Asset-Price Redistribution

Andreas Fagereng
BI

Matthieu Gomez
Columbia

Émilien Gouin-Bonenfant
Columbia

Martin Holm
University of Oslo

Benjamin Moll
LSE

Gisle Natvik
BI

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Rising asset prices

U.S. Nonfinancial Corporate Businesses
Rising asset prices ... relative to income = rising valuations

U.S. Nonfinancial Corporate Businesses
Welfare consequences of rising asset valuations?

- Rising asset valuations had large effects on distribution of wealth

Q: But what are welfare consequences of rising valuations?

- Answer is not obvious. Two polar views regarding effect of $P \uparrow$:
  1. Shift of real resources towards wealthy (Piketty-Zucman, Saez-Yagan-Zucman)
  2. Welfare-irrelevant paper gains (Cochrane, Krugman)
What We Do: Theory

• Formula for money-metric welfare gains/losses (= compensating variation) from asset price changes here for case of one asset and no borrowing

\[
\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \text{Discounting}_t \times (\text{Asset sales}_{it} \times \text{Price deviation}_t) + \ldots
\]

• price deviations holding cashflows constant \( \Rightarrow \) pure valuation effects
• envelope theorem \( \Rightarrow \) first-order approximation
• +... captures other effects (collateral, GE,..) but 1st term is always there

• **Two main lessons:** higher valuations ...

  1. benefit sellers, not holders

  2. are purely redistributive in terms of welfare (for every seller there is a buyer)

• Implication: both polar positions from previous slide are wrong
What We Do: Empirics

• Implement as sufficient statistic using Norwegian admin data (1994–2019)

\[
\text{Welfare Gain}_i = \sum_{t=0}^{T} \text{Discounting}_t \left( \sum_{k=1}^{K} \left( \text{Asset sales}_{ikt} \times \text{Price deviation}_{kt} \right) + \text{Borrowing}_{it} \times \text{Rate deviation}_t \right) + \text{Terms from generalizations}
\]

• measure net asset sales, borrowing (housing, stocks, debt, deposits)
• measure price deviations = deviations from constant price-dividend ratios (Gordon growth model) to isolate valuation effect

• Document large redistributive effects of rising asset valuations
  • in cross section
  • from young towards old
  • from poor towards wealthy
Rising asset valuations generate large welfare gains & losses
Example: large redistribution from young to old...
... mostly due to house price changes
Plan

1. Theory: Intuition in two-period model

2. Theory: Sufficient statistic in full dynamic model with multiple assets

3. Empirics: implementation using Norwegian administrative data

4. Empirics: redistribution across households

5. Empirics: generalizations of baseline sufficient statistics approach
Intuition in two-period model

- Periods $t = 0$ and $t = 1$, endowments $Y_0$ and $Y_1$
- Can trade shares $N$ at time $t = 0$ that pay dividend $D$ at time $t = 1$

$$V = \max_{\{C_0, C_1\}} U(C_0) + \beta U(C_1)$$

$$C_0 + (N_0 - N_{-1})P_0 = Y_0, \quad C_1 = Y_1 + N_0D_1$$

**Comparative static:** effect of $P_0$ on welfare $V$?

$$dV = U'(C_0) \times (N_{-1} - N_0) \times dP_0$$

- Rising asset prices benefit sellers ($N_{-1} - N_0 > 0$), not holders ($N_{-1} > 0$)
- Note: $D_1$ held constant, else additional term $+ \frac{\beta U'(C_1)}{U'(C_0)} N_0 dD_1$
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- **Comparative static:** effect of $P_0$ on welfare $V$?

$$\frac{dV}{U'(C_0)} = (N_{-1} - N_0) \times \frac{dP_0}{\text{asset sales}} \times \frac{dD_1}{\text{price deviation}}$$

- Rising asset prices benefit sellers ($N_{-1} - N_0 > 0$), not holders ($N_{-1} > 0$)
- Note: $D_1$ held constant, else additional term $+ \frac{\beta U'(C_1)}{U'(C_0)} N_0 \, dD_1$
Intuition in two-period model

- Rising asset prices benefit sellers \((N_{-1} - N_0 > 0)\), not holders \((N_{-1} > 0)\)

- How can initial holders not benefit from \(P_0 \uparrow\)? Two counteracting effects:
  - \((t = 0)\) High initial return \(R_0 = P_0 / P_{-1} \uparrow\)
  - \((t = 1)\) Low future returns \(R_1 = D_1 / P_0 \downarrow\)

- For sellers, high initial returns dominate ...

