

Asset-Price Redistribution*

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Abstract

Over the last several decades, there has been a large increase in asset valuations across many asset classes. While these rising valuations had important effects on the distribution of wealth, little is known regarding their effect on the distribution of *welfare*. To make progress on this question, we develop a sufficient statistic for the money-metric welfare effect of deviations in asset valuations (i.e., changes in asset prices keeping cash flows fixed). This welfare effect depends on the present value of an individual's *net asset sales* rather than asset holdings: higher asset valuations benefit prospective sellers and harm prospective buyers. We estimate this quantity using panel microdata covering the universe of financial transactions in Norway from 1994 to 2019. We find that the rise in asset valuations had large redistributive effects: it redistributed from the young towards the old and from the poor towards the wealthy.

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1 Introduction

The last few decades have seen large increases in asset prices across many asset classes. A substantial share of this increase is accounted for by declining asset discount rates rather than increasing cash flows.¹ These rising asset valuations had large effects on the distribution of wealth. This raises the question: what are the *welfare* consequences of such valuation effects? Who wins and who loses from declining asset discount rates?

One view is that any rise in asset prices represents an actual shift of resources towards the wealthy, and should be taxed as such (e.g., [Piketty and Zucman, 2014](#); [Saez et al., 2021](#)).² An opposite view is that a rise in asset prices due to lower discount rates (as opposed to higher cash flows) simply generate “paper gains”, with no effect on actual income and therefore welfare (e.g., [Cochrane, 2020](#); [Krugman, 2021](#)).³ Which (if any) of these two opposing views is correct?

To make progress on this question, we develop a sufficient statistic approach that quantifies the individual (money metric) welfare gain associated with a change in asset valuations (i.e. a change in asset prices due to changes in discount rates rather than cash flows).⁴ We operationalize this approach by using Norwegian administrative panel data on asset transactions from 1994 to 2019 and quantify the distribution of welfare gains over this time period.

We ask the following question: how much does a given individual value a deviation in the trajectory of asset prices? Importantly, we consider the effect of a deviation in asset prices while keeping dividends fixed, thereby capturing deviations in asset discount rates. The answer to this question is given by the following formula (here for the case of one asset – the extension to multiple assets is straightforward):

$$\text{Welfare Gain}_i = \sum_{t=0}^T R^{-t} (\text{Sales}_{it} \times \text{Price Deviation}_t), \quad (1)$$

where i denotes the individual, T is the length of the sample period, $R > 1$ is a discount rate, Sales_{it} are the net sales of the asset by the individual in year t , and Price Deviation_t is the deviation of the price of the asset from a baseline scenario. In words, the welfare gain equals the net present value (NPV) of the trading profits due to asset price deviations which are, for instance, positive for asset sellers when asset prices rise. The welfare gain is in dollar terms

¹See, for example, [Farhi and Gourio, 2018](#), [Greenwald et al., 2019](#), [Van Binsbergen, 2020](#), or [Knox and Vissing-Jorgensen, 2022](#) for empirical evidence.

²For example, [Piketty and Zucman \(2014\)](#) write “Because wealth is always very concentrated [... a] high [wealth-to-income ratio] implies that the inequality of wealth, and potentially the inequality of inherited wealth, is likely to play a bigger role for the overall structure of inequality in the twenty-first century than it did in the postwar period. This evolution might reinforce the need for progressive capital taxation.”

³[Cochrane \(2020\)](#) writes “much of the increase in ‘wealth inequality’ [...] reflects higher market values of the same income flows, and indicates nothing about increases in consumption inequality”. [Krugman \(2021\)](#) discusses the hypothetical effect of declining interest rates on large fortunes in 19th-century England and writes “So since the ownership of land, in particular, was concentrated in the hands of a narrow elite, would falling interest rates and rising land prices have meant increased inequality? Clearly not. [...] The paper value of their estates would have gone up, but so what? The distribution of income wouldn’t have changed at all.”

⁴Throughout this paper we will use the term “rising asset valuations” to mean increases in asset prices without an increase in the corresponding cash flows; that is, rising asset prices due to declining asset discount rates.

and corresponds to the individual willingness to pay for the deviation in asset prices.⁵ The formula follows from an application of the envelope theorem and thus holds for small price deviations, a point we discuss in more detail below. More general versions of the formula feature additional terms, for example capturing the welfare gains of asset price changes operating via collateral effects.

The formula for the welfare gain in (1) generates two main insights. First, what matters are asset *transactions*, not asset holdings. Intuitively, higher asset valuations are good news for prospective sellers (those with $\text{Sales}_{it} > 0$) and bad news for prospective buyers (those with $\text{Sales}_{it} < 0$). A particularly interesting case is an individual who owns assets but does not plan to buy or sell (i.e., $\text{Sales}_{it} = 0$). For such an individual, rising asset valuations are merely “paper gains”, with no effect on welfare.⁶

Second, changes in asset valuations are *purely redistributive*. Keeping cash flows fixed, higher asset prices redistribute (money metric) welfare from buyers to sellers. But since for every seller there is a buyer, summing the welfare gains in formula (1) across all parties and counterparties of financial transactions in the economy implies that these aggregate to zero.⁷ This aggregation result holds across all participants in asset markets, and not just the aggregate household sector. Because households trade with other sectors of the economy, namely foreigners and the government, the household sector as a whole may benefit, but necessarily at the expense of another sector.

It is useful to contrast these results with the two polar views described earlier. The first view posited that higher asset valuations redistribute toward existing asset holders. Our formula shows that, it is *sellers* that benefit, not *holders*: if asset holders never sell, they do not benefit from the unrealized capital gains generated by the price deviation. The second view held that all (or at least most) of rising asset valuations are irrelevant for welfare. As our formula shows, this is only true if assets are not traded (e.g., in an economy with a representative agent). But when heterogeneous individuals buy and sell assets like they do in the real world, fluctuations in asset prices do generate welfare gains and losses. In short, both polar views are incomplete.

As we show in the paper, the formula easily extends to multiple assets including bonds and long-lived assets subject to transaction costs (e.g., housing). Our key contribution is an empirical implementation of this welfare formula for the Norwegian economy. We compute welfare gains and losses due to the observed asset price path for the time period 1994 to 2019 comparing it with a baseline where asset prices grew in tandem with dividends. Formally, we

⁵Our welfare gain corresponds to the individual willingness to pay for a small (infinitesimal) price deviation. Because it corresponds to a first-order approximation, it can alternatively be seen as an equivalent variation or a compensating variation (see Mas-Colell et al., 1995). Alternatively, our welfare gain can be seen as the deviation in the individual value function caused by the price deviation divided by marginal utility of consumption.

⁶As already mentioned, more general versions of formula (1) feature additional terms. Some of these are non-zero even for individuals who neither buy nor sell, for example welfare gains operating via collateral effects.

⁷This result arises because our measure of welfare gains is a money metric, i.e., it is measured in dollars but is silent on the value of these extra dollars to the individual or to a social planner. It is therefore also silent on the desirability of an asset price change from the point of view of social welfare. Whenever sellers and buyers systematically have different marginal utilities of consumption (or more generally social welfare weights), then the effect of a price deviation on social welfare can be positive or negative.

compute the price deviation in (1) as the relative difference between the actual price-dividend ratio PD_t and a baseline price-dividend ratio \overline{PD} :

$$\text{Price Deviation}_t = \frac{PD_t - \overline{PD}}{PD_t}. \quad (2)$$

This is motivated by the fact that, across many asset classes, fluctuations in price-dividend ratios are mostly driven by fluctuations in discount rates.⁸ For our application, we use the 1992–1996 average price-dividend ratio as the baseline (i.e., a 5-year window around the beginning of the sample). Importantly, all of the variables in (1) and (2) are readily observable in our data. Price deviations in Norway have been particularly large for real estate (i.e., house prices have grown much faster than rents) and debt (i.e., real interest rates have declined sharply).

Our main findings are as follows. First, rising asset valuations have had large redistributive effects. While the average individual-level money metric welfare gain is around \$10,000, it is $-\$198,000$ at the 1st percentile and $\$286,000$ at the 99th percentile (in 2011 dollars). As a fraction of total wealth (i.e., financial wealth plus human wealth), the average welfare gain is -0.1% , while it is -31% at the 1st percentile and 28% at the 99th percentile. Importantly, the distribution of welfare gains differs substantially from the distribution of revaluation gains (defined as the discounted sum of asset holdings times the changes in asset valuations), which are positive for almost everyone (18.9% on average).

Second, we quantify the amount of redistribution *across cohorts*. Overall, we find a large amount of redistribution from young to old. For instance, the average welfare gain is approximately $-\$15,000$ for individuals aged 15 and younger at the end of 1993 (Millennials), and around $\$24,000$ for individuals aged 30 and older at the end of 1993 (Baby boomers). This is primarily due to the fact that the young are net buyers of housing. Declining interest rates of mortgage debt offset the welfare losses of the young due to rising house prices, but do so only partially.

Third, we quantify the amount of redistribution *across the wealth distribution*. We rank adults according to their total initial wealth (measured at the end of 1993) within cohort and find that welfare gains have been concentrated at the top of the wealth distribution. The wealthiest 1% experienced on average a $\$52,000$ welfare gain while the corresponding number is nearly zero at the 10th percentile, reflecting the fact that the wealthy tend to be net sellers of housing and equity. However, and perhaps surprisingly, average welfare gains track total wealth almost one-for-one along most of the wealth distribution: the average welfare gain as a fraction of total wealth remains approximately constant from the 20th through the 80th percentile, at around 1.7%. This reflects the fact that transactions are roughly proportional to wealth in that part of the wealth distribution.

We then quantify the amount of redistribution *across sectors* of the economy: households,

⁸While our main results compute welfare gains relative to a baseline scenario with constant price-dividend ratios, thereby capturing pure valuation effects, formula (1) can also be used to compute welfare gains relative to other baseline scenarios. For example, we may instead be interested in computing the welfare gains and losses of asset-price changes due to cash flow changes. In this case, our formula is still correct but we would also want to take into account the direct effect of cash flows on individual welfare (an additional additive term).

the government, and foreigners. As discussed above, the average individual in Norway (i.e., in the household sector) experienced a small, but positive, welfare gain of roughly \$10,000. This corresponds exactly to the “welfare loss” for the consolidated government sector (i.e., government plus central bank and sovereign wealth fund). The reason is that Norwegians are net debtors while the government is a net creditor to an almost identical extent (via Norway’s sovereign wealth fund). As a result, declining interest rates have benefited the household sector at the expense of the government. The government budget constraint implies that the household sector will eventually have to bear the cost of this “government welfare loss” through lower net transfers.

Finally, we generalize our sufficient statistic and discuss its interpretation in more general environments. Taking advantage of the flexibility of the envelope theorem, we consider a number of model extensions and explain how these affect our main welfare gain formula. Building on these theoretical results, we then empirically implement versions of our sufficient statistic to address what we view as the most important omissions of our main empirical exercise: collateral effects, incomplete markets, second-order effects due to the large observed asset price changes, and valuation changes beyond the end of our sample period. These generalizations affect our estimated welfare gains and losses quantitatively but not qualitatively. We also discuss the interpretation of our sufficient statistic in more general environments, in particular when asset prices are determined in general equilibrium.

Literature. Our paper contributes to several strands of literature. In recent decades, there has been a sustained rise in valuations across many asset classes (e.g., [Piketty and Zucman, 2014](#), [Farhi and Gourio, 2018](#), [Greenwald et al., 2019](#)). As a response to this trend, a growing literature focuses on understanding the effect of rising asset prices (and declining interest rates) on wealth inequality (e.g., [Kuhn et al., 2020](#); [Gomez, 2016](#); [Wolff, 2022](#); [Gomez and Guin-Bonenfant, 2020](#); [Cioffi, 2021](#); [Catherine et al., 2020](#); [Greenwald et al., 2021](#)). Relative to this literature, our contribution is to study the heterogeneous effect of rising asset prices on welfare.⁹ More broadly, we contribute to a large literature that uses microdata to study the heterogeneity in saving and portfolio choices over the life cycle (e.g., [Feiveson and Sabelhaus, 2019](#); [Calvet et al., 2021](#); [Black et al., 2022](#)) and along the wealth distribution (e.g., [Bach et al., 2017](#); [Fagereng et al., 2019](#); [Mian et al., 2020](#); [Bach et al., 2020](#)).

The effect of asset prices on welfare is studied by [Dávila and Korinek \(2018\)](#), who emphasize the pecuniary externalities that arise in an environment with financial constraints. Relative to this paper, our contribution is to develop an empirical framework to measure a money-metric notion of welfare gains and losses and to implement it using household-level transaction data. Our formula for welfare gains is also related to [Auclert \(2019\)](#) who derives the welfare and consumption effects of deviations in interest rates. Relatedly, [Greenwald et al. \(2021\)](#) stress that the welfare effect of a permanent decline in interest rates can be measured

⁹Our theoretical results build on [Moll \(2020\)](#) who studied a two-period model similar to that in Section 2.1. Our result that the welfare of an individual who never buys or sells an asset is unaffected when the asset’s price changes is related to (but different from) a result by [Sinai and Souleles \(2005\)](#) that an individual with an infinite expected residence spell is insulated from house price risk.

as the duration mismatch between consumption and income which they estimate using U.S. data. While there is a profound connection between the two approaches, our sufficient statistic has two main advantages for our application.¹⁰ First, it allows us to consider the welfare effect of arbitrary valuation changes across asset classes, rather than the ones induced by a uniform change of discount rates in all asset classes. Second, it allows us to measure welfare gains using financial transactions, which we observe directly, rather than in terms of the path of consumption and income, which is typically harder to observe. Finally, our focus on the heterogeneous welfare effect of asset price fluctuations connects this paper to [Doepke and Schneider \(2006\)](#), who study the redistributive effect of inflation episodes using data from the Survey of Consumer Finances, and [Glover et al. \(2020\)](#), who analyze inter-generational redistribution during the Great Recession using a calibrated model.

More generally, our paper is related to a large asset pricing literature on the role of discount rate shocks. One key finding in the literature is that discount rate shocks account for most of asset price fluctuations (a seminal paper is [Campbell and Shiller, 1988](#)). The distinction between cash flow and discount rate shocks has important implications on portfolio allocation (e.g., [Merton, 1973](#), [Campbell and Viceira, 2002](#), [Campbell and Vuolteenaho, 2004](#), [Catherine et al., 2022](#)). Relative to these papers, we examine the effect of discount rate shocks on *welfare*, both theoretically and empirically.

Finally, our argument that rising asset valuations benefit sellers and not asset holders has some historical precedent in the works of [Paish \(1940\)](#), [Kaldor \(1955\)](#) and [Whalley \(1979\)](#) which were, in turn, part of a debate in the public finance literature whether unrealized capital gains are a form of income and should therefore be taxed ([Haig, 1921](#); [Simons, 1938](#)).¹¹

Roadmap. This paper is organized as follows. In Section 2, we present our theoretical results for the welfare effect of a deviation in asset prices. In Section 3, we discuss our sufficient statistic approach using administrative data from Norway. We report our estimates for the redistributive effects of asset price changes within the household sector in Section 4 and between the household, government, and foreign sectors in Section 5. We discuss generalizations of our sufficient statistic approach in Section 6. Section 7 concludes.

2 Theory

This section presents our sufficient statistic approach. To focus on the intuition, we first examine the welfare effect of asset price deviations in a two-period model with only one asset in Section 2.1. We generalize the result to an infinite horizon model with multiple assets and adjustment costs in Section 2.2. We discuss interpretations of our sufficient statistic approach in Section 2.3. Finally, we discuss a number of important extensions such as collateral constraints,

¹⁰We discuss the precise mapping between the two approaches in Appendix A.2.1.

¹¹For example, [Kaldor \(1955\)](#) writes: “We may now turn to the other type of capital appreciation which [comes] without a corresponding increase in the flow of real income accruing from that wealth. [...] In] so far as a capital gain is realized and spent [...] the benefit derived from the gain is equivalent to that of any other casual profit. If however it is not so realized, there is clearly only a smaller benefit.”

idiosyncratic and aggregate risk, finite lives and bequest, as well as assets in the utility function in Section 2.4.

2.1 Intuition in a two-period model

Time is discrete with two time periods $t = 0, 1$. Individuals have time separable preferences with a differentiable utility function $U(\cdot)$ that is increasing and strictly concave and a subjective discount factor $\beta < 1$. Individuals receive labor income Y_0 at time 0 and Y_1 at time 1. There is one asset available for trading at time $t = 0$ with price $P_0 > 0$, which pays a dividend $D_1 > 0$ at time 1. Moll (2020) analyzes a similar two-period environment.

Individual problem. Denote by C_t the consumption of the individual at time t and N_t the number of shares owned at the end of period t . Given initial asset holdings N_{-1} , the problem of the individual is to choose consumption and asset holdings to maximize welfare

$$V = \max_{\{C_0, C_1, N_0\}} U(C_0) + \beta U(C_1), \quad (3)$$

subject to the following budget constraints:

$$C_0 + (N_0 - N_{-1})P_0 = Y_0, \quad (4)$$

$$C_1 = N_0 D_1 + Y_1. \quad (5)$$

These budget constraints say that, in each period t , consumption plus net asset purchases (the left hand side) must equal income (the right hand side).¹²

Comparative static with respect to prices. We are interested in the welfare effect of a small change in the price of the asset, holding dividends fixed; that is, the welfare effect of a pure *valuation change*. To do so, we consider an infinitesimal change in the price of the asset dP_0 , holding its dividend D_1 constant.¹³

Since the price P_0 only appears in the budget constraint (4) at time $t = 0$, the envelope theorem states that

$$dV = U'(C_0)(N_{-1} - N_0) dP_0. \quad (6)$$

The effect of a rise in P_0 is given by the marginal utility of consumption at $t = 0$, $U'(C_0)$, times the extent to which it relaxes the budget constraint at $t = 0$, namely asset sales $N_{-1} - N_0$. Intuitively, a rise in the price of the asset benefits individuals who plan to sell the asset (i.e., $N_0 < N_{-1}$) and hurts individuals who plan to buy the asset (i.e., $N_0 > N_{-1}$). In particular, a rise in the price of the asset does not affect individuals who do not plan to trade (i.e., $N_0 =$

¹²Recall that the environment has only two periods, with no market for transactions at time $t = 1$ (alternatively, the price of the asset is zero at $t = 1$). We consider the multi-period case below, in which case we add appropriate terminal conditions.

¹³To put this more precisely, it is useful to adopt the asset pricing perspective that the asset price at $t = 0$ is the present discounted value of future cash flows: $P_0 = D_1/R_1$ where R_1 is the asset required rate of return, which we take as exogenous. An increase in the price P_0 without a change in the dividend D_1 is then equivalent to a fall in the required rate of return R_1 . We develop this general point in Appendix A.2.1.

N_{-1}): for those individuals, the rise in the price of the asset is merely a “paper gain” with no corresponding effect on consumption and thus welfare.

Importantly, the comparative static in equation (6) holds the dividend D_1 constant (as we want to capture the welfare effect of a pure valuation change). If instead the asset’s dividend increased $dD_1 > 0$ together with the asset price increase, the comparative static in equation (6) would have an extra term $\beta U'(C_1)N_0 dD_1$, reflecting that higher dividends benefit asset holders.

Welfare versus wealth gains. The result in equation (6) may be surprising at first. How can an asset holder (i.e., $N_{-1} > 0$) not benefit from a rise in prices given that the market value of his or her initial wealth $N_{-1}P_0$ increases? The reason is that, while a rise in P_0 increases the initial return on the asset at time $t = 0$, it also *decreases* the future return of holding the asset until $t = 1$ by lowering the asset’s dividend yield. As a result, only individuals whose holdings decline over time (i.e., sellers) benefit from a rise in asset valuation.

To see this formally, denote R_t the return of the asset at time t ; that is $R_0 = P_0/P_{-1}$ and $R_1 = D_1/P_0$. Note that a rise in P_0 increases R_0 , via a higher capital gain, but *decreases* R_1 , via a lower dividend yield:

$$\frac{dR_0}{dP_0} = 1/P_{-1} > 0, \quad \frac{dR_1}{dP_0} = -R_1/P_0 < 0. \quad (7)$$

The welfare effect of this change in asset returns can be written as the marginal utility of consumption times the change in asset returns every period:¹⁴

$$\begin{aligned} dV &= \underbrace{U'(C_0) \times N_{-1}P_{-1} \times dR_0}_{\text{contribution of return at } t=0} + \underbrace{\beta U'(C_1) \times N_0P_0 \times dR_1}_{\text{contribution of return at } t=1} \\ &= U'(C_0)N_{-1}dP_0 - U'(C_0)N_0dP_0 \\ &= U'(C_0)(N_{-1} - N_0)dP_0, \end{aligned} \quad (8)$$

where the second line is obtained by combining the Euler equation $\beta U'(C_1) = R_1^{-1}U'(C_0)$ with (7). This alternative derivation highlights that the welfare effect (6) can be seen as the sum of two terms: the first term, $U'(C_0)N_{-1}dP_0$, accounts for the positive effect of a rise in P_0 on today’s return (through a higher capital gain) while the second term, $-U'(C_0)N_0dP_0$, accounts for the negative effect of a rise in P_0 on tomorrow’s return (via a lower dividend yield). For an individual who does not trade, the two terms offset each other: as a result, a change in asset prices has no effect on welfare.

In short, to capture the welfare effect of a rise in asset prices, it is important to capture not only its positive effect on today’s return but also its negative effect on tomorrow’s return. In our empirical exercise, we will document that the *welfare* effect of a deviation in asset prices—the left-hand side of equation (8)—is often very different from its *wealth* effect—the first term on the right-hand side of equation (8).

¹⁴This follows from rewriting the budget constraints (4) and (5) as $C_0 + A_0 = R_0A_{-1} + Y_0$ and $C_1 = R_1A_0 + Y_1$, where $A_t \equiv N_tP_t$, and calculating the welfare effect of deviations in R_0 and R_1 using the envelope theorem.

Another intuition for this point is to consider the present-value budget constraint obtained by combining (4) and (5)

$$C_0 + \frac{C_1}{D_1/P_0} = Y_0 + \frac{Y_1}{D_1/P_0} + N_{-1}P_0. \quad (9)$$

which states that the present-value of consumption (discounted at $R_1 = D_1/P_0$) must equal the present value of income plus initial wealth. This present-value constraint shows clearly that the argument that a higher asset price P_0 benefits asset holders by increasing the market value of initial wealth $N_{-1}P_0$ ignores the offsetting effect on the return going forward, $R_1 = D_1/P_0$, and hence on the present values of both consumption and income.¹⁵ We now flesh out this point in more detail graphically.

Graphical intuition. Building on [Whalley \(1979\)](#), we now provide a graphical intuition for equation (6). We represent the individual's optimization as a standard problem of intertemporal choice: maximize utility (3) subject to a present-value budget constraint, here given by (9). Figure 1 shows the standard budget constraint and indifference curve, with the slope of the former given by (the negative of) the asset return $R_1 = D_1/P_0$.¹⁶

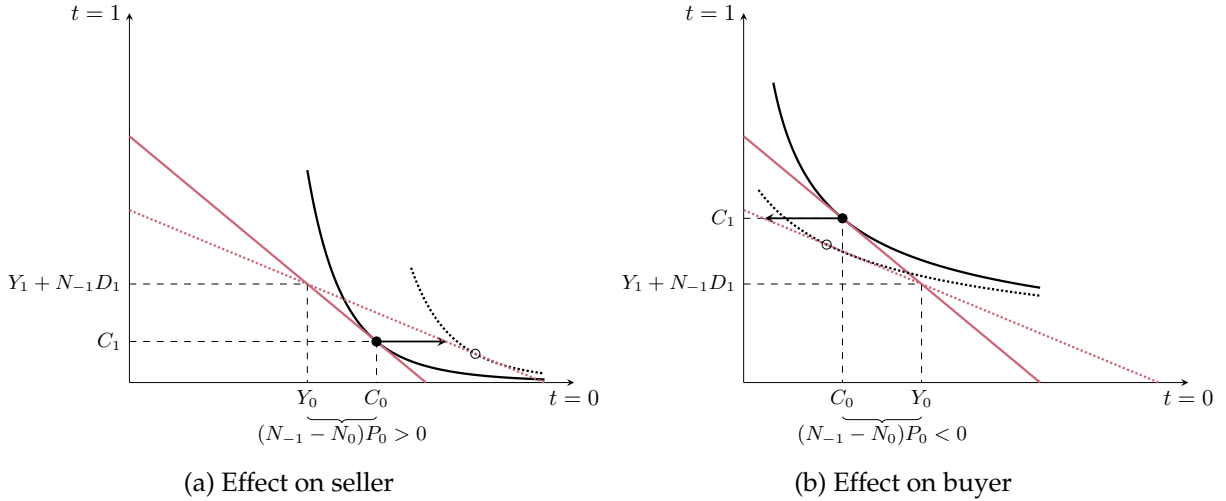


Figure 1: Welfare effect of a rise in the asset price P_0 (two-period model)

Notes. Figure 1 graphically analyzes the effect of an increase in the asset price P_0 on the welfare of a seller (panel a) and that of a buyer (panel b). In both panels, the present-value budget constraint goes through the endowment point $C_0 = Y_0$ and $C_1 = Y_1 + N_{-1}D_1$ and has slope $-D_1/P_0$ (see footnote 16). The solid budget constraint and indifference curve correspond to the allocation at the initial asset price and the dotted lines are those at the new, higher price. When the asset price P_0 increases, the budget constraint rotates through the endowment point and becomes shallower. The seller's welfare increases (panel a) and the buyer's welfare decreases (panel b).

Consider the welfare consequences of a rise in the asset price P_0 for a hypothetical seller (panel a) and buyer (panel b). In both panels, the solid budget constraint and indifference curve correspond to the allocation at the initial asset price and the solid lines are those at the new, higher price. When the asset price P_0 rises, the budget constraint rotates through the endowment point and becomes shallower (the slope is $-D_1/P_0$).

¹⁵See the paragraph [Derivation of welfare gains formula from present-value budget constraint](#) in Appendix A.1 for a similar intuition in an infinite horizon model.

¹⁶ To see that the slope of the present-value budget constraint (9) is $-D_1/P_0$ and that the endowment point is given by $C_0 = Y_0$ and $C_1 = Y_1 + N_{-1}D_1$ as in Figure 1, it is useful to write it as $C_1 = \frac{D_1}{P_0}(Y_0 - C_0) + Y_1 + N_{-1}D_1$.

Panel (a) depicts the case of an individual selling the asset at $t = 0$ (i.e., $N_{-1} - N_0 > 0$) so that optimally chosen initial consumption exceeds initial labor income $C_0 > Y_0$. Panel (b) considers the case of a buyer. The figure shows clearly that the seller ends up on a higher indifference curve (her welfare increases) whereas the buyer ends up on a lower indifference curve (her welfare decreases).¹⁷

2.2 Baseline model

For the sake of intuition, the previous section focused on the case of a two-period economy with only one asset. We now extend our formula to an multi-periods economy with multiple assets and adjustment costs (henceforth the “baseline model”), which is key to bringing the theory to the data. We will examine a number of additional extensions in Section 2.4.

Financial markets. There is a sequence of liquid one-period bonds B_t with a face value of one and price $Q_t > 0$ available for trading. Note that holding a one-period bond is equivalent to investing in a deposit account with an interest rate $R_{t+1} = 1/Q_t$ between time t and $t + 1$. Denote by $R_{0 \rightarrow t} = R_1 \cdot R_2 \cdots R_t$ the cumulative return of the liquid asset between time 0 and t . There are also K long-lived assets $N_{k,t}$ available for trading (e.g., housing, stocks, private businesses, and long-term bonds). Each share of asset k is a claim to a stream of dividends $\{D_{k,t}\}_{t=0}^{\infty}$, with price $P_{k,t} > 0$ at the end of period t . The asset’s return between t and $t + 1$ is thus $R_{k,t+1} \equiv (D_{k,t+1} + P_{k,t+1})/P_{k,t}$.

We assume that trading these long-lived assets is subject to adjustment costs which may be large or small depending on the asset. Some assets, such as houses and privately-traded equity, are illiquid and the adjustment costs capture this illiquidity. We assume that adjustment costs are continuous but allow them to be kinked (non-differentiable) to capture infrequent adjustment and inaction regions (as in Bertola and Caballero, 1990 or Kaplan et al., 2018). For other assets, such as publicly traded equity, the adjustment costs—which may be arbitrarily small but positive—are instead a technical assumption that is necessary in our deterministic setup. In short, they allow different assets to have different returns without generating the possibility of infinite profits via arbitrage.¹⁸ Specifically, to buy a quantity of shares $N_{k,t} - N_{k,t-1}$ of asset k at time t , the individual will have to pay $\chi_k(N_{k,t} - N_{k,t-1})$ in adjustment costs. While the particular functional form does not matter for the effect of asset price changes on welfare at the first order (i.e., for infinitesimally small price deviations), it will matter for higher-order effects, as discussed in Section 6.

Individual problem. Individuals have time-separable preferences with a differentiable utility function $U(\cdot)$ that is increasing and strictly concave and a subjective discount factor $\beta \in$

¹⁷In fact, our notion of money-metric welfare gain $dV/U'(C_0)$ corresponds, at the first-order, to the horizontal distance between the initial C_0 and the new budget line (as indicated by the solid arrows). Indeed, this distance corresponds to the extent to which C_0 needs to adjust following the rise in asset price if C_1 (and therefore N_0) is held constant: $\Delta C_0 = (N_{-1} - N_0)\Delta P_0$.

¹⁸In the stochastic environment discussed in Section 2.4, adjustment costs can be zero when different assets have different risk profiles thereby making them imperfect substitutes. Alternatively, assets may be imperfect substitutes due to the non-monetary benefits of owning them (e.g., owning a house versus renting it) as in Appendix A.3.5.

$(0, 1)$. They receive labor income $Y_t > 0$ at time t and we denote by B_t the holdings of the one period bonds and by $N_{k,t}$ those of asset k at the end of period t . Individuals take asset prices as given and choose an optimal path of consumption and asset holdings,

$$V = \max_{\{C_t, B_t, \{N_{k,t}\}_k\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t), \quad (10)$$

subject to initial holdings B_{-1} and $\{N_{k,-1}\}_k$, as well as a sequence of budget constraints

$$C_t + \sum_{k=1}^K (N_{k,t} - N_{k,t-1}) P_{k,t} + B_t Q_t + \sum_{k=1}^K \chi_k (N_{k,t} - N_{k,t-1}) = \sum_{k=1}^K N_{k,t-1} D_{k,t} + B_{t-1} + Y_t. \quad (11)$$

As in the two-period model, the budget constraint simply says that consumption plus net purchases of financial assets (the left-hand side) must equal total income in each period t (the right-hand side), where total income is the sum of dividend income, fixed income, and labor income. Because of the infinite horizon setup, we also assume the following technicality conditions: a lower bound on the price of one-period bonds $\liminf_{T \rightarrow \infty} Q_T > 0$, a no-bubble condition $\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} P_{k,T} = 0$, a bound on asset holdings $N_{k,t} \in \Theta_k$ where Θ_k is compact, as well as a no-Ponzi condition $\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} (B_T Q_T + \sum_{k=1}^K N_{k,T} P_{k,T}) \geq 0$. Lemma 5 in Appendix A.1 spells out the implied present-value budget constraint whereas Section 2.4 covers the finite-horizon case. Finally, we assume that there exists one and only one solution $\{C_t, B_t, \{N_{k,t}\}_{k=1}^K\}_{t=0}^{\infty}$ which is continuous with respect to prices at $\{Q_t, \{P_{k,t}\}_{k=1}^K\}_{t=0}^{\infty}$.

Welfare gain. We are interested in the effect of a change in asset prices on welfare. Formally, we consider a deviation of the path of asset prices, denoted by $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^{\infty}$, holding everything else constant. We assume that the deviation does not explode over time, i.e. that it satisfies the no-bubble condition $\lim_{t \rightarrow \infty} R_{0 \rightarrow t}^{-1} dQ_t = \lim_{t \rightarrow \infty} R_{0 \rightarrow t}^{-1} dP_{k,t} = 0$.¹⁹

Denote by dV the effect of the price deviation on welfare defined in (10). We define the *money metric welfare gain* as the change in welfare scaled by the marginal utility of consumption at time $t = 0$

$$\text{Welfare Gain} \equiv dV / U'(C_0). \quad (12)$$

This welfare gain is in units of consumption and has the interpretation of an individual's willingness to pay for this particular price deviation. For brevity we will often refer to this quantity simply as "welfare gain" but it is important to keep in mind that it is a money metric, i.e. it is silent on the value of these extra resources to the individual.

Totally differentiating the definition of welfare (10) gives the following expression for the welfare gain:

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} dC_t = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_t, \quad (13)$$

where the second equality uses the Euler equation; that is, $\beta^t R_{0 \rightarrow t} U'(C_t) / U'(C_0) = 1$. This equation says that our measure of welfare gain can be seen as the present value of the con-

¹⁹As discussed in Appendix A.2.1, this is equivalent to a deviation in the required return of the asset.

sumption changes caused by the price deviation.²⁰

We now express the welfare gain from a deviation in the path of asset prices.

Proposition 1 (Welfare Gain). *The welfare gain implied by a price deviation $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^\infty$ is*

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t \right). \quad (14)$$

The proposition, proven in Appendix 1, says the welfare gain corresponds to the net present value (NPV) of the trading profits due to the deviation in the path of asset prices. As in the two-period model, the welfare gain depends on whether the individual is a buyer or seller of assets. The key insight is that the welfare gain associated with deviations in asset prices depends on financial *transactions* rather than *holdings*. Note, however, that for the liquid asset, transactions and holdings coincide given that the asset must be continuously rolled over. Thus, declining interest rates (i.e., $dQ_t > 0$) benefit individuals holding short-term debt (i.e., $B_t < 0$) because lower debt payments relax their budget constraint.

Finally, note that the adjustment cost function does not appear in the welfare formula. This is a direct implication of the envelope theorem, which says that the changes in adjustment costs are second-order for welfare.²¹ Similarly, how the individual allocates trading profits each period—to consume or to invest—has no bearing on welfare.²² Finally, we note that the welfare gain formula in Proposition 1 can be derived using either the sequence of period budget constraints (11) or the corresponding present-value constraint (see Appendix A.1).

Aggregation. We now describe an important aggregation result. Suppose that the economy is populated by $i = 1, 2, \dots, I$ individuals who trade assets with each other. We denote by $\{B_{it}, \{N_{i,k,t}\}_{k=1}^K\}_{t=0}^\infty$ the sequence of asset holdings of individual i .

Corollary 2 (Aggregation). *Suppose that initial prices $\{Q_t, \{P_{k,t}\}_k\}_{t=0}^\infty$ clear all asset markets, i.e., asset sales and purchases add up to zero for each asset class. Welfare gains implied by a price deviation $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^\infty$ aggregate to zero and thus price deviations are purely redistributive:*

$$\sum_{i=1}^I (N_{i,k,t-1} - N_{i,k,t}) = 0, \text{ all } k \quad \text{and} \quad \sum_{i=1}^I B_{i,t} = 0 \quad \implies \quad \sum_{i=1}^I \text{Welfare Gain}_i = 0.$$

Corollary (2) is intuitive. For instance, when asset prices rise, sellers benefit, but market clearing implies that for every seller there is an offsetting buyer that is hurt. Hence, the welfare gains must aggregate to zero over the full population. In fact, market clearing implies that

²⁰In turn, these consumption changes can be decomposed into income and substitution effects, where income effects sum up to welfare gains while substitution effects sum up to zero (see Appendix A.1).

²¹To apply the envelop theorem we assumed that the solution of the optimization problem was locally continuous with respect to prices. While this does not rule out kinked adjustment costs (as in Kaplan et al., 2018), this does rule out adjustment cost functions that lead to discrete adjustments in response to infinitesimal changes in prices.

²²This is because, at the optimum, the agent is indifferent between consuming versus investing a marginal dollar. We discuss this point more precisely in the paragraph **Fixed holdings versus reoptimization** in Appendix A.1.

welfare gains aggregate to zero for each asset class.²³ This result highlights a key difference between wealth gains and welfare gains: while a rise in asset prices leads to positive wealth gains in aggregate (as long as the asset is in positive net supply), it does not lead to aggregate welfare gains. In terms of welfare, asset price changes are therefore purely redistributive.

