Lecture 3

The Textbook Heterogeneous Agent Model
Overview of Current Research

Macroeconomics EC442

Benjamin Moll

London School of Economics, Fall 2020

Recall: Plan for remaining lectures before break

1. [DONE] Income fluctuation problem a.k.a. consumption-saving problem with idiosyncratic labor income risk in partial equilibrium
   - Ethan already covered this
2. [DONE] Numerical dynamic programming a.k.a. numerical solution of Bellman equations
   - application: numerical solution of income fluctuation problem
3. Textbook heterogeneous agent model: Aiyagari-Bewley-Huggett
   - income fluctuation problem, embedded in general equilibrium
4. Overview of some current research
   - business cycles with heterogeneous agents (idiosyncratic + aggregate risk): Den Haan & Krusell-Smith
   - Heterogeneous Agent New Keynesian (HANK) models
   - Why is the wealth distribution so skewed?

Plan for rest of lecture

1. Refresher: neoclassical growth model

2. Key differences between representative and heterogeneous agent models

3. Textbook heterogeneous agent model: Aiyagari-Bewley-Huggett

4. Overview of some current research

5. From cross-section to aggregates: the “missing intercept problem”

Refresher:
Neoclassical Growth Model

Growth Model in Discrete Time

- Preferences: representative household with utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

- Technology:

$$y_t = f(k_t), \quad c_t + i_t = y_t$$
$$k_{t+1} = i_t + (1-\delta)k_t, \quad c_t \geq 0$$

- Endowments: $\hat{k}_0$ units of capital at $t = 0$

- Pareto optimal allocation solves

$$V(\hat{k}_0) = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.}$$
$$k_{t+1} = f(k_t) + (1-\delta)k_t - c_t, \quad k_0 = \hat{k}_0$$

Growth Model in Continuous Time

- Preferences: representative household with utility function

$$\int_0^\infty e^{-\rho t} u(c(t))dt$$

  $\rho \geq 0$ = discount *rate* (as opposed to β = discount *factor*)

- Technology:

$$y(t) = f(k(t)), \quad c(t) + i(t) = y(t)$$

$$\dot{k}(t) = i(t) - \delta k(t), \quad c(t) \geq 0, \quad k(t) \geq 0$$

- Endowments: $\hat{k}_0$ of capital at $t = 0$

- Pareto optimal allocation solves

$$V(\hat{k}_0) = \max_{\{c(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(c(t))dt \quad \text{s.t.}$$

$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t), \quad k(0) = \hat{k}_0$$

Optimality Condition: Euler Equation

- Discrete time

$$\lambda_t = \beta \lambda_{t+1}(f'(k_{t+1}) + 1 - \delta) \quad \text{where} \quad \lambda_t = u'(c_t)$$

or equivalently

$$u'(c_t) = \beta u'(c_{t+1})(f'(k_{t+1}) + 1 - \delta)$$

- Continuous time

$$\dot{\lambda}(t) = (\rho + \delta - f'(k(t)))\lambda(t) \quad \text{where} \quad \lambda(t) = u'(c(t)) \qquad (*)$$

- Derivation of $(*)$: Hamiltonian

$$\mathcal{H} = u(c) + \lambda[f(k) - \delta k - c]$$

- Question: how many of you know how go from Hamiltonian to $(*)$?

Steady State

- Steady state: “if you start there you stay there”
  - look for k^*, c^*, λ^* such that this is true, e.g. if $k_t = k^*$ then also $k_{t+1} = k^*$
  - in particular, in Euler equation set $\lambda_t = \lambda_{t+1}$ or $\dot{\lambda}(t) = 0$
- Discrete time: steady state capital stock solves

$$1 = \beta(f'(k^*) + 1 - \delta) \tag{DSS}$$

- Continuous time: steady state capital stock solves

$$f'(k^*) = \rho + \delta \tag{CSS}$$

- Note: this is the same equation
  - define discrete-time discount rate $\rho = 1/\beta - 1$
  - then (DSS) reduces to (CSS)

Infinitely-elastic steady state capital supply

- Recall condition for k^* (as usual $\rho = 1/\beta - 1$)

$$\frac{1}{\beta} = f'(k^*) + 1 - \delta \quad \Leftrightarrow \quad f'(k^*) = \rho + \delta$$