- For buyers, low future returns dominate
Graphical intuition: welfare effect of $P_0 \uparrow$

A seller’s investment decision

$$\text{slope} = -\frac{D_1}{P_0}$$

$Y_1 + N_{-1}D_1$

$C_{1}^*$

$(N_{-1} - N_0)P_0 > 0$

A buyer’s investment decision

$$\text{slope} = -\frac{D_1}{P_0}$$

$Y_1 + N_{-1}D_1$

$C_{1}^*$

$(N_{-1} - N_0)P_0 < 0$
Graphical intuition: welfare effect of $P_0 \uparrow$

Effect of $P_0 \uparrow$ on seller

Effect of $P_0 \uparrow$ on buyer

$P_0 \uparrow$ and so $\frac{D_1}{P_0} \downarrow$

$(N_{-1} - N_0)P_0 > 0$

$(N_{-1} - N_0)P_0 < 0$
Full dynamic model with multiple assets

- Infinite horizon, no risk

- One-period bond \( \{ B_{i,t} \}_{t=0}^{\infty} \) with prices \( \{ Q_t \}_{t=0}^{\infty} \) (deposits)
  - one-period return: \( R_{t+1} = 1/Q_t \)
  - return from 0 to \( t \): \( R_{0\rightarrow t} \equiv R_1 \cdot R_2 \cdots R_t \)

- \( K \) long-duration assets \( \{ N_{ik,t} \}_{t=0}^{\infty} \), prices \( \{ P_{k,t} \}_{t=0}^{\infty} \) & dividends \( \{ D_{k,t} \}_{t=0}^{\infty} \)
  - adjustment cost \( \chi_{ik}(N_{ik,t} - N_{ik,t-1}) \), potentially kinked (inaction)
  - asset returns: \( R_{k,t+1} \equiv \frac{D_{k,t+1} + P_{k,t+1}}{P_{k,t}} \)
Extensions: see paper, show you two of these today

1. Borrowing and collateral constraints
2. Incomplete markets
3. Bequests
4. Consolidating businesses with their owners.
5. Government sector
6. Taxes on assets
7. Housing and wealth in the utility function
8. General equilibrium
Welfare gains/losses in full dynamic model

• Households solve

\[ V_i = \max_{\{C_{i,t}, B_{i,t}, \{N_{ik,t}\}_{k=1}^{K}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \quad \text{s.t.} \]

\[ C_{i,t} + \sum_{k=1}^{K} (N_{ik,t} - N_{ik,t-1}) P_{k,t} + B_{i,t} Q_t + \sum_{k=1}^{K} \chi_{ik} = \sum_{k=1}^{K} N_{ik,t-1} D_{k,t} + B_{i,t-1} + Y_{i,t} \]

• **Proposition:** welfare effect of perturbation \( \{dP_{1,t}, \ldots, dP_{K,t}, dQ_t\}_{t=0}^{\infty} \) is

\[ dV_i = U'(C_{i,0}) \times \sum_{t=0}^{\infty} R_{0\rightarrow t}^{-1} \left( \sum_{k=1}^{K} (N_{ik,t-1} - N_{ik,t}) dP_{k,t} - B_{i,t} dQ_t \right) \]

Welfare Gain$_i$

• As in two-period model, rising asset prices benefit net sellers

... but portfolio choice + timing of purchases also matters
Aggregation

- **Corollary:** Suppose that initial prices clear all asset markets, i.e. asset sales and purchases add up to zero for each asset class. Then

\[ \sum_{i=1}^{I} \text{Welfare Gain}_i = 0 \]

so that asset price deviations are purely redistributive.
Extension: general equilibrium

- Claim: in GE, our formula does not capture full welfare effect but rather welfare effect working through equilibrium asset price changes
- Fundamental drivers of asset prices: vector \( z_t = \bar{z}_t + \theta \Delta z_t \)
- Equilibrium prices: \( \Gamma_t(\theta) = \{ \{ P_{k,t}(\theta), D_{k,t}(\theta) \}_k, Q_t(\theta), Y_t(\theta) \} \)
- Welfare \( V(\{ \Gamma_t(\theta) \}_{t=0}^\infty, \theta) \). Hence

\[
\frac{dV}{d\theta} = \sum_{t=0}^{\infty} \left( \sum_{k=1}^{K} \frac{\partial V}{\partial P_{k,t}} \frac{\partial P_{k,t}}{\partial \theta} + \frac{\partial V}{\partial Q_t} \frac{\partial Q_t}{\partial \theta} \right) d\theta
\]

**Welfare gain through asset prices = our main formula**

\[
+ \sum_{t=0}^{\infty} \left( \sum_{k=1}^{K} \frac{\partial V}{\partial D_{k,t}} \frac{\partial D_{k,t}}{\partial \theta} + \frac{\partial V}{\partial Y_t} \frac{\partial Y_t}{\partial \theta} \right) d\theta
\]

**Welfare gain through dividends, labor income, ...**

\[
+ \frac{\partial V}{\partial \theta} d\theta
\]