Our result says that welfare gains aggregate to zero. It is important to recall, however, that individual welfare gains are money metric gains as defined in (12), i.e. they are measured in dollars but are silent on the value of these extra dollars to the individual. Put differently, the result says nothing about the desirability of an asset price deviation from the point of view of a social planner. In particular, the effect of a price deviation on social welfare can be positive or negative, depending on whether the welfare weights assigned by the planner to individuals covary positively or negatively with their money metric gains.²⁴ What our aggregation result says is simply that the social planner could, in principle, undo the effect of asset price changes on social welfare, by redistributing resources from individuals with money metric gains to those with money metric losses.

Finally, the result hinges on a set of strong assumptions that we will relax later. As we will discuss below (Section 2.4), welfare gains no longer aggregate to zero in case of collateral constraints, market incompleteness, or when individuals trade with other entities (e.g. the government and foreigners). Similarly, welfare gains no longer aggregate to zero for non-infinitesimal changes in prices, as asset markets typically do not clear outside of equilibrium prices. Still, this is an important baseline to keep in mind, and we will treat the fact that welfare gains aggregate to zero as an accounting identity guiding our measurement of welfare gains in the data.

2.3 Discussions

We now discuss the interpretation of our sufficient statistic approach.

Equivalence with deviations in discount rates. In the baseline model, we have specified exogenous paths for dividends and asset prices $\{D_{k,t}, P_{k,t}\}_{t=0}^{\infty}$. Another approach, often favored in the asset pricing literature, is to treat asset prices as determined by exogenous paths for a sequence of dividends and discount rates $\{D_{k,t}, R_{k,t+1}\}_{t=0}^{\infty}$ where $R_{k,t+1}$ represents the required rate return for holding the asset between t and $t+1$. Under the no-bubble condition, asset prices are then obtained as the discounted value of dividends; that is, $P_{k,t} = \sum_{s=1}^{\infty} (R_{k,t+1} \cdots R_{k,t+s})^{-1} D_{k,t+s}$.²⁵ This formula highlights that changes in asset prices must come from either changes in dividends or changes in asset discount rates.

²³Note that this is true even if the aggregate number of shares evolves over time. For example, when entrepreneurs issue new equity shares to investors, higher valuations benefits entrepreneurs at the expense of investors.

²⁴More precisely, the change in social welfare is $\sum_{i=1}^I \lambda_i U'(C_{i0}) \times \text{Welfare Gain}_i$, where λ_i is the Pareto weight assigned to individual i . The term $\lambda_i U'(C_{i0})$ can be seen as a marginal welfare weight (see, e.g., Saez and Stantcheva, 2016; Dávila and Schaab, 2022). In Appendix C.1, we will plot the change in social welfare for different sets of marginal welfare weights.

²⁵The no-bubble condition is $\lim_{t \rightarrow \infty} (R_{k,1} \cdots R_{k,t})^{-1} P_t = 0$.

With this in mind, our thought experiment of changing asset prices while keeping dividends unchanged should be understood as changing the asset discount rates. In particular, Proposition 1 should be understood as examining the welfare effect of changes in asset prices *due to changes in discount rates* (as opposed to changes in dividends). We give the mathematical mapping between deviations in asset prices and deviations in asset discount rates in Lemma 6 in Appendix 2.3.

This alternative interpretation relates our results to Auclert (2019) and Greenwald et al. (2021), who study the welfare effect of changes in interest rates and show that it depends on the duration mismatch between consumption and income. We formalize the relationship between the two results in Appendix A.2.1: intuitively, an individual whose income has a shorter duration than their consumption needs to save by purchasing assets over time, and, hence, is hurt by higher asset prices (i.e., lower interest rates).

General equilibrium. So far, we have considered a partial equilibrium environment, where asset price deviations are exogenous. We now discuss the interpretation of our sufficient statistic in a general equilibrium environment, where asset prices are endogenously determined by supply and demand forces. In this case, our baseline welfare gain formula should be interpreted as capturing the welfare effect of a deviation in these fundamental forces that operates through asset prices.

More precisely, denote the fundamental drivers of the economy by the sequence of vectors $\{z_t\}_{t=0}^{\infty}$. This vector could contain determinants of asset prices such as demographic trends, technological growth, monetary policy, and so on. The thought experiment that we consider in this context is to perturb the path of z_t and trace out its effect on welfare, both through asset prices *and* through income (i.e., dividend and labor income).

Formally, let $z_t = \bar{z}_t + \theta \Delta z_t$, where θ is a scalar that indexes deviations from a baseline scenario $\{\bar{z}_t\}_{t=0}^{\infty}$ and $\{\Delta z_t\}_{t=0}^{\infty}$ determines the direction of these deviations. We denote the equilibrium prices and income by $\{\{P_{k,t}(\theta)\}_{k=1}^K, Q_t(\theta)\}_{t=0}^{\infty}$ and $\{\{D_{k,t}(\theta)\}_{k=1}^K, Y_t(\theta)\}_{t=0}^{\infty}$, emphasizing their dependence on θ . A small change $d\theta$ induces deviations in asset prices and income $dP_{k,t} = (\partial P_{k,t}/\partial\theta) d\theta$, $dQ_t = (\partial Q_t/\partial\theta) d\theta$, $dD_{k,t} = (\partial D_{k,t}/\partial\theta) d\theta$, $dY_t(\theta) = (\partial Y_t/\partial\theta) d\theta$. The following proposition, which we prove in Appendix A.2.2, expresses the welfare gain associated with a perturbation $d\theta$.

Proposition 3 (Welfare Gain in GE). *The welfare gain due to a perturbation $d\theta$ is*

$$\text{Welfare Gain} = \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t \right)}_{\text{Welfare gain through asset prices (Proposition 1)}} + \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K N_{k,t} dD_{k,t} + dY_t \right)}_{\text{Welfare gain through income}}. \quad (15)$$

The first term captures the welfare gain due to deviations in asset prices, which is the same as in Proposition 1. The second term captures the welfare gain due to deviations in income (i.e., dividends and labor income), which is simply the present value of the change in income associated with the fundamental shock $d\theta$. The formula highlights the key difference between

the welfare effect of income and asset price changes: while welfare increases one-for-one with changes in the present value of income (the second term), they only benefit from an increase in asset prices if they sell. [Del Canto et al. \(2023\)](#) uses a similar formula to decompose the welfare effects of inflationary shocks in the U.S. (which impact both asset prices and income).

As a concrete example of redistribution in general equilibrium, in [Appendix A.2.2](#) we solve an overlapping generation model with a single long-lived asset (as in [Samuelson, 1958b](#)). In the model, the old are the natural sellers and the young are the natural buyers. We simulate the total welfare effect of a demand shock (rise in patience) as well as a supply shock (rise in productivity) that both yield an equilibrium increase in the price of the asset. We then provide closed-form expressions for the welfare gains coming from both the change in asset prices and the change in income.

Finally, our empirical application considers the impact of declining asset discount rates on the welfare of Norwegian households, a decline that has arguably been a global phenomenon. Whenever the drivers of this decline do not directly impact the income of Norwegian households—for instance if the driver is a saving glut from abroad—then our baseline formula (14) captures the total welfare effect of these drivers.²⁶

Ex-ante versus ex-post welfare. In the baseline model, we made the simplifying assumption that deviations in asset prices are deterministic (i.e., that they are known at time $t = 0$). In [Appendix A.2.3](#), we consider the case where deviations in asset prices are stochastic; that is, where individuals do not have perfect foresight over future asset price deviations. In this case, we show that our welfare gain formula (14) can be interpreted as the “ex-post” welfare effect of a realized sequence of small asset price shocks around a deterministic economy.²⁷ We discuss ex-ante welfare effects in a fully stochastic economy shortly (see the paragraph [Idiosyncratic and aggregate risk](#)).

2.4 Extensions

The baseline model is deliberately stylized and abstracts from a number of potentially important features of the real world. Before we bring our theory to the data, we consider a number of model extensions. In the rest of this section, we briefly summarize how the extension affects our welfare gain formula (14) as well as its interpretation. [Appendix A.3](#) provides a rigorous treatment of each model extension.

Borrowing constraints and collateral effects. In the baseline model, individuals can take unrestricted positions in any asset (i.e., long and short). In reality, there are limits on these positions, for instance on how much uncollateralized debt an individual can obtain. More generally, the interest rate charged by a lender may increase with the debt level and decrease with the net worth of the borrower. In [Appendix A.3.1](#), we consider an extension of the baseline model

²⁶In practice, the trend of declining interest rates and rising asset prices amongst developed economies is often thought to be the result of an increased demand for saving from abroad ([Bernanke, 2005](#)), population aging ([Auclert et al., 2020](#)), or rising inequality ([Mian et al., 2021](#)).

²⁷See [Samuelson \(1958a\)](#), or more recently [Bhandari et al. \(2021\)](#), for similar uses of small-noise expansions.

with an upward-sloping interest rate schedule that may also be a function of collateral values (a “credit surface” in the language of Geanakoplos, 2016). We show that “hard” borrowing limits (and collateral constraints) can be seen as a limiting case in which the upward-sloping interest rate schedule becomes vertical at some debt level so that we nest these specifications.

We show that such borrowing constraints affect our welfare gain formula in two ways. First, whenever individuals are on the upward-sloping part of the interest rate schedule, the standard Euler equation does not hold (i.e., the marginal utility of consumption today exceeds discounted marginal utility tomorrow times the interest rate). Hence, the rate at which future net asset purchases must be discounted in the welfare gain formula (14) is higher than the cumulative return $R_{0 \rightarrow t}$ on the liquid asset. In this case, our welfare measure will tend to overestimate the contribution of future price deviations on welfare. In our empirical implementation, we do not attempt to measure individual-specific welfare-relevant discount rates, but our choice of discount rate is meant to be conservative (i.e., higher than the interest rate on bank deposits).

Second, the tightness of the borrowing constraint, or more generally the interest-rate schedule, may directly depend on the price of an asset $P_{k,t}$, as in collateral constraint models (e.g., Kiyotaki and Moore, 1997; Miao and Wang, 2012; Mian et al., 2013). In this case, the welfare gain formula (14) has an additional term that accounts for the effect of asset prices on the tightness of the borrowing constraint. In collateral constraint models, higher asset prices tend to be welfare-improving because they allow constrained individuals to better smooth their consumption or because they allow individuals to borrow at lower interest rates.²⁸ The appendix also discusses the case of individuals borrowing against higher asset valuations rather than selling the appreciated assets, for example for tax avoidance reasons or due to asset illiquidity. In Section 6.1, we quantify the contribution of an empirically-relevant form of the collateral effect: the dependence of mortgage interest rates on the loan-to-value ratio.

Idiosyncratic and aggregate risk. In the model above, we assumed that the baseline economy was deterministic. In Appendix A.3.2, we consider an extension where the baseline economy is stochastic (e.g., idiosyncratic labor income risk or aggregate dividend income risk). We then examine the ex-ante welfare gain due to a deviation in asset prices (the deviation can itself be stochastic).²⁹ It is given by the expected path of net asset sales times the price deviation *under the individual’s risk-neutral probability measure* (i.e., the objective probability measure tilted by the individual growth of marginal utility).

Put differently, the baseline formula for the welfare gain changes in two ways. The first is that, in a stochastic world, what matters for (ex-ante) welfare is the *expected* path of net asset sales, not the realized one. The second is that this expectation is under the individual’s risk-neutral measure, which tilts the objective measure by the growth of the individual’s marginal

²⁸Dávila and Korinek (2018) provide a theoretical treatment of pecuniary externalities in collateral constraint models. Analogous to our results they isolate two pecuniary externalities: a redistributive externality that depends on asset transactions and a collateral externality which is non-zero whenever collateral constraints bind.

²⁹As above, welfare gain corresponds to the willingness to pay for the deviation in prices at $t = 0$, before the risk is realized.

utility. This adjustment reflects the fact that individuals care about certain states of the world more than others. To be concrete in the context of idiosyncratic labor income risk, consider the ex-ante welfare gain of young individuals who face uncertainty over their future paths of labor income, and who plan to buy houses only if they are successful in the labor market. From today’s perspective, these individuals “care more” about the states of the world in which their income is low, as their marginal utility of consumption will be higher in these states. Hence, their expected housing purchases are lower under the risk-neutral probability measure than under the objective measure.

Formally, we show that the (ex-ante) welfare gain formula with risk features two terms (here for the case of one asset):

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \mathbb{E}_0 [(N_{t-1} - N_t) dP_t] + \sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_t)}{U'(C_0)}, (N_{t-1} - N_t) dP_t \right). \quad (16)$$

The first term is the expected path of net asset sales times the price deviation. The second term is the covariance between the growth rate of marginal utility and net asset sales times price deviations. It says that an agent benefits from higher asset prices when the growth of their marginal utility covaries positively with their net asset sales.

While the first term (on the right-hand side) averages to zero across agents, the second term does not in the presence of uninsurable idiosyncratic risk (as individuals no longer equalize the growth of their marginal utilities in a given state of nature). Using the terminology in [Dávila and Schaab \(2022\)](#), the fact that the second term does not aggregate to zero with incomplete markets corresponds to the “risk-sharing” component of a deviation in asset prices. In [Section 6.2](#), we estimate the adjustment due to uninsurable idiosyncratic income risk for each cohort in our sample.

Finite lives and bequests. In the baseline model, we abstract from life-cycle considerations, inter-generational linkages and bequests. In practice, bequests have been shown to be an important determinant of consumption and saving decisions ([De Nardi, 2004](#)). In [Appendix A.3.3](#), we consider an extension of the baseline model where individuals have finite lives and receive utility from giving assets to their heirs via a “warm glow bequest function”. We do not specify the functional form of the bequest function, hence nesting both altruistic models and other ad-hoc specifications.

Finite lives by themselves (i.e., without bequests) do not change our welfare formula in any way. Specifically, while a finitely-lived individual without a bequest motive optimally sells off all assets before she dies, it is still true that the welfare gain from changing asset valuations equals the present discounted value of future asset sales times price deviations just like in [Proposition 1](#).

In contrast, adding bequests does change the formula in two ways. First, the effect of a price deviation dP_t on welfare matters through the number of shares sold ($N_{k,t-1} - N_{k,t} + \text{Net inheritance}_{k,t}$), not the decrease in holdings ($N_{k,t-1} - N_{k,t}$) alone. Intuitively, if an individual inherits a house and immediately sells it—such that holdings of housing are unchanged

$N_{k,t-1} = N_{k,t}$ but housing sales equal Net inheritance $_{k,t}$ —higher house prices benefit the individual. Conversely, if an individual inherits a house and plans to live in it forever—such that holdings increase from $N_{k,t-1}$ to $N_{k,t} = N_{k,t-1} + \text{Net inheritance}_{k,t}$ but housing sales are zero—higher house prices are irrelevant for the individual’s welfare (the inheritance itself of course still benefits the individual in absolute terms, just not in a way that is dependent on house prices).³⁰ This distinction is easy to deal with empirically, since we observe financial transactions directly, not just changes in holdings.

Second, the welfare gain formula has an additional term that accounts for the change in net inheritance as a result of asset prices. The idea is that individuals may decide to adjust the quantity of assets that they give to their heirs in response to an asset price change. In our empirical implementation, we assume that this term is zero. To be more concrete, in the context of housing, our assumption says that changes in asset prices do not affect the physical quantity of real estate (e.g., square meters) parents leave to their children. Note that, as a result, a rise in asset prices means that the *value* of a positive inheritance increases even though the quantity of assets inherited is unchanged.

Financial transactions done by businesses. In the baseline model, individuals directly own and trade financial assets. In reality, individuals typically own businesses that themselves own and trade financial assets (this includes, in particular, debt issued by businesses as well as share repurchases). In Appendix A.3.4, we show that the sufficient statistic formula still holds after allocating the transactions done by these businesses to their ultimate owners. Intuitively, it is irrelevant for welfare whether financial transactions are done directly by the individuals or indirectly through the businesses that they own. Similarly, it is irrelevant whether a business pays out dividends or repurchases its own shares; what matters instead is its income stream (earnings minus investment). As a result, we will take into account the indirect financial transactions done by businesses owned by each individual when implementing our sufficient statistic.

Housing and wealth in utility function. In the baseline model, individuals only get utility from consumption and thus do not care about asset ownership *per se*. In reality, individuals may also care about the quantity of assets they own. An important example is owning a house and living in it which generates a direct utility flow. Other examples include preferences for social status or power. In Appendix A.3.5, we consider an extension of the baseline model where assets enter the utility function directly. We show that, if only the *quantity* of assets enters the utility function (as is natural in the housing case), this “joy of ownership” channel does not affect our welfare gain formula. It is only when individuals directly care about the *market value* of their assets (rather than simply the *quantity* of assets they own) that the welfare

³⁰It is also worth noting that higher house prices hurt individuals who did not inherit a house but are planning to buy one in the future. Thus, even if an individual inherits a house and plans to live in it forever, this individual still benefits from higher house prices *relative* to these other individuals (the inheritance may mean that the individual no longer needs to buy an expensive house).

gain formula gains an additional term.³¹ We do not attempt to quantify this channel in our empirical implementation — we focus instead on quantifying the change in welfare resulting exclusively from changes in consumption.

Government sector. When individuals only trade assets with each other, the individual welfare gains of asset price deviations aggregate to zero (see Corollary 2). The logic is that for every individual selling an asset, there is an offsetting individual purchasing it. In practice, however, individuals routinely trade assets with non-individual entities, such as the government. For example, if individuals are net buyers of government bonds, a change in the interest rate on government debt leads to a redistribution of resources from the government towards individuals.

In Appendix A.3.6, we study an extension of the baseline model with a government that taxes and makes transfers and is allowed to run surpluses and deficits (subject to a no-Ponzi condition as in the individual problem). We do not assume that the government maximizes a social welfare function and instead make a weaker assumption on cost minimization (i.e., the marginal return of investing in the different assets is equalized). We obtain two main results.

First, relative to the individual welfare gain formula in the baseline model, there is an additional term that accounts for the present value of changes in net government transfers. The idea is that, in general, the government will adjust taxes and transfers in response to a change in asset prices. In our empirical exercise, we will not estimate how a deviation in asset prices affects individual-specific net transfers.

Second, summing over all individuals, we show that aggregate present value of changes in net government transfers is precisely equal to the “welfare gain of the government” (i.e., equation (14) in the baseline model). This is intuitive and follows directly from the government budget constraint. For instance, if the government is a borrower and its cost of borrowing increases (i.e., negative government welfare gain), then it means that there are less resources available for making net transfers to individuals.

Taxes on assets. In the baseline model, individuals pay no taxes on either their asset holdings, asset transactions or income generated by these assets. In Appendix A.3.7, we consider an extension with four types of taxes: wealth taxes, asset transaction taxes, taxes on dividend income, and taxes on interest income.

The presence of taxes changes our baseline formula in Proposition 1 in three noteworthy ways. First, whereas Proposition 1 implied that it is asset transactions and not asset holdings that matter for welfare gains from asset price changes, holdings do matter whenever there is a wealth tax (i.e., a tax on the market value of asset holdings). In particular, whenever asset prices increase, asset holders experience a welfare loss. Second, a transaction tax reduces asset sellers’ welfare gains from rising asset prices because the after-tax asset price faced by sellers increases by less than the pre-tax price. However, it also *increases* asset buyers’ welfare losses

³¹Such preferences are explored, formally or informally, by Smith (1759), Weber and Kalberg (1958), Bakshi and Chen (1996), Carroll (1998), Roussanov (2010), or Piketty et al. (2013).

from rising asset prices because the after-tax asset price faced by buyers increases by *more* than the pre-tax price. Third and related, both transaction and wealth taxes introduce aggregate welfare losses for the household sector as a whole, which benefit the government. Finally, though unsurprisingly, the presence of dividend income taxes leave welfare gains from asset-price changes unaffected.

3 Methodology

We now discuss how we bring the theory to the data in order to estimate the distribution of welfare gains (caused by asset price changes) across individuals. We first describe our methodology. We then describe the combination of administrative and publicly-available data from Norway to quantify our sufficient statistic formula. A more detailed description can be found in Appendix B.

3.1 Implementation and sufficient statistic

We now discuss how we bring the theory to the data in order to estimate the distribution of welfare gains due to rising asset prices (i.e., declining discount rates) across individuals.

First-order approximation. Proposition 1 gives a formula for the infinitesimal welfare gain associated with an arbitrary infinitesimal deviation in prices $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^\infty$. We use this formula to obtain a first-order approximation of the welfare effect of a non-infinitesimal deviation in the price of different asset classes $\{\Delta Q_t, \{\Delta P_{k,t}\}_k\}_{t=0}^\infty$:

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} - B_t \Delta Q_t \right). \quad (17)$$

The approximation is accurate for small price deviations. In Section 6.3, we argue that our approximation captures well the non-infinitesimal change in consumer surplus as, in our sample, the real quantity of transactions in asset class k , $N_{k,t-1} - N_{k,t}$, seems unresponsive to deviations in prices.³²

Price deviations. Formula (17) gives the welfare effect of a deviation in asset prices $\Delta P_{k,t}$ relative to a baseline path for asset prices. In our empirical exercise, we take as the baseline path a world in which asset prices grow at the same rate as dividends; that is, a world in which the price-dividend ratio is constant. Hence, our approach answers the following question: what are the welfare gains of the realized path of asset prices compared to a baseline scenario in

³²If, instead, the *value* of these transactions, $(N_{k,t-1} - N_{k,t})P_{k,t}$ was unresponsive to changes in asset prices, a better approximation of the non-infinitesimal change in consumer surplus would be

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) P_{k,t} \Delta \log P_{k,t} - B_t Q_t \Delta \log Q_t \right).$$

See Section 6.3 (specifically Equation 26) for a formal definition of non-infinitesimal changes in consumer surplus.

which they grew in tandem with dividends? This is a natural baseline because it corresponds to the trajectory of asset prices when asset discount rates and expected dividend growth rates are constant over time (Campbell and Shiller, 1988).

Empirically, fluctuations in price-dividend ratios are mostly driven by fluctuations in future asset discount rates rather than in future expected dividend growth (see Campbell and Shiller, 1988 for the case of U.S. equity and Kuvshinov, 2023 for a similar finding across many asset classes and countries).^{33,34} Assuming that all of the rise in the price-dividend ratios in our sample comes from a decline in asset discount rates (rather than an increase in expected dividend growth), our approach can be interpreted as answering the following question: what are the welfare gains of the rise in asset prices due to declining discount rates?

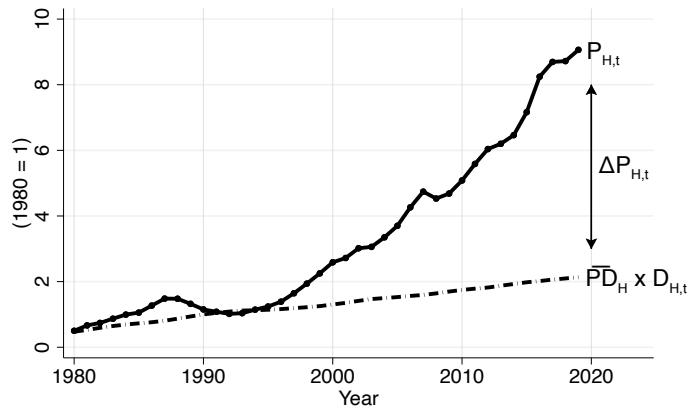


Figure 2: Graphical representation of the price deviation $\Delta P_{H,t}$

Notes. Figure 2 plots the house price index in Norway from Norges Bank's project on Historical Monetary Statistics (solid line) as well as the rental price index from *Statistics Norway* (dashed line). Both are normalized to one in 1980. The difference between the two can be interpreted as a deviation $\Delta P_{H,t}$ between the realized price path $P_{H,t}$ and a counterfactual price path with constant price-to-rent ratio $\overline{PD}_H \times D_{H,t}$.

Formally, we denote by $PD_{k,t} \equiv P_{k,t}/D_{k,t}$ the price-dividend ratio for asset class k . Given a baseline value \overline{PD}_k , we consider the following price deviation

$$\Delta P_{k,t} = P_{k,t} - \overline{PD}_k \times D_{k,t}. \quad (18)$$

As a motivating example, Figure 2 plots the index of house prices in Norway together with the index of house rents. Starting around the mid-1990s, the price of housing has grown faster than rents. In this case, the price deviation corresponds to the difference between realized prices $\{P_{H,t}\}_{t=0}^T$ and the counterfactual price path associated with a constant price-to-rent ratio $\{\overline{PD}_H \times D_{H,t}\}_{t=0}^T$. Equation (18) can also be written as

$$\frac{\Delta P_{k,t}}{P_{k,t}} = \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}},$$

i.e. the price deviation in relative terms equals the relative difference between the actual price-

³³Formally, this happens if (log) dividends follow a random walk. See Appendix B.1 for more details.

³⁴This is true at the level of an asset class but not necessarily for individual assets (Vuolteenaho, 2002).

dividend ratio PD_t and a baseline price-dividend ratio \overline{PD} . This is equation (2) in the introduction. For the liquid asset, we consider a deviation of the price of one-period bonds from a constant baseline value \overline{Q} (i.e., $\Delta Q_t = Q_t - \overline{Q}$).

Finite time horizon. While the formula (17) depends on of all transactions done by the individual, in any empirical application we only observe price deviations and financial transactions over a finite sample period.

Our simple solution to this issue will be to only do the summation from $t = 0$ to $t = T$, where T denotes the length of the sample period. In this case, the sufficient statistic should be interpreted as the welfare effect of asset price deviations up to time T . This truncation is inconsequential if either (i) the price deviation reverts to zero after T or (ii) if there is no trade after year T . More generally, if the price deviation remains positive after T , truncation overestimates the welfare gain for individuals who tend to buy financial assets after the truncation time T , while underestimating the welfare gain for individuals who tend to sell after T . Still, note that the bias due to truncation averages to zero in the full population, since there are as many sales as there are purchases after time T .

To fix ideas, consider the effect of truncation in a simple example: an individual buys N_s units of housing at time $s < T$ and resells them at time $t > T$. Proposition 1 tells us that the net welfare gain of these transactions is $N_s \left(R_{0 \rightarrow t}^{-1} dP_t - R_{0 \rightarrow s}^{-1} dP_s \right)$. However, a researcher observing transactions up to time T will estimate a welfare gain of $-N_s R_{0 \rightarrow s}^{-1} dP_s$ (i.e. a welfare loss) thereby underestimating the true welfare gains by $N_s R_{0 \rightarrow t}^{-1} dP_t = N_s R_{0 \rightarrow t}^{-1} P_t (dP_t / P_t)$. Conceptually, this bias depends on three distinct forces: (i) how large the truncation time T is (ii) how large the discount rate is relative to the baseline growth of house prices (i.e., how quickly $R_{0 \rightarrow t}^{-1} P_t$ decays to zero as $t \rightarrow \infty$, and (iii) how persistent are house price deviations after T (i.e., how large dP_t / P_t is for $t > T$).

As an alternative to truncating the infinite sum (17), we also construct hypothetical price deviations and financial transactions after year T in Section 6. We show that these alternative measures give similar results to our truncated measure under a wide range of scenarios about the path of future asset prices. This comes from the fact that we observe a relatively long time sample ($T = 25$ years).

Sufficient statistic. Combining the first-order approximation of welfare gains (17) with the empirical price deviations (18) and truncating the formula at time horizon T , we obtain a sufficient statistic for the individual-level welfare gain of realized price deviations that we can take to the data:

$$\text{Welfare Gain} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) P_{k,t} \times \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}} - B_t Q_t \times \frac{Q_t - \overline{Q}}{Q_t} \right). \quad (19)$$

This formula forms the core of our empirical implementation using administrative data.³⁵ It is a sufficient statistic in the sense that it depends only on data on financial transactions

³⁵This corresponds to the combination of formulas (1) and (2) in the introduction, generalized to multiple assets.

$(N_{k,t-1} - N_{k,t})P_{k,t}$ and B_tQ_t , as well as valuation ratios $PD_{k,t}$ and Q_t , which are observable — in particular, it does not require researchers to understand what drives these financial transactions over time or even how they react to deviations in asset prices.³⁶

Sample and asset classes. We estimate equation (19) using data covering the 1994–2019 period. The reference year (i.e., $t = 0$) is 1994 and the sample length (T) is therefore 25 years. Our data cover the universe of individuals in Norway who were at least 18 years old for at least one year in the 1994–2019 period. We consider four asset classes: housing, debt, deposits, and equity, which correspond to the four main asset classes traded by Norwegian individuals. Note that we do not need to account for fully illiquid forms of wealth such as human wealth and defined-benefit pensions since they are not traded (i.e., they have no market price).

Given this, we estimate our sufficient statistic as follows:

$$\begin{aligned}
\text{Welfare Gain} &= \sum_{k \in \{\text{housing, debt, deposit, equity}\}} \text{Welfare Gain}_k, \\
\text{Welfare Gain}_{\text{housing}} &= \sum_{t=0}^{25} R^{-t} (N_{H,t-1} - N_{H,t}) P_{H,t} \times \frac{PD_{H,t} - \overline{PD}_H}{PD_{H,t}}, \\
\text{Welfare Gain}_{\text{debt}} &= \sum_{t=0}^{25} R^{-t} (-B_{M,t} Q_{M,t}) \times \frac{Q_{M,t} - \overline{Q}_M}{Q_{M,t}}, \\
\text{Welfare Gain}_{\text{deposit}} &= \sum_{t=0}^{25} R^{-t} (-B_{D,t} Q_{D,t}) \times \frac{Q_{D,t} - \overline{Q}_D}{Q_{D,t}}, \\
\text{Welfare Gain}_{\text{equity}} &= \sum_{t=0}^{25} R^{-t} (N_{E,t-1} - N_{E,t}) P_{E,t} \times \frac{PD_{E,t} - \overline{PD}_E}{PD_{E,t}},
\end{aligned} \tag{20}$$

where \overline{PD}_H , \overline{Q}_M , \overline{Q}_D , and \overline{PD}_E represent the average valuation of housing, debt, deposits, and equity (respectively) over 1992–1996.^{37,38}

Our empirical implementation (20) also assumes that the discount rate in equation (19) is constant, $R_t = R$ and hence $R_{0 \rightarrow t}^{-1} = R^{-t}$. We set the discount rate to 5% (i.e., $R = 1.05$), which roughly corresponds to the average of the deposit and mortgage rates in a five-year window around the start of our sample.

Computing these welfare gains requires data on valuation ratios for each asset class (to compare the actual valuations to a baseline) as well as the market value of financial transactions (at the individual level). We now discuss each component separately.

³⁶The latter is only true because we compute welfare gains at the first-order. See section 6.3 for an discussion of second-order effects.

³⁷Relative to formula (19), we split the total amount of one-period bonds into two terms: $B_{D,t}Q_{D,t}$, the amount held in deposits, and $B_{M,t}Q_{M,t}$, the amounts held in debt, which is negative if individuals are net borrowers.

³⁸Note we use the same price deviation $(PD_{k,t} - \overline{PD}_k)/PD_{k,t}$ for all assets within an asset class. Hence, the welfare gain for asset class k should be interpreted as the welfare gain due to a common deviation in the relative price of all assets within this asset class (the one given by the deviation in the aggregate price-dividend ratio of the asset class).

3.2 Data on valuations

We rely on publicly available data sources for asset prices. For interest rates on debt and deposits (i.e., the inverse of the price of one-period bonds Q in the theory), we use *Statistics Norway*'s database on interest rates on loans and deposits offered by banks and mortgage companies.³⁹ More than 90 percent of Norwegian mortgage debt in our sample has adjustable interest rates so that year-to-year variation in bank-level interest rates immediately affects individuals' interest costs.⁴⁰ Put differently, given that mortgage debt is mostly floating rate, we interpret the outstanding balance of the mortgage as a negative position in one-year bonds.

For the price-to-rent ratio in the Norwegian housing market (i.e., the price-dividend ratio $PD_{H,t} = P_{H,t}/D_{H,t}$ in the theory), we combine data from different sources. The most detailed data is produced by *Eiendomsverdi* (EV), a private company that collects data on the housing market. Their data comes from registries of housing transactions, rental brokers, and the main Norwegian housing rental market place, *Finn.no*. However, EV's price-to-rent ratio is only available starting in 2012. We therefore combine two other indices, one for house prices and one for housing rents, to obtain our price-to-rent series in the years before 2012. The rental index comes from *Statistics Norway*, and is part of the official Consumer Price Index. The house price series comes from Norges Bank's project on Historical Monetary Statistics *Eitrheim and Erlandsen* (2005).⁴¹ As these two series are indices, we scale their ratio so that in 2012, it equals EV's measure of the price-to-rent ratio.⁴² In the results that follow, we use our constructed series for the years prior to 2012 and EV's series after 2012.

We now turn to equity valuation (i.e., the price-dividend ratio for equity $PD_{E,t} = P_{E,t}/D_{E,t}$ in the theory). As explained in Appendix A.3.4, we focus on a valuation ratio for the overall corporate sector (i.e. unlevered equity). We measure it as the ratio between an aggregate measure of enterprise value (i.e., market value of equity plus debt) and the total cash flows distributed to equity and debt holders among publicly-listed non-financial Norwegian firms using data from *Worldscope*.⁴³ Note that, unlike the price-dividend ratio, our equity valuation ratio is unaffected by the relative importance of dividend payouts versus share repurchases as well as firms' capital structure (i.e., debt versus equity financing). We account for the fact

³⁹These data are available on *Statistics Norway*'s website <https://www.ssb.no/en/statbank/table/08175/>.

⁴⁰Mortgage contracts in Norway typically are annuity loans with 25-year repayment schedules. When interest rates change, the payment schedule adjusts so that the sum of monthly debt repayment and interest costs remains constant at a new level throughout the remaining period of the contract. Such adjustments happen frequently, normally whenever the Central Bank policy rate changes.

⁴¹This house price index is in turn obtained from combining data by the Norwegian Real Estate Broker's Association, the private consulting firm Econ Poyry, and listings at the main platform for house transactions *Finn.no*. Norges Bank updates these data regularly and provides them online, currently at <https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/House-price-indices/>.

⁴²Importantly, because all these three data series exist after 2012, we can use this most recent period to validate that our constructed price-to-rent series for the years before 2012 tracks the high-quality EV series after 2012. Indeed, we find no substantial difference between using EV's price-to-rent ratio or using our constructed alternative based on publicly available data for the years after 2012.

⁴³We use a valuation ratio for Norwegian firms, as opposed to foreign firms, as Norwegians mostly own and sell domestic equity (more precisely, Norwegians' holdings in domestic equity account for 100% of their private equity holdings and 71.9% of their public equity holdings). Note that this contrasts with the Norwegian government, which mainly owns and buys foreign equity. In Appendix D, we will discuss how using separate price indices for domestic and foreign equity changes our estimates of welfare gains at the sectoral level.

that firms have financial liabilities besides equity (such as debt for most firms and deposits for private banks) by allocating these indirectly-held assets to the equity holders (see Appendix A.3.4 for more details on the theoretical motivation and B.3.2 for more details on our implementation).

Figure 3 plots the yield of each asset class over time (i.e., $1/Q_t$ for debt and deposits and $D_{k,t}/P_{k,t}$ for long-lived assets $k \in \{H, E\}$), which are the inverse of the valuation ratios in Equation (20). The notches next to the vertical line marking the year 1993 correspond to our baseline values for each asset class. All yields decline substantially over time (i.e., valuations increase). On average, over our time sample, the housing yield fell by 7.5 pp., mortgage interest rates by 2.5 pp., deposit interest rates by 1.3 pp., and the equity yield by 0.5 pp. In particular, note that the equity yields have decreased a bit less in Norway relative to the rest of the world.

To compute the welfare gains of asset price deviations, Equation (20) requires a measure of the relative difference between valuations at time t and their average baseline value (i.e., their averages over the 1992–1996 period). Figure A1 in Appendix B visualizes these price deviations.

