definition of firm profits $\Pi = An - wn - \kappa v$ into the household budget constraint $c = wn + \kappa v + \Pi$ we have $c = wn + \kappa v + An - wn - \kappa v = An$. It is important here that we have assumed that the cost of posting vacancies κv is a transfer that is paid from firms to households. If this were not the case, posting vacancies would eat up productive resources and the market clearing condition would be $c + \kappa v = An$.

market clearing condition (8) into (7). Further, we have assumed that the matching function takes the form $\mu(\theta) = \bar{\mu}\theta^{\eta-1}$. These equations pin down three unknowns (w, n, θ).

To understand the properties of an equilibrium, we manipulate these equations further. First, rewrite (10) as

$$w = \phi A + (1-\phi)\gamma A n \equiv W(n) \tag{12}$$

Note that $W'(n) > 0$ so that the wage is higher the lower is unemployment, $1-n$. This is because workers have more bargaining power if unemployment is low. Second, rewrite (11) and

$$\theta = \left(\frac{n - n_0}{\bar{\mu}(1-n)}\right)^{1/\eta} \equiv \Theta(n) \tag{13}$$

Note that $\Theta'(n) > 0$ and that for tightness to be positive and finite the solution needs to satisfy $n_0 < n < 1$. Next, substitute (12) and (13) into (9) to get

$$A - W(n) = \frac{\kappa}{\mu(\Theta(n))} \tag{14}$$

or using the function form for $\mu(\theta)$,

$$A - W(n) = (\kappa/\bar{\mu})\Theta(n)^{1-\eta}$$

This is one equation in one unknown, n. In the lecture we graphed this equation, which we used to get the comparative statics discussed further below.

Comment: Nice feature of DMP model: notion of unemployment corresponds closely to question of U.S. Bureau of Labor statistics: "Have you been actively looking for a job over the past four weeks?"

Existence and Uniqueness of Equilibrium: Substituting from the definitions of $W(n)$ and $\Theta(n)$, (12) and (13), we get

$$A(1-\phi)(1-\gamma n) = (\kappa/\bar{\mu})\left(\frac{n-n_0}{\bar{\mu}(1-n)}\right)^{(1-\eta)/\eta}.$$

Define

$$\text{LHS}(n) \equiv A(1-\phi)(1-\gamma n), \quad \text{RHS}(n) \equiv (\kappa/\bar{\mu})\left(\frac{n-n_0}{\bar{\mu}(1-n)}\right)^{(1-\eta)/\eta}.$$

We have that

$$\text{RHS}(n) > 0 \text{ for } n > n_0, \quad \text{RHS}'(n) > 0, \quad \text{RHS}(n_0) = 0, \quad \text{RHS}(1) = \infty.$$

We further have

$$\text{LHS}(n) > 0 \text{ for } 0 \leq n < 1/\gamma, \quad \text{LHS}'(n) < 0, \quad \text{LHS}(0) = A(1-\phi), \quad \text{LHS}(1/\gamma) = 0.$$

Therefore the two curves, RHS(n) and LHS(n), will always intersect each other exactly once and hence the equilibrium exists and is unique (see the graph in the lecture slides)

It is also clear that equilibrium employment will always be below employment in the frictionless case from Section 2 $n^* = 1/\gamma$. As discussed the frictionless case also obtains when the recruiting cost $\kappa = 0$.

3.4 Comparative Statics

See graphs in lecture:

$$\frac{\partial n}{\partial A} > 0, \quad \frac{\partial n}{\partial \bar{\mu}} > 0, \quad \frac{\partial n}{\partial \kappa} < 0, \quad \frac{\partial n}{\partial \gamma} < 0$$

The second result, $\partial n / \partial \bar{\mu} > 0$, follows because

$$\text{RHS}(n) = (\kappa/\bar{\mu})\Theta(n)^{1-\eta} = (\kappa/\bar{\mu})\left(\frac{n-n_0}{\bar{\mu}(1-n)}\right)^{(1-\eta)/\eta} = \kappa\bar{\mu}^{-1/\eta}\left(\frac{n-n_0}{1-n}\right)^{(1-\eta)/\eta}$$

which is decreasing in $\bar{\mu}$.

3.5 Unemployment Insurance

A further interesting comparative static is the question: how does an increase in unemployment insurance affect the level of unemployment in an economy? This question can be answered through a simple extension. Denote the level of unemployment benefits by b. The utility of a household that has a fraction n of employed workers and a fraction $1-n$ of unemployed workers is:

$$E(n) = U(wn + (1-n)b) - V(n) = \log(wn + (1-n)b) - \gamma n$$

$$E'(n) = U'(c)(w-b) - V'(n) = \frac{w-b}{c} - \gamma = \frac{w-b-\gamma c}{c}$$

Point: same as an increase in the disutility of working! As before, the firm will never pay a wage above A. But now the lowest wage the worker will ever accept is higher by b. Therefore the wage must satisfy:

$$\frac{V'(n)}{U'(c)} + b < w < A$$
$$\gamma c + b < w < A$$

Similarly to above, the Nash bargaining solution then has the form:

$$w = \phi A + (1 - \phi)[\gamma c + b]$$

i.e. an increase in b works in the same way as an increase in the workers' marginal rate of substitution (recall that this is their "threat point" below which they don't accept a wage). Therefore, an increase in b increases wages. Using the graphical analysis above, one can then show that unemployment must increase $\partial n / \partial b < 0$.