**Direct effect**
Extension: collateral constraints $- B_{i,t} \leq \theta N_{i,t} P_t$

- With collateral constraint, money-metric welfare gains are

$$Welfare \; Gain_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left( (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) + \sum_{t=0}^{\infty} \frac{\beta^t \mu_{i,t}}{U'(C_{i,0})} \theta N_{i,t} dP_t$$

where $\mu_{i,t}$ is Lagrange multiplier on collateral constraint

- Alternatively, household-specific rate schedule $Q_{i,t} = Q_t F(N_{i,t} P_t, B_{i,t})$:
Extension: collateral constraints $-B_{i,t} \leq \theta N_{i,t} P_t$

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![Diagram showing interest rate vs. debt with different curvature schedules]
Extension: collateral constraints $- B_{i,t} \leq \theta N_{i,t} P_t$

- With collateral constraint, money-metric welfare gains are

\[
\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left( (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t \right) + \sum_{t=0}^{\infty} \frac{\beta^t \mu_{i,t}}{U'(C_{i,0})} \theta N_{i,t} dP_t
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\]

- Later: implement by taking interest rate schedule from data
Empirics
Implementation

1. Replace infinitesimal changes $dP_{k,t}$ by discrete changes $\Delta P_{k,t}$
   • robustness: second-order effects

2. Consider deviations in prices away from constant price-dividend ratios

\[
\Delta P_{k,t} = P_{kt} - \overline{PD}_k \times D_{k,t} \iff \frac{\Delta P_{k,t}}{P_{k,t}} = \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}}
\]
Implementation

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   \]

![Graph showing the relationship between PD_H x D_Ht and PHt with the percentage change (ΔPHt) and its percentage change (ΔP_Ht) indicated.]}
Implementation

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   - robustness: second-order effects

2. Consider deviations in prices away from constant price-dividend ratios

\[
\Delta P_{k,t} = P_{kt} - \bar{P}D_k \times D_{k,t} \quad \Leftrightarrow \quad \frac{\Delta P_{k,t}}{P_{k,t}} = \frac{PD_{k,t} - \bar{P}D_k}{PD_{k,t}}
\]

3. Truncate formula at finite date $T$ (end of sample)
   - robustness: extrapolate trading and price deviations beyond $T$ (later)
Implementation

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   \]

3. Truncate formula at finite date $T$ (end of sample)
   - robustness: extrapolate trading and price deviations beyond $T$ (later)
   - $\Rightarrow$ Formula we implement empirically

\[
\text{Welfare gain}_i \approx \sum_{t=0}^{T} R_{0 \rightarrow t} \left( \sum_{k=1}^{K} (N_{ik,t-1} - N_{ik,t}) P_{k,t} \times \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}} - B_{i,t} Q_t \times \frac{Q_t - \overline{Q}}{Q_t} \right)
\]
Data on Holdings and Transactions

- Administrative data covering the universe of Norwegians over 1993–2019
- Focus on 4 broad asset categories that cover most of household wealth
  1. deposits (15%)
  2. debt (mortgage, student loan, ..., −35%)
  3. equity (individual stocks, mutual funds, private businesses, ..., 10%)
  4. housing (110%)

- For deposits/debt, we only need to measure holdings
- For equities/housing, we use data on individual transactions
- Take into account indirect transactions/holdings through equity ownership
Rising valuations, declining yields in all asset classes

Gross real interest rate (debt/deposits); Rents/Price (housing); Cashflows/EV (equity)
Data on housing and equity transactions
Data on debt and deposits
Rising asset valuations generate large welfare gains & losses.
Large gains and losses as % of initial wealth

In theory, we have

\[
\text{Welfare gain} \over \text{Total wealth} = \frac{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_t}{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_t} = \text{welfare gain as a share of lifetime consumption}
\]
Welfare vs wealth gains (revaluation gains)

Revaluation gain \( i \)  
\[ R_{0 \rightarrow t}^{-1} \sum_{k=1}^{K} N_{ik,t-1} P_{ik,t-1} \, d \left( \frac{P_{k,t}}{P_{k,t-1}} \right) \]  
(recall asset returns \( \frac{D_{k,t} + P_{k,t}}{P_{k,t-1}} \))
Joint distribution of welfare and revaluation gains

Quintile of welfare gains

Quintile of revaluation gains

<table>
<thead>
<tr>
<th>Quintile of welfare gains</th>
<th>Quintile 1</th>
<th>Quintile 2</th>
<th>Quintile 3</th>
<th>Quintile 4</th>
<th>Quintile 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quintile 1</td>
<td>0.1%</td>
<td>3.7%</td>
<td>12.7%</td>
<td>3.3%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>6.5%</td>
<td>3.9%</td>
<td>3.2%</td>
<td>4.4%</td>
<td>4.0%</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>5.0%</td>
<td>4.4%</td>
<td>12.7%</td>
<td>4.4%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>3.8%</td>
<td>1.2%</td>
<td>3.7%</td>
<td>6.2%</td>
<td>2.9%</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>4.7%</td>
<td>0.7%</td>
<td>4.0%</td>
<td>4.4%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>
Redistribution from the young to the old