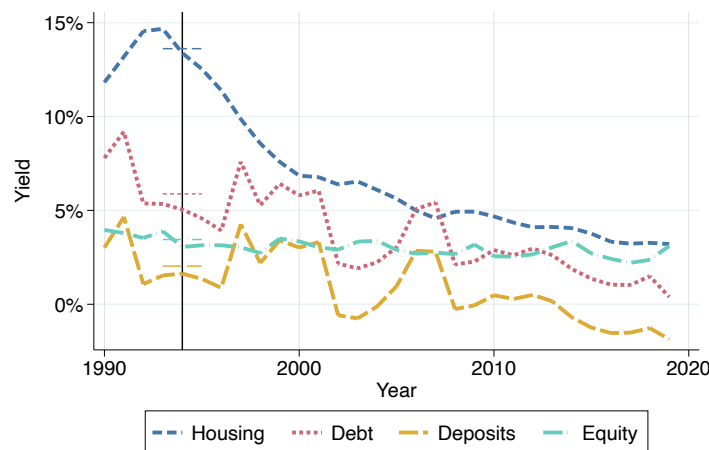


Figure 3: Evolution of yields in Norway

Notes. Figure 3 plots the yield of each asset class over time, i.e., the inverse of the valuation ratios in equation (20). For debt and deposit, the yield corresponds to the average real interest rate on mortgages and debt, respectively, as estimated by *Statistics Norway*. The housing yield corresponds to the rent-to-price ratio (see text for details). The equity yield corresponds to the aggregate ratio of cash flows to enterprise value amongst publicly-listed Norwegian firms from *Worldscope*.

3.3 Microdata on holdings and transactions

We combine data from a variety of Norwegian administrative registries that cover the universe of Norwegians from the end of 1993 to the end of 2019. These data come with identifiers at the individual, household, and firm level, as well as information on parent-children links. In particular, we use registries for individual tax payments, holdings of equity shares (listed and unlisted corporations), private business balance sheets, and housing transactions. Flow variables are measured annually, whereas assets and liabilities are valued at the end of the year. The data are uncensored (i.e., no top coding), and the only sources of attrition are mortality and emigration. The income and wealth data are largely third-party reported (i.e., employers and

financial intermediaries) and scrutinized by the tax authority as they are used for (income and wealth) tax purposes.

Holdings. On individual balance sheets, we separately observe bank deposits, bond holdings (corporate, sovereign, mutual, and money market funds), debt, vehicles (cars and boats), stock mutual funds, publicly-listed and private businesses, housing and other forms of estate holdings. The values of the holdings of these asset classes are available starting from the end of 1993.

In principle, we observe each individual's holdings. However, while financial holdings are *registered* at the individual level, they are *taxed* at the household level. The reported allocation of assets between individuals within the household is therefore somewhat arbitrary and can vary substantially from year to year. To compute a consistent measure of individual holdings across time, we therefore aggregate holdings at the household-level and distribute it equally across adult household members.⁴⁴

We construct five main variables that cover most of financial wealth: "debt" (mortgages, student loans, and unsecured credit); "deposits" (bank deposits and bonds); "housing" (principal residence, secondary homes, and recreational estates); "private business equity" (equity in private businesses); "public business equity" (listed stocks and stock funds). All of these variables are recorded at market value at the end of the year, except for private business equity, which is a tax assessed value (i.e., the value reported to the tax authority, which is typically higher than the book value of equity, see Appendix B.3.2). For housing, we use a valuation approach that combines transaction data and registered housing characteristics to estimate a value for each house in every year (see Fagereng et al., 2020 for details on the valuation methodology). Note that this will only matter when reporting our welfare gains relative to total wealth.

Some individuals own private businesses. These firms hold assets and liabilities directly, but in many cases also own shares in other firms. To properly account for individuals' ownership, we must therefore include their indirect asset positions held through private businesses. Our procedure is as follows. For each individual, we compute their direct and indirect ownership of private businesses. For instance, if a individual owns 80% of firm A, which in turn holds 50% of firm B, then the individual owns 80% of firm A and 40% of firm B. Moreover, firm B might hold 25% of firm C, which then implies that the individual owns 10% of C. We compute each individual's indirect ownership by going through ten such layers of firm holdings. Equipped with these ownership shares and private firms' balance sheets, as well as publicly available data on public firms' balance sheets, we allocate holdings and transactions done by firms to their ultimate owners (see Appendix B.3.2 for details). The key idea is that it is equivalent to purchase an asset directly or indirectly via a firm that one owns. In Appendix A.3.4, we describe precisely how theory guides our consolidation of firms' financial transactions to individuals.

⁴⁴Our definition of a *household* is either a single individual, or a married or cohabitant (with children) couple. Each offspring older than 18 years living with its parents is a separate household.

Our notion of welfare gain can be interpreted as the present value of the deviation in consumption due to the deviation in asset prices (see Equation 13). It would therefore be natural to express it as a share of the present value of consumption. However, we do not observe consumption directly in our sample. Instead, in some exercises, we will scale the welfare gain by “total wealth”, which is defined as the sum of financial wealth (i.e., debt, deposits, housing, and equity) and human wealth (i.e., the present value of earned income, defined as future labor income plus net government transfers received between 1994 and 2019, discounted at 5% annually). We also set the minimum value of earned income to twice the base amount in the social security system.⁴⁵

Appendix Table A2 summarizes the data. Throughout the paper, we express all values in real terms (2011 Norwegian Krone using the CPI) and then convert them to US dollars using a fixed exchange rate of 5.607. In Appendix B.3.1, we show that our aggregated microdata matches publicly-available data on individual wealth by asset very closely.

Transactions. Equation (20) highlights the fact that we need data on *holdings* for debt and deposits, and *net transactions* for housing and equity.

For housing, we observe the annual value of market transactions in the housing market at the individual level. Thus, net transactions in housing are directly observed. For public equities, we observe holdings at the beginning and end of the year and a price index. We then compute a measure of unrealized capital gains by assuming that all transactions are in the same direction and uniformly distributed within a year. Net transactions are thus constructed as the change in market value minus imputed capital gains. The price index used for imputation differs between assets. For listed stocks, the method differs depending on the available information. Starting in 2005, we have information on individual stock ownership and use market prices on individual stocks to impute capital gains. Before 2005, we lack information on individual stock ownership and use capital gains from the Financial Accounts to impute capital gains on listed stocks at the individual level. We also use capital gains from the Financial Accounts to impute individual capital gains for mutual funds.

For equity in private businesses, we impute the value of transactions using the data on ownership shares described earlier. In particular, if we see that a individual owns 50% of a private business in a given year and 25% the next year, this implies that the individual sold a 25% stake of the business.⁴⁶ In Appendix B.3.2, we describe this methodology in detail. Private business equity transactions are extremely rare and not quantitatively important. As a result, private business owners are not meaningfully exposed to private equity valuation changes. It is worth stressing the fact that, even in a world in which business owners never sell their stakes in their businesses, they are still exposed to asset price changes via the financial transactions made by the firms that they own. For instance, if the interest rate on debt declines, then the owner of a private business that has a lot of debt will incur a positive welfare gain. This is

⁴⁵As with financial holdings, an individual’s human wealth is computed based based on his or her household’s human wealth.

⁴⁶Alternatively, the business might have issued new equity, leading to a dilution of existing owners. In terms of welfare exposure to equity prices, those two scenarios are equivalent (see Appendix A.3.4).

particularly important for individuals at the top of the wealth distribution, as they hold a lot of assets through their private firms.

Bequest events pose two challenges when computing net transactions. First, housing transactions may be problematic at the time of death. In most cases, when an individual dies, the estate is transferred to the heirs. In this case, the heirs sell the property and net transactions are computed correctly. But in a few cases, parts of the estate is sold after death but before it is transferred to the heirs. In this case, we allocate the transaction to the living children of the deceased, in accordance with the Norwegian inheritance law.⁴⁷

Second, because our imputation of net transactions in equity is based on changes in holdings net of imputed capital gains, a bequest event may be problematic because transfers of wealth may be counted as transactions. For example, if one individual gives 100 equity shares to another individual, this should not be reported as a purchase by the recipient nor as a sale by the giver. To address this issue, we allocate all imputed equity transactions of givers to recipients when there is a bequest event. A bequest event is defined as any transfer reported in the inheritance tax registry (both inter vivo and at death).⁴⁸

4 Redistribution within the household sector

We now estimate our sufficient statistic (20) for all Norwegians who were at least 18 years old at some point between 1994 and 2019. More precisely, we describe the heterogeneity in welfare gains across individuals in Section 4.1, across cohorts in Section 4.2, and across the wealth distribution in Section 4.3.

4.1 Redistribution across individuals

Transactions. We start by documenting the heterogeneity in financial transactions. Table 1 reports summary statistics for transactions across the population, computing them every year and averaging them across all years in our sample. Compared to Table A2, we also include indirect transactions via firms owned by individuals.

Housing transactions are very lumpy, while most people hold debt and deposits. The magnitude of equity transactions is much smaller than housing transactions, which reflects the fact that housing holdings dominate equity holdings for Norwegian individuals (see Table A2). Also, deposits are negative (and debt is positive) for a substantial fraction of the population. This comes from the fact that we report *consolidated* holdings and transactions: individuals that own equity in financial firms (e.g. banks) indirectly hold long positions in debt and short positions in deposits. Finally, financial transactions do not average exactly to zero: as we will discuss below, this reflects the fact that individuals in our sample also trade with the Norwegian government and the rest of the world.

⁴⁷By the letter of the law, inheritance is split equally between all direct descendants unless explicitly specified otherwise in a will.

⁴⁸Before 2014, there was an inheritance tax in Norway and the tax authority collected information on sender, receiver, and the amount transacted. However, this register does not contain information on the types of assets transferred.

Table 1: Summary statistics on transactions (net purchases in thousands of dollars)

Asset type	Average	S.D.	p1	p10	p25	p50	p75	p90	p99
Housing	0.93	116.52	-190.18	0.00	0.00	0.00	0.09	0.90	220.37
Debt	-73.40	3071.63	-603.04	-221.96	-127.44	-36.03	-0.03	9.72	351.94
Deposits	19.41	1863.55	-158.78	-1.88	1.01	7.55	28.71	76.62	339.48
Equity	-0.57	590.32	-31.47	-0.82	-0.07	0.00	0.04	1.36	33.19

Notes. All numbers are in thousands of 2011 US dollars.

Welfare gains. Figure 4 reports the histogram for total welfare gains. As predicted in Section 2, the average welfare gain is close to zero. However, there is substantial heterogeneity: the welfare gain is $-\$198,000$ at the 1st percentile and $\$286,000$ at the 99th percentile, with an interquartile range of $\$32,000$. There is a large mass around zero, which reflects the fact that consumption remains close to income for a large fraction of individuals. As already mentioned, financial transactions within the household sector do not average to zero in our sample. As a result, welfare gains do not average to zero either: they average to $\$10,000$, which is slightly positive. In Section 5, we will show that this positive welfare gain corresponds to a welfare loss of the Norwegian government as well as foreigners. The Kelly skewness of the distribution is fairly small, at 0.06, reflecting the fact that the distribution of welfare gains is fairly symmetrical around its mean.⁴⁹

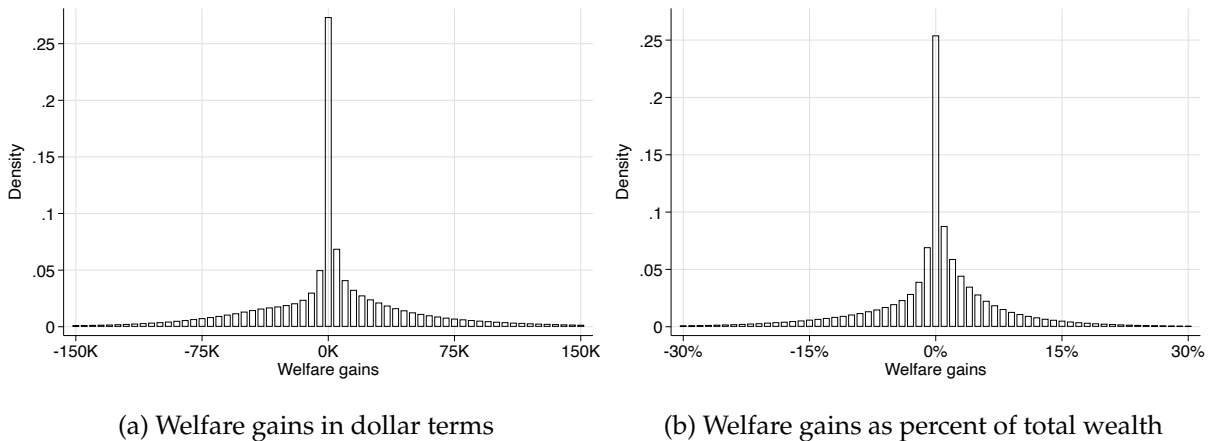


Figure 4: Histogram of welfare gains

Notes. This figure plots the density of individual welfare gains, as defined in (20), across individuals in Norway. More precisely, the figure plots the relative mass of individuals within equally spaced bins of welfare gains (width of $\$1,000$). Panel (a) plots welfare gains in levels (in 2011 US dollars) while panel (b) plots welfare gains normalized by initial wealth, where initial wealth is defined as the sum of financial wealth and human capital at the end of 1993 (i.e., the present value of labor income earned and government benefits received from 1994 to 2019).

To understand which asset class contributes the most to redistribution, Table 2 decomposes the average welfare gain in different percentile groups of the welfare gain distribution. More precisely, for each percentile group, the table reports the average welfare gain, as well as the average welfare gain due to each asset class. Housing is by far the asset class that generates the most redistribution. This comes from the fact that, even though housing transactions tend

⁴⁹Kelly skewness is defined as $(p90 + p10 - 2 \times p50) / (p90 - p10)$ where $p10$, $p50$, and $p90$ are the 10th, 50th and 90th percentiles of the distribution under consideration.

to be smaller than debt or deposits holdings (Table 1), the price deviations associated with housing are much larger than the price deviations associated with debt and deposits (Figure A1). Nevertheless debt is also an important (and almost always positive) contributor, with a relatively large magnitude both at the top and at the bottom of the welfare gain distribution. Similarly, deposits make a very small and almost always negative contribution. Welfare gains due to equity are small, reflecting the fact that there are fewer equity transactions in our sample (Table 1) and that the run-up in equity prices was smaller than the run-up in house prices (see Figure A1).

Table 2: Decomposition of welfare gains by percentile groups

Asset	Average	Average by percentile groups of welfare gains					
		p0-1	p1-10	p10-50	p50-90	p90-99	p99-100
Housing	-5.0	-322.1	-108.1	-19.7	2.7	64.9	447.0
Debt	16.9	-202.2	19.2	9.0	16.0	38.7	193.6
Deposits	-2.4	68.6	-4.1	-3.0	-2.1	-4.5	-15.7
Equity	0.2	-98.7	-1.9	-0.2	0.0	0.7	67.8
Total	9.7	-554.4	-95.0	-13.8	16.7	99.8	692.8

Notes. For each percentile group of welfare gains, the table reports the average welfare gain, as well as the average welfare gain due to each asset class, as defined in (20). All numbers are in thousands of 2011 US dollars.

Welfare gains as percent of total wealth. We now evaluate the dispersion of welfare gains relative to total wealth, defined as the sum of financial and human wealth (see Section 3.3). As discussed in Section 2 welfare gains can be interpreted as the present value of the change in consumption due to the deviation in asset prices (see Equation 13). As a consequence, this *normalized* version of welfare gains can be interpreted as the relative change in consumption due to asset price deviations.⁵⁰ In this exercise and the ones below, we winsorize total wealth at the bottom 1% within each cohort (to limit the influence of observations with very small total wealth).

Figure 4 shows significant heterogeneity in welfare gains, even after normalizing by initial wealth. The normalized welfare gain is -31% at the 1st percentile and 28% at the 99th percentile, with an interquartile range of 5.0% . While the Kelly skewness of the distribution is close to zero (-0.06), reflecting a symmetric distribution, the kurtosis of the distribution is 10, reflecting a larger mass in the tails relative to the normal distribution.

Social planner. Our money metric notion of welfare gain represents the cash transfer that would make each individual indifferent between the baseline asset valuations and the realized ones. As discussed in Section 2, one can aggregate these individual welfare gains to compute the welfare gain of a hypothetical social planner, using individual-specific Pareto weights. As an example, in Appendix C.1, we compute the welfare gain of a social planner who aggregates individual utilities given by iso-elastic utility functions with curvature parameter γ . We find

⁵⁰Another way to interpret this number is that it corresponds to the relative increase in consumption every period that would be welfare equivalent to the change in asset prices (see Lucas, 1987).

that the social planner welfare gain turns negative for high enough γ , reflecting the fact that rising asset prices redistributed from the poor to the wealthy.

Revaluation gains. We now compare *welfare* gains with *revaluation* gains, defined as the (present value of the) effect of a deviation in asset prices on wealth:

$$\text{Revaluation Gain} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \sum_{k=1}^K N_{k,t-1} P_{k,t-1} \Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right), \quad (21)$$

where we define $\Delta(P_{k,t}/P_{k,t-1}) \equiv (P_{k,t}/P_{k,t-1})(\Delta P_{k,t}/P_{k,t} - \Delta P_{k,t-1}/P_{k,t-1})$ as the deviation in the capital gains component $P_{k,t}/P_{k,t-1}$ of asset returns caused by the price deviation $\{\Delta P_{k,t}\}_{t \geq 0}$.

Welfare gains are different from revaluation gains. This is because revaluation gains only capture the positive effect of rising valuations on returns through higher capital gains, while welfare gains also take into account the negative effects of higher valuations on returns through lower dividend yields. In particular, revaluation gains systematically overestimate welfare gains in a time of inflated asset prices. We derive a formal expression for the difference between welfare and revaluation gains in Appendix C.2.

Figure 5 compares the histograms of welfare and revaluation gains, both normalized by initial wealth. As discussed above, welfare gains are centered around zero (-0.1% on average). In contrast, revaluation gains are centered around a large positive value (18.9% on average). This reflects the fact that revaluation gains are positive for all asset holders while welfare gains are only positive for asset sellers.

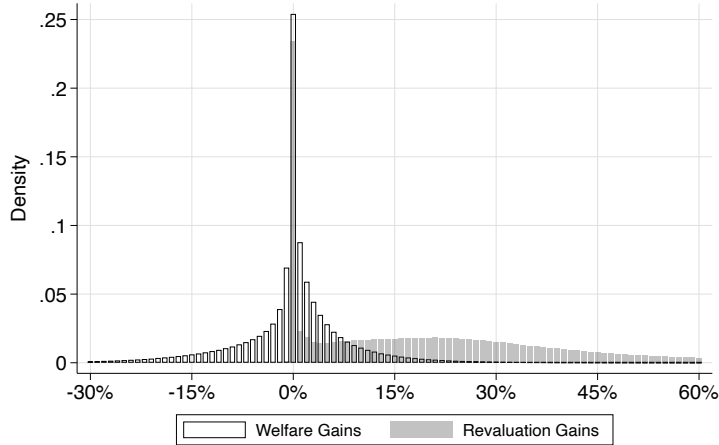


Figure 5: Histogram of normalized welfare gains versus normalized revaluation gains

Notes. The figure plots the density of welfare gains defined in (20), in black lines, and the density of revaluation gains defined in (21), in grey shading, across individuals in Norway. Welfare and revaluation gains are normalized by initial wealth, defined as the sum of financial wealth and human capital at the end of 1993 (e.g. the present value of labor income earned and government benefits received from 1994 to 2019).

Do individuals with higher revaluation gains also tend to have higher welfare gains? To answer this question, we now focus on the *ordinal* relationship between the two variables. We find that the Spearman's rank correlation between welfare gains and revaluation gains is 0.17, which shows that there is a substantial difference between those who get richer from the rise in

asset prices and those who truly benefit from it.⁵¹ Some individuals with large asset positions buy and hence lose in welfare terms; conversely, others with small positions sell and hence win. Appendix Figure A4 plots a heatmap for the joint density of ranks of welfare gains and ranks of revaluation gains.

4.2 Redistribution across cohorts

In the previous section, we documented a large amount of heterogeneity in welfare gains across individuals. We now focus on describing the heterogeneity in welfare gains across one observable characteristic: the age of each individual at the end of 1993 (or, alternatively, the cohort he or she belongs to). Indeed, the existing literature on household finance has documented large differences in portfolio holdings over the life cycle (e.g., Flavin and Yamashita, 2011; Cocco et al., 2005). This heterogeneity may naturally generate heterogeneity in financial transactions, and, therefore, in welfare gains.

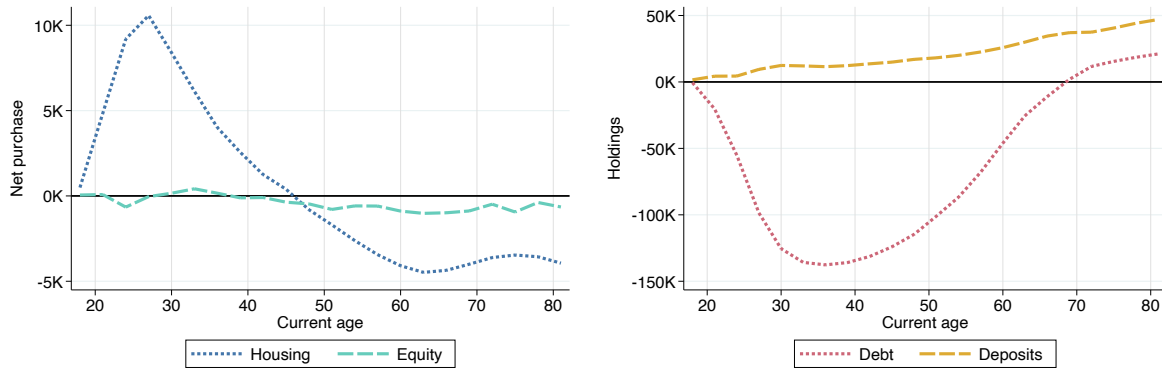
Transactions. Figure 6a plots the average (consolidated) financial transactions in equity and housing by age. Importantly (though unsurprisingly), younger individuals tend to be net buyers of housing and equity whereas older individuals tend to be net sellers. Figure 6b plots the average holdings of debt and deposits by age, as they also enter the sufficient statistic (20). Younger individuals hold a large amount of debt, primarily mortgage debt.

Welfare gains. Figure 6c plots the average welfare gain for different cohorts, indexed by individuals' age at the end of 1993. The main pattern is that welfare gains are negative for the young and positive for the old meaning that rising asset prices redistributed from the young towards the old. This is consistent with standard life cycle models of savings: the young save for retirement by purchasing financial assets while the old sell their financial assets to consume.

Quantitatively, the average welfare gain is approximately $-\$20,000$ for individuals below 15 years old in 1993 (Millennials), and around $\$20,000$ for individuals above 50 years old in 1993 (Baby boomers). The figure also decomposes welfare gains into the contribution of each asset class which reveals interesting patterns. On the one hand, higher house prices redistribute from young to old, as the young tend to buy houses from the old. On the other hand, lower mortgage rates redistribute from old to young, as the young tend to borrow from the old.⁵² Overall, the effect of higher house prices dominates the effect of lower mortgage rates for two reasons. First, and most importantly, the housing yield decreased more than the interest rate on debt (see Figure 3). Second, as young people build equity in their houses, they decrease their mortgage balances over time, which means that they benefit relatively less from the decline in mortgage rates as they age.

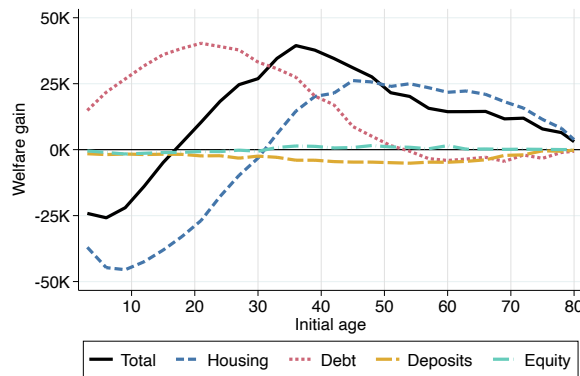
⁵¹The Spearman's rank correlation between *normalized* welfare and revaluation gains is also low, at 0.16.

⁵²As we discuss in Section 5, the household sector as a whole is a net debtor. Therefore, the young do not borrow only from the old, but also from another sector of the economy, which turns out to be the government sector. Also note that, while lifecycle mortgage balances peak around age 30 (Figure 6b), the welfare effect of lower mortgage rates is highest for individuals who are 20 years old in 1993 (Figure 6c). This is due to two forces: (i) mortgage rates are mostly flat at the beginning of our sample and only start declining in 2001 (Figure 3), and (ii) this cohort spends a longer amount of time with mortgage debt than the older cohorts aged around 30 in 1993 (Figure 6b).



(a) Housing and equity transactions

(b) Debt and deposits



(c) Welfare gains

Figure 6: Financial transactions and welfare gains by age group

Notes. Figure 6a and 6b plot (consolidated) financial transactions per capita by age, averaged across all years in our sample. More precisely, for each asset class and year in our sample, we compute the average transaction within groups of individuals with a given age at the end of that year. We then average this quantity across all years in our sample. Figure 6c plots the average welfare gain (20) for individuals in each cohort. Cohorts are indexed by the age of individuals at the end of 1993. All numbers are in 2011 US dollars.

4.3 Redistribution across wealth percentiles

A growing literature has emphasized that rising asset valuations affect the distribution of wealth (e.g., Kuhn et al., 2020; Gomez, 2016; Greenwald et al., 2021). A natural question is: are these revaluation gains actually welfare gains? To answer this question, we compare revaluation and welfare gains across percentiles of the initial wealth distribution at the end of 1993. More precisely, we rank individuals according to their total initial wealth *within their cohort*. We then compare average revaluation and welfare gains at these different percentiles.

Transactions. Figure 7a plots the average consolidated transactions of equity and housing across different percentiles of the wealth distribution. To make it more easily comparable across different percentiles, we normalize average transactions by the total wealth at the end of 1993 at each percentile. The key observation is that richer individuals tend to be net sellers of equity while poorer individuals tend to be net buyers. In contrast, housing net purchases are mildly positive across most of the wealth distribution (consistent with the mildly positive aggregate housing net purchases by households – see Table 3).

Figure 7b plots the consolidated holdings of debt and deposits across the wealth distribu-

tion. As a proportion of financial wealth, the level of debt decreases (in absolute value) with the level of wealth while the level of deposits increases. The negative value of deposits at the top 1% reflects the fact that richer individuals tend to hold more equity, and, as a result, they indirectly hold negative positions in deposits through their ownership of Norwegian banks. Finally, the top 1% holds little debt on a consolidated basis.⁵³

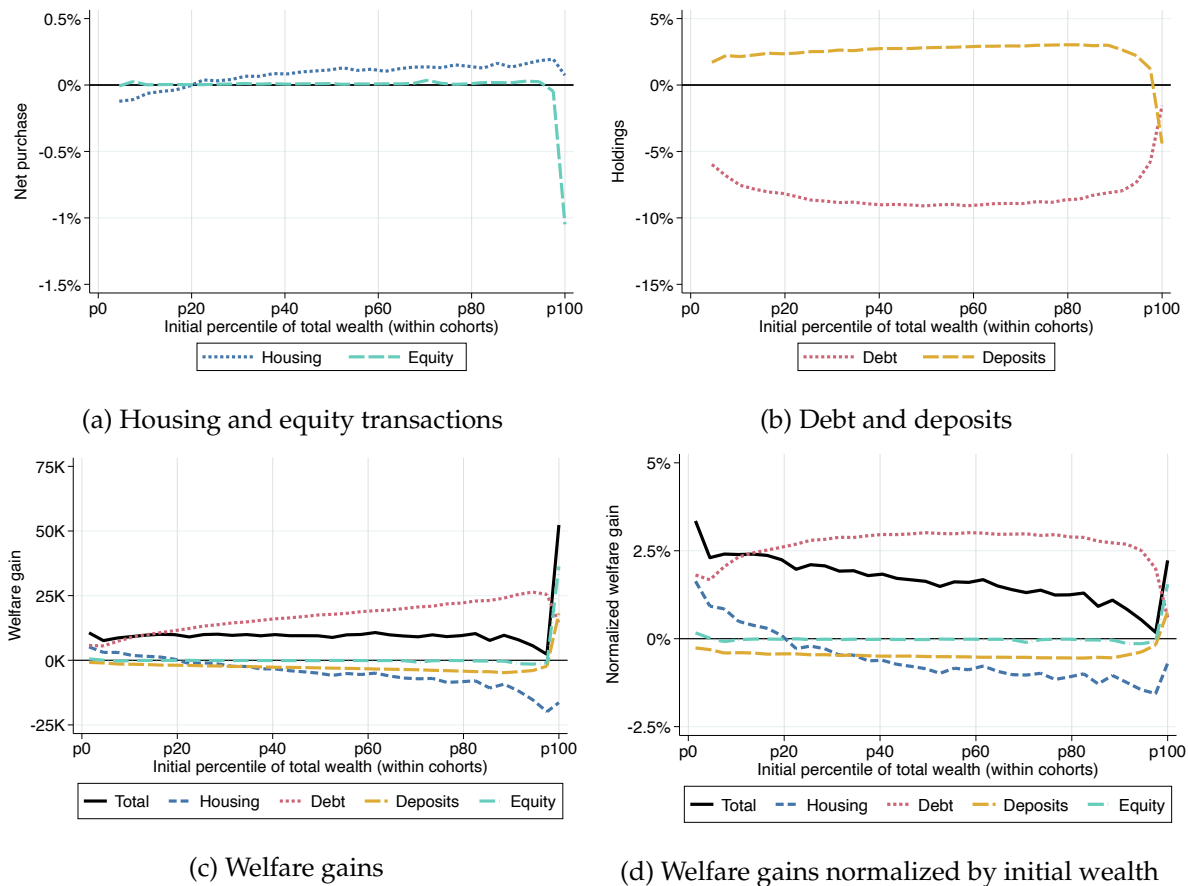


Figure 7: Financial transactions and welfare gains by wealth percentile

Notes. Figure 7a and 7b plot net transactions per capita data by initial wealth, averaged over across years, and divided by total wealth measured at the end of 1993. Figure 7c plots the average welfare gain, as defined in (20), for each wealth percentile in Norway. Figure 7c plots welfare gains *normalized* by the average total wealth measured at the end of 1993 in each percentile. Wealth percentiles are constructed by ranking individuals within each cohort with respect to total wealth, defined as the sum of financial wealth and human capital at the end of 1993 (e.g. the present value of labor income earned and government benefits in our sample).

Welfare gains. Figure 7c plots the average welfare gains at different wealth percentiles. Welfare gains increase with total wealth: the top 1% experienced on average a \$52,000 welfare gain, while the corresponding number is \$11,000 at the bottom 1%. Figure 7d plots welfare gains normalized by the average total wealth in each percentile. The main pattern is that normalized welfare gains tend to be stable across the wealth distribution, except for the top 1%. Individuals in the top 1% of their cohort experience a normalized welfare gain of roughly 2.2%, which is higher than the population average of 1.5%. Moreover, most of the relatively higher

⁵³While richer individuals issue debt through their ownership in non-financial businesses, they also buy this debt through their ownership in financial businesses.

welfare gains for the top 1% comes from equity, which reflects the fact that they tend to be net sellers in this asset class.

Revaluation gains. Finally, Figure 8 contrasts revaluation and welfare gains. Similarly to welfare gains, revaluation gains increase with top percentiles, which reflects the importance of revaluations for the rise in wealth inequality. However, the figure show that the magnitude of revaluation gains (46.7% of total wealth for the top 1%) is much bigger than the magnitude of welfare gains (2.2% of total wealth for the top 1%). Put differently, only a small part of these revaluation gains is relevant for welfare.

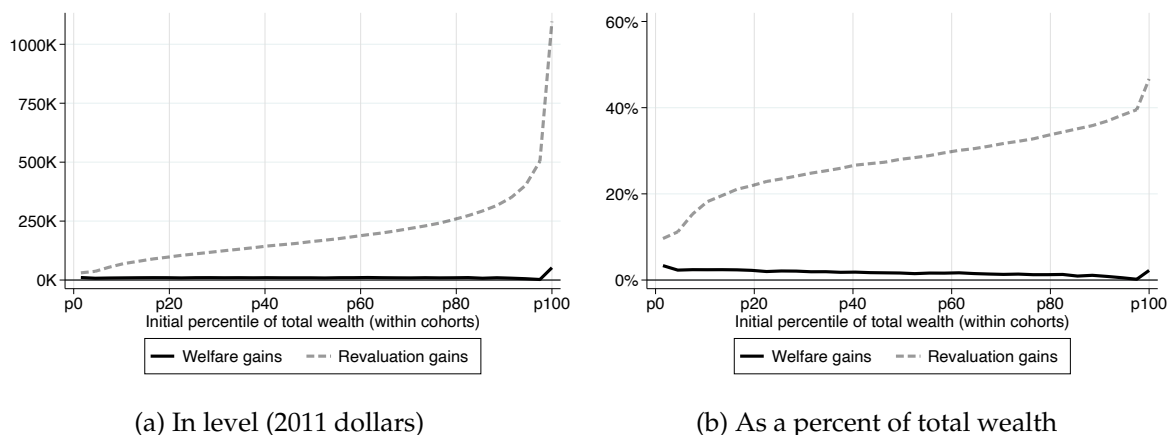


Figure 8: Welfare and revaluation gains across wealth percentiles

Notes. This figure plots the average welfare and revaluation gains, as defined in (20), for each wealth percentile in Norway. Figure (a) reports the two quantities in level (dollar terms) while Figure (b) reports the two quantities normalized by total wealth measured at the end of 1993. Wealth percentiles are constructed by ranking individuals within each cohort with respect to total wealth, defined as the sum of financial wealth and human capital at the end of 1993 (e.g. the present value of labor income earned in our sample).

5 Redistribution across sectors

As discussed in the previous section, welfare gains do not aggregate to zero within the household sector. The reason is that individuals trade with other non-household entities, such as the government and foreigners. We now conduct a systematic investigation of welfare gains across sectors. This is particularly important in Norway given the scale of the sovereign wealth fund, which purchases domestic and foreign assets on behalf of Norwegian households.

More precisely, we group all entities in the economy into three sectors: households (H), the government (G), and foreigners (F). The key accounting identity that we use is that every asset bought by one sector must be sold by another sector. With this in mind, it is immediate that in a multisector economy, Corollary 2 becomes

$$\text{Welfare Gain}_H + \text{Welfare Gain}_G + \text{Welfare Gain}_F = 0, \quad (22)$$

where the sector-level welfare gain is defined analogously to Equation (20). In words, a positive welfare gain for the household sector must be exactly offset by a welfare loss in another

sector. We first present the data in Section 5.1 and we discuss the results in Section 5.2.

5.1 Data sources

We use publicly available data from the *Financial Accounts*, which cover all holdings and transactions of financial assets in the Norwegian economy starting from 1995. For our analysis, we combine the government sector with the central bank and the non-profit sector. Importantly, this means that our government sector includes the *Government Pension Fund of Norway*, which is financed by income taxes on the energy (oil and gas) sector. It is composed of the Government Pension Fund Global — which invests in foreign assets — and the Government Pension Fund Norway — which is smaller and invests in domestic and Scandinavian assets.⁵⁴

Consistent with what we do in the microdata, we also consolidate the domestic business sector with its ultimate owners (i.e., we assign the financial assets owned by the domestic business sector to their ultimate owners). Hence, we are left with three sectors: the household sector, the government sector, and the foreign sector (rest of the world).

Real estate is a real asset rather than a financial asset, which means that housing holdings and transactions are not recorded in the Financial Accounts. We therefore augment the Financial Accounts with between-sector housing holdings and transactions, which we construct by aggregating the housing transaction registry data described in Section 3.3. The resulting dataset covers the total amount of asset holdings and transactions for three sectors (households, government, and foreigners) and four asset classes (housing, deposits, debt, equity) over the 1995–2019 period. See Appendix D for more details on the data construction.

5.2 Results

Transactions. Before we quantify the welfare gains by sector, we briefly discuss the main pattern of housing and equity transactions as well as debt and deposit holdings across sectors, as reported in Table 3.

The annual levels of net housing purchases across sectors are very low (less than \$1,000 per capita in absolute value). The reason is that most housing transactions are within the household sector, with minimal transactions between sectors. Regarding equity purchases, households have a positive but small level of net equity purchase on average. In contrast, the government is a net buyer of foreign equities via the sovereign wealth fund described above. Those transactions are quite large, and amount to more than \$5,000 per capita per year.