- Can think of this in terms of demand and supply of capital
- Will draw demand-supply diagram with k on x-axis and r on y-axis
- Demand: capital demand $k^d(R)$ satisfies

$$f'(k) = R$$

  This is a nice, well-behaved downward-sloping demand curve

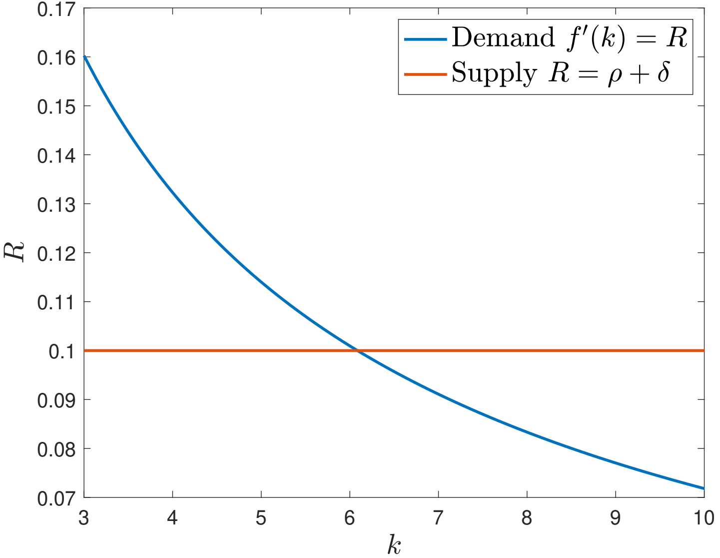
- Supply: capital supply $k^s(R)$ satisfies

$$R = \rho + \delta$$

  This is an infinitely-elastic supply curve! Intuition in 3 slides.

- This infinite elasticity = important property of growth model

Infinitely-elastic steady state capital supply



Capital Demand: Derivation

- Recall representative firm's optimality condition

$$F_k(k_t, h_t) = R_t$$

- Defining $f(k) := F(k, 1)$ and using $h_t = 1$

$$f'(k_t) = R_t$$

- And in particular in steady state:

$$f'(k^*) = R$$

- This defines a downward-sloping capital demand curve $k^d(R)$

Capital Supply: Derivation

- Euler equation for capital

$$u'(c_t) = \beta(R_{t+1} + 1 - \delta)u'(c_{t+1})$$

- In steady state

$$1 = \beta(R + 1 - \delta)$$

- Therefore the steady state rental rate must equal

$$R = \frac{1}{\beta} - 1 + \delta = \rho + \delta \qquad (*)$$

- This is an infinitely-elastic supply curve! Intuition:
  - if $\beta(R + 1 - \delta) > 1$, households would accumulate $k = \infty$

    $$\beta(R_{t+1} + 1 - \delta) > 1 \quad \Rightarrow \quad c_{t+1} > c_t$$

  - if $\beta(R + 1 - \delta) < 1$, households would accumulate 0

    $$\beta(R_{t+1} + 1 - \delta) < 1 \quad \Rightarrow \quad c_{t+1} < c_t$$

  - any equilibrium with $0 < k^* < \infty$ has to feature $(*)$

Supply and demand in terms of interest rate r

- Sometimes people also write this in terms of the steady-state interest rate r rather than rental rate R
  - recall that alternative "decentralization" = firms own and accumulate capital (and firms in turn owned by households)
- Demand: capital demand $k^d(r)$ satisfies

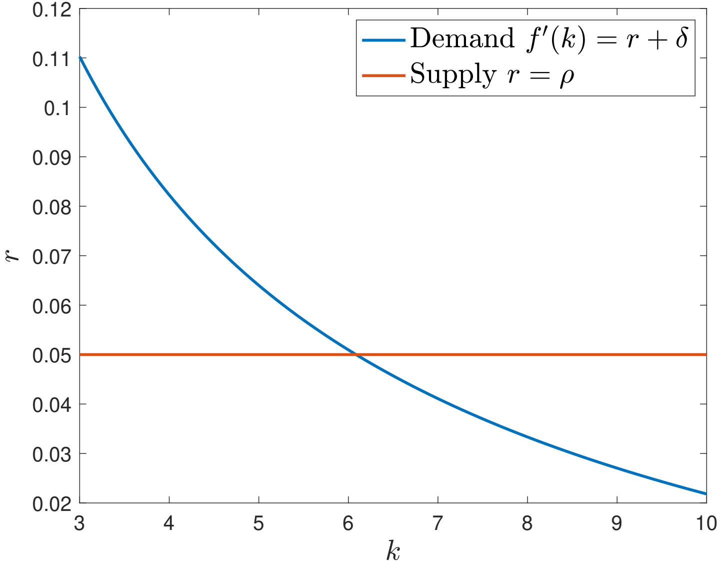
$$f'(k) = r + \delta$$

  This is a nice, well-behaved downward-sloping demand curve
- Supply: capital supply $k^s(r)$ satisfies