![Graph showing welfare gain over initial age](image)
Redistribution from the young to the old

![Graph showing redistribution from the young to the old.](image-url)

- **Legend:**
  - Black: Total
  - Blue: Housing
  - Red: Debt
  - Orange: Deposits
  - Cyan: Equity

- **Axes:**
  - Y-axis: Welfare gain (in thousands of dollars)
  - X-axis: Initial age (in years)

- **Key Observations:**
  - The graph illustrates the distribution of welfare gains across different age groups.
  - Initially, there is a notable redistribution from younger to older generations.
  - The impact varies significantly by category (e.g., housing, debt, deposits, equity).

- **Data Points:**
  - The data points are not explicitly listed, but the graph visually represents the redistribution for different age groups and financial categories.

- **Interpretation:**
  - The redistribution trends suggest a significant transfer from younger to older individuals, particularly noticeable in the early years of the age range.
  - The disparity in gains is substantial, highlighting the economic implications of such a redistribution strategy.

- **Conclusion:**
  - The redistribution from the young to the old has significant economic implications, with notable disparities across different financial categories and age groups.
Redistribution from the poor to the rich

![Graph showing redistribution from the poor to the rich](image-url)
Welfare vs revaluation gains across wealth distribution

Figure: In level (2011 dollars)
Redistribution across sectors

- Households welfare gains aggregate to $\approx 10K$ per capita

- Who is the losing counterparty?

  \[
  \text{Welfare Gain}_{\text{Households}} = -\text{Welfare Gain}_{\text{Government}} - \text{Welfare Gain}_{\text{Foreigners}}
  \]
Redistribution across sectors

Welfare gains for different sectors:
- **Households**: Various welfare gains and losses, with negative values indicating losses.
- **Government**: Shows significant negative welfare gains, indicating losses.
- **Foreigners**: Shows a mixed picture with some gains and losses.

The chart includes categories such as Total, Housing, Debt, Deposits, and Equity, indicating how welfare gains are distributed across these sectors.
Generalizations of baseline sufficient statistics approach
Collateral effects

- Recall extension: interest rate schedule $Q_{i,t} = Q_t F(B_{i,t}, N_{i,t}P_t)$

\[
\text{Welfare Gain}_i = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left( (N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \frac{dQ_t}{Q_t} \right) \\
+ \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left( -B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t} dP_t \right)
\]

- Estimate second term by measuring effect of LTV on mortgage rates
Mortgage interest rates increase with loan-to-value ratio
Collateral effects

- Welfare gains due to collateral effect
  - $\beta = 0.0025$
  - $\beta = 0.005$
  - $\beta = 0.01$

- Initial age
  - $\beta = 0$
  - $\beta = 0.0025$
  - $\beta = 0.005$
  - $\beta = 0.01$
Valuations changes beyond end of our sample period

- We extend our baseline formula to account for future valuation changes:

\[
\text{Welfare Gain}_i \approx \sum_{t=0}^{T} R_{0 \rightarrow t}^{-1} (N_{i,t-1} - N_{i,t}) \Delta P_t + \sum_{t=T+1}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,t-1} - N_{i,t}) \Delta P_t
\]

- Estimate second term assuming: for \( t \geq T \)

\[
\frac{\Delta P_t}{P_t} = \frac{PD_t - \overline{PD}_t}{PD_t} \quad \text{with} \quad \log \left( \frac{PD_t}{PD} \right) = \phi^{t-T} \log \left( \frac{PD_T}{PD} \right), \quad \phi < 1
\]

\( N_{a,t-1} - N_{a,t} = N_{a,T-1} - N_{a,T} \) where \( a = \text{age} \)
Valuations changes beyond end of our sample period

- $\phi = 0$
- $\phi = 0.9$
- $\phi = 0.95$
- $\phi = 0.98$
Conclusion

- Simple framework to quantify welfare effects of asset price deviations
- Framework can be extended to take into account collateral effects, incomplete markets, …
- Application to Norway (1994–2019)
  1. large heterogeneity in welfare gains across households
  2. welfare gains ≠ revaluation gains
  3. redistribution from young to old and from poor to rich
  4. negative “welfare gain” for government ⇒ future net transfers ↓
- Could apply in other contexts, e.g. collapsing asset prices in recessions
- Optimal taxation? (Aguiar-Moll-Scheuer)