Table 3 reveals that the household sector has a large amount of debt. Most of it is household debt (mainly mortgages), but some of it is corporate debt, which individuals indirectly hold through their ownership of businesses. While households, on net, hold debt securities as liabil-

⁵⁴Over our sample period, the Government Pension Fund Global fund's value grew from approximately zero in 1997 to approximately one 1B\$ in 2019. Its portfolio mandate first prescribed 40 percent equities and 60 percent fixed income assets. In 2007 this was changed to 60 percent equities. In 2010, the fund's portfolio was extended to real estate with a 5 percent weight, and the fixed income share was cut to 35. A fiscal policy rule states that the expected real rate of return, first 4% and since 2017 3%, of the current fund value can be spent over the national budget each year. As the fund grew over our sample period, so did government spending. Details regarding the fund's mandate and investment strategy are provided at <https://www.nbim.no/en/the-fund/how-we-invest>.

Table 3: Transactions (net purchases) across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	0.92	-0.38	-0.54	0.00
Debt	-74.38	65.11	9.27	0.00
Household debt	-65.21	24.81	40.40	0.00
Corporate debt	-26.81	-9.95	36.75	0.00
Government debt	6.38	-38.62	32.24	0.00
Foreign debt	11.26	88.86	-100.12	0.00
Deposits	20.38	-7.95	-12.43	0.00
Corporate deposits	15.57	-11.23	-4.35	0.00
Government deposits	0.67	-1.95	1.28	0.00
Foreign deposits	4.14	5.23	-9.36	0.00
Equity	0.63	6.98	-7.61	0.00
Corporate equity	-0.62	-0.65	1.27	0.00
Foreign equity	1.25	7.63	-8.88	0.00

Notes. All numbers are in thousands of 2011 US dollars, and divided by the population of Norway. Averages over 1995-2019. "Household debt" is debt taken by households (mostly mortgages); "Corporate debt" is debt issued by the corporate sector (i.e., bonds and bank loans); "Foreign debt" contains all debt issued by foreigners (e.g. foreign corporate entities, foreign households, and foreign governments); "Corporate deposits" is deposits issued by private banks; "Government deposits" is central bank reserves; "Corporate equity" is equity issued by corporations; "Foreign equity" is equity issued by foreign corporations.

ities (i.e., they are indebted), the government, on net, holds debt securities as assets (i.e., they are lenders). In fact, the debt level of households is approximately equal to the government's net holding of debt securities (roughly \$50,000 per capita). The foreign sector only holds a small amount of debt on net. While households do not borrow directly from the government, the effect is the same in terms of welfare redistribution: a decline in interest rates redistribute from the government towards households.⁵⁵

A similar pattern holds for deposits, although the magnitudes are much smaller. The household sector is a net holder of deposits (including indirectly via its ownership of businesses), while the government and foreign sector hold these deposits as liabilities. The reason is that deposits are a liability for the financial sector, and since the government includes the central bank, and foreigners are important holders of financial business equity, they are ultimately liable for interest payments on these deposits.

Welfare gains. Figure 9 presents the welfare gains across sectors, where all numbers are expressed per capita (i.e., scaled by the number of individuals in Norway in 1994). We use the same welfare gain formula and valuation ratios as before. One caveat is that we implicitly assign the same price deviation for foreign and Norwegian assets (more on this shortly).

The household sector has a positive welfare gain of roughly \$9,000 per capita. Breaking down the welfare gain by asset class, we find a large positive contribution of debt (\$18,000) and

⁵⁵Most of household debt is mortgages, who are then securitized into mortgage bonds by private banks. Then, these bonds are for the most part sold to domestic pension funds as well as foreigner. However, foreigners also issue a large amount of debt that is held by the sovereign wealth fund. This explains why the net foreign debt position is close to zero in Table 3. The sovereign wealth fund's holding of foreign bonds then account for most of the government's net holding of debt securities, while a small fraction are held by other public pension funds that invest domestically. The main domestic public pension funds are *Folketrygdfondet* and *Kommunenenes Landspensjonskasse* (see Bank (2021) for an overview of Norway's financial system).

a small contribution of deposits (−\$3,000). Equity transactions make a negligible contribution (−\$1,000) and housing transactions a more important one (−\$5,000). The positive welfare gain of the household sector is therefore mostly due to declining interest rates, which have been beneficial to households since they are net debtors (i.e., their debt exceeds their bank deposits).



Figure 9: Welfare gains across sectors

Notes. Figure 9 contains the welfare gain for each sector of the economy, as well as the contribution of each asset class. To make it comparable to the other figures in our paper, the aggregate welfare gain of each sector is divided by the number of individuals in Norway. Units are 2011 US dollars.

If the household sector has experienced a positive welfare gain, who is the counterparty that experienced a welfare loss? For the most part, it was the government. As discussed earlier, the government is a net saver and is thus hurt by declining interest rates. Overall, the welfare loss of the government is negative, with a large contribution of debt and equity. As reported in Table 3, this comes from the fact that the government is a net holder of debt and a net purchaser of equity. In contrast, the contributions of deposits and housing for welfare gains are negligible (<\$2,000 in absolute value). Appendix D.2 also discusses the breakdown of welfare gains *within* asset class (see Table A4).

The fact that the Norwegian government is hurt by rising asset prices and declining interest rates can seem surprising from a U.S. perspective. In the U.S., the government is a net debt issuer, and so it tends to benefit from a rise in asset prices at the expense of households and foreigners, who hold its debt. The same effect holds true in Norway: as shown in Appendix Table A4, if we restrict ourselves to the debt issued by the government (i.e. the row “Government debt”), the rise in asset prices does benefit the government at the expense of households and foreigners. However, this effect is swamped by the fact that the Norwegian government holds a large amount of debt issued by households and foreigners: this is why the government is ultimately hurt by rising asset prices and declining interest rates.

As discussed in Section 2.4, the welfare loss of the government represents a loss of real resources available for net transfers to the household sector. While it is beyond our paper’s scope to quantify how the Norwegian government has adjusted (and will adjust) net transfers in response to persistently lower interest rates and higher asset prices, it is entirely possible that the very individuals who experienced welfare losses (i.e., the young) will also be the ones

to bear the brunt of future reductions in government transfers such as pension benefits.

In all of these exercises we use the same price deviation for foreign assets as for domestic assets (that from Section 3). This assumption was innocuous when computing the average welfare gain within the household sector as most of the financial transactions between Norwegians are transactions of domestic assets. However, this assumption becomes more restrictive when discussing welfare gains across sectors, as the Norwegian government buys a large amount of foreign assets. Price deviations for foreign equity and foreign debt may differ from the ones for Norwegian (domestic) assets. In Appendix D.3, we estimate price deviation series separately for domestic and foreign assets, and in Appendix Table A5, we use those indices to compute welfare gains across the different sectors. Overall, the results are very similar.

6 Generalizations of the baseline sufficient statistic approach

We now implement a number of generalizations of our baseline statistic approach.

6.1 Collateral effects

We now build on our collateral effects extension in Section 2.4 and Appendix A.3.1 to quantify the welfare gains from asset price deviations that operate via collateral effects. We focus on the welfare effect of rising housing values operating via lower mortgage interest rates: a higher house price decreases an individual's loan-to-value ratio and allows the individual to borrow at a lower interest rate.

To capture this intuition, we assume that the individual-specific mortgage interest rate increases linearly with the individual's loan-to-value ratio $LTV_{i,t}$ defined as its mortgage debt (in absolute value) divided by its house value. Denoting the corresponding slope coefficient by β and using our convention to model debt as a one-period bond with value Q (so that the gross interest rate is $1/Q$), we assume that the individual-specific mortgage bond price $Q_{i,M,t}$ is given by

$$Q_{i,M,t} = Q_{M,t} e^{-\beta \times LTV_{i,t}}, \quad (23)$$

where $Q_{M,t}$ is a "reference" mortgage bond price. When $\beta > 0$ a rise in housing values decreases the loan-to-value ratio and hence the mortgage interest rate.

Using the sufficient statistic in the presence of collateral constraints (see Proposition 8 in Appendix A.3.1), we add the following term to the welfare gain formula (20) that we then bring to the data:⁵⁶

$$\text{Welfare Gain}_{i,\text{collateral}} = \sum_{t=0}^{25} R^{-t} (-B_{i,M,t} Q_{i,M,t}) \times \beta \times LTV_{i,t} \times \frac{PD_{H,t} - \overline{PD}_H}{PD_{H,t}}. \quad (24)$$

To quantify (24), we need an estimate of β . Figure 10a reports a binned scatter-plot which contains the mortgage interest rate of an individual plotted against the loan-to-value ratio. A clear positive relationship is visible: as loan-to-value ratios increase from 0 to 100%, mortgage

⁵⁶See Appendix E.1 for a derivation.

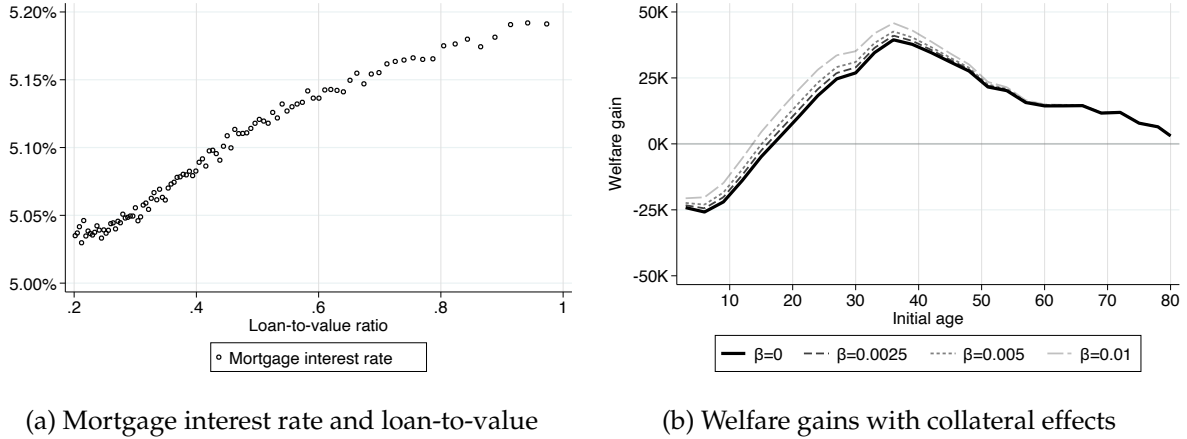


Figure 10: Mortgage interest rates, housing values, and collateral effects

Notes. Figure 10a contains a binned scatter plot of the interest rate on mortgages and the loan-to-value ratio across individuals over the years 1994–2019. To do this figure, we demean the individual-specific interest rate and loan-to-value ratios by their average values each year, adding back the average interest rate over 1994–2019. Each dot represents a percentile of the sample, ranked according to their loan-to-value ratio. Figure 10b plots the welfare gain including the collateral effect adjustment term. The welfare gain with $\beta = 0$ is the same as in Figure 6c while the welfare gain with $\beta > 0$ accounts for the effect of collateral constraints by adding the term (24). Units are 2011 US dollars.

interest rates increase by around 0.2 pp. from around 5% to 5.20%. In Appendix E.1, we estimate this relationship more formally using panel regressions and obtain values for β between 0.0025 and 0.005, depending on the controls included. The interpretation is that a 10 pp. higher loan-to-value ratio is associated with a 0.025 pp. to 0.05 pp. (2.5 to 5 basis points) higher mortgage interest rate. Note, however, that this estimated coefficient might underestimate the true extent of the collateral effect, for instance if the loan-to-value ratio contains measurement errors. For this reason, we also collect direct evidence of an interest rate schedule posted by a Norwegian bank, which suggests a higher value of $\beta \approx 0.01$, i.e. that a 10 pp. increase in the loan-to-value ratio implies a 0.1 pp. (10 basis points) rise in the interest rate.

Figure 10b reports the welfare gains across cohorts including the contribution of the collateral constraint channel (24). Given the uncertainty regarding the value of β , we report results for a range of values $\beta \in \{0, 0.0025, 0.005, 0.01\}$, where the case $\beta = 0$ corresponds to the baseline welfare gain formula (i.e., same welfare gains as in Figure 6c). Overall, the welfare gains associated with the collateral effect are small. Notice that the young are the ones who benefit the most, due to the fact that they hold larger mortgage balances on average (see Figure 6b). A rise in house prices thus leads to a decline in interest payments via a lower loan-to-value, which disproportionately benefits individuals with high mortgage balances.⁵⁷

In Appendix E.1, we also examine the dispersion of the “collateral welfare gains” in (24) across individuals (i.e., including heterogeneity within cohorts). We document sizable dispersion: while collateral welfare gains are only \$5,000 on average, they increase to \$67,000 for the top 1% of individuals most impacted via this channel. Despite this sizable dispersion, the

⁵⁷While we do not discuss aggregation in the context of the collateral effects extension, banks charging lower mortgage interest rates in response to higher home values may also generate some losers, in particular bank shareholders who indirectly hold mortgage debt as an asset. An offsetting effect is that lower loan-to-value ratios may lower bank monitoring costs so that bank shareholders may not be impacted much overall.

inferred welfare gains due to collateral effects remain small *relative* to the large baseline welfare gains (i.e., the ones due to rising asset prices benefiting prospective sellers and harming prospective buyers, as reported in Table 2). The reason is that even our largest estimate for $\beta = 0.01$ implies that a 50 pp. decrease in the loan-to-value ratio reduces the mortgage interest rate by 0.5 pp. This remains small compared to the 2.5 pp. overall decline in the reference interest rate over our sample period.

6.2 Idiosyncratic labor income risk

We now quantify the contribution of incomplete markets and uninsurable idiosyncratic labor income risk on the individual-specific welfare gain associated with asset price deviations. As discussed in Section 2.4, incomplete markets result in an adjustment term equal to the discounted sum of future covariances between the growth rate of marginal utility and asset sales times price deviations – see equation (16).

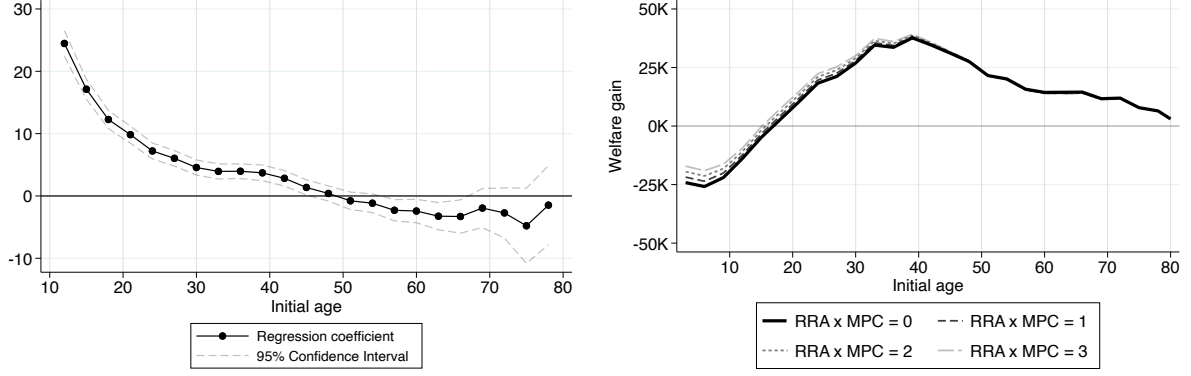
To estimate this incomplete markets adjustment term in the data, we need to make additional assumptions. Appendix E.2 shows that, assuming CRRA utility, we can use a log-linear approximation to marginal utility to write this term as (here for the case of one asset):

$$\sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_t)}{U'(C_0)}, N_{t-1} - N_t \right) dP_t \approx \text{RRA} \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \text{cov}_0 (\log C_t, N_t - N_{t-1}) dP_t, \quad (25)$$

where RRA is relative risk aversion. If we additionally assume a constant marginal propensity to consume (MPC) out of permanent labor income shocks, this adjustment term becomes $\text{RRA} \times \text{MPC} \times \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \text{cov}_0 (\log \bar{Y}_t, N_t - N_{t-1}) dP_t$ where MPC is this constant MPC and where $\log \bar{Y}_t$ is the permanent component of labor income. The key empirical objects are therefore the covariances between future (log) permanent labor income and future asset savings at each horizon $t \geq 0$. In Appendix E.2, we describe how we use a regression framework to estimate these covariances separately for each cohort and each asset class.

Figure 11a reports, for each cohort, the regression coefficient of net housing purchases at the ten-year horizon, $(N_{10} - N_9)P_{H,10}$, on log permanent income, $\log Y_{10}$, conditional on information in 1994. The coefficient is positive for cohorts that were below 50 years old in 1994, and close to zero afterwards. This says that, within each cohort, individuals who end up earning more than peers (i.e. others with similar initial characteristics) tend to purchase more housing. This is a force that will dampen the (ex-ante) welfare loss associated with rising house prices, given that housing purchases disproportionately occur in states of the world where individuals have high income and low marginal utility.

Figure 11b reports the welfare gains across cohorts including the incomplete market adjustment term (24) for different combinations of values for the risk aversion parameter RRA and the MPC. Usual values for risk aversion range from one to three and standard incomplete market models generate MPCs out of permanent income shocks close to, but below, one and definitely above one half (see e.g. Carroll, 2009). Therefore, we consider values for $\text{RRA} \times \text{MPC}$ between zero and three. Overall, we find that the incomplete market adjustment term dampens the welfare losses of the young. However, the magnitudes are small in comparison to our



(a) Regressing housing purchases on (log) income (b) Welfare gains with incomplete markets

Figure 11: Idiosyncratic labor income risk and incomplete markets

Notes. Figure 11a plots the regression coefficient of net housing purchase on (log) permanent income at the ten-year horizon, conditional on information in 1994; that is, $\beta_{10} = \text{cov}_0(\log Y_{10}, (N_{10} - N_9)P_{H,10}) / \text{Var}_0(\log Y_{10})$ for each cohort. Figure 11b plots the welfare gain including the incomplete markets adjustment term. The welfare gain with $\text{MPC} \times \text{RRA} = 0$ is the same as in Figure 6c while the welfare gain with $\text{MPC} \times \text{RRA} > 0$ accounts for the effect of incomplete markets by adding the covariance term (25). Units are 2011 US dollars.

baseline results, unless one makes aggressive assumptions on the value of MPCs and RRAs.

6.3 Second-order approximation

Proposition 1 characterized welfare gains from infinitesimal asset-price deviations and therefore holds to first order. However, the empirical price deviations are substantial suggesting that higher-order effects may be important.

We now consider the effect of a non-infinitesimal deviation in asset prices ΔQ_t and $\{\Delta P_{k,t}\}_{k=1}^K$ on welfare. To do so, we consider intermediate economies in which asset prices are given by $Q_t(\theta) = Q_t + \theta \Delta Q_t$ and $P_{k,t}(\theta) = P_{k,t} + \theta \Delta P_{k,t}$, where $\theta \in [0, 1]$ indexes the size of the price deviation. The case $\theta = 0$ correspond to the baseline economy while $\theta = 1$ corresponds to the fully perturbed one. The change in consumer surplus due to the deviations ΔQ_t and $\{\Delta P_{k,t}\}_{k=1}^K$ then equals the integral of the infinitesimal welfare gains as θ goes from 0 to 1:⁵⁸

$$\text{Welfare Gain} = \int_0^1 \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(\theta) \left(\sum_{k=1}^K (N_{k,t-1}(\theta) - N_{k,t}(\theta)) dP_{k,t}(\theta) - B_t(\theta) dQ_t(\theta) \right), \quad (26)$$

where $B_t(\theta)$, $\{N_{k,t}(\theta)\}_{k=1}^K$ denote the demand for assets in the economy indexed by θ .

Using a trapezoidal approximation, we then obtain a second-order approximation for this

⁵⁸Alternatively, one could define welfare gains as the equivalent variation (resp. the compensating variation) corresponding to the change in asset prices. In this case, one would get the same expression (26), with $B_t(\theta)$, $\{N_{k,t}(\theta)\}_{k=1}^K$ now denoting the Hicksian demands holding constant welfare at $\theta = 0$ (resp. $\theta = 1$). While these concepts are the same at the first-order, they differ at higher-orders.

change in consumer surplus:⁵⁹

$$\text{Welfare Gain} \approx \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (1/2) \left(\sum_{k=1}^K \left(N_{k,t-1} - N_{k,t} + \frac{\Delta(N_{k,t-1} - N_{k,t})}{2} \right) \Delta P_{k,t} - \left(B_t + \frac{\Delta B_t}{2} \right) \Delta Q_t \right), \quad (27)$$

where $\Delta B_t = B_t(1) - B_t(0)$ and $\Delta N_{k,t} = N_{k,t}(1) - N_{k,t}(0)$ for $1 \leq k \leq K$ denote the change in asset demands between the economy indexed by $\theta = 1$ and the one indexed by $\theta = 0$.

In contrast to the first-order approximation in (17), this second-order approximation takes into account asset transactions responding to asset price changes (e.g., portfolio reshuffling). In particular, welfare gains are higher for individuals who sell in response to increasing asset prices, $\Delta(N_{k,t-1} - N_{k,t}) > 0$ when $\Delta P_{k,t} > 0$, or for those who buy in response to declining prices, $\Delta(N_{k,t-1} - N_{k,t}) < 0$ when $\Delta P_{k,t} < 0$.⁶⁰

The empirical implementation of this second-order approximation requires additional assumptions: in contrast to the first-order approximation (17), we now need to specify what financial transactions would be if valuations had remained at their 1994 level. One way to do so would be to specify a parametric form for the utility function, a parametric form for the adjustment cost functions, as well as individuals' beliefs about future asset prices.

Instead, we make the simple assumption that, had valuations remained at their 1994 level, the quantity of transactions of a 30-year old in 2019 would be the same as the transactions of 30-year olds in 1994. Formally, we assume that the counterfactual transactions of individuals of age a at time $t > 0$ are given by:⁶¹

$$N_{a,k,t} - N_{a,k,t-1} + \Delta(N_{a,k,t} - N_{a,k,t-1}) = N_{a,k,0} - N_{a,k,-1}, \quad B_{a,t} + \Delta B_{a,t} = G^t B_{a,0}, \quad (28)$$

where $G = 1.01$ denotes the real per-capita growth rate of the economy in our sample period. The assumption here is that the policy functions for asset holdings (as a function of prices) of Norwegian individuals have remained the same over time (or, equivalently, that the rise in asset prices is driven by factors outside of the household sector).

We now examine how these counterfactual transactions differ from actual transactions. Figure 12a compares the actual and counterfactual housing and equity transactions for different age groups. We see that the two quantities are very close, simply because real net housing and equity purchases have remained roughly constant over time. Figure 12b compares the actual and counterfactual debt balances. Net debt (debt minus deposits) has increased much more rapidly than one could expect from the growth of the economy. Intuitively, the young must now borrow more in order to finance the purchase of houses whose values have grown faster than the economy. Overall however, we find that counterfactual transactions are relatively

⁵⁹For any functions $f(\theta), g(\theta)$ and $h(\theta)$ we have $\int_0^\epsilon f(\theta)g(\theta)dh(\theta) \approx f(\epsilon/2) \frac{g(0)+g(\epsilon)}{2} (h(\epsilon) - h(0))$ at the second-order in ϵ . To see this note that both sides of the formula have the same first and second derivatives at $\epsilon = 0$. Setting $f(\theta) = R_{0 \rightarrow t}^{-1}(\theta)$, $g(\theta) = N_{k,t-1}(\theta) - N_{k,t}(\theta)$, and $h(\theta) = P_{k,t}(\theta)$ gives (27).

⁶⁰Martínez-Toledano (2022) empirically studies the implications of "timing the market" and portfolio reshuffling for the evolution of wealth inequality.

⁶¹We write (28) in terms of observables as follows: multiplying both sides by $P_{k,t} = PD_{k,t}D_{k,t} = PD_{k,t}G^tD_{k,0} = \frac{G^t PD_{k,t}}{PD_{k,0}} P_{k,0}$ we obtain $[N_{a,k,t} - N_{a,k,t-1} + \Delta(N_{a,k,t} - N_{a,k,t-1})] P_{k,t} = \frac{G^t PD_{k,t}}{PD_{k,0}} (N_{a,k,0} - N_{a,k,-1}) P_{k,0}$, where the right-hand side is now observable.

similar to actual transactions, which suggests that second-order effects are likely moderate.

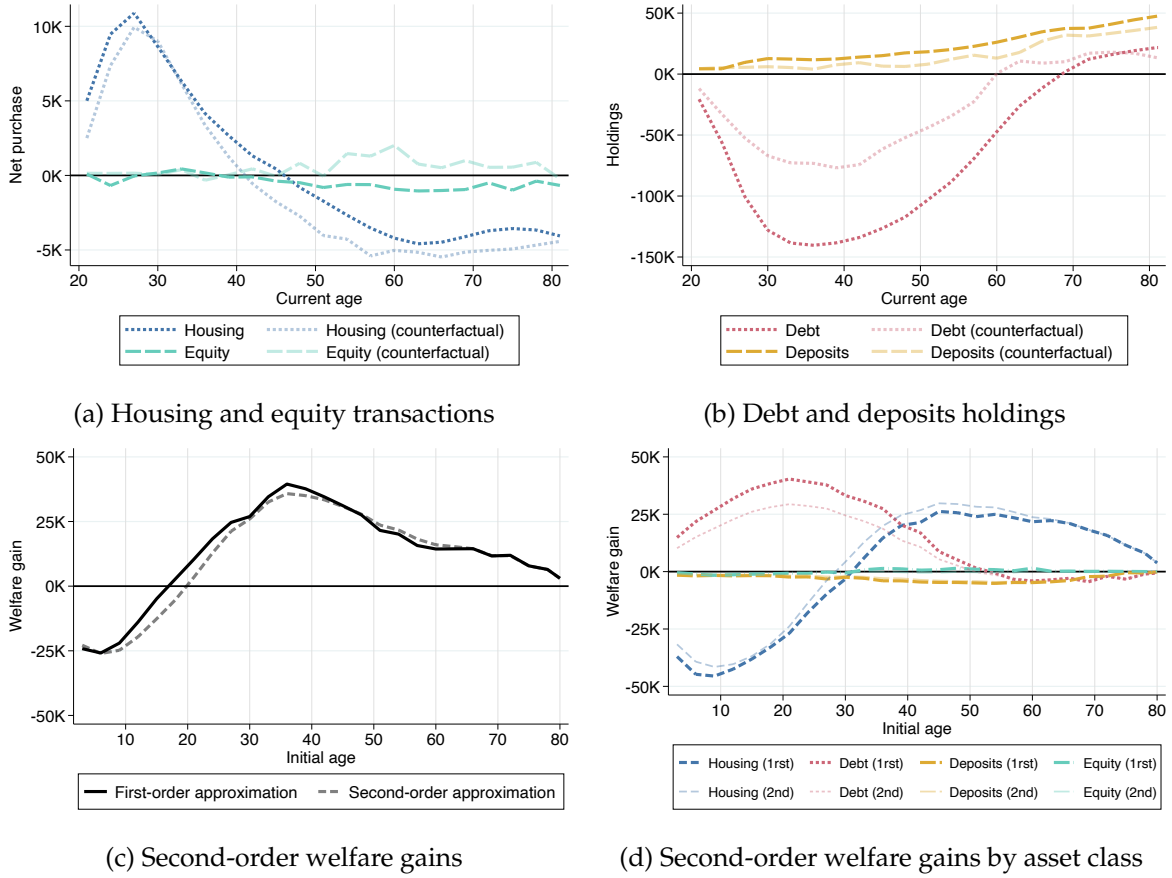


Figure 12: Transactions and welfare gains at the second order

Notes. Figure 12a and 12b compare actual transactions by age as well as the counterfactual transactions if valuations had remained at their baseline level. More precisely, Figure 12a plots $(N_{k,t} - N_{k,t-1})P_{k,t}$ and $(N_{k,t} - N_{k,t-1} + \Delta(N_{k,t} - N_{k,t-1}))P_{k,t}$, averaged by year and age at the end of the year, for $k \in \{H, E\}$. Figure 12b plots $B_t Q_t$ and $(B_t + \Delta B_t)Q_t$, averaged by year and age at the end of the year. Figure 12c plots the average welfare gain at the first order and at the second order for individuals in each cohort (indexed by their age at the end of 1994) while Figure 12d plots it asset class by asset class. The second-order approximation is constructed using the assumption that, if valuations were back to their level of 1994, individuals would trade the same quantity of assets as in 1994. Units are 2011 US dollars.

Figure 12c plots the second-order welfare gains computed by using these counterfactual financial transactions in equation (27) and confirms this intuition: the overall effect of the second-order adjustment is small and the results are quantitatively similar to those using our first-order approximation. One interesting effect is that the second-order adjustment decreases the welfare gains of younger individuals. This is because, as we have discussed, low mortgage rates provide an important offsetting effects for home buyers who are hurt by rising house prices. If house prices had remained at their initial values, the young would have lower mortgage balances and, as a result, they would benefit less from any change in mortgage rates (see Figure 12d for a plot of the second-order correction by asset class).

6.4 Valuation changes beyond the end of our sample period

Our measure of welfare gains in Proposition 1 expresses the welfare gains as the present value of all future transactions, multiplied by the path of future price deviations. However, as dis-

cussed in Section 3, we only apply our formula on a finite sample that ends in year 2019 ($T = 25$). Therefore, our formula should be interpreted as the welfare gain associated with price deviations equal to zero after 2019 (i.e., assuming that valuations revert to the baseline in which asset prices grow at the same rate as dividends after 2019).

How important is this truncation for our results? To examine this question, we recompute our welfare gains with different assumptions about the behavior of asset prices after 2019. More precisely, we assume that, after the end of the sample, valuations revert back to their baseline level according to a mean reversion parameter $\phi \in [0, 1]$. Formally, we assume that the valuation of asset class k at $t > T$ is given by:⁶²

$$\log (PD_{k,t}/\overline{PD}_k) = \phi^{t-T} \log (PD_{k,T}/\overline{PD}_k), \quad \log (Q_t/\overline{Q}) = \phi^{t-T} \log (Q_T/\overline{Q}), \quad (29)$$

where $PD_{k,T}$ denotes the asset valuation in year 2019 and \overline{PD}_k denotes the baseline level of the asset valuation defined in Section 3. Our baseline summary statistic, which considers asset price deviations that stop after T , can be seen as the limit case $\phi = 0$. Figure 13a plots the series of house prices obtained using this methodology up to 2060, for values of ϕ between 0 and 1. Note that, in all scenarios, we assume that housing valuations ultimately revert back to their initial value ($\phi < 1$), consistent with the fact that asset valuations are stationary processes (Campbell and Shiller, 1988).

To implement the sufficient statistic formula, we also need to predict individuals' transactions in future years. To do so, we assume that the number of assets sold by a given cohort in a given year will equal the number of assets sold by the cohort with the same age in 2019, after adjusting for economic growth. See Appendix E.3 for details. This assumption is motivated by the fact that the quantity of transactions by age groups has remained remarkably stable over our sample period, as discussed above (Section 6.3).

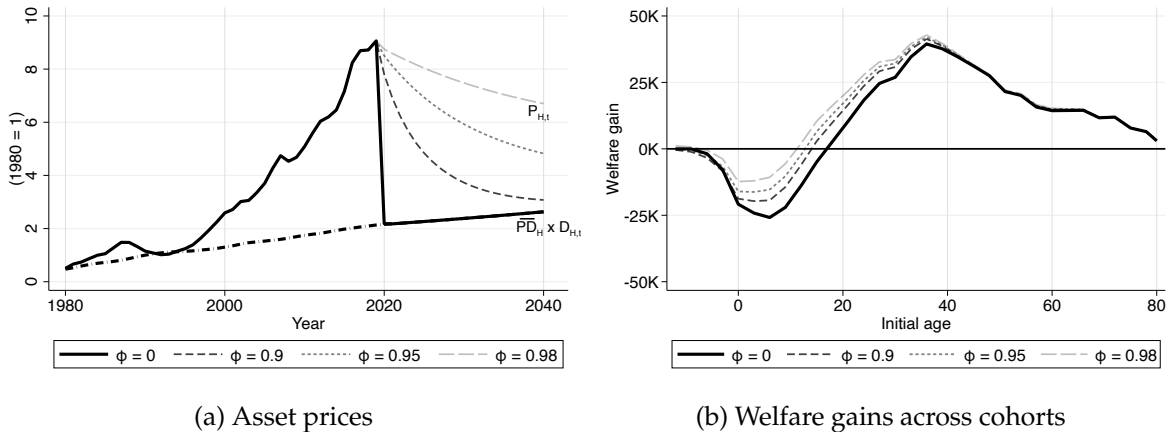


Figure 13: Asset prices and welfare gains with valuation changes beyond 2019

Notes. Figure 13a plots the path of future house prices for different values of ϕ , constructed using (29). Figure 13b plots the average welfare gain in each cohort with different assumptions about the path of asset prices. Units are 2011 US dollars.

Figure 13 plots our estimated values for the average welfare gain in each cohort for different

⁶²See Campbell (2018) for an example of such a AR(1) specification for the logarithmic price-dividend ratio.

values of ϕ . As ϕ increases, two things happen. First, the graph of welfare gains shifts to the left. Intuitively, a high ϕ means that aging individuals sell more assets at elevated prices beyond the end of our sample period thereby increasing their welfare gains. However, this comes at the expense of young generations, unborn in 1994, who will ultimately purchase these assets. Second, the graph of welfare gains shifts up. This is because, as we have shown in the sectoral analysis in Section 5, individuals benefit on net from the rise in asset prices because they hold a positive amount of debt in the aggregate. As ϕ increases, higher valuations last for a longer time, which means that the average welfare gain per capita increases. However, doing the same exercise for sectoral welfare gains would reveal that this comes at the cost of a decrease in the total welfare gains for the government so that welfare gains still aggregate to zero (Corollary 2). Appendix Figure A7 decomposes the welfare gains by asset class. The decomposition shows that, as ϕ increases, most of the higher welfare gains in the population comes from lower interest rates on debt.

7 Conclusion

The main contribution of our paper is to provide a simple framework to quantify the welfare effects of fluctuations in asset valuations. Two economic ideas lie at the core of our sufficient statistic approach. First, rising asset valuations benefit prospective sellers and harm prospective buyers. Second, because there is a seller for every buyer, changes in asset valuation are also purely redistributive. We implement our sufficient statistic formula using administrative data on financial transactions to quantify welfare gains and losses in Norway for the years 1994 to 2019.

Our empirical implementation generates four main findings. First, the rise in asset valuations had large redistributive effects, i.e., they resulted in significant welfare gains and losses. At the same time, welfare gains differed substantially from naïvely calculated revaluation gains; in particular, individuals with the highest revaluation gains were not necessarily the ones with the highest welfare gains. Second, rising asset prices redistributed across cohorts, with the old benefiting at the expense of the young. Third, they redistributed across the wealth distribution, from the poor toward the wealthy. Fourth, they also redistributed across sectors: declining interest rates benefited households at the expense of the government.

Recent work building on our methods suggests that our sufficient statistic approach may also prove useful in other contexts. [Del Canto et al. \(2023\)](#) study the welfare impact of inflationary oil shocks and monetary expansions on U.S. households and implement the corresponding welfare formulae using micro data on household consumption, asset holdings and labor income. Similarly, [Crawley and Gamber \(2023\)](#) study the welfare consequences of the large asset-price and interest-rate changes on U.S. households over the time period 2021 to 2023 rather than the longer-run trends considered here. Another valuable exercise would be to systematically quantify the welfare consequences of higher-frequency asset-price booms and busts that the literature has emphasized as important drivers of wealth inequality dynamics ([Kuhn et al., 2020](#); [Martínez-Toledano, 2022](#); [Gomez, 2016](#); [Cioffi, 2021](#)).