$$r = \rho$$

  This is an infinitely-elastic supply curve! Intuition:
  - if $r > \rho$, households would accumulate $k = \infty$
  - if $r < \rho$, households would accumulate 0
  - any equilibrium with $0 < k^* < \infty$ has to feature $r = \rho$

Supply and demand in terms of interest rate r

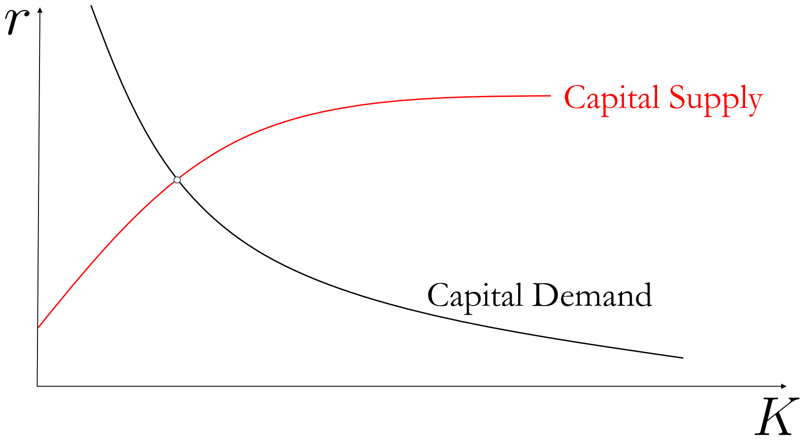


Rep vs Heterog Households:
Key Differences

Four key differences between RA and HA models

1. Wealth distribution
    - RA: degenerate or indeterminate stationary distribution
    - HA: non-degenerate stationary distribution
2. Long-run capital supply
    - RA: infinite elasticity
    - HA: finite elasticity
3. Borrowing constraints, marginal propensities to consume (MPCs)
    - RA: low MPCs
    - HA: potentially high MPCs
4. Welfare theorems
    - RA (for this point = growth model): typically hold
    - HA: typically do not hold

Key difference 2: long-run capital supply in HA models



The Textbook Heterogeneous Agent Model

Aiyagari-Bewley-Huggett

From income fluctuation problem to agg capital supply

1. [DONE] Individuals are subject to exogenous income shocks. These shocks are not fully insurable because of the lack of a complete set of Arrow-Debreu contingent claims

2. [DONE] There is only a risk-free asset (i.e., and asset with non-state contingent rate of return) in which the individual can save/borrow, and that the individual faces a borrowing (liquidity) constraint

3. [DONE] A continuum of such agents subject to different shocks will give rise to a wealth distribution

4. Integrating wealth holdings across all agents will give rise to an aggregate supply of capital

Aggregate Capital Supply

- For a given interest rate r, we can compute stationary distribution $g(a, y; r)$. Since g is a density, it satisfies:

$$g(a, y) \geq 0, \qquad \sum_j \int_{\underline{a}}^{\infty} g(a, y_j; r) da = 1$$

- Note: dist will typically have mass points e.g. at $\underline{a}$ so we should really treat dist as measure and write $\sum_j \int_{\underline{a}}^{\infty} G(da, y_j; r) = 1$ etc
    - my notation will simply ignore this
    - numerical g is not function anyway (vector or via simulation)

- Compute aggregate savings in stationary distribution:

$$A(r) = \sum_j \int_{\underline{a}}^{\infty} a g(a, y_j; r) da$$

- When $r = -1$ (discrete time), no-one saves so $A(-1) = \underline{a}$
- When $r = \beta^{-1} - 1$ (equivalently, $r = \rho$), assets explode: $A(r) \to \infty$

Precautionary Savings

- Intuition for why savings diverge when $1 + r = \beta^{-1}$ (equivalently $r = \rho$): Precautionary savings

- Households have three motives for saving in this model:
  1. Inter-temporal motive: difference between $1 + r$ and β
  2. Smoothing motive: concavity of utility function
  3. Precautionary motive: either (i) presence of