Finally, the result that rising asset prices benefit asset sellers rather than asset holders raises a number of questions for optimal tax theory. It suggests that taxing wealth or unrealized capital gains (as under the Wyden “Billionaires Income Tax” proposal) may be undesirable from a normative perspective. When asset prices rise, such taxes can redistribute “in the wrong direction”: they hit not only individuals who benefit in welfare terms (those who sell their assets) but also those whose welfare is unaffected or declines (those who do not sell or perhaps even buy). Are there other forms of taxes that are closer to optimal? Perhaps the existing practice of taxing capital gains on realization (i.e., when a sale occurs) is preferable? Answering such questions requires studying environments with changing asset prices using the tools from public finance. Ongoing work by [Aguiar et al. \(2022\)](#) takes some steps in this direction.

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Appendix

A Appendix for Section 2

A.1 Welfare gains and consumption changes

Proof of Proposition 1

Proof of Proposition 1. To provide some intuition, we first provide an heuristic derivation using the Lagrangian corresponding to the individual optimization problem and totally differentiating with respect to the sequence of asset prices $\{Q_t, \{P_{k,t}\}_1^K\}_{t=0}^\infty$. We then provide a more rigorous proof that uses a perturbation of this sequence indexed by a perturbation parameter θ as well as a version of the envelope theorem due to [Oyama and Takenawa \(2018\)](#).

Heuristic derivation. The Lagrangian associated with the optimization problem is

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^t U(C_t) \\ &+ \sum_{t=0}^{\infty} \lambda_t \left(\sum_{k=1}^K N_{k,t-1} D_t + B_{t-1} + Y_t - C_t - \sum_{k=1}^K (N_{k,t} - N_{k,t-1}) P_{k,t} - \sum_{k=1}^K \chi_k (N_{k,t} - N_{k,t-1}) - B_t Q_t \right). \end{aligned} \quad (30)$$

The first-order condition for C_t is $\beta^t U'(C_t) = \lambda_t$ while the first-order condition for B_t is $\lambda Q_t = \lambda_{t+1}$. Assuming that the value function is differentiable, we can write the infinitesimal change in the value function in terms of the infinitesimal change in the Lagrangian:

$$\begin{aligned} dV &= \sum_{t=0}^{\infty} \left(\sum_{k=1}^K \frac{\partial \mathcal{L}}{\partial P_{k,t}} dP_{k,t} + \frac{\partial \mathcal{L}}{\partial Q_t} dQ_t \right) \\ &= \sum_{t=0}^{\infty} \lambda_t \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t \right) \\ &= \lambda_0 \sum_{t=0}^{\infty} (Q_0 \dots Q_{t-1}) \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t \right) \\ &= U'(C_0) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t \right). \end{aligned}$$

The third equality uses the first-order conditions for B_t while the fourth equality uses the first-order conditions for C_t as well as the definition of the cumulative return $R_{0 \rightarrow t}^{-1} = Q_0 \dots Q_{t-1}$.

Formal derivation. Consider a deviation in asset prices in the direction $\{\Delta Q_t, \{\Delta P_{k,t}\}_1^K\}_{t=0}^\infty$. Consider a parameter $\theta \in [0, 1]$ indexing the size of the perturbation:

$$Q_t(\theta) \equiv Q_t + \theta \Delta Q_t, \quad P_{k,t}(\theta) = P_{k,t} + \theta \Delta P_{k,t}$$

The optimization problem takes the form $V(\theta) = \max_x f(x, \theta)$ where $x = \{B_t, \{N_{k,t}\}_{k=1}^K\}_{t=0}^\infty$, and

$$f : (x, \theta) \mapsto \sum_{t=0}^{\infty} \beta^t U \left(\sum_{k=1}^K N_{k,t-1} D_{k,t} + B_{t-1} + Y_t - \sum_{k=1}^K (N_{k,t} - N_{k,t-1}) P_{k,t} - B_t Q_t - \sum_{k=1}^K \chi_k (N_{k,t} - N_{k,t-1}) \right).$$

Note that f is continuous in x , and that its derivative with respect to θ is

$$\partial_{\theta} f(x, \theta) = \sum_{t=0}^{\infty} \beta^t U'(C_t) \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} - B_t \Delta Q_t \right),$$

which is continuous in x and θ . Under this set of assumptions, Proposition 2.1 in [Oyama and Takenawa \(2018\)](#) gives that V is differentiable at 0 and $V'(0) = \partial_{\theta} f(x^*, 0)$, where x^* denote the optimal solution of the maximization problem at $\theta = 0$. Using the expression for $\partial_{\theta} f$ above gives:

$$\begin{aligned} V'(0) &= \sum_{t=0}^{\infty} \beta^t U'(C_t) \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} - B_t \Delta Q_t \right) \\ &= U'(C_0) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} - B_t \Delta Q_t \right). \end{aligned}$$

where the second line uses the Euler equation. This concludes the proof as $dV = V'(0) d\theta$, $dQ_t = \Delta Q_t d\theta$, and $dP_{k,t} = \Delta P_{k,t} d\theta$ for $1 \leq k \leq K$. \square

Fixed holdings versus reoptimization. As shown in (13), our measure of welfare gains corresponds to the present value of the change in consumption due to the change in asset prices; that is

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_t.$$

In turn, one can decompose the change in consumption in each period, dC_t , as the sum of the change in consumption that would obtain if asset holdings were fixed, $dC_t^{\text{fixed holdings}}$, and a residual that captures the effect of reoptimization of asset holdings, $dC_t^{\text{reoptimization}}$.

Differentiating the budget constraint (11) gives:

$$\begin{aligned} dC_t^{\text{fixed holdings}} &\equiv \sum_{k=1}^K (N_{k,t-1} - N_{k,t-1}) dP_{k,t} - B_t dQ_t, \\ dC_t^{\text{reoptimization}} &\equiv \sum_{k=1}^K (dN_{k,t-1} - dN_{k,t-1}) P_{k,t} - Q_t dB_t \\ &\quad + \sum_{k=1}^K D_{k,t} dN_{k,t-1} - \sum_{k=1}^K \chi_k (N_{k,t} - N_{k,t-1}) (dN_{k,t} - dN_{k,t-1}) + dB_{t-1}, \end{aligned} \tag{32}$$

with $dC_t = dC_t^{\text{fixed holdings}} + dC_t^{\text{reoptimization}}$. Moreover, note that the present value of the consumption response, holding fixed holdings, is equal to the welfare gain. This means that, conversely, the present value of the consumption responses due to the reoptimization of holdings is zero.

$$\begin{aligned} \text{Welfare Gain} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_t^{\text{fixed holdings}}, \\ 0 &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_t^{\text{reoptimization}}. \end{aligned} \tag{33}$$

This is a consequence of the envelope theorem: re-optimizing asset holdings in response to asset price changes only has a second-order effect on welfare.

Income versus substitution effect. Another way to decompose the change in consumption in each period due to the change in asset prices, dC_t , is to decompose it into an income and a substitution effect, where the income effect dC_t^{income} is the response of consumption to a change in initial income that is welfare-equivalent to the change in prices, and where the substitution effect $dC_t^{\text{substitution}}$ is the response of consumption to the change in prices holding welfare constant.

We now give an analytical expression for each effect in the two-period model of Section 2.1.

Lemma 4. *In the two-period model, the response of consumption to an asset-price deviation is:*

$$\begin{aligned} dC_0 &= \underbrace{MPC_0 \times (N_{-1} - N_0) dP_0}_{dC_0^{\text{income}}} && \underbrace{-\psi(C_1) MPC_0 \times C_1 R_1^{-1} d \log R_1}_{dC_0^{\text{substitution}}} \\ dC_1 &= \underbrace{(1 - MPC_0) R_1 \times (N_{-1} - N_0) dP_0}_{dC_1^{\text{income}}} && \underbrace{+\psi(C_1) MPC_0 \times C_1 d \log R_1}_{dC_1^{\text{substitution}}}, \end{aligned} \quad (34)$$

where $MPC_0 \equiv \left(1 + R_1^{-1} \frac{C_1 \psi(C_1)}{C_0 \psi(C_0)}\right)^{-1}$ is the marginal propensity to consume out of income at time $t = 0$ and $\psi(C) \equiv -U'(C)/(U''(C)C)$ denotes the (local) elasticity of intertemporal substitution.

The expressions in (34) are similar to those in Auclert (2019). Note that the substitution effect of a higher asset price (i.e., a lower asset return) is positive at $t = 0$ and negative at $t = 1$. One can check that, consistently with the definition of the income and substitution effects, we have:

$$\begin{aligned} \text{Welfare Gain} &= dC_t^{\text{income}} + R_1^{-1} dC_1^{\text{income}}, \\ 0 &= dC_0^{\text{substitution}} + R_1^{-1} dC_1^{\text{substitution}}. \end{aligned} \quad (35)$$

While this is similar to (33), note that income and substitution effects typically differ from the fixed and reoptimization effects defined in (32).

Proof of Lemma 4. Differentiating budget constraint at $t = 0$ and $t = 1$ gives

$$\begin{aligned} dC_0 + (N_0 - N_{-1}) dP_0 + dN_0 P_0 &= 0 \\ dC_1 &= D_1 dN_0. \end{aligned}$$

Substituting out dN_0 gives the consolidated (differentiated) budget constraint:

$$dC_0 + \frac{P_0}{D_1} dC_1 = (N_{-1} - N_0) dP_0. \quad (36)$$

We now turn to the Euler equation:

$$\beta \frac{U'(C_1)}{U'(C_0)} = \frac{P_0}{D_1}.$$

Taking logs and differentiating gives

$$\frac{U''(C_1)}{U'(C_1)} dC_1 - \frac{U''(C_0)}{U'(C_0)} dC_0 = \frac{dP_0}{P_0}. \quad (37)$$

Hence, we obtain a system of two equations, (36) and (37), and two unknowns, dC_0 and dC_1 . In matrix

form, this gives:

$$\begin{pmatrix} 1 & \frac{P_0}{D_1} \\ -\frac{U''(C_0)}{U'(C_0)} \frac{U'(C_1)}{U''(C_1)} & 1 \end{pmatrix} \begin{pmatrix} dC_0 \\ dC_1 \end{pmatrix} = \begin{pmatrix} (N_{-1} - N_0) dP_0 \\ \frac{U'(C_1)}{U''(C_1)} \frac{dP_0}{P_0} \end{pmatrix}.$$

Solving this system gives

$$\begin{pmatrix} dC_0 \\ dC_1 \end{pmatrix} = \frac{1}{1 + \frac{U''(C_0)}{U'(C_0)} \frac{U'(C_1)}{U''(C_1)} \frac{P_0}{D_1}} \begin{pmatrix} 1 & -\frac{P_0}{D_1} \\ \frac{U''(C_0)}{U'(C_0)} \frac{U'(C_1)}{U''(C_1)} & 1 \end{pmatrix} \begin{pmatrix} (N_{-1} - N_0) dP_0 \\ \frac{U'(C_1)}{U''(C_1)} \frac{dP_0}{P_0} \end{pmatrix}.$$

Using the definition $R_1 = D_1/P_0$ and $\psi(C) = -U'(C)/(U''(C)C)$, this simplifies to

$$\begin{pmatrix} dC_0 \\ dC_1 \end{pmatrix} = \frac{1}{1 + R_1^{-1} \frac{C_1 \psi(C_1)}{C_0 \psi(C_0)}} \begin{pmatrix} 1 & -R_1^{-1} \\ \frac{C_1 \psi(C_1)}{C_0 \psi(C_0)} & 1 \end{pmatrix} \begin{pmatrix} (N_{-1} - N_0) dP_0 \\ \psi(C_1) C_1 d \log R_1 \end{pmatrix}. \quad (38)$$

We now discuss how to separate the total consumption response into an income and substitution effect. By definition, the income effect is the response of consumption to a change in income at $t = 0$ that is welfare-equivalent to the change in prices; that is, $dY_0 = (N_{-1} - N_0) dP_0$. It can be obtained by setting $d \log R_1$ to zero in (38). Conversely, the substitution effect is defined as the response of consumption to a change in price dP_0 holding welfare constant. It can be obtained by setting dP_0 to zero in (38). One can easily see that the income and substitution effects aggregate to the total consumption response. \square

Present-value budget constraint. We here derive the present-value budget constraint implied by the period budget constraints (11) and the no-Ponzi condition stated in the main text. At the optimum, the no-Ponzi condition holds with equality and is therefore given by

$$\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} \left(B_T Q_T + \sum_{k=1}^K N_{k,T} P_{k,T} \right) = 0. \quad (39)$$

To derive the present-value budget constraint, it is useful to derive a period budget constraint in terms of the individual's total wealth

$$A_t = B_t Q_t + \sum_{k=1}^K N_{k,t} P_{k,t}.$$

Using this definition, (11) can be rewritten as

$$C_t + A_t = Y_t + R_t A_{t-1} + \sum_{k=1}^K (R_{k,t} - R_t) N_{k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{k,t} - N_{k,t-1}), \quad (40)$$

where the reader should recall that $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ is the return of asset k and $R_{t+1} = 1/Q_t$ is the return on the one-period bond. The right-hand side of this constraint says that the individual's capital income equals total wealth times the bond return $R_t A_{t-1}$ plus the excess return from holding long-lived assets $\sum_{k=1}^K (R_{k,t} - R_t) N_{k,t-1} P_{k,t-1}$ minus any adjustment costs from transacting these. The no-Ponzi condition (39) becomes

$$\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} A_T = 0. \quad (41)$$

We then have:

Lemma 5. *The present-value budget constraint implied by the sequence of budget constraints (11) and the no-*

Ponzi condition (39) is

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_t = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left[Y_t + \sum_{k=1}^K (R_{k,t} - R_t) N_{k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{k,t} - N_{k,t-1}) \right] + R_0 A_{-1}, \quad (42)$$

where $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ is the return of asset k , $R_{t+1} = 1/Q_t$ is the return on the one-period bond, and where $A_{-1} = \sum_{k=1}^K N_{k,-1} P_{k,-1} + B_{-1} Q_{-1}$ is exogenously given initial wealth.

Proof of Lemma 5. Recursive forward substitution of the sequence of budget constraints (40) (eliminating A_0, A_1 and so on) yields

$$R_{0 \rightarrow T}^{-1} A_T + \sum_{t=0}^T R_{0 \rightarrow t}^{-1} C_t = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left[Y_t + \sum_{k=1}^K (R_{k,t} - R_t) N_{k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{k,t} - N_{k,t-1}) \right] + R_0 A_{-1},$$

where we have used the definition of the cumulative return $R_{0 \rightarrow t} = R_1 \cdot R_2 \cdot \dots \cdot R_t$. Taking the limit as $T \rightarrow \infty$ and imposing the no-Ponzi condition (41) yields (42). \square

For completeness, we note briefly that there are a number of alternative (but equivalent) ways of writing the present-value budget constraint, for example one other option is

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_t = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left[Y_t + \sum_{k=1}^K (N_{k,t-1} D_t + (N_{k,t-1} - N_{k,t}) P_{k,t} - \chi(N_{k,t} - N_{k,t-1})) \right] + B_{-1}.$$

Derivation of welfare gains formula from present-value budget constraint. While the proof of Proposition 1 uses the sequence of period budget constraints, our welfare gains formula can also be derived from the corresponding present-value constraint. We here explain the connection between the two proof strategies. To explain the logic in the most transparent fashion and avoid cluttered notation, we here consider a version of the baseline model without bonds and only one long-lived asset N_t without adjustment costs. Everything generalizes to the case covered in our baseline model, i.e. a bond and K long-lived assets that are potentially subject to adjustment costs.

In this one-asset version, individuals have utility $\sum_{t=0}^{\infty} \beta^t U(C_t)$, the sequence of budget period constraints

$$C_t + P_t(N_t - N_{t-1}) = Y_t + D_t N_{t-1}, \quad t = 0, 1, \dots \quad (43)$$

the asset's return is $R_{t+1} = (D_{t+1} + P_{t+1})/P_t$, and the no-Ponzi condition is $\lim_{T \rightarrow \infty} R_{0 \rightarrow T} N_T P_T = 0$ where $R_{0 \rightarrow t} = R_1 \cdot R_2 \cdot \dots \cdot R_t$. Following the same steps as in Proposition 1, it is then easy to show that our welfare gains formula is:

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{t-1} - N_t) dP_t. \quad (44)$$

An alternative proof strategy instead focuses on the present-value budget constraint corresponding to (43). The corresponding maximization problem is

$$V = \max_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_t = N_{-1}(P_0 + D_0) + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Y_t.$$

We now show how to derive (44) from this formulation. Along the way we derive two alternative expressions for the welfare gains formula (equations (45) and (45) below). Both are instructive and carry important economic intuition.

The Lagrangian corresponding to the maximization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t U(C_t) + \mu \left(N_{-1}(P_0 + D_0) + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (Y_t - C_t) \right)$$

where μ is the Lagrange multiplier on the present-value budget constraint. Using the envelope condition to compute dV and that $\mu = U'(C_0)$, the welfare gain $dV/U'(C_0)$ from a deviation in asset prices $\{dP_t\}_{t=0}^{\infty}$ and the associated deviation in asset returns $\{dR_{t+1}\}_{t=0}^{\infty}$ equals

$$\text{Welfare Gain} = N_{-1} dP_0 + \sum_{t=0}^{\infty} (Y_t - C_t) dR_{0 \rightarrow t}^{-1}. \quad (45)$$

This equation says that the welfare gain from an asset-price change is the change in wealth, $N_{-1} dP_0$, plus an extra term $\sum_{t=0}^{\infty} (Y_t - C_t) dR_{0 \rightarrow t}^{-1}$. This formulation of the welfare-gains formula clarifies that, when asset discount rates change, price deviations not only result in a revaluation of initial wealth $N_{-1} dP_0$ but also affect the discounting in the present-value budget constraint.

To derive the second alternative formulation of our welfare gains formula, we now write this extra term as:

$$\begin{aligned} \sum_{t=0}^{\infty} (Y_t - C_t) dR_{0 \rightarrow t}^{-1} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (Y_t - C_t) \left(- \sum_{s=1}^t d \log R_s \right) \\ &= - \sum_{t=1}^{\infty} \left(\sum_{s=t}^{\infty} R_{0 \rightarrow s}^{-1} (Y_s - C_s) \right) d \log R_t \\ &= \sum_{t=1}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{t-1} P_{t-1} d \log R_t \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_t P_t d \log R_{t+1} \end{aligned}$$

where the third equality uses the consolidated budget constraint from the point of view of time t . Plugging this expression into (45) gives

$$\text{Welfare Gain} = N_{-1} dP_0 + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_t P_t d \log R_{t+1}. \quad (46)$$

The intuition is that welfare gains depend on prices today but also returns going forward.

Finally, to get back our main welfare formula (44), differentiate the definition of the asset return

$$d \log R_{t+1} = R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t}.$$

Plugging into (46) we have

$$\text{Welfare Gain} = N_{-1} dP_0 + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_t P_t \left(R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t} \right) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{t-1} - N_t) dP_t,$$

where the second equality uses that $R_{0 \rightarrow t+1}^{-1} dP_{t+1} \rightarrow 0$ as $t \rightarrow \infty$, and we have therefore recovered our main welfare gains formula (44).

A.2 Discussions

A.2.1 Equivalence between deviations in prices and deviations in discount rates

We mentioned in the main text that deviation in asset prices, holding dividends constant, are equivalent to deviations in discount rates. We now express this idea more formally. Consider a sequence of dividends $\{D_{k,t}\}_{t=0}^{\infty}$ and a sequence of discount rates $\{R_{k,t}\}_{t=0}^{\infty}$. Assuming the no-bubble condition, the price of the asset is then given by

$$P_{k,t} = \sum_{s=0}^{\infty} \frac{D_{k,t+s+1}}{R_{k,t+1} \cdots R_{k,t+s+1}}$$

This formula makes clear that, in the absence of dividend changes, price changes must come from changes in discount rates. The following lemma gives the formal mapping between the two.⁶³

Lemma 6. *Holding dividends constant, a given deviation in the path of asset discount rates $\{dR_{k,t}\}_{t=0}^{\infty}$ generates the following deviation in asset prices $\{dP_{k,t}\}_{t=-1}^{\infty}$:*

$$\frac{dP_{k,t}}{P_{k,t}} = - \sum_{s=0}^{\infty} \frac{(R_{k,t+1} \cdots R_{k,t+s})^{-1} P_{k,t+s}}{P_{k,t}} \cdot \frac{dR_{k,t+s+1}}{R_{k,t+s+1}}. \quad (47)$$

This equation gives the deviation in asset prices generated by a given deviation path in discount rates. In spirit, this equation is very similar to [Campbell and Shiller \(1988\)](#)'s log linearized present value identity, which relates fluctuations in the price-dividend ratio to fluctuations in future returns. We now briefly discuss the connection between the two results. In the particular case where the price-dividend ratio is constant on the baseline path, we have

$$R_{k,t+1} = \frac{D_{k,t+1} + P_{k,t+1}}{P_{k,t}} = \left(1 + \frac{D_{k,t+1}}{P_{k,t+1}}\right) \frac{P_{k,t+1}}{P_{k,t}} = \frac{1}{\rho} \frac{P_{k,t+1}}{P_{k,t}},$$

where $\rho \equiv 1/(1 + D_{k,t+1}/P_{k,t+1})$. Plugging this into (47) gives:

$$d \log P_{k,t} = - \sum_{s=0}^{\infty} \rho^s d \log R_{k,t+s+1}$$

which corresponds to [Campbell and Shiller \(1988\)](#)'s present value identity when cash flows are not moving.

Proof of 6. Differentiating the definition of returns $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ gives

$$dR_{k,t+1} = \frac{dP_{k,t+1} - R_{k,t+1} dP_{k,t}}{P_{k,t}}.$$

This equation can be used to express the deviation in prices today as a function of the deviation in returns and prices tomorrow:

$$dP_{k,t} = R_{k,t+1}^{-1} P_{k,t} dR_{k,t+1} + R_{k,t+1}^{-1} dP_{k,t+1}$$

⁶³See also [Knox and Vissing-Jorgensen \(2022\)](#) for a similar result.

Iterating forward gives

$$dP_{k,t} = \sum_{s=0}^{\infty} (R_{k,t+1} \dots R_{k,t+s+1})^{-1} P_{k,t+s} dR_{k,t+s+1}.$$

Dividing by $P_{k,t}$ gives (b). □

Relationship with Auclert (2019). Auclert (2019) examines the effect of a one-time perturbation in the path of interest rates on consumption and welfare. We now discuss how this result relates to our Proposition 1. Consider an economy where, at time $t = 0$, individuals can trade bonds of all maturities. Denote Q_h the price of the bond with maturity $h \geq 1$. That is, the long-term interest rate between 0 and h is $R_{0 \rightarrow h} = 1/Q_h$.

As in the baseline model, the individual receives labor income Y_t at time t and they initially own N_{-1} shares of a long lived asset that pays a sequence of dividends $\{D_t\}_{t=0}^{\infty}$. The individual chooses consumption and holdings to maximize utility

$$V = \max_{\{C_t, N_t, B_t\}_0^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t),$$

with the following sequence of budget constraints

$$\begin{aligned} C_0 + \sum_{h=1}^{\infty} B_h Q_{h0} &= N_{-1} D_0 + Y_0 && \text{for } t = 0, \\ C_t &= N_{-1} D_t + B_h + Y_t && \text{for } t \geq 1, \end{aligned}$$

where B_h denotes the number of bonds with maturity h bought at time $t = 0$. Proposition 1 states that the welfare effect of a perturbation in the price of bonds with different maturities depends on transactions:

$$\begin{aligned} \text{Welfare Gain} &= - \sum_{h=1}^{\infty} B_h dQ_{h0} \\ &= \sum_{h=1}^{\infty} (N_{-1} D_h + Y_h - C_h) dQ_{h0} \\ &= \sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (C_h - Y_h - N_{-1} D_h) d \log R_{0 \rightarrow h}. \end{aligned}$$

This corresponds to Appendix formula (A.37) in Auclert (2019).

In the special case in which the perturbation is a level shift in the yield curve (i.e., $d \log R_{0 \rightarrow h} = h d \log R$ for $h > 1$), the formula simplifies to

$$\text{Welfare Gain} = \left(\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (C_h - Y_h - N_{-1} D_h) h \right) d \log R.$$

This formula expresses the welfare gain of a permanent rise in interest rate on welfare, as a share of total wealth, as the difference between the duration of consumption and the duration of income, where ‘‘duration’’ is defined as the value-weighted time to maturity of a sequence of cash flows (see Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021).⁶⁴

⁶⁴More specifically, the duration of consumption is $\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} \frac{C_h}{\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} C_h} h$ while the duration of income is $\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} \frac{Y_h + N_{-1} D_h}{\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (Y_h + N_{-1} D_h)} h$.

A.2.2 General equilibrium

Proof of Proposition 3. The proof follows the proof of Proposition 1, except that we now allow the deviation θ to impact income. The Lagrangian is the same as in the baseline model (30). Assuming that the value function is differentiable, we can write the infinitesimal change in the value function in terms of the infinitesimal change in the Lagrangian:

$$\begin{aligned} dV &= \sum_{t=0}^{\infty} \left(\sum_{k=1}^K \frac{\partial \mathcal{L}}{\partial P_{k,t}} dP_{k,t} + \frac{\partial \mathcal{L}}{\partial Q_t} dQ_t + \sum_{k=1}^K \frac{\partial \mathcal{L}}{\partial D_{k,t}} dD_{k,t} + \frac{\partial \mathcal{L}}{\partial Y_t} dY_t \right) \\ &= \sum_{t=0}^{\infty} \lambda_t \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t + \sum_{k=1}^K N_{k,t-1} dD_{k,t} + dY_t \right) \\ &= U'(C_0) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{k,t-1} - N_{k,t}) dP_{k,t} - B_t dQ_t + \sum_{k=1}^K N_{k,t-1} dD_{k,t} + dY_t \right), \end{aligned}$$

where the third equality is obtained by combining the first-order conditions for B_t and C_t . \square

We now present an overlapping generation model in the spirit of Samuelson (1958b) with a single long-lived asset (i.e., land). The goal is to clarify the meaning of our welfare gain formula in a general equilibrium. We use the model to simulate a rise in asset prices due to either demand or supply shocks.

Environment. Consider an economy where at each year $t \geq -1$, a new cohort of measure one is born. Households in cohort t have a subjective discount factor $\beta_t \in (0, 1)$ and are endowed with a share $1 - \alpha$ of aggregate income Y_t when young and zero when old. The cohort born at $t = -1$ is endowed with a long-lived asset that pays a stream of positive dividends $\{D_t\}_{t \geq 0} = \{\alpha Y_t\}_{t \geq 0}$. The parameter $\alpha \in (0, 1)$ has the interpretation of the capital share. Denote the (ex-dividend) price of the asset at time t as P_t and the one-period holding return on the asset by $R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}$. Denote N_t the share of the asset owned by cohort t at the end of period t .

Household problem. The problem of the young in period $t \geq 0$ is

$$\begin{aligned} V_t &= \max_{C_t, C'_t, N_t} (1 - \beta_t) \log(C_t) + \beta_t \log(C'_t), \\ \text{s.t. } & C_t + N_t P_t = (1 - \alpha) Y_t \\ & C'_t = N_t (\alpha Y_{t+1} + P_{t+1}) \end{aligned}$$

where C_t and C'_t denote respectively consumption when young and old. The Lagrangian is

$$\mathcal{L}_t = (1 - \beta_t) \log(C_t) + \beta_t \log(C'_t) + \lambda_t ((1 - \alpha) Y_t - C_t - N_t P_t) + \lambda'_t (N_t (\alpha Y_{t+1} + P_{t+1}) - C'_t).$$

The optimal consumption level when young is a fixed fraction of labor income

$$C_t = (1 - \beta_t)(1 - \alpha) Y_t.$$

For the initial old, the solution is simply given by $C_{-1} = (1 - \alpha) Y_{-1}$ and $C'_{-1} = N_{-1} (\alpha Y_0 + P_0)$ and we define.

Equilibrium. At every period $t \geq 0$, the asset market clearing condition is

$$N_t = N_{t-1} = 1,$$

which says that the purchases of the young N_t must equal the sales of the old N_{t-1} , which is normalized to one. Using the optimal consumption of the young combined with their budget constraint and the market clearing condition, we have that the equilibrium price is

$$P_t = (1 - \alpha)\beta_t Y_t.$$

In equilibrium, we therefore have that changes in prices can be decomposed into the contribution of demand and supply shocks

$$\frac{dP_t}{P_t} = \underbrace{\frac{d\beta_t}{\beta_t}}_{\text{Demand shock}} + \underbrace{\frac{dY_t}{Y_t}}_{\text{Supply shock}}.$$

Comparative statics. We now do a comparative static on β_0 and Y_0 and decompose the resulting welfare gains using Proposition 3. We call the generations born at $t = -1$ and $t = 0$ are respectively “the old” and “the young”. To map Proposition 3 to this model, we use the fact that, in equilibrium: labor income of the young is $(1 - \alpha)Y_0$, dividend income of the old is αY_0 , asset sales of the young equal -1 , and asset sales of the old equal $+1$. Using these facts, we obtain the following expressions for the welfare gain of the old and young associated with small shocks $d\beta_0$ and dY_0 .

$$\begin{aligned} \text{Welfare Gain}_{\text{old}} &= dP_0 + \alpha dY_0, \\ \text{Welfare Gain}_{\text{young}} &= -dP_0 + (1 - \alpha) dY_0, \\ \text{Welfare Gain}_{\text{aggregate}} &= 0 + dY_0. \end{aligned}$$

As in Proposition 3, welfare gains are the sum of two terms. The first term is the contribution of changes in asset prices, which has opposite sign for the young (who buy the asset) and the old (who sell the asset). The second term is the contribution of changes in income, which affect the labor income of the young and the dividend income of the old.

Table A1: Welfare gain decomposition of transitory shocks in GE

Shock	Through prices	Through income	Total
Demand shock ($d\beta > 0, dY_0 = 0$)			
Old	+	0	+
Young	-	0	-
Aggregate	0	0	0
Supply shock ($d\beta = 0, dY_0 > 0$)			
Old	+	+	+
Young	-	+	+
Aggregate	0	+	+

Notes. “Total” is the sum of welfare effect within group; “Aggregate” is the sum of a welfare effect across groups; “+” means positive, “-” means negative.

Table A1 summarizes the results of the comparative static experiment. For the demand shock experiment, we consider $d\beta_0 > 0$ and $dY_0 = 0$. For the demand shock experiment, we consider $d\beta_0 = 0$ and $dY_0 > 0$. In both experiments, the welfare gain due to asset prices is the same: positive for the old, negative for the young, and zero in aggregate. In the case of demand shocks, there is no change in income. In contrast, in the case of supply shocks, the direct welfare effect of rising income is positive for both the old and the young. Notice that two different combinations of demand and supply shocks

($dY, d\beta$) that generate the same equilibrium increase in prices dP will have the same negative welfare effect through asset prices, but an ambiguous total welfare gain (i.e., either positive or negative).

A.2.3 Ex-ante versus ex-post welfare

In the case where asset price deviations are stochastic, we now show that our expression for welfare gains can be interpreted as the “ex-post” welfare gains around a deterministic economy.

Stochastic processes. For simplicity, we consider a two-asset version of the baseline model. We assume that prices are given by $P_t(\theta) = \bar{P}_t + \theta u_t$ and $Q_t(\theta) = \bar{Q}_t + \theta v_t$. The sequence $\{\bar{P}_t, \bar{Q}_t\}_{t=0}^{\infty}$ is deterministic while the sequence $\{u_t, v_t\}_{t=0}^{\infty}$ is stochastic. As in the main text, denote $R_{0 \rightarrow t}(\theta) = 1/(Q_1(\theta) \dots Q_t(\theta))$. For a given change $d\theta$, the resulting (stochastic) deviation in asset prices is $dP_t = u_t d\theta$ and $dQ_t = v_t d\theta$.

Individual problem. For a given parameter value θ , the individual problem is

$$V = \max_{\{C_t, N_t, B_t\}_0^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right], \quad (48)$$

subject to budget constraints at each period $t \geq 0$

$$C_t + (N_t - N_{t-1})(P_t + B_t Q_t + \chi(N_t - N_{t-1})) = Y_t + N_{t-1} D_t + B_{t-1}. \quad (49)$$

Ex-post welfare Define ex-post welfare (a random variable) as

$$W(\theta) = \sum_{t=0}^{\infty} \beta^t U(C_t(\theta)).$$

We define ex-post welfare gains as the effect of a small change $d\theta$ around zero on ex-post welfare:

$$\text{Ex-Post Welfare Gain} \equiv \frac{W'(0) d\theta}{U'(C_0(0))}. \quad (50)$$

Proposition 7. *In the stochastic environment (48) and (49), the ex-post welfare gain is*

$$\text{Ex-Post Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}(0) - N_t(0)) dP_t - B_t(0) dQ_t \right),$$

Proof of Proposition 7. Differentiating the expression for ex post welfare with respect to θ and evaluating at $\theta = 0$:

$$\begin{aligned} W'(0) &= \sum_{t=0}^{\infty} \beta^t U'(C_t(0)) C'_t(0) \\ &= U'(C_0(0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) C'_t(0), \end{aligned}$$

using the fact that the Euler equation is satisfied at $\theta = 0$. Differentiating the budget constraint gives

$$\begin{aligned} W'(0) &= U'(C_0(0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}(0) - N_t(0)) P_t'(0) - B_t(0) Q_t'(0) \right) \\ &\quad + U'(C_0(0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) (B_t'(0) Q_t(0) - B_{t-1}'(0)) \\ &\quad + U'(C_0(0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}'(0) - N_t'(0)) P_t(0) + \chi'(N_t - N_{t-1})(N_t'(0) - N_{t-1}'(0)) - N_{t-1}'(0) D_t(0) \right) \end{aligned}$$

As usual, the second and third equations are zero due to fact holdings where optimal at time $t = 0$ and so this simplifies to

$$W'(0) = U'(C_0(0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}(0) - N_t(0)) u_t - B_t(0) v_t \right).$$

Multiplying by $d\theta$ and dividing by $U'(C_0(0))$ gives the result. \square

This proposition implies the following first-order approximation for the effect of stochastic asset prices on realized welfare:

$$\frac{W(\theta) - W(0)}{U'(C_0(0))} \approx \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}(0) - N_t(0)) (P_t(\theta) - P_t(0)) - B_t(0) (Q_t(\theta) - Q_t(0)) \right),$$

which is valid at the first-order in θ . Another approximation can be obtained with *realized* asset purchases along the stochastic path:

$$\frac{W(\theta) - W(0)}{U'(C_0(0))} \approx \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(0) \left((N_{t-1}(\theta) - N_t(\theta)) (P_t(\theta) - P_t(0)) - B_t(\theta) (Q_t(\theta) - Q_t(0)) \right),$$

since $(N_{t-1}(\theta) - N_t(\theta))\theta \approx (N_{t-1}(0) - N_t(0))\theta$ and $B_t(\theta)\theta \approx B_t(0)\theta$ at the first order in θ . This is the one that we will bring to the data as we observe ex-post transactions rather than ex-ante ones. See Section 6.3 for a discussion on welfare gains at the second-order.

A.3 Model extensions

A.3.1 Borrowing constraints and collateral effects

We now examine the welfare effect of price deviations in the presence of borrowing and collateral constraints. We implement these by means of individual-specific interest rate schedules that potentially depend on asset prices. For simplicity, we consider a two-asset version of the baseline model:

$$V_i = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \quad (51)$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_{i,t} + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + B_{i,t-1} + N_{i,t-1}D_t, \quad (52)$$

and an individual-specific interest rate schedule

$$Q_{i,t} = Q_t F(B_{i,t}, N_{i,t}, P_t). \quad (53)$$

The function F captures the interest rate schedule that an individual faces (i.e., recall that the gross interest rate is the inverse of the bond price $Q_{i,t}$). We allow $Q_{i,t}$ to depend on how much the individual borrows B_t , the value of its asset holdings $N_{i,t}P_t$ and the bond price Q_t which now has the interpretation as a reference interest rate (for instance a “prime rate”). We assume that F is differentiable. Geanakoplos (2016) termed this type of interest rate schedule a “credit surface.”

We interpret the tightness of the borrowing constraint as the local curvature of the interest rate schedule, which is standard in the literature on financial constraints (see Section 3 of Farre-Mensa and Ljungqvist, 2016 for a detailed discussion). The local curvature faced by individual i at time t is $-\partial \log Q_{i,t} / \partial B_{i,t}$. It quantifies the sensitivity of individual-level interest rates to debt. Similarly, the dependence of the interest rate schedule on the value of asset holdings $N_{i,t}P_t$ allows us to capture the idea that asset price changes may loosen or tighten borrowing constraints. In particular when $\partial Q_{i,t} / \partial (N_{i,t}P_t) > 0$ a higher asset price increases the individual’s asset value and allows the individual to issue bonds at a higher price $Q_{i,t}$, i.e., to borrow at a lower interest rate $Q_{i,t}^{-1}$. This captures the key idea in collateral constraint models that increasing asset prices loosen these constraints.⁶⁵

Hard borrowing constraints as a limiting case. One extreme case of financial constraints is a (locally) vertical interest rate schedule, which implies that the individual is unable to borrow more (or alternatively faces an interest rate of infinity). We now show that borrowing limits of the form

$$-B_{i,t} \leq G(N_{i,t}P_t) \quad (55)$$

can be obtained as a limiting case of our setup.⁶⁶ Note that this includes both ad-hoc borrowing limits of the form $-B_{i,t} \leq \phi$, where $\phi > 0$ is a borrowing limit, as well as collateral constraints of the form $-B_{i,t} \leq \kappa N_{i,t}P_t$ where $\kappa \in [0, 1]$ is a maximum leverage ratio.

We can view this type of constraint as the limit of an interest rate schedules of the form (53) with

$$F(B, NP) = 1 - e^{-\theta(B+G(NP))}, \quad (56)$$

as $\theta \rightarrow \infty$. Indeed, this specification entails that $Q_{i,t}$ is given by:

$$\begin{aligned} Q_{i,t} &= Q_t \left(1 - e^{-\theta(B_{i,t}+G(N_{i,t}P_t))} \right) \\ &\rightarrow Q_t 1_{-B_{i,t} \leq G(N_{i,t}P_t)} \quad \text{as } \theta \rightarrow \infty. \end{aligned} \quad (57)$$

When $\theta \rightarrow \infty$, individuals can borrow at the reference interest rate Q_t^{-1} whenever the borrowing limit $-B_{i,t} \leq G(N_{i,t}P_t)$ is satisfied, but they face an interest rate of infinity when the borrowing limit binds (i.e., they can not borrow more). The key takeaway is that our framework can approximate general borrowing and collateral constraints using a differentiable interest rate schedule F .

Welfare gains formula with collateral effects. We now obtain an expression for the welfare gains in the presence of borrowing constraints and collateral effects.

⁶⁵All our results would also go through with a more general individual-specific interest rate schedule

$$Q_{i,t} = F(Q_t, N_{i,t}, B_{i,t}, P_t) \quad (54)$$

that depends separately on the individual holdings (N_t, B_t) and market prices (P_t, Q_t) . However, we focus on (53) as it captures the economics of borrowing and collateral constraints in a more parsimonious fashion.

⁶⁶As in Footnote 65, we could also have a more general limiting borrowing constraint of the form $-B_{i,t} \leq G(N_{i,t}, P_t)$.

Proposition 8. *In the presence of borrowing constraints and collateral effects captured by the interest rate schedule (53), the welfare gain is*

$$\begin{aligned} \text{Welfare Gain}_i = & \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \frac{dQ_t}{Q_t} \right) \\ & + \underbrace{\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(-B_{i,t} \frac{\partial Q_{i,t}}{\partial(N_{i,t}P_t)} N_{i,t} dP_t \right)}_{\text{collateral effect}}. \end{aligned} \quad (58)$$

In the case of a hard borrowing constraint, (56) with $\theta \rightarrow \infty$, the welfare gain formula becomes

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) + \underbrace{\sum_{t=0}^{\infty} \frac{\beta^t \mu_{i,t}}{U'(C_{i,0})} G'(N_{i,t}P_{i,t}) N_{i,t} dP_t}_{\text{collateral effect}}, \quad (59)$$

where $\mu_{i,t}$ satisfies $U'(C_{i,t})Q_t = \beta U'(C_{i,t+1}) + \mu_{i,t}$ and $\mu_{i,t}$ has the interpretation of the Lagrange multiplier on the implied borrowing constraint (55).

Formula (58) differs from the welfare gain formula in the baseline model (14) in two respects. First, marginal rates of substitutions $\beta^t U'(C_{i,t})/U'(C_{i,0})$ are no longer equalized across individuals due to the fact that they face different interest rates, so the discount rate in the welfare gain formula is individual-specific. Second, the contribution of debt holdings $B_{i,t}$ on welfare now contains the effect of deviations in the reference interest rate dQ_t as well as the price of the long-lived asset dP_t on the individual-specific interest rate $Q_{i,t}$. When $\partial Q_{i,t}/\partial(N_{i,t}P_t) > 0$ indebted individuals (those with $B_{i,t} < 0$) experience an additional welfare gain $-B_{i,t} \partial Q_{i,t}/\partial(N_{i,t}P_t) N_{i,t} dP_t > 0$ from rising asset prices because they can borrow at a lower interest rate.

In the limiting case of a hard borrowing constraint, the interpretation of the formula (59) is similar. In particular it still differs from the baseline formula (14) in two respects. What differs is the second term that captures the effect of the asset price on the tightness of the borrowing constraint which is positive whenever the Lagrange multiplier $\mu_{i,t}$ is positive, i.e. whenever the constraint binds. Intuitively, whereas in (58) rising asset prices are welfare-improving because they allow the individual to borrow at a lower interest rate (the term involving $\partial Q_{i,t}/\partial(N_{i,t}P_t)$), in (59) they are welfare-improving because they allow a borrowing-constrained individual to borrow more (the term involving $\mu_{i,t} G'(N_{i,t}P_t)$). In both cases, rising asset prices are welfare-improving because they allow the individual to better smooth her consumption over time.

Proof of Proposition 8. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i = & \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \\ & + \sum_{t=0}^{\infty} \lambda_{i,t} (Y_{i,t} + N_{i,t-1} D_{i,t} + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1}) P_t - B_{i,t} Q_t F(B_{i,t}, N_{i,t} P_t) - \chi(N_{i,t} - N_{i,t-1})). \end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned}
dV_i &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(Q_{i,t} \frac{dQ_t}{Q_t} + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right), \\
&= U'(C_{i,0}) \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(Q_{i,t} \frac{dQ_t}{Q_t} + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right),
\end{aligned}$$

where the last line uses the first-order condition with respect to $C_{i,t}$, $\beta^t U'(C_{i,t}) = \lambda_{i,t}$, for $t \geq 0$.

Using the definition of the welfare gain in (12) yields

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(Q_{i,t} \frac{dQ_t}{Q_t} + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right).$$

Rearranging gives the first formula in the proposition.

We now prove the second formula in the Proposition. For the case of the interest rate schedule (56) satisfying (57), as $\theta \rightarrow \infty$, the derivative $\partial F(B_{i,t}, N_{i,t} P_t) / \partial (N_{i,t} P_t)$ becomes unbounded (the interest rate schedule becomes vertical) and hence it is uninformative to directly consider this limit in the formula for the welfare gain. However, it is still possible to derive an alternative characterization. To this end, consider the first-order condition for $B_{i,t}$, $\partial \mathcal{L}_i / \partial B_{i,t} = 0$:

$$U'(C_{i,t}) \left(Q_{i,t} + B_{i,t} Q_t \frac{\partial F(B_{i,t}, N_{i,t} P_t)}{\partial B_{i,t}} \right) = U'(C_{i,t+1}),$$

which, in turn, can be rewritten as

$$U'(C_{i,t}) Q_{i,t} = \beta U'(C_{i,t+1}) + \mu_{i,t} \tag{60}$$

where

$$\mu_{i,t} \equiv -U'(C_{i,t}) B_{i,t} Q_t \frac{\partial F(B_{i,t}, N_{i,t} P_t)}{\partial B_{i,t}} \tag{61}$$

is the wedge in the Euler equation due to the upward-sloping interest rate schedule (56). Note that this wedge is akin to a Lagrange multiplier. In fact, this statement is exact in the limit as $\theta \rightarrow \infty$ in which the interest rate schedule enforces a hard constraint: in this case, the Euler equation is $U'(C_{i,t}) Q_{i,t} = \beta U'(C_{i,t+1}) + \mu_{i,t}$ where $\mu_{i,t}$ is precisely the Lagrange multiplier on (55).

Given this, the proof proceeds by rewriting the collateral effects term in (58) in terms of $\mu_{i,t}$ defined in (61) and then taking the limit as $\theta \rightarrow \infty$. From (58) the welfare gain due to collateral effects is

$$\begin{aligned}
\text{Welfare gain}_{i,\text{collateral}} &= \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \\
&= \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(-B_{i,t} Q_t \frac{\partial F(B_{i,t}, N_{i,t} P_t)}{\partial N_{i,t} P_t} N_{i,t} dP_t \right) \\
&= \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(-B_{i,t} Q_t \frac{\partial F(B_{i,t}, N_{i,t} P_t)}{\partial B_{i,t}} G'(N_{i,t} P_{i,t}) N_{i,t} dP_t \right) \\
&= \sum_{t=0}^{\infty} \frac{\beta^t \mu_{i,t}}{U'(C_{i,0})} G'(N_{i,t} P_{i,t}) N_{i,t} dP_t.
\end{aligned}$$

where the second equality uses (53), the third equality uses that the specific interest rate schedule (56)

satisfies $\partial F(B, NP)/\partial(NP) = \partial F(B, NP)/\partial B \times G'(NP)$, and the fourth equality uses (61). Plugging this rewritten collateral effects term back into (58) we have

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \frac{dQ_t}{Q_t} \right) + \sum_{t=0}^{\infty} \frac{\beta^t \mu_{i,t}}{U'(C_{i,0})} G'(N_{i,t} P_{i,t}) N_{i,t} dP_t.$$

Finally, we take $\theta \rightarrow \infty$ so that the interest rate schedule converges to (57). In this case, in the relevant range $-B_{i,t} \leq G(N_{i,t} P_t)$, we have $Q_{i,t} \rightarrow Q_t$ and, therefore, we obtain (59). \square

Relation between welfare gains and borrowing responses to asset price changes. When asset valuations increase, individuals often respond by consuming more. Rather than selling the asset whose valuation has increased, individuals may finance their increased consumption by increasing their borrowing (at the expense of lower future consumption). We now briefly discuss under what circumstances this effect shows up in our welfare gains formula with collateral effects.

In short, it depends on whether the individual is collateral constrained or not:⁶⁷ when the individual is unconstrained so that the original consumption plan was optimal, the effect is still there but has a second-order impact on welfare and is therefore not captured by our formula; in contrast, when the individual is constrained, the effect has a first-order impact on welfare and therefore shows up in our formula. One interesting possibility that is likely relevant in practice is that real-world features like asset illiquidity or the preferential tax treatment of borrowing relative to asset sales may push individuals into collateral constraints, in which case our formula captures the effect of borrowing responding to asset-price changes.

The key observation for explaining this result is that borrowing is about consumption smoothing over time: by borrowing an individual can increase her consumption today at the expense of lower consumption in the future (at the time the loan is repaid). When borrowing constraints bind so that the individual's initial consumption profile was suboptimal, increased borrowing against higher asset valuations has a first-order impact on welfare. In contrast, when borrowing constraints do not bind so that the initial consumption profile was optimal to begin with, the individual may still change her borrowing in response to changing asset prices (an intertemporal substitution effect) but this will have a second-order welfare impact and is therefore not picked up by our formula based on the envelope theorem (see the discussion in Appendix A.1).

To see this formally, let us consider the case of an individual facing a hard borrowing constraint; that is, an individual maximizing (51) subject to (52) and (55).⁶⁸ For the same model but *without* a binding borrowing constraint, we showed in Appendix A.1 that one interpretation of our welfare gains formula is as the present value of consumption changes holding constant asset holdings – see equation (32). We now repeat the same construction in the model with a borrowing constraint. Differentiating the budget constraint (52) we have that the consumption response to asset price changes holding constant asset holdings is

$$dC_{i,t}^{\text{fixed holdings}} = (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t,$$

⁶⁷In the case of the interest rate schedule (53) whether the individual has an asset position $(B_{i,t}, N_{i,t})$ such that $\partial F(B_{i,t}, N_{i,t} P_t)/\partial B_{i,t} > 0$.

⁶⁸We here consider the case with hard borrowing constraints because the argument is somewhat simpler. But an analogous argument applies to the case of the interest rate schedule (53).

Hence the corresponding welfare gains are

$$\begin{aligned} \text{Welfare gains}_i^{\text{fixed holdings}} &= \frac{dV_i^{\text{fixed holdings}}}{U'(C_{i,0})} = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} dC_{i,t}^{\text{fixed holdings}} \\ &= \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) \end{aligned}$$

Importantly, this formula now differs from the expression for welfare gains obtained in the “hard borrowing constraint” case (59): there is no collateral effects term. Hence, this derivation shows that what we call “collateral effect” in (59) can be interpreted as the first-order welfare gains due to the change in borrowing from loosening borrowing constraints. The key observation is: when the borrowing constraint is slack the individual may reoptimize her borrowing and hence the timing of her consumption plan in response to asset price changes but this has a second-order welfare effect precisely because the individual’s original consumption plan was optimal to begin with; the flip side is that, when the individual is constrained, such re-optimization instead has a first-order welfare effect.

As already noted, one interesting possibility is that real-world features like asset illiquidity may push individuals into collateral constraints. To this end, consider the case where adjustment costs $\chi(N_{i,t} - N_{i,t-1})$ are large, i.e., selling the asset $N_{i,t}$ is very costly and hence the asset is illiquid. In this case, individuals may be rich but nevertheless borrowing constrained – the “wealthy hand-to-mouth of Kaplan and Violante (2014). When the asset price P_t increases, such individuals may want to increase their spending. If the asset-price increase relaxes the borrowing constraint, they will do so by borrowing more. A similar logic applies when the tax system favors borrowing over selling assets, e.g. because selling the asset may require paying a capital gains tax. One interesting special case is individuals borrowing against their assets to consume as part of a “buy, borrow, die” tax avoidance strategy.⁶⁹ Just like in the case of asset illiquidity, individuals may want to borrow as much as possible and may therefore run into collateral constraints precisely *because* of the differential tax treatment of borrowing and asset sales. Asset-price changes will then have first-order welfare effects even for individuals who do not sell any of their assets.⁷⁰ Finally, yet another similar case would be a world in which rich individuals get direct utility from ownership of the asset (as in Appendix A.3.5): also in this case, individuals may not want to sell their assets precisely *because* of the utility benefit from ownership and may instead be pushed into collateral constraints. In all these cases, our formula would pick up the first-order effect from asset price increases via relaxing these collateral constraints.

A.3.2 Idiosyncratic and aggregate risk

We now examine the welfare effect of price deviations in the presence of uninsurable idiosyncratic labor income risk (the case of aggregate risk on dividend income, prices, or even price deviations can be treated similarly and leads to the same expression for welfare gains). To be precise, suppose that the individual-specific labor income sequence $\{Y_{i,t}\}_{t=0}^{\infty}$ follows a stochastic process. For simplicity, we consider a two-asset version of the baseline model. We study an ex-ante concept of welfare, where the

⁶⁹One main reason individuals use a “buy, borrow, die” strategy is step-up in basis at death. This feature of the U.S. tax system (and some other countries) means that dying without ever having sold an asset and passing it on to an heir greatly reduces the heir’s capital gains tax bill if he or she sells the inherited asset.

⁷⁰Rising asset prices will also have an effect on the *relative* welfare of asset sellers who use the strategy relative to those who do not. When asset prices rise, asset sellers who do not use the “buy, borrow, die” strategy pay higher capital gains taxes which attenuates their welfare gain (as in Appendix A.3.7). In contrast, this attenuation effect is smaller (or non-existent) for individuals who use the strategy because they pay less (or no) capital gains taxes in the first place.

expectation is taken over the realization of labor income risk:

$$V_i = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right],$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + N_{i,t-1}D_t.$$

Proposition 9. *In the presence of labor income risk, the individual welfare gain of individual i is*

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) \right].$$

Proof. It is understood that $C_{i,t}$ and $N_{i,t}$ are functions of the full history of shocks up to t , which we denote s_i^t . We now make that dependence explicit. Moreover, we denote the probability that history s^t occurs by $\pi(s_i^t)$. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^{\infty} \beta^t \pi(s_i^t) U(C_{i,t}(s_i^t)) \\ &+ \sum_{t=0}^{\infty} \pi(s_i^t) \lambda_{i,t}(s_i^t) \left(Y_{i,t}(s_i^t) + N_{i,t-1}(s_i^t) D_t - C_{i,t}(s_i^t) - (N_{i,t}(s_i^t) - N_{i,t-1}(s_i^t)) P_t - \chi(N_{i,t} - N_{i,t-1}) \right). \end{aligned}$$

The first order condition for B is $\beta^t U'(C_{i,t}(s_i^t)) = \lambda(s_i^t)$, which implies the usual Euler equation

$$\mathbb{E}_0 \left[\beta^t \frac{U'(C_{i,t})}{U'(C_{i,0})} R_{0 \rightarrow t} \right] = 1. \quad (62)$$

Using the definition of welfare gain, we obtain

$$\begin{aligned} \text{Welfare Gain}_i &= \frac{1}{U'(C_{i,0})} dV_i \\ &= \frac{1}{U'(C_{i,0})} \sum_{t=0}^{\infty} \pi(s_i^t) \lambda_{i,t}(s_i^t) \left((N_{i,t-1}(s_i^t) - N_{i,t}(s_i^t)) dP_t - B_{i,t}(s_i^t) dQ_t \right) \\ &= \sum_{t=0}^{\infty} \pi(s_i^t) \frac{\beta^t U'(C_{i,t}(s_i^t))}{U'(C_{i,0})} \left((N_{i,t-1}(s_i^t) - N_{i,t}(s_i^t)) dP_t - B_{i,t}(s_i^t) dQ_t \right) \\ &= \sum_{t=0}^{\infty} \mathbb{E}_0 \left[\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) \right]. \end{aligned}$$

The second equality uses the envelope theorem, the third equality uses the first-order condition for B and the third uses the definition of expectation. \square

Welfare gain under the risk-neutral measure. We now use a change of measure to obtain a simpler expression for the welfare gain. Let the individual-specific risk-neutral measure be define as

$$\mathbb{E}_0^{Q_i} [X_t] \equiv \mathbb{E}_0 \left[\left(R_{0 \rightarrow t} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \right) X_t \right],$$

where X_t is a random variable (i.e., it is a function of the history s^t). It is immediate that

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \mathbb{E}_0^{Q_i} [(N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t].$$

While this formula says that welfare gains depends on future net asset sales, there are two differences with the baseline welfare gain formula. The first is that, in a stochastic world, what matters for the (ex-ante) welfare gain is the expected path of net asset sales. The second is that individuals care about certain states of the world more than others: the expectation is under the risk-neutral measure, which tilts objective measure by the growth of their marginal utility.

The expected net asset sales in the risk-neutral probability can be written as the sum of two terms:

$$\begin{aligned} \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \mathbb{E}_0^{Q_i} [(N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t]}_{\text{Welfare Gain (Expectation in risk-neutral measure)}} &= \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \mathbb{E}_0 [(N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t]}_{\text{Expectation in objective measure}} \\ &+ \underbrace{\sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right)}_{\text{Covariance term}}. \end{aligned} \quad (63)$$

A.3.3 Finite lives and bequests

We now examine the welfare effect of asset price deviations when individuals have finite lives. Mostly for pedagogical reasons, we first consider a case in which individuals have no offspring and hence die with exactly zero assets. We then move on to more realistic cases in which individuals leave bequests to their children as well as making inter-vivos transfers.

Finite lives. We consider an individual who is born at time 0 and dies at time T with no offspring and hence no reason to leave bequests. For simplicity, we restrict ourselves to a two-asset version of the baseline model. More precisely, the individual solves the following optimization problem:

$$V = \max_{\{C_t, N_t, B_t\}} \sum_{t=0}^T \beta^t U(C_t),$$

subject to budget constraints at each period $t = 0, 1, \dots, T$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + N_{t-1}D_t + B_{t-1},$$

and with terminal holdings $N_T \geq 0$ and $B_T \geq 0$. Note that these constraints will bind at the optimum, that is the individual will optimally die with zero assets, i.e. she will sell off her assets as she approaches the end of life.⁷¹ These sales then show up in our welfare gains formula as the next proposition shows:

⁷¹Alternatively, we could impose the (weaker) terminal condition that terminal wealth is non-negative; that is, $N_T P_T + B_T Q_T \geq 0$. In this alternative formulation, the individual is allowed to die with debt $B_T Q_T < 0$ in which case an asset sale $N_T P_T$ would happen right after the individual's death so as to pay off this debt (e.g. conducted by the bank or executor of the individual's will). Differentiating the corresponding Lagrangian

$$\mathcal{L} = \sum_{t=0}^T \beta^t U(C_t) + \sum_{t=0}^T \lambda_t (Y_t + N_{t-1}D_t + B_{t-1} - C_t - (N_t - N_{t-1})P_t - B_t Q_t - \chi(N_t - N_{t-1})) + \mu(N_T P_T + B_T Q_T)$$

Proposition 10. For an individual with a finite life of length T and no offspring, the welfare gain is exactly the same as formula (14) for an infinitely-lived individual (except the T in the summation):

$$\text{Welfare Gain} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t). \quad (64)$$

Intuitively, the individual optimally sells off all of her assets before she dies. When asset valuations rise, this generates a welfare gain. While finite lives result in a different time path for optimal asset transactions, the way these asset transactions show up in our welfare gains formula is exactly the same as in the case with infinitely-lived individuals.

Proof of Proposition 10. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^T \beta^t U(C_t) \\ &+ \sum_{t=0}^T \lambda_t (Y_t + N_{t-1}D_t + B_{t-1} - C_t - (N_t - N_{t-1})P_t - B_tQ_t - \chi(N_t - N_{t-1})) \\ &+ \mu_N N_T + \mu_B B_T, \end{aligned}$$

where μ_N and μ_B are the Lagrange multipliers on the terminal conditions $N_T \geq 0$ and $B_T \geq 0$. Totally differentiating the welfare function using the envelope theorem and following the same steps as in the proof of Proposition 1

$$\begin{aligned} dV &= \sum_{t=0}^T \frac{\partial \mathcal{L}}{\partial P_t} dP_t + \sum_{t=0}^T \frac{\partial \mathcal{L}}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^T \lambda_t (-(N_t - N_{t-1}) dP_t - B_t dQ_t) \\ &= U'(C_0) \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t) \end{aligned}$$

where the second equality uses that the terms $\mu_N N_T$ and $\mu_B B_T$ in the Lagrangian do not depend on asset prices $\{P_t, Q_t\}_{t=0}^T$. \square

To summarize, the formula for an individual with a finite life and no offspring is exactly the same as as the formula for an infinitely-lived individual. We now turn to the more realistic case where individuals leave bequest to their offspring.

and using the first-order conditions for N_T and B_T , $\lambda_T P_T = \mu P_T$ and $\lambda_T Q_T = \mu Q_T$, would lead to the formula

$$\text{Welfare Gain} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t) + R_{0 \rightarrow T}^{-1} (N_T dP_T + B_T dQ_T).$$

This formula is the same as (64) in Proposition 10, with the only difference that it also takes into account the additional asset sales at time T right after the person's death. Put another way, the formula still says that welfare gains equal the PDV of asset sales times price deviations. Intuitively, it is easy to see that it makes no conceptual difference whether the person makes these sales before she dies (which is the formulation in the main text or someone else makes these sales right after she dies (which is the formulation in this footnote).

Bequests. We now consider an individual with finite life but with a “warm-glow” bequest motive. The individual solves the following problem:

$$V = \max_{\{C_t, N_t, B_t\}} \sum_{t=0}^T \beta^t U(C_t) + \beta^T F(N_T, B_T, \{P_t\}_{t=T}^\infty, \{Q_t\}_{t=T}^\infty),$$

subject to budget constraints at each period $t = 0, 1, \dots, T$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1})1_{0 < t < T} = Y_t + N_{t-1}D_t + B_{t-1}$$

and initial holdings (themselves inherited from the individual’s parents) N_{-1} and B_{-1} . The bequest function $F(\cdot)$ governs the “warm glow” utility that individuals receive from leaving assets to their children. Note that it is allowed to depend on all prices, and therefore nests the altruistic model, where $F(\cdot)$ would correspond to the value function of the children. Finally, to simplify, we assume that there are no adjustment costs at $t = 0$ and at $t = T$.

To compute our welfare gain formula (i.e., Equation 14 in the baseline model), we want to exclude the “warm glow” utility associated with bequest and focus only on the utility change due to consumption: otherwise, we would be double-counting the welfare effect of a bequest event (i.e., positive welfare effect for both the parents and children). Hence, the measure of welfare gain we consider is $dV^*/U'(C_0)$ where $V^* \equiv \sum_{t=0}^T \beta^t U(C_t)$.

Proposition 11. *With finite lives and bequest, the welfare gain is*

$$\begin{aligned} \text{Welfare Gain} &= \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t) \\ &\quad + (P_0 + D_0) dN_{-1} + Q_0 dB_0 - R_{0 \rightarrow T}^{-1} (P_T dN_T + Q_T dB_T). \end{aligned}$$

Proof. The FOCs for the optimization problem are:

$$\begin{aligned} \beta^t U'(C_t) &= \lambda_t && (\partial \mathcal{L} / \partial C_t = 0) \\ \lambda_t Q_t &= \lambda_{t+1} && (\partial \mathcal{L} / \partial B_t = 0 \text{ for } t < T) \\ \lambda_t (P_t + \chi'(N_t - N_{t-1})1_{0 < t < T}) &= \lambda_{t+1} (D_{t+1} + P_{t+1} + \chi'(N_{t+1} - N_t)1_{0 < t < T}) && (\partial \mathcal{L} / \partial N_t = 0 \text{ for } t < T) \end{aligned}$$

Because we are using a special notion of welfare V^* , which only includes the utility of consumption rather than the warm glow utility, we cannot use the envelope theorem. Instead, we use a more direct approach which start by differentiating V^* :

$$dV^* = \sum_{t=0}^T \beta^t U'(C_t) dC_t = \sum_{t=0}^T \lambda_t dC_t$$

We then differentiate the budget constraint every period allows us to substitute out the sequence $\{dC_t\}_{t=0}^T$:

$$\begin{aligned} dV^* &= \sum_{t=0}^T \lambda_t \left((N_{t-1} - N_t) dP_t - B_t dQ_t \right. \\ &\quad \left. + dN_{t-1} D_t + dB_{t-1} + (dN_{t-1} - dN_t) P_t - Q_t dB_t - \chi'(N_t - N_{t-1})(dN_t - dN_{t-1}) 1_{0 < t < T} \right) \\ &= \sum_{t=0}^T \lambda_t \left((N_{t-1} - N_t) dP_t - B_t dQ_t \right) + \sum_{t=0}^T \lambda_t (dB_{t-1} - Q_t dB_t) \\ &\quad + \sum_{t=0}^T \lambda_t \left((D_t + P_t + \chi'(N_t - N_{t-1})) 1_{0 < t < T} dN_{t-1} - (\chi'(N_t - N_{t-1}) 1_{0 < t < T} + P_t) dN_t \right). \end{aligned}$$

We can simplify the expression using the FOCs for asset holdings:

$$\begin{aligned} dV^* &= \sum_{t=0}^T \lambda_t \left((N_{t-1} - N_t) dP_t - B_t dQ_t \right) + \lambda_0 dB_{-1} - \lambda_T Q_T dB_T \\ &\quad + \lambda_0 (P_0 + D_0) dN_{-1} - \lambda_T P_T dN_T. \end{aligned}$$

Finally, using the FOC for consumption $\beta^t U'(C_t) = \lambda_t$ gives

$$\begin{aligned} dV^* &= U'(C_0) \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left((N_{t-1} - N_t) dP_t - B_t dQ_t \right) + U'(C_0) \left(dB_{-1} - R_{0 \rightarrow T}^{-1} Q_T dB_T \right) \\ &\quad + U'(C_0) \left((D_0 + P_0) dN_{-1} - R_{0 \rightarrow T}^{-1} P_T dN_T \right). \end{aligned}$$

Dividing by $U'(C_0)$ gives the expression for welfare gains. \square

The resulting welfare gain formula differs from the one in the baseline model due to the fact that inheritance/bequests may react to asset prices. More precisely, beyond the effect due to net asset sales, the present value of the change in consumption for an individual increases if they inherit more from their parents (i.e., $(D_0 + P_0) dN_{-1} + dB_{-1} > 0$) or if they decide to leave fewer bequests to their children (i.e., $P_T dN_T + dB_T < 0$).

In the data, our baseline assumption is that $dN_{-1} = dB_{-1} = dB_T = dQ_T = 0$; that is, that the *quantity* of inherited and bequeathed assets does not respond to asset prices. For instance, in the context of housing, this says that changes in asset prices do not affect the physical quantity of real estate (e.g., square meters) parents leave to their children.

This assumption means that we overestimate the welfare gain of parents who bequeathed more assets to their children in response to rising asset prices (e.g., to compensate them for their welfare loss). Conversely, this means that we underestimate the welfare gains of parents who bequeathed fewer assets to their children in response to rising asset prices (e.g., by borrowing more against the value of their houses, leaving larger outstanding balances to their children). We think of our baseline assumption as a realistic middle ground between these two situations.

In the absence of such assumption, note that one can still estimate welfare gains at the *dynasty* level by aggregating welfare gains of all individuals across generations: indeed, changes in the quantity of assets inherited/bequeathed aggregate to zero within a dynasty. Note that the resulting formula for dynastic welfare gains is equivalent to the infinite-horizon setup obtained in the main text.

Inter-vivos transfers. We now augment the previous model to allow for inter-vivos transfers. The results are broadly similar but they highlight that what matters for welfare gain is the sequence of transactions done by individuals rather their changes in asset holdings.

We consider an individual who lives for T period who may receive or give a bequest at any time. Denote $I_{N,t}^+$ and $I_{B,t}^+$ (respectively $I_{N,t}^-$ and $I_{B,t}^-$) the number of long-duration assets and bonds inherited (resp. bequeathed) in period t . The individual solves

$$V = \max_{\{C_t, N_t, B_t, I_{N,t}^+, I_{B,t}^+, I_{N,t}^-, I_{B,t}^-\}} \sum_{t=0}^T \beta^t \left(U(C_t) + F(I_{N,t}^-, I_{B,t}^-, \{P_t\}_{t=T}^\infty, \{Q_t\}_{t=T}^\infty) \right),$$

subject to budget constraints at each period $t = 0, 1, \dots, T$

$$C_t + (N_t - N_{t-1})P_t + B_tQ_t + \chi(N_t - N_{t-1}) = Y_t + N_{t-1}D_t + B_{t-1} + I_{N,t}P_t + I_{B,t}Q_t,$$

where $I_{B,t} \equiv I_{B,t}^+ - I_{B,t}^-$ is the net inheritance received in bonds at time t and $I_{N,t} \equiv I_{N,t}^+ - I_{N,t}^-$ is the net inheritance received in the long duration asset at time t . To reflect the fact that transfers now happen inter-vivos, we assume that $N_{-1} = B_{-1} = 0$ and $N_T = B_T = 0$. As above, we use as a measure of welfare $V^* \equiv \sum_{t=0}^T \beta^t U(C_t)$.

Proposition 12. *With finite lives and inter-vivos transfers, the welfare gain is*

$$\begin{aligned} \text{Welfare Gain} &= \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{t-1} - (N_t - I_{N,t})) dP_t - (B_t - I_{B,t}) dQ_t) \\ &\quad + \sum_{t=0}^T R_{0 \rightarrow t}^{-1} (P_t dI_{N,t} + Q_t dI_{B,t}). \end{aligned}$$

The proof of the proposition follows exactly the same steps as the proof of Proposition 11 and so we omit it for brevity. As in Proposition 11, the welfare gain comprises two terms. The first line of the formula corresponds to the welfare gain due to asset sales. The second line in the formula corresponds to the welfare gain due to the change in bequest/inheritance.

Note that, with inter-vivos transfers, what matters for welfare is “actual” asset sales $N_{t-1} - (N_t - I_{N,t})$: the change in asset holdings that is not accounted for by inheritance or bequest. Intuitively, older generations only benefit from a rise in house prices if they actually sell their houses (not if they bequest them), and, conversely, younger generations are only hurt by a rise in house prices if they actually buy their houses (not if they inherit them). Consistently with this idea, when feasible, we focus on actual asset transactions rather than changes in asset holdings when applying our sufficient statistic to the data.⁷²

A.3.4 Financial transactions done by businesses

In the baseline model, we examined the welfare effect of changes in the path of the price of an asset $\{P_t\}_{t=0}^\infty$ holding constant its dividends $\{D_t\}_{t=0}^\infty$. However, this assumption does not seem adapted to businesses that themselves buy and sell financial assets, as changes in asset prices will typically affect their dividend payments. This appendix explains how we adapt our methodology to take into account such financial transactions by businesses. For example, it explains the reasoning behind our empirical measure for the equity valuation ratio used in Section 3.2 which is unaffected by share repurchases and is capital-structure neutral.

The case of share repurchase. It is useful to start with an example in which a business can only make one type of financial transaction: repurchase its own shares. Formally, consider a business that

⁷²This is possible for housing, which accounts for most of asset transactions in Norway. However, it is not possible for debt, deposits, and equity due to data limitations.

produces an income stream (i.e. earnings minus investment) $\{\Pi_t\}_{t=0}^{\infty}$ from its fundamental (e.g., non-financial) operations. These cash flows are distributed to shareholders through both dividends and share repurchases:

$$\Pi_t = \mathcal{N}_{t-1}D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t)P_t \quad (65)$$

where D_t denotes the business dividends per share, P_t denotes the share price, and $\mathcal{N}_t = \sum_{i=1}^I N_{i,t}$ denotes the total amount of outstanding shares with $N_{i,t}$ denoting individual ownership shares of the firm's investors $i = 1, \dots, I$. When $\mathcal{N}_t < \mathcal{N}_{t-1}$ the business is repurchasing its own shares. From this equation it is already apparent that share repurchases and dividend payments are equivalent means of distributing cash flows $\{\Pi_t\}_{t=0}^{\infty}$ to shareholders as a whole (more on this shortly). As discussed above, the presence of share repurchases implies that changes in share prices will mechanically affect the path of dividends $\{D_t\}_{t=0}^{\infty}$, as higher share prices will force the firm to either spend more cash to buy the same amount of shares (which reduces dividends per share in the current period) or to buy fewer shares with the same amount of cash (which reduces dividends per shares in future periods).

Let us consider the budget constraint of an individual i who, for simplicity, can only invest in the business:

$$(N_{i,t} - N_{i,t-1})P_t = N_{i,t-1}D_t + Y_{i,t} - C_{i,t}. \quad (66)$$

When the business repurchases its shares (i.e., $\mathcal{N}_t < \mathcal{N}_{t-1}$) this results in an income stream $(N_{i,t-1} - N_{i,t})P_t$ for those individual selling their shares to the business. Denoting by $s_{i,t} \equiv N_{i,t}/\mathcal{N}_t$ the individual's ownership share of the business, we can combine the individual and business budget constraints, (66) and (65), to obtain:

$$(N_{i,t} - N_{i,t-1} + s_{i,t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t))P_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}.$$

Denoting by $M_t \equiv \mathcal{N}_t P_t$ the market value of the business, we obtain:

$$(s_{i,t} - s_{i,t-1})M_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}. \quad (67)$$

This budget constraint has the same form as (66), except that (i) the dividend per share D_t is replaced by the income stream from operations Π_t , (ii) the price per share P_t is replaced by the market value of the firm M_t , and (iii) the number of shares held by the individuals $N_{i,t}$ is replaced by the ownership share in the business $s_{i,t}$. An alternative viewpoint on this consolidated budget constraint is to consider the return to investing in the business. As usual, the return implied by the non-consolidated budget constraint is $R_{t+1} \equiv (D_{t+1} + P_{t+1})/P_t$, i.e., the return is the sum of dividend yield and capital gains. Multiplying and dividing by \mathcal{N}_t , we have

$$R_{t+1} \equiv \frac{\mathcal{N}_t D_{t+1} + \mathcal{N}_t P_{t+1}}{\mathcal{N}_t P_t} = \frac{\mathcal{N}_t D_{t+1} + (\mathcal{N}_t - \mathcal{N}_{t+1})P_{t+1} + \mathcal{N}_{t+1}P_{t+1}}{\mathcal{N}_t P_t} = \frac{\Pi_{t+1} + M_{t+1}}{M_t}$$

where the last equality uses (65) and the definition of the market value $M_t \equiv \mathcal{N}_t P_t$. Just like the consolidated budget constraint (67), writing the return as $R_{t+1} = (\Pi_{t+1} + M_{t+1})/M_t$ again makes clear that what ultimately matters are the business's cash flows $\{\Pi_t\}_{t=0}^{\infty}$ and its market value $\{M_t\}_{t=0}^{\infty}$ and not whether cash flows are distributed to shareholders via dividend payouts or share repurchases.

In our baseline model we examined the welfare effect of changes in the path of the price of an asset $\{P_t\}_{t=0}^{\infty}$ holding constant its dividends $\{D_t\}_{t=0}^{\infty}$. The consolidated budget constraint (67) makes clear that, in the presence of share repurchases, the correct analogous experiment is instead to consider deviations in the market value of the business, $\{M_t\}_{t=0}^{\infty}$, holding constant its income stream $\{\Pi_t\}_{t=0}^{\infty}$. In

particular, for investors as a whole, it is irrelevant whether the business increases its dividend payments or share repurchases; what matters instead is whether the firm's income stream changes $\{\Pi_t\}_{t=0}^{\infty}$.

Using a similar reasoning as in Proposition 1, we get:

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (s_{i,t-1} - s_{i,t}) dM_t, \quad (68)$$

Hence, in the presence of share repurchases, what matters for welfare is not the number of shares $N_{i,t}$ directly traded by the individual, but the overall change in his/her ownership share $s_{i,t}$ in the business. In particular, note that individual welfare gains still aggregate to zero, as ownership shares always aggregate to one in the population. Similarly, and as already noted, what matters is not deviations in the share price P_t holding constant dividends D_t but deviations in the market value $M_t = \mathcal{N}_t P_t$ holding constant the income stream Π_t .

One way to understand expression (68) is to consider the case of a business that repurchases a given fraction of its shares every period. A rise in valuations benefits individuals who sell shares to the business while hurting the owners of the business as the business needs to spend more cash to purchase the same number of shares. The two effects compensate exactly for individuals who sell a fraction of their holdings equal to the fraction of outstanding shares purchased by the business, i.e. who have $s_{i,t} = s_{i,t-1}$. On the other hand, for individuals who do not sell any of their shares to the business, $N_{i,t} = N_{i,t-1}$ so that $\mathcal{N}_{t-1} - \mathcal{N}_t > 0$ implies $s_{i,t} = N_{i,t}/\mathcal{N}_t < N_{i,t-1}/\mathcal{N}_{t-1} = s_{i,t-1}$, only the second effect is operational and hence those individuals lose from higher valuations.

The case of arbitrary financial transactions. We now consider the more general case of a business that can all the financial transactions that can be done by individuals: every period, the business can (i) repurchase its own shares (ii) buy and sell one-period bonds, and (iii) buy and sell K financial assets. The business budget constraint is:

$$\Pi_t + \sum_k \mathcal{N}_{k,t-1} D_{k,t} + \mathcal{B}_{t-1} = \mathcal{N}_{t-1} D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t) P_t + \sum_k (\mathcal{N}_{k,t} - \mathcal{N}_{k,t-1}) P_{k,t} + \mathcal{B}_t Q_t, \quad (69)$$

where, as above, Π_t denotes the income stream of a business from its non-financial operations, D_t denotes dividends per share and \mathcal{N}_t denotes the total amount of outstanding shares. The new part is $\mathcal{N}_{k,t}$, which denotes asset holdings in asset k , and \mathcal{B}_t , which denotes bond holdings.

Let us consider an individual investing in K financial assets, one-period bonds, as well as in the business. Dropping i subscripts for notational simplicity, the individual budget constraint is

$$(N_t - N_{t-1})P_t + \sum_k (N_{k,t} - N_{k,t-1})P_{k,t} + B_t Q_t = N_{t-1}D_t + \sum_k N_{k,t-1}D_{k,t-1} + B_{t-1} + Y_t - C_t. \quad (70)$$

Combining it with the business budget constraint (69) gives the following consolidated budget constraint:

$$\begin{aligned} & (N_t - N_{t-1} + s_{t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t)) P_t + \sum_k (N_{k,t} - N_{k,t-1} + s_{t-1}(\mathcal{N}_{k,t-1} - \mathcal{N}_{k,t})) P_{k,t} + (B_t + s_{t-1}B_t) Q_t \\ & = s_{t-1}\Pi_t + \sum_k (N_{k,t-1} + s_{t-1}\mathcal{N}_{k,t-1}) D_{k,t} + (B_{t-1} + s_{t-1}B_{t-1}) + Y_t - C_t, \end{aligned}$$

where, as above, $s_t \equiv N_t/\mathcal{N}_t$ denotes the individual ownership share in the business.

We can simplify this expression after denoting $\tilde{N}_{k,t} \equiv N_{k,t} + s_t \mathcal{N}_{k,t}$ the individual's consolidated shares in asset k of the individuals through its ownership of the business, $\tilde{B}_t \equiv B_t + s_t B_t$ the individual's

consolidated bond holdings, and $\tilde{M}_t \equiv \mathcal{N}_t P_t - \mathcal{B}_t Q_t - \sum_k \mathcal{N}_{k,t} P_{k,t}$ the market value of the firm exclusive of financial assets:

$$(s_t - s_{t-1})\tilde{M}_t + \sum_k (\tilde{N}_{k,t} - \tilde{N}_{k,t-1})P_{k,t} + \tilde{B}_t Q_t = s_{t-1}\Pi_t + \sum_k \tilde{N}_{k,t-1}D_{k,t} + \tilde{B}_{t-1} + Y_t - C_t. \quad (71)$$

This has the same form as (70), except that (i) D_t , the business dividend per share of the business, is replaced by Π_t , the business income stream from its non-financial operations, (ii) P_t , the business price per share, is replaced by \tilde{M}_t , the market value of its fundamental (non-financial) component, (iii) N_t , the number of shares held by the individual, is replaced by s_t , his/her ownership share in the business, and (iv) individual asset holdings in financial assets and one period bonds, $\{N_{k,t}\}_k$ and B_t , are replaced by their consolidated ones, $\{\tilde{N}_{k,t}\}_k$ and \tilde{B}_t .

This budget constraint allows us to consider the welfare effect of a deviation in the market value of the fundamental component of a business, \tilde{M}_t , holding constant its income stream Π_t , together with our usual deviations in asset prices $\{P_{k,t}\}_{k=0}^K, Q_t$:

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left((s_{t-1} - s_t) d\tilde{M}_t + \sum_k (\tilde{N}_{k,t-1} - \tilde{N}_{k,t}) dP_{k,t} - \tilde{B}_t dQ_t \right)$$

There are two main takeaways of this formula relative to (68). First, when measuring individual financial transactions, we should also account for all of the indirect transactions done through the businesses that they own (i.e. $\tilde{N}_{t-1} - \tilde{N}_t$ instead of $N_{t-1} - N_t$). Second, when measuring deviations in business valuations, we should only consider deviations in the market value of their non-financial components (i.e. $d\tilde{M}_t$ instead of dM_t). Put differently, this formula tells us to split businesses between their financial and non-financial components, and assign their financial components to individuals who ultimately own them.

A.3.5 Housing and wealth in the utility function

We now examine the welfare effect of price deviations in the presence of “assets in the utility function” (i.e., joy of asset ownership). For simplicity, we consider a two-asset version of the baseline model:

$$V = \max_{\{C_t, N_t, B_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t, F(N_t, P_t)),$$

subject to budget constraints at each period $t \geq 0$

$$C_t + (N_t - N_{t-1})P_t + B_t Q_t + \chi(N_t - N_{t-1}) = Y_t + B_{t-1}.$$

We assume that $U(\cdot, \cdot)$ is strictly increasing and concave in both arguments. The function F governs the sensitivity of flow utility to asset ownership. For instance, if $F(N_t, P_t) = \bar{F}$, then the model coincides with the baseline (i.e., assets ownership does not affect flow utility directly). If $F(N_t, P_t) = N_t$, then individuals value the quantity of assets that they own directly, but not their market value. This is the natural assumption in the case of housing. If $F(N_t, P_t) = P_t N_t$, then individuals value the market value of their wealth directly.

Proposition 13. *In the presence of assets in the utility function, the welfare gain is*

$$\text{Welfare Gain} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t) + \sum_{t=0}^{\infty} \beta^t \frac{U_F(C_t, F(N_t, P_t))}{U_C(C_t, F(N_t, P_t))} F_P(N_t, P_t) dP_t.$$

Proof of Proposition 13. The Lagrangian associated with the individual problem is

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t U(C_t, F(N_t, P_t)) \\ &+ \sum_{t=0}^{\infty} \lambda_t (Y_t + N_{t-1}D_t + B_{t-1} - C_t - (N_t - N_{t-1})P_t - B_tQ_t - \chi(N_t - N_{t-1})).\end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned}dV &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \lambda_t (-(N_t - N_{t-1}) dP_t - B_t dQ_t) + \sum_{t=0}^{\infty} \beta^t U_F(C_t, F(N_t, P_t)) F_P(N_t, P_t) dP_t \\ &= U_{C_0}(C_0, F(N_0, P_0)) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{t-1} - N_t) dP_t - B_t dQ_t) + \sum_{t=0}^{\infty} \beta^t U_F(C_t, F(N_t, P_t)) F_P(N_t, P_t) dP_t,\end{aligned}$$

where the last line uses the FOC with respect to B_t , $\lambda_t Q_t = \lambda_{t+1}$, as well as the FOC with respect to C_0 , $U_C(C_0, F(N_0, P_0)) = \lambda_0$. \square

Relative to the welfare gain formula in the baseline model (i.e., Equation 14), the formula has an additional term, which accounts for the direct effect of price deviations on utility. Note that, when flow utility only depends on the quantity of assets, not their market value (i.e., $F_P = 0$), the welfare gain formula coincides with the formula in the baseline model.

A.3.6 Government sector

We now examine the welfare effect of price deviations in the presence of government transfers. For simplicity, we consider a two-asset version of the baseline model. Suppose that the government makes targeted transfers to individuals $i \in \{1, \dots, I\}$, where $T_{i,t}$ denotes the net amount of resources transferred from the government to individual i at time t . The individual problem is now given by

$$V_i = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + T_{i,t} + N_{i,t-1}D_t + B_{i,t-1},$$

We assume that the government can trade both assets and thus faces, at each period $t \geq 0$, the following budget constraint:

$$(N_{G,t} - N_{G,t-1})P_t + B_{G,t}Q_t = N_{G,t-1}D_t + B_{G,t-1} - \sum_{i=1}^I T_{i,t} - \chi(N_{G,t} - N_{G,t-1}), \quad (72)$$

where, for simplify, χ is assumed to be differentiable. We do not fully specify the government problem, but we assume that the government's portfolio choice satisfies the following cost-minimization condition

$$Q_t^{-1} = \frac{D_{t+1} + P_{t+1} - \chi'(N_{G,t+1} - N_{G,t})}{P_t + \chi'(N_{G,t} - N_{G,t-1})}, \quad (73)$$

at every $t \geq 0$. The idea is that the government minimizes the cost of borrowing (or alternatively maximizes the return on saving) by adjusting portfolio shares until the marginal return on the long-lived asset (net of adjustment costs) is equalized with the bond return.

The following proposition characterizes the welfare gain in the presence of government transfers.

Proposition 14. *In the presence of government transfers, the welfare gain of individual i is*

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dT_{i,t}.$$

Moreover, the aggregate contribution of deviations in government transfers $dT_{i,t}$ to individual welfare is

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \sum_{i=1}^I dT_{i,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t).$$

Proof of Proposition 14. The welfare gain formula follows immediately from the envelope theorem, as in the baseline model. This proof focuses on the second equation. Differentiating the government budget constraint (72), we obtain

$$\begin{aligned} \sum_{i=1}^I dT_{i,t} &= (N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t \\ &\quad - (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} + (D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t-1} - Q_t dB_{G,t} + dB_{G,t-1}. \end{aligned}$$

The sum of aggregate net transfer deviations discounted using the liquid asset return is

$$\begin{aligned} \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \sum_{i=1}^I dT_{i,t} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} \\ &\quad + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t-1} \\ &\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Q_t dB_{G,t} + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dB_{G,t-1} \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} \\ &\quad + \sum_{t'=-1}^{\infty} R_{0 \rightarrow t'+1}^{-1} (D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1}) dN_{G,t'} \\ &\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Q_t dB_{G,t} + \sum_{t'=-1}^{\infty} R_{0 \rightarrow t'+1}^{-1} dB_{G,t'} \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) \\ &\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\chi'(N_{G,t} - N_{G,t-1}) + P_t - Q_t (D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1}) \right) dN_{G,t} \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t). \end{aligned}$$

The second equality uses a change of variables $t' \equiv t - 1$. The third equality uses the fact that $R_{0 \rightarrow t+1}^{-1} = R_{0 \rightarrow t}^{-1} Q_t$ as well as $dN_{G-1} = dB_{G-1} = 0$. The fourth equality uses the cost-minimization assumption (73). \square

The formula for the welfare gain of individual i differs from the one in the baseline model since it

includes the present-value of deviations in net government transfers. The reason is that the government might respond to a change in asset prices by adjusting net transfers. Moreover, the second part of Proposition 14 states that the discounted sum of aggregate net transfers to the household sector is equal to the “welfare gain of the government”. Note that we obtain this result without making assumptions on the objective of the government. It is merely a consequence of the budget constraint of the government.

A.3.7 Taxes on assets

We now examine the welfare effect of asset price changes in the presence of various taxes on assets. We consider: (i) a non-linear wealth tax $\tau_{W,t}$ on the market value of wealth $N_{t-1}P_t$, (ii) a non-linear transaction tax $\tau_{\chi,t}$ on the market value of asset sales $(N_{t-1} - N_t)P_t$, (iii) a dividend income tax τ_D on dividend income $N_{t-1}D_t$, and (iv) a linear tax $\tau_{Q,t}$ on interest income or equivalently on the cost of buying bonds B_tQ_t . Individuals maximize

$$V = \max_{\{C_t, N_t, B_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t),$$

subject to budget constraints at each period $t \geq 0$

$$\begin{aligned} C_t + (N_t - N_{t-1})P_t + \chi(N_t - N_{t-1}) + \tau_{\chi,t}((N_{t-1} - N_t)P_t) + B_tQ_t(1 + \tau_Q) + \tau_{W,t}(N_{t-1}P_t) + \tau_{D,t}(D_tN_{t-1}) \\ = Y_t + N_{t-1}D_t + B_{t-1}. \end{aligned}$$

Here the functions $\tau_{\chi,t}(\cdot)$, $\tau_{W,t}(\cdot)$, $\tau_{D,t}(\cdot)$ are general non-linear and potentially time-dependent tax functions. This allows us to capture a number of features of real-world tax systems. For example, transaction taxes often apply on both sales and purchases (i.e. $\tau_{\chi,t}(\cdot)$ may be positive and increasing when $N_{t-1} - N_t > 0$, positive and decreasing when $N_{t-1} - N_t < 0$ and zero when $N_{t-1} - N_t = 0$). Similarly, there are often large exemption levels, in particular for wealth taxes $\tau_{W,t}(\cdot)$. In contrast, we restrict the tax on interest income to be linear with tax rate $\tau_{Q,t}$ so as to preserve an Euler equation that is independent of bond holdings B_t . Finally, we assume that the tax functions $\tau_{\chi,t}(\cdot)$, $\tau_{W,t}(\cdot)$ are differentiable.

Proposition 15. *In the presence of taxes on wealth, asset sales, and interest income, $\tau_{W,t}$, $\tau_{\chi,t}$ and $\tau_{Q,t}$, the welfare gain is*

$$\begin{aligned} \text{Welfare Gain} = \sum_{t=0}^{\infty} \tilde{R}_{0 \rightarrow t}^{-1} \left((N_{t-1} - N_t) \left(1 - \tau'_{\chi,t}((N_t - N_{t-1})P_t) \right) dP_t \right. \\ \left. - \tau'_{W,t}(N_{t-1}P_t)N_{t-1} dP_t - B_t(1 + \tau_{Q,t}) dQ_t \right). \end{aligned}$$

The presence of taxes changes our baseline formula in Proposition 1 in three noteworthy ways. First, whereas Proposition 1 implied that it is asset transactions and not asset holdings that matter for welfare gains from asset price changes, holdings do matter whenever there is a wealth tax (i.e., a tax on the market value of asset holdings). In particular, whenever asset prices increase, $dP_t > 0$, asset holders experience a welfare loss $\tau'_{W,t}(N_{t-1}P_t)N_{t-1} dP_t$.

Second, a transaction tax reduces asset sellers’ welfare gains from rising asset prices because the after-tax asset price faced by sellers increases by less than the pre-tax price

$$0 < \left(1 - \tau'_{\chi,t}((N_t - N_{t-1})P_t) \right) dP_t < dP_t \quad \text{when} \quad N_t - N_{t-1} > 0 \text{ and } dP_t > 0.$$

However, it also *increases* asset buyers’ welfare losses from rising asset prices because the after-tax asset

price faced by buyers increases by *more* than the pre-tax price

$$0 < dP_t < \left(1 - \tau'_{\chi,t}((N_t - N_{t-1})P_t)\right) dP_t \quad \text{when } N_t - N_{t-1} < 0 \text{ and } dP_t > 0.$$

Third and related, both transaction and wealth taxes introduce aggregate welfare losses for the household sector as a whole. Finally, though unsurprisingly, the presence of dividend income taxes $\tau_{D,t}$ leaves welfare gains unaffected.

Proof of Proposition 15. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t U(C_t) + \sum_{t=0}^{\infty} \lambda_t (Y_t + N_{t-1}D_t + B_{t-1} - C_t - (N_t - N_{t-1})P_t - \chi(N_t - N_{t-1}) \\ & - \tau_{\chi,t}((N_{t-1} - N_t)P_t) - B_t Q_t(1 + \tau_{Q,t}) - \tau_{W,t}(N_{t-1}P_t) - \tau_{D,t}(D_t N_{t-1})). \end{aligned}$$

The first-order condition for B_t is

$$\lambda_{t+1} = \lambda_t \tilde{Q}_t \quad \text{where } \tilde{Q}_t = Q_t(1 + \tau_{Q,t})$$

is the after-tax bond price. The infinitesimal change in the value function is given by the infinitesimal change in the Lagrangian:

$$\begin{aligned} dV &= \sum_{t=0}^{\infty} \left(\frac{\partial \mathcal{L}}{\partial P_t} dP_t + \frac{\partial \mathcal{L}}{\partial Q_t} dQ_t \right) \\ &= \sum_{t=0}^{\infty} \lambda_t \left((N_{t-1} - N_t) dP_t - \tau'_{\chi,t}((N_{t-1} - N_t)P_t)(N_{t-1} - N_t) dP_t \right. \\ &\quad \left. - \tau'_{W,t}(N_{t-1}P_t)N_{t-1} dP_t - B_t(1 + \tau_{Q,t}) dQ_t \right) \\ &= \sum_{t=0}^{\infty} \lambda_t \left((N_{t-1} - N_t) \left(1 - \tau'_{\chi,t}((N_t - N_{t-1})P_t)\right) dP_t - \tau'_{W,t}(N_{t-1}P_t)N_{t-1} dP_t - B_t(1 + \tau_{Q,t}) dQ_t \right) \\ &= U'(C_0) \sum_{t=0}^{\infty} \tilde{R}_{0 \rightarrow t}^{-1} \left((N_{t-1} - N_t) \left(1 - \tau'_{\chi,t}((N_t - N_{t-1})P_t)\right) dP_t \right. \\ &\quad \left. - \tau'_{W,t}(N_{t-1}P_t)N_{t-1} dP_t - B_t(1 + \tau_{Q,t}) dQ_t \right). \end{aligned}$$

where the third equality uses the Euler equation for B_t which implies $\lambda_t = U'(C_0) \tilde{R}_{0 \rightarrow t}^{-1}$ with $\tilde{R}_{0 \rightarrow t} = (\tilde{Q}_0 \dots \tilde{Q}_{t-1})^{-1}$. \square

B Appendix for Section 3

B.1 Price deviations relative to baseline price-dividend ratio

As explained in Section 3.1 when implementing Proposition 1, we construct the empirical price deviations $\Delta P_{k,t}$ as deviations of asset prices from those that would arise under a constant price-dividend ratio (see equation 18 and Figure 2). We now show that, under the assumption that dividends follow a random walk, price deviations around a constant price-dividend ratio can be interpreted as deviations around a constant value for discount rates. We first treat the deterministic case and then the stochastic case.

Deterministic case. Consider the case in which dividends are deterministic and grow at a constant rate:

$$D_{t+s} = D_t G^s. \tag{74}$$

Under this constant-growth assumption, the price of an asset is

$$\begin{aligned} P_t &= \sum_{s=1}^{\infty} R_{t \rightarrow t+s}^{-1} D_{t+s} \\ &= D_t \sum_{s=1}^{\infty} R_{t \rightarrow t+s}^{-1} G^s. \end{aligned} \quad (75)$$

When discount rates are constant, $R_t = \bar{R}$ for all t with $\bar{R} > G$, this simplifies to

$$P_t = D_t \times \overline{PD} \quad \text{with} \quad \overline{PD} = \frac{G}{\bar{R} - G}, \quad (76)$$

i.e., the price-dividend ratio is constant and the price grows at the same rate as dividends. This is the original ‘‘Gordon growth model’’, studied in [Gordon and Shapiro \(1956\)](#). This shows that variations of the price-dividend ratio over time correspond to variations in discount rates.

In our exercise, we construct price deviations as deviations of asset prices from a baseline with a constant price-dividend ratio, $\Delta P_t = (PD_t - \overline{PD}) \times D_t$. Combining (75) and (76), the difference in prices is:

$$\Delta P_t = \left(\sum_{s=1}^{\infty} (R_{t \rightarrow t+s}^{-1} - R^{-s}) G^s \right) D_t. \quad (77)$$

Hence, deviations in asset prices around a constant price-dividend ratio can be interpreted as deviations around a constant discount rate.

Stochastic case. We now show that the same ideas hold in a stochastic environment, which also allows for a connection with the Campbell-Shiller decomposition. Denote by $d_t = \log D_t$ the logarithm of dividends and by $r_t = \log R_t$ the logarithm of realized return. Assume that the logarithm of dividend growth $d_{t+1} - d_t$ follows a stationary process with average g and that discount rates $\mathbb{E}_t[r_{t+1}]$ follow a stationary process with average $\bar{r} > g$.

The Campbell-Shiller approximation for the log price-dividend ratio $pd_t = \log(P_t/D_t)$ gives:

$$pd_t = \frac{\kappa}{1 - \rho} + \underbrace{\sum_{s=0}^{\infty} \rho^s \mathbb{E}_t [d_{t+1+s} - d_{t+s}]}_{\text{part due to expected cash flows}} - \underbrace{\sum_{s=0}^{\infty} \rho^s \mathbb{E}_t [r_{t+1+s}]}_{\text{part due to discount rates}}, \quad (78)$$

This expresses current prices in terms of future expected dividends and future discount rates. For a derivation, see for example [Campbell \(2018, Section 5.3.1\)](#).

By analogy with the deterministic case, we now assume that the logarithm of dividends follows a random walk:

$$d_{t+1} - d_t = g + u_{t+1}, \quad \mathbb{E}_t[u_{t+1}] = 0. \quad (79)$$

Equation (78) becomes:

$$pd_t = \frac{\kappa + g}{1 - \rho} - \sum_{s=0}^{\infty} \rho^s \mathbb{E}_t [r_{t+1+s}]. \quad (80)$$

The key takeaway is that, under the assumption that dividends follow a random walk, fluctuations in the price-dividend ratio reflect fluctuations in discount rates.

More precisely, the average log price dividend ratio in this economy, denoted \overline{pd} is:

$$\overline{pd} = \frac{\kappa + g}{1 - \rho} - \sum_{s=0}^{\infty} \rho^s \bar{r}. \quad (81)$$

Table A2: Individual wealth at the end of 1993

Asset type	Average	S.D.	p10	p25	p50	p75	p90	p99
Total wealth	595.04	738.81	193.78	365.59	565.07	749.55	961.18	1735.21
Financial wealth	125.66	652.69	-9.78	12.17	88.75	171.08	276.41	765.60
Housing	130.57	218.21	0.00	0.00	109.78	182.39	272.24	625.35
Debt	-43.79	131.81	-110.81	-64.19	-20.54	0.00	0.00	0.00
Deposits	22.62	93.97	0.05	1.24	6.66	22.78	56.39	211.83
Public equity	2.70	431.42	0.00	0.00	0.00	0.00	0.18	24.42
Private equity	9.20	419.12	0.00	0.00	0.00	0.00	0.00	121.65
Human wealth	469.37	309.10	99.36	249.14	468.71	631.70	797.67	1346.44
NPV labor income	349.07	337.65	0.00	20.80	331.00	560.15	737.05	1280.53
NPV transfers	120.30	105.78	15.93	34.63	90.86	182.29	266.56	442.71

Notes. The table displays the summary statistics for individual wealth as of December 31st 1993. The total number of observations is 3,270,273. Values are reported in thousands of 2011 US dollars. Each statistic is computed for each variable separately.

Hence, the difference between the price and our baseline price, $\Delta P_t = (e^{p^d_t} - e^{\bar{p}^d})D_t$, can be written as:

$$\Delta P_t = \left(e^{-\sum_{s=0}^{\infty} \rho^s \mathbb{E}_t [r_{t+1+s} - \bar{r}]} - 1 \right) e^{\bar{p}^d} D_t. \quad (82)$$

This is the stochastic analogue to expression (77): under the assumption that dividends follow a random walk, fluctuations in asset prices due to changes in the price-dividend ratio capture fluctuations in discount rates.

B.2 Data on asset prices

Figure A1 plots price-deviations for our four asset classes: housing, equity, debt, and deposits.

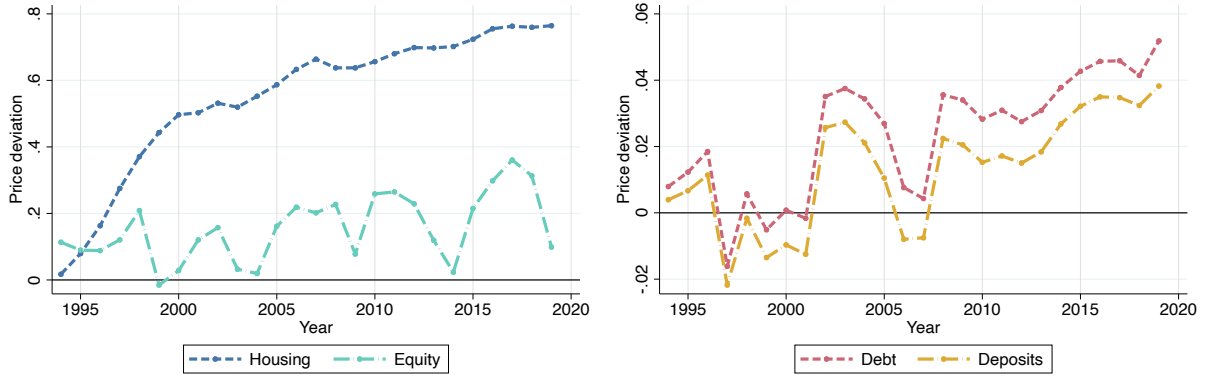


Figure A1: Price deviations for four asset classes in Norway

B.3 Microdata on holdings and transactions

B.3.1 Summary statistics and validation

Table A2 reports summary statistics on the balance sheet of Norwegian individuals at the end of 1993 (start of our sample).

Figure A2 compares the aggregate value of individuals' net assets for each asset category in the microdata as well as the ones reported in the Financial Accounts. Overall, the microdata aligns closely with the Financial Account data. The only notable discrepancies are public equity which is higher in

the microdata than in the National accounts after 2010, and mutual fund equity which is higher in the Financial Accounts than in our microdata throughout our sample period.

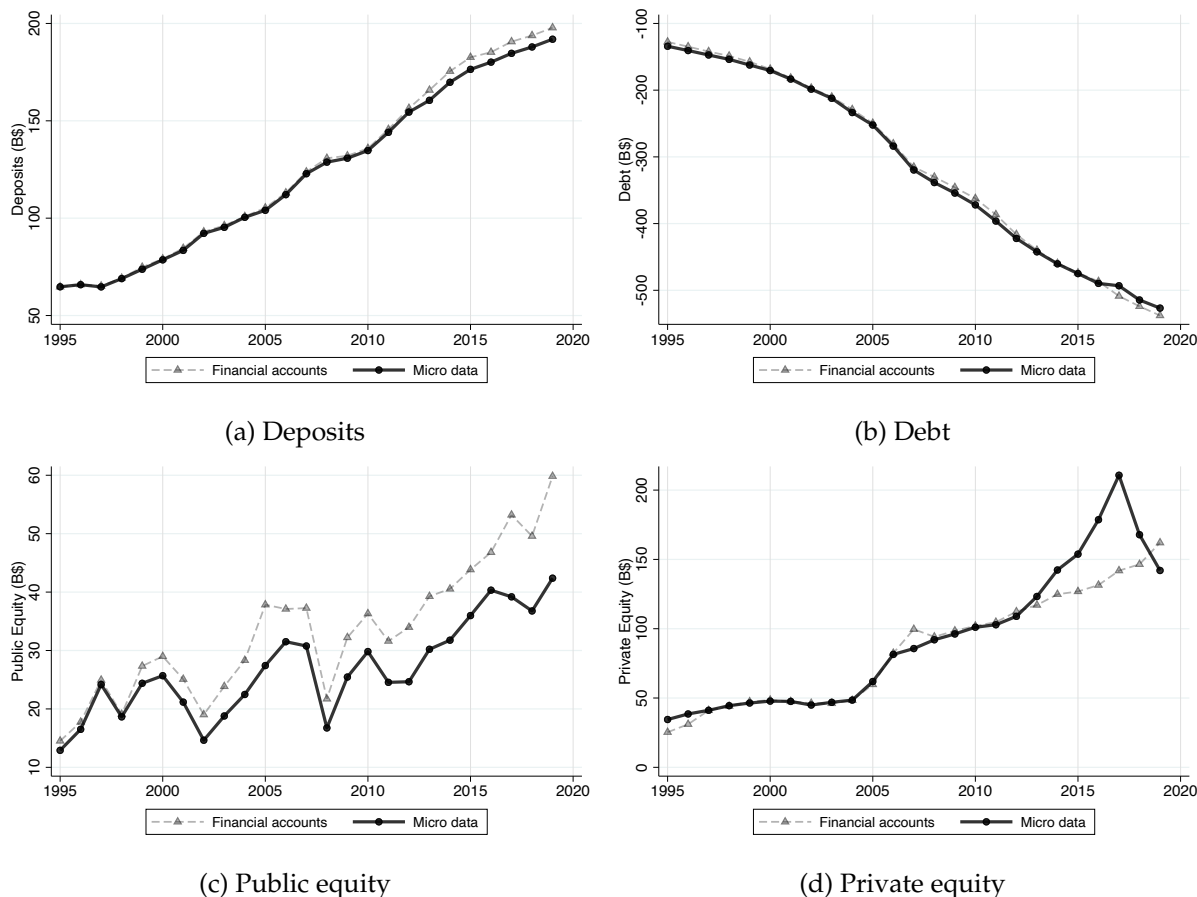


Figure A2: Aggregated administrative microdata versus the Financial Accounts (Holdings)

B.3.2 Imputing indirect holdings and transactions

Individuals who own firms are indirectly exposed to asset price changes through the asset holdings and transactions of the firms they own. We now describe how we impute these indirect holdings and transactions.

Private businesses. Starting in 2005, our data contains information on the ownership of limited liability businesses. The data contains information on the number of shares owned by an individual or a firm, and the market price if that exists. In addition, we observe the total number of shares issued by a company.

We first compute the direct ownership share of firm j by an individual or firm i . We obtain this number by dividing the number of shares held by owner i by the number of shares outstanding in firm j (i.e., the total number of shares issued by the firm minus the shares held by the firm itself). More precisely, the direct ownership share of a owner i in firm $j \neq i$ is

$$s_{ij} \equiv \frac{N_{ij}}{\sum_{i \neq j} N_{ij}},$$

where N_{ij} denotes the number of shares held by an owner i in firm j .

In our sample, a substantial fraction of businesses are owned by other businesses. For example, a common structure among wealthy individuals is to have one umbrella private holding company that owns several holding companies operating in different sectors. Our goal is to allocate the financial transactions done by all of these businesses to their ultimate owner. Formally, denote s_{ij}^n the ownership share of individual i in firm j through n (and exactly n) intermediate firm layers. When $n = 0$, this corresponds to our direct ownership share $s_{ij}^0 = s_{ij}$. For $n > 0$, we can compute the ownership shares of individual i in firm j at level n recursively:

$$s_{ij}^n \equiv \sum_k s_{ik}^{n-1} s_{kj}.$$

Finally, we obtain the consolidated ownership share of an individual by aggregating the ownership shares at all levels $n \geq 0$.⁷³

$$\tilde{s}_{ij} = \sum_{n=0}^{\infty} s_{ij}^n$$

In practice, we only compute indirect ownership shares up to $n = 10$ as indirect ownership shares are close to zero past that point.

Using these ownership shares, we construct an individual-level measure of private business book equity, which we define as the book value of a firm's assets minus net financial assets.⁷⁴ We only use book equity to compute the value of private businesses transactions, which we describe shortly. More generally, we rely on the tax assessed value of private business equity, which we observe over the full sample (i.e., starting in 1994).

Table A3 reports the average value of indirect holdings and transactions as a fraction of the tax assessed value of the equity in the firm over the 2005-2019 period. Private firms have, on average, positive net leverage (i.e., debt exceeds deposits). Moreover, private firms hold a significant amount of housing and (publicly-traded) stocks on their balance sheet, with a small amount of transactions every year. Before 2005, we do not observe the balance sheet of private firms, hence we do not have data on indirect holdings and transactions. From 1994 to 2004, we therefore impute indirect holdings and transactions by using the values in Table A3 multiplied by the tax assessed value of equity.

Table A3: Indirect holdings through private businesses (share of tax assessed value, 2005–2019 average)

Asset class	Holdings	Transactions
Deposits	0.40	–
Debt	1.11	–
Housing	0.65	–0.03
Stocks	0.16	–0.00

To measure the net transactions in private business equity for individual i in firm j in year t , we use the formula

$$\text{private equity transaction}_{ijt} = (\tilde{s}_{ij,t+1} - \tilde{s}_{ij,t}) \times \text{book equity}_{jt} \times Q,$$

⁷³ Formally, denote Ω the matrix of ownership within firms, that is, $\Omega_{ij} = s_{ij}$ for $i \neq j$ and $\Omega_{ij} = 0$ for $i = j$. Then, the vector of consolidated ownership of an individual i with direct ownership shares $\mathbf{s}_i = (s_{ij})_j$ is given by $(I - \Omega')^{-1} \mathbf{s}_i = \sum_{n=0}^{\infty} (\Omega')^n \mathbf{s}_i$.

⁷⁴ For instance, suppose that a firm has \$2 of assets, which includes \$1 of stocks, and \$0.25 of debt outstanding. The net financial assets of the firm is then $\$1 - \$0.25 = \$0.75$. Book equity is then $\$2 - \$0.75 = \$1.25$.

where, as above, $\tilde{s}_{ij,t}$ denotes the ownership share of individual i in firm j at time t . If the firm does not exist at time t and enters at time $t + 1$, we set the net transactions in private business equity to zero. Note that this formula automatically accounts for equity issuance. For instance, when a firm issues equity to finance its growth, the existing owners get diluted (i.e., their ownership share declines). In terms of exposure to asset price changes, this is equivalent to the owners selling equity shares.

The term Q represents the ratio between the market value of private business equity and its book value. While we do not observe Q directly, we set it to a value of 0.80, which corresponds to the aggregate tax assessed value of private business equity to aggregate book value of private business equity, averaged over the 2005-2019 period. Before 2005, we do not observe ownership shares and therefore set private equity transactions to zero.

Public businesses. Finally, we impute indirect holdings and transactions due to the ownership of publicly-traded stocks using a different methodology. We start from the indirect holdings and transactions of individuals through their ownership of the aggregate corporate sector, as reported in the Financial Accounts (see Appendix D for more details). We then subtract the aggregate indirect holdings and transactions due to their ownership of private businesses, as computed above. We therefore obtain residually the indirect aggregate holdings and transactions of public businesses that must be allocated to individuals. We then allocate these indirect holdings and transactions to individuals, for every year in our sample, in proportion to their equity holdings of public firms.

C Appendix for Section 4

C.1 Planner welfare

In this section, we detail how one can use our measure of welfare gains, estimated at the individual level, to compute the aggregate welfare gain from the point of view of the social planner. Consider a social planner that weighs the utility of each individual by λ_i . The effect of deviation in asset prices on the planner's welfare is:

$$\text{Social Planner's Welfare Gain} = \sum_i \lambda_i U'(C_{i0}) \times \text{Welfare Gain}_i.$$

where Welfare Gain_i represents the (money metric) welfare gain of each individual i and $\lambda_i U'(C_{i0})$ can be interpreted as the social planner's marginal welfare weights on each individual i . As discussed in [Saez and Stantcheva \(2016\)](#), such a representation encompasses more general social welfare functions. The point we want to make here is that, because we measure welfare gains at individual level, one could use our empirical results to compute the social planner's welfare gain given an arbitrary set of marginal welfare weights.

To show an example of this, we consider the special case in which the social planner's marginal welfare weights are proportional to individuals' total wealth (as measured in 1994) at the power $-\gamma$:

$$\text{Social Planner's Welfare Gain} = \sum_i \frac{(\text{Total Wealth}_i)^{-\gamma}}{\sum_i (\text{Total Wealth}_i)^{-\gamma}} \times \text{Welfare Gain}_i. \quad (83)$$

We think this special case is a natural one because, when Pareto weights equal one for each individual (i.e., $\lambda_i = 1$) and individual utilities have a Constant Relative Risk Aversion γ (i.e., $U(C) = C^{1-\gamma}/(1-\gamma)$), marginal welfare weights are given by $\lambda_i U'(C_{i0}) = C_{i0}^{-\gamma}$. While we do not observe the initial consumption of each individual in year 1995, one natural proxy is their total wealth in year 1995, as

consumption is proportional to total wealth in a wide range of consumption models. Finally, note that we can scale marginal welfare weights so that they sum up to one in the population. This allows us to interpret the social planner's welfare gain in dollar term: a social planner's welfare gain of $\$X$ means that the planner is indifferent between this and giving $\$X$ to each individual.

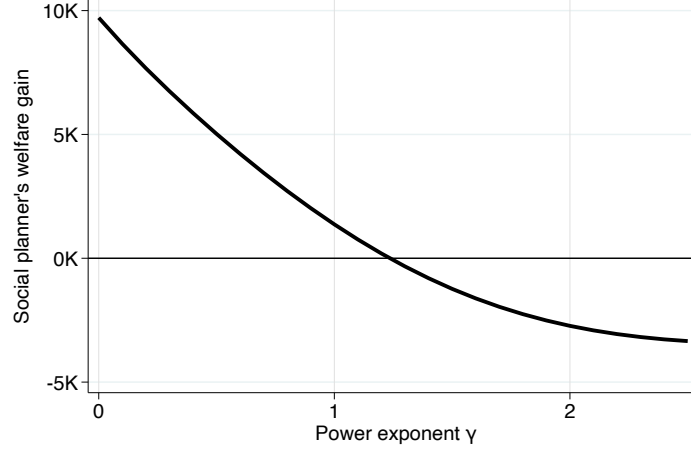


Figure A3: Social planner's welfare gain (83) as a function of γ

Figure A3 plots the social planner's welfare gain as a function of γ following (83). When $\gamma = 0$, the social planner's welfare gain is simply the average welfare gain in the population, which is roughly $\$10,000$. As γ increases, however, the social planner's welfare gain decreases and ultimately becomes negative. This reflects the fact that, as γ increases, the planner weighs more and more the welfare gains of poorer individuals and, as discussed in Section 4.3, rising asset prices redistributed from the poor to the wealthy.

C.2 Welfare and revaluation gains

We now explain the revaluation gains defined in (21) in the main text and how these differ from our baseline welfare gains formula. We first consider infinitesimal price deviations $\{dP_{k,t}\}_{t \geq 0}$ and then discuss non-infinitesimal deviations $\{\Delta P_{k,t}\}_{t \geq 0}$ as in equation (21).

Infinitesimal Deviations. We now discuss the relationship between welfare and revaluation gains due to infinitesimal price deviations.

Proposition 16. Consider an asset $1 \leq k \leq K$ and a sequence of price deviations $(dP_{k,t})_{t \geq 0}$. We have:

$$\underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{k,t-1} - N_{k,t}) dP_{k,t}}_{\text{Welfare gain}} = \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} P_{k,t-1} d\left(\frac{P_{k,t}}{P_{k,t-1}}\right)}_{\text{Revaluation gain}} + \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{k,t-1} P_{k,t-1} \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} d\left(\frac{D_{k,t}}{P_{k,t-1}}\right)}_{\text{Effect of price deviations on dividend yields}}. \quad (84)$$

The proposition decomposes the welfare effect of the deviation in asset prices (the left-hand side in Equation 84) into two terms. The first term ("revaluation gains") corresponds to the positive effect of a rise in asset prices on returns through higher capital gains. The second term corresponds to the negative effect of higher prices on returns through lower dividend yields.

This generalizes the intuition of the two-period model in a model with infinite horizon and multiple assets. The key message is that, following a rise in asset prices, revaluation gains overestimate welfare gains because they only take into account the positive effect of rising prices on capital gains without taking into account their negative effects going forward through lower dividend yields.

Finally note that the capital gains deviation that enters the revaluation gain in (84) can also be written as

$$d\left(\frac{P_{k,t}}{P_{k,t-1}}\right) = \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{dP_{k,t}}{P_{k,t}} - \frac{dP_{k,t-1}}{P_{k,t-1}} \right). \quad (85)$$

Proof of Proposition 16. Using summation by parts on the sequence $(R_{0 \rightarrow t}^{-1} dP_{k,t})_{t \geq 0}$ and $(N_{k,t})_{t \geq 0}$, the welfare gain for asset k can be rewritten as:

$$\begin{aligned} \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{k,t-1} - N_{k,t}) dP_{k,t} &= \sum_{t=0}^{\infty} N_{k,t-1} \left(R_{0 \rightarrow t}^{-1} dP_{k,t} - R_{0 \rightarrow t-1}^{-1} dP_{k,t-1} \right) \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} P_{k,t-1} \frac{dP_{k,t} - R_t dP_{k,t-1}}{P_{k,t-1}}. \end{aligned} \quad (86)$$

This equation highlights a duality between measuring welfare gains as the present value of sales interacted with price deviations (the left-hand-side) and the present value of asset holdings interacted with return deviations (the right-hand side).

To see why $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ can be interpreted as the deviation in returns, note that we have:

$$\begin{aligned} \frac{dP_{k,t} - R_t dP_{k,t-1}}{P_{k,t-1}} &= \frac{dP_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} dP_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}} dP_{k,t-1} - R_t dP_{k,t-1}}{P_{k,t-1}} \\ &= \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{dP_{k,t}}{P_{k,t}} - \frac{dP_{k,t-1}}{P_{k,t-1}} \right) + \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t \right) \frac{dP_{k,t-1}}{P_{k,t-1}} \\ &= d\left(\frac{P_{k,t}}{P_{k,t-1}}\right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} \left(-\frac{D_{k,t}}{P_{k,t-1}} \right) \frac{dP_{k,t-1}}{P_{k,t-1}} \\ &= d\left(\frac{P_{k,t}}{P_{k,t-1}}\right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} d\left(\frac{D_{k,t}}{P_{k,t-1}}\right), \end{aligned} \quad (87)$$

where the third line uses the definition of the return of asset k at time t $R_{k,t} \equiv (D_{k,t} + P_{k,t})/P_{k,t-1}$. This decomposes $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ into a part due to the deviation in capital gains (the first term in the RHS) and a part due to the deviation in dividend yields (the second part in the RHS). In the particular case where $R_{k,t} = R_t$ (no adjustment costs), we have $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1} = d(P_{k,t}/P_{k,t-1}) + d(D_{k,t}/P_{k,t-1})$; that is, $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ corresponds exactly to the deviation in the return of asset k . The proposition obtains by combining (86) with (87). \square

Non-infinitesimal deviations. In our empirical application, we measure welfare and revaluation gains using non-infinitesimal price changes. We now derive give a counterpart of Proposition 16 above for non-infinitesimal price deviations.

Corollary 17. Consider an asset $1 \leq k \leq K$ and a sequence of non-infinitesimal changes in prices $(\Delta P_{k,t})_{t \geq 0}$.

We have:

$$\begin{aligned}
\underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{k,t-1} - N_{k,t}) \Delta P_{k,t}}_{\text{Welfare gain}} &= \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} P_{k,t-1} \Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right)}_{\text{Revaluation gain}} \\
&+ \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{k,t-1} P_{k,t-1} \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t \right) \frac{\Delta P_{k,t-1}}{P_{k,t-1}}}_{\text{Effect of price deviations on dividend yields}}.
\end{aligned} \tag{88}$$

where we define

$$\Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right) \equiv \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\Delta P_{k,t-1}}{P_{k,t-1}} \right) \tag{89}$$

as the deviation in the capital gains component $P_{k,t}/P_{k,t-1}$ of asset returns caused by the price deviation $\{\Delta P_{k,t}\}_{t \geq 0}$.

Note that $\Delta(P_{k,t}/P_{k,t-1})$ defined in (89) is the natural discrete counterpart to $d(P_{k,t}/P_{k,t-1})$ in (85).

Proof of Corollary 17. The proof follows the same steps as the proof of Proposition 16, except with non-infinitesimal price deviations. Using summation by parts:

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} P_{k,t-1} \frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

In turn, we can write

$$\frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}} = \frac{\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

Plugging into the previous equation gives

$$\begin{aligned}
\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{k,t-1} - N_{k,t}) \Delta P_{k,t} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} \left(\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} \right) \\
&+ \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{k,t-1} \left(\frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} - R_t \Delta P_{k,t-1} \right).
\end{aligned}$$

Rearranging gives the result. Finally, note that, as price deviations become infinitesimal, each term in the formula converges to the respective term in Proposition 16. \square

Rank correlation Figure A4 plots the joint density of the rank of welfare and revaluation gains. More precisely, it reports the fraction of individuals within each quintile of welfare and revaluation gains. It can be seen as a discretized representation of the copula between the two variables.

D Appendix for Section 5

D.1 Consolidating financial accounts

Definitions. The *Financial Accounts* are produced by Statistics Norway and provide consistent measures of stocks and flows in financial markets. We use Table 10788, which provides annual data on (i) financial assets and liabilities by sector and (ii) financial transactions between sectors. We consider the following sectors of the economy:

1. Households (14);

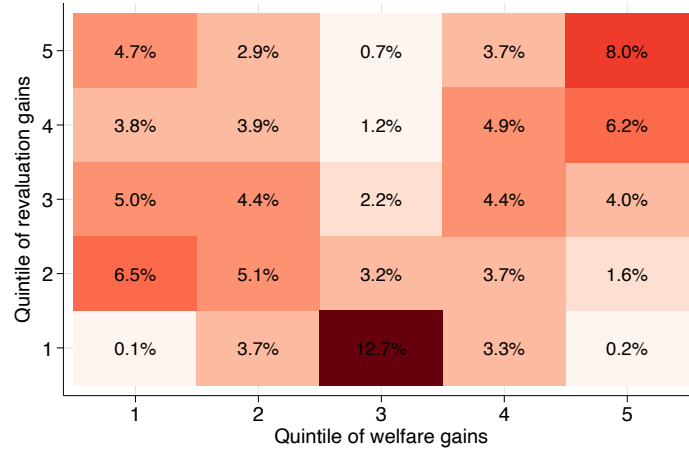


Figure A4: Heatmap of welfare and revaluation gains

Notes. The figure plots an heatmap of welfare and revaluation gains. More precisely, the figure reports the fraction of individuals within 5×5 quantiles of welfare and revaluation gains. By definition of quintiles, numbers within each row (or column) aggregate to 20%.

2. Government (121, 13, 15);
3. Foreigners (2).
4. Corporations
 - 4.1 Non financial corporations (11)
 - 4.2 Monetary financial institutions (122-123)
 - 4.3 Non-MM investment funds (124)
 - 4.4 Other financial institutions (125-127)
 - 4.5 Insurance corporations and pension funds (128-129)

The numbers in parentheses denote the sector codes from the Financial Accounts that we aggregate. Note that our definition of “Government” includes the central bank as well as the non-profit sector (i.e., institutions that serve the domestic household sector).

We consider the following asset categories:

1. Deposits (22);
2. Loans and debt securities (30, 40);
3. Public equity shares (511);
4. Private equity shares (512);
5. Fund equity shares (520);
6. Other (10, 21, 519, 610–800).

The numbers in parentheses denote the line items from the Financial Accounts that we aggregate. The category “other” contains assets that are either quantitatively unimportant or illiquid. We can further decompose each asset category using the identity of the sector issuing the security (e.g., public equity shares issued by the corporate sector versus the foreign sector).

Incorporating housing transactions. Housing is a real asset rather than a financial asset, which means that it is not included in the Financial Accounts. For our analysis, we augment the Financial Accounts by aggregating the housing transaction registry data described in Section 3.

Consolidating the corporate sector. We consolidate the different sectors constituting the corporate sector to their ultimate owner (i.e., either households, the government, or foreigners) by using the exact formula provided in Footnote 73. The consolidation process therefore adjusts the measures of holdings and transactions by households, the government, and foreigners by accounting for their indirect holdings and transactions through their ownership of the corporate sector. Note that this consolidation maintains the Financial Accounts identities. In particular, financial transactions remain in zero sum.

D.2 Welfare gains across sectors by asset class

Table A4 reports the detailed welfare gains asset class by asset class, including a breakdown within asset class (i.e., equity is the sum of domestic corporate equity and foreign corporate equity). Note

Table A4: Welfare gains across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	-4.7	1.9	2.7	0.0
Debt	16.4	-15.6	-0.8	0.0
Household debt	14.7	-5.5	-9.2	0.0
Corporate debt	5.8	2.8	-8.6	0.0
Government debt	-1.3	8.4	-7.1	0.0
Foreign debt	-2.7	-21.4	24.1	0.0
Deposits	-2.5	1.0	1.5	0.0
Corporate deposits	-1.8	1.4	0.4	0.0
Government deposits	-0.1	0.3	-0.2	0.0
Foreign deposits	-0.6	-0.7	1.2	0.0
Equity	-0.9	-10.8	11.7	0.0
Corporate equity	0.8	0.7	-1.5	0.0
Foreign equity	-1.7	-11.4	13.2	0.0
Total	8.4	-23.4	15.0	0.0

that welfare gains sum up to zero within each asset class, by construction, and that the welfare gain per capita in the household sector is very similar to the one estimated in our microdata (see Table 2). The small difference is due the fact that our microdata does not aggregate exactly to the Norwegian Financial Accounts (see Appendix B.3.1), as well as the fact that our microdata starts in 1994 while the Norwegian Financial Accounts only start in 1995.

D.3 Heterogeneous price indices

We now estimate different price indices for foreign debt and equity. We get valuation for foreign debt using the OECD average 3-year government bond yield (series from Global Financial Data). We obtain valuation for equity using the ratio of enterprise value to total firm payout using the universe of firms from Worldscope.

Figure A5 plots the price deviations using domestic versus foreign assets. One can see that the two valuations are very similar, which suggests that the rise in asset prices in Norway is similar to the general rise in valuations in other countries. The main difference is that the value of equity increased faster in the rest of the world compared to Norway.

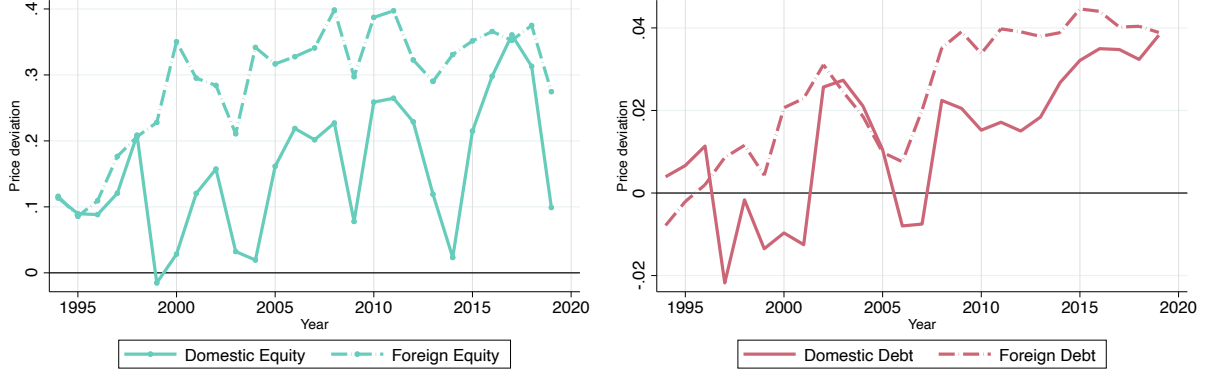


Figure A5: Price deviations for domestic and foreign assets

Table A5 reports the welfare gains use this foreign price indices for debt, deposit, and equity. Compared to Table A4, the rows that change are indicated by a dagger † sign. Overall, one can see that we obtain very similar result. The main difference is that the welfare gains of the Norwegian government are more negative in magnitude, which reflects the fact that they disproportionately purchased foreign equity, whose valuation increased more than the valuation of domestic equity.

Table A5: Welfare gains across sectors using heterogeneous price indices

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	-4.7	1.9	2.7	0.0
Debt [†]	16.7	-14.9	-1.8	0.0
Household debt	14.7	-5.5	-9.2	0.0
Corporate debt	5.8	2.8	-8.6	0.0
Government debt	-1.3	8.4	-7.1	0.0
Foreign debt [†]	-2.5	-20.7	23.1	0.0
Deposits [†]	-3.0	0.4	2.5	0.0
Corporate deposits	-1.8	1.4	0.4	0.0
Government deposits	-0.1	0.3	-0.2	0.0
Foreign deposits [†]	-1.0	-1.3	2.3	0.0
Equity [†]	-0.6	0.8	-0.1	0.0
Corporate equity	0.8	0.7	-1.5	0.0
Foreign equity [†]	-1.5	0.1	1.4	0.0
Total [†]	8.4	-11.8	3.4	0.0

E Appendix for Section 6

E.1 Collateral effects

We first provide a derivation for the empirical formula for the contribution of collateral effects on welfare gains given in Equation (24). According to Proposition 8, the welfare gain due to the collateral effect is:

$$\text{Welfare gain}_{i,\text{collateral}} = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right).$$

With the functional form $F(B_{i,t}, N_{i,t}, P_t) = \exp\left(\beta \frac{B_{i,t}}{N_{i,t}P_t}\right)$, we get

$$\text{Welfare gain}_{i,\text{collateral}} = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} (-B_{i,t} Q_{i,t}) \times \beta \frac{-B_{i,t} Q_{i,t}}{N_{i,t} P_t} \frac{dP_t}{P_t}.$$

Hence, a first-order approximation for the welfare gain due to the actual deviation in asset prices is:

$$\text{Welfare gain}_{i,\text{collateral}} \approx \sum_{t=0}^{\infty} R^{-t} (-B_{i,t} Q_{i,t}) \times \beta \text{LTV}_{i,t} \frac{PD_t - \overline{PD}}{PD_t},$$

using the approximation $\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} = R^{-t}$ (as discussed in Section 2.4) and the definition of loan-to-value $\text{LTV}_{i,t} \equiv \frac{-B_{i,t} Q_{i,t}}{N_{i,t} P_t}$.

Regression evidence. We now describe how we estimate β in (24), which governs the importance of the collateral effect on welfare. We start from the full sample of individuals over the 1994–2019 period. To compute the implied mortgage interest rate, we first compute the interest costs by outstanding debt, both of which are readily available in our data. We compute the interest rate and loan-to-value of individual i at time t as

$$\text{Interest rate}_{i,t} = \frac{\text{Interest costs}_{i,t}}{\frac{1}{2}\text{Debt}_{i,t-1} + \frac{1}{2}\text{Debt}_{i,t}}, \quad \text{LTV}_{i,t} = \frac{\text{Debt}_{i,t}}{\text{Housing value}_{i,t}},$$

where $\text{Debt}_{i,t} \equiv -B_{i,M,t} Q_{i,M,t}$.

To estimate β in (23), we use the approximation $\text{Interest rate}_{i,t} \approx -\log Q_{i,t}$ and estimate a regression of the form

$$\text{Interest rate}_{i,t} = \alpha_t + \beta \times \text{LTV}_{i,t-1} + u_{i,t},$$

where α_t is a year fixed-effect, and $u_{i,t}$ is an error term. We remove observations with a loan-to-value lower than 0.2 as (i) the interest rate estimate is imprecise for low debt levels and (ii) these low values are more likely to be driven by consumer debt rather than mortgage debt (empirically, we actually find that interest rates decrease between 0 and 0.2).

Specification (1) in Table A6 reports the results. The implied value of β is approximately 0.0025: a 10 pp. increase in the loan-to-value ratio implies a 0.025 pp. (2.5 basis point) increase in the interest rate.

One concern with the regression evidence is that there could be (potentially time-varying) omitted variables that affect the mortgage interest rate beyond the long-to-value ratio. For instance, some groups of the population could be more likely to shop around for the most competitive mortgage interest rate. Therefore, we also estimate a specification with age dummies and education groups as controls. Specification (2) in Table A6 reports the results. The implied value of β is approximately 0.005: a 10 pp. increase in the loan-to-value ratio implies a 0.05 pp. (5 basis point) increase in the interest rate.

Evidence from banks interest rate schedules. An additional concern with the regression evidence is that the presence of measurement error in the loan-to-value variable will generate an attenuation bias (i.e., bias the estimate of β towards zero). We therefore provide external evidence on the relationship between loan-to-value and mortgage interest rate using posted interest rate schedules published by banks. In Norway, it appears that the largest banks do not provide this data on their websites. However, some smaller banks present the interest rate they charge on a mortgage as a function of the loan-to-value ratio. Table A7 presents an example of such a schedule from *Bulder Bank*.

Using the last four rows of Table A7 and using the midpoints of the loan-to-value range, we obtain an linear slope of $\beta = 0.011$, which means that a 10 pp. increase in the loan-to-value ratio implies a

Table A6: Regression of mortgage interest rate on loan-to-value

Mortgage interest rate	(1)	(2)
Loan-to-value	0.00239*** (0.00003)	0.00459*** (0.00002)
Year fixed effects	✓	✓
Age and education controls		✓
Sample size	26,876,068	28,876,068
R^2	0.507	0.520

Notes. Standard errors in parentheses (*** $p < 0.01$). The education groups are: “less than high school”, “high school”, “college”.

Table A7: Example of interest rate schedule

Loan-to-value	Interest rate
< 50%	3.33%
50 – 55%	3.41%
55 – 60%	3.51%
60 – 65%	3.56%
65 – 70%	3.60%
70 – 75%	3.64%

Notes. Extracted on October 26 2022 from Bulder Bank’s website (<https://www.bulderbank.no/priser>).

11 basis point rise in the interest rate. This is roughly four times as large as the regression evidence without controls and twice as large as the regression evidence with controls. For robustness, we plot the effect of collateral using these three distinct values in Figure 10b.

Histograms. In the main text, we focus on average welfare gains due to the collateral effect *across* cohorts. However, there is also an important heterogeneity in the collateral effect *within* cohorts. In particular, individuals with a higher mortgage debt disproportionately benefit from the collateral effect. We now examine the welfare gains due to the collateral effect at the individual effect.

Table A8 reports the average welfare gains due to the collateral effect in the population, as well as in six percentile bins, for each of our three $\beta \in \{0.0025, 0.005, 0.01\}$. Looking at the row with $\beta = 0.01$, the average welfare gain due to the collateral effect is approximately \$5,000 in the population. This is a bit lower than the average baseline welfare gain in the population, which is \$10,000 (Table 2).

Similarly to debt holdings, welfare gains due to the collateral effect are right-skewed. As shown in Table A8, they become as high as \$67,000 for the top 1% of most affected individuals. Note that this is remains much smaller than the top 1% of baseline welfare gains, which is \$693,000 (Table 2).

Figure A6 plots the histogram of welfare gains due to the collateral effect across individuals in Norway. In all cases, to avoid scaling issue, we do not plot the density at zero (i.e., observations with zero mortgage debt), as they account for roughly 50% of our observations (see Table A8). Naturally, higher values of β are associated with a higher dispersion of welfare gains, as a high β magnifies the effect of a given change in loan-to-values of the interest rate.

Table A8: Welfare gains due to collateral effect

Value for β	Average	Average by percentile groups					
		p0-1	p1-10	p10-50	p50-90	p90-99	p99-100
$\beta = 0.0025$	1.3	0.0	0.0	0.0	1.5	5.4	16.6
$\beta = 0.005$	2.5	0.0	0.0	0.1	2.9	10.8	33.3
$\beta = 0.01$	5.0	0.0	0.0	0.1	5.9	21.7	66.6

Notes. For each percentile group of welfare gains due to the collateral effect, the table reports the average welfare gain due to the collateral effect (24). All numbers are in thousands of 2011 US dollars.

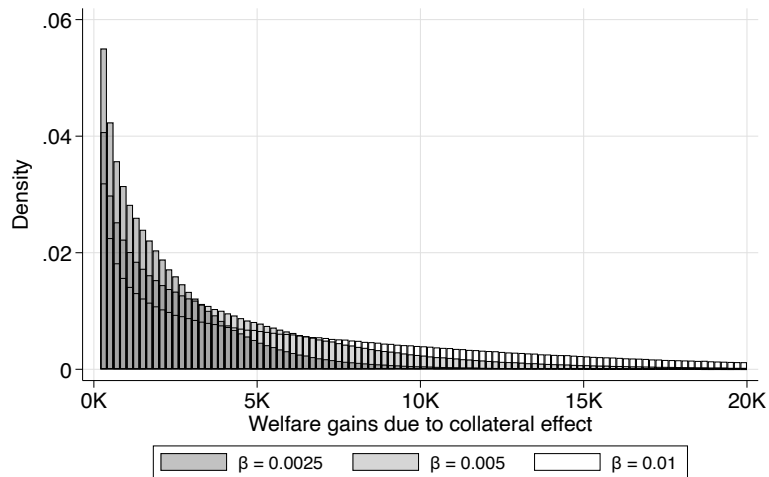


Figure A6: Histogram of welfare gains due to the collateral effect

Notes. This figure plots the density of welfare gains due to the collateral effect (24), with $\beta = \{0.0025, 0.005, 0.01\}$, across individuals in Norway. More precisely, the figure plots the relative mass of individuals within equally-spaced bins of welfare gains (width of \$100). For the sake of legibility, we do not report the relative mass of individuals with a welfare gain between \$0 and \$100, as approximately half of the population has no mortgage debt. All numbers are in thousands of 2011 US dollars.

E.2 Idiosyncratic labor income risk

We now describe how we estimate the sequence of covariances in (25), which governs the importance of the incomplete markets on welfare.

Approximation of covariance term. We now provide an approximation for the covariance term in Equation (63). Assuming CRRA utility, we have that $U'(C_{i,t}) = C_{i,t}^{-\gamma}$. This implies

$$\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} = \frac{\beta^t C_{i,t}^{-\gamma}}{C_{i,0}^{-\gamma}}$$

In turn, using a first-order approximation for $\log C_{i,t}$ around $c^* = \frac{1}{\gamma} \log \mathbb{E}_0 [C_{i,t}^{-\gamma}]$ gives:

$$\begin{aligned} \frac{\beta^t C_{i,t}^{-\gamma}}{C_{i,0}^{-\gamma}} &= \frac{\beta^t e^{-\gamma \log C_{i,t}}}{C_{i,0}^{-\gamma}} \\ &= \frac{\beta^t \mathbb{E}_0 [C_{i,t}^{-\gamma}] (1 - \gamma (\log C_{i,t} - c^*))}{C_{i,0}^{-\gamma}} \\ &\approx R_{0 \rightarrow t}^{-1} (1 - \gamma (\log C_{i,t} - c^*)), \end{aligned}$$

where the last line uses the Euler equation. Plugging this result into the expression for the covariance gives

$$\begin{aligned} &\sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) \\ &\approx -\gamma \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \text{cov}_0 (\log C_{i,t}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t). \end{aligned}$$

This is equation (25) in the main text (there for the case of one asset, i.e., with $B_{i,t} = 0$).

Under the additional assumption that $\log C_{i,t} - \mathbb{E}_0 [\log C_{i,t}] \approx \text{MPC}(\log \bar{Y}_{i,t} - \mathbb{E}_0 [\log \bar{Y}_{i,t}])$, where $\bar{Y}_{i,t}$ denotes the permanent component of labor income, we have that

$$\begin{aligned} &\sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) \\ &\approx -\gamma \times \text{MPC} \times \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \text{cov}_0 (\log \bar{Y}_{i,t}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) \end{aligned}$$

where we have used that $\text{cov}_0 (\mathbb{E}_0 [\log C_{i,t}], (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) = 0$ and similarly $\text{cov}_0 (\mathbb{E}_0 \log Y_{i,t}, (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) = 0$. This is the expression for the adjustment cost stated in the text just below (25) in the main text.

Measurement of permanent labor income $\bar{Y}_{i,t}$. We measure the permanent component of labor income $\bar{Y}_{i,t}$ using a lagged three-year moving average of net non-financial income (i.e., labor income plus net government transfers, as we use to compute human wealth), which gives us a proxy for permanent income shocks.

Regression framework. For each asset class k (i.e., debt, deposits, equity, housing) and individual i , denote the welfare relevant notion of asset sales at time t as $S_{i,k,t}$. For debt and deposits, it is holdings-based — that is, $S_{i,k,t} = B_{i,t} Q_t$ — while for housing and equity, it is transactions-based — that is,

$S_{i,k,t} = (N_{i,k,t} - N_{i,k,t-1})P_{k,t}$. For each cohort c , asset class k , and horizon t , we estimate the following cross-sectional regressions for all individuals i in cohort c :

$$S_{i,k,t} = \alpha_{c,k,t} + \beta_{c,k,t} \log \bar{Y}_{i,t} + x'_{i,0} \theta_{c,k,t} + u_{i,k,t}, \quad (90)$$

where $\log \bar{Y}_{i,t}$ denotes the logarithm of our proxy of permanent labor income, and $x_{i,0}$ is a vector of controls that includes: (i) highest lifetime education achievement (i.e., “less than high school”, “high school”, “college” dummies), (ii) deciles of within-cohort financial wealth at $t = 0$ (i.e., ten dummies), (iii) and permanent income at $t = 0$. For individuals who were less than 25 years old at the beginning of the sample, the vector of controls only contains the highest lifetime education achievement.

For each regression, we residualize both $S_{i,k,t}$ and $\log \bar{Y}_{i,t}$ at the 1% level. To convert the estimated coefficient $\beta_{k,c,t}$ into the (conditional) covariance between log labor income and asset sales, we need to scale it up by the variance of $\log \bar{Y}_{i,t}$, residualized against the controls $x_{i,0}$, which we denote $\sigma_{k,c,t}^2$.

E.3 Valuation changes beyond the end of our sample period

To implement the sufficient statistic formula, we also need to predict individuals’ transactions in future years. As discussed in the main text, we assume that the number of assets sold by a given cohort in a given year will equal the number of assets sold by the cohort with the same age in 2019, after adjusting for economic growth. Formally, we assume that the aggregate transactions of individuals of age a at time $t > T$ are given by:

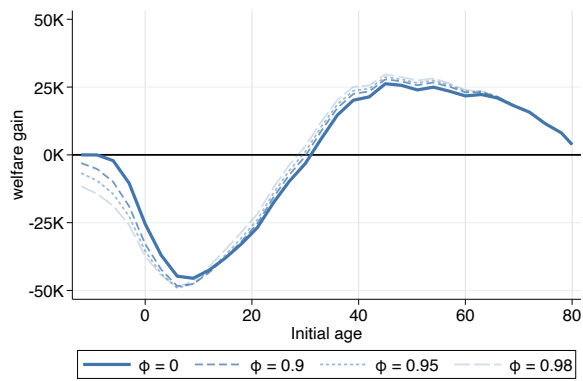
$$N_{a,k,t} - N_{a,k,t-1} = N_{a,k,T} - N_{a,k,T-1}, \quad B_{a,t} = G^{t-T} B_{a,T}, \quad (91)$$

where $G = 1.01$ denotes the predicted real per-capita growth rate of the economy (which corresponds to the per-capita growth rate of Norway’s GDP over our time sample). We then use that $P_{k,t} = PD_{k,t} D_t = PD_{k,t} G^{t-T} D_T = \frac{G^{t-T} PD_{k,t}}{PD_{k,T}} P_{k,T}$ to write this in terms of observables

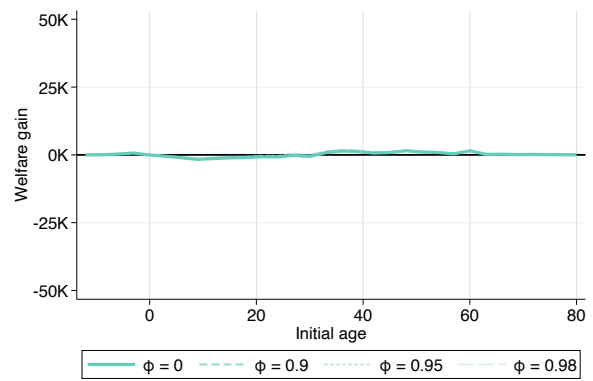
$$(N_{a,k,t} - N_{a,k,t-1})P_{k,t} = \frac{G^{t-T} PD_{k,t}}{PD_{k,T}} (N_{a,k,T} - N_{a,k,T-1})P_{k,T}, \quad B_{a,t} = G^{t-T} B_{a,T},$$

where, for $t > T$, the price-dividend ratio $PD_{k,t}$ is given by (29).

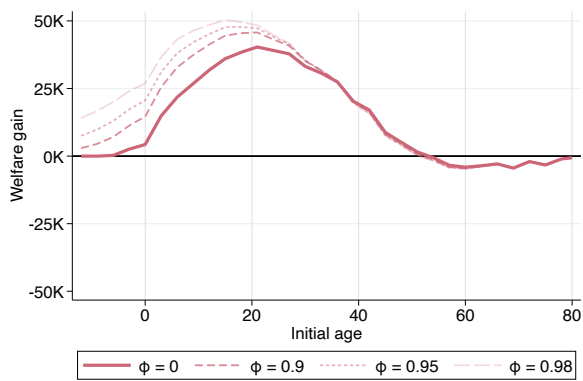
Figure A7 decomposes the welfare gains by asset class. As ϕ increases, most of the higher welfare gains in the population comes from lower interest rates on debt.



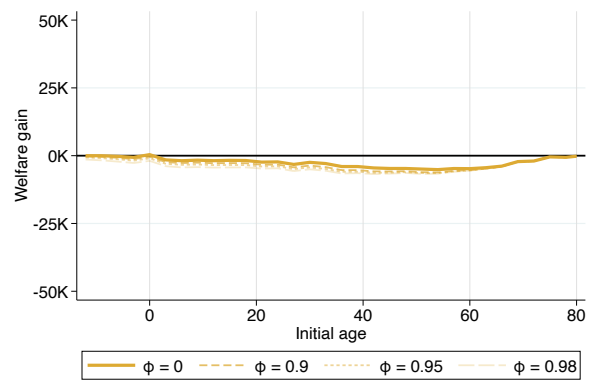
(a) Housing



(b) Equity



(c) Debt



(d) Deposits

Figure A7: Welfare gain depending on the behavior of asset prices in the future, asset class by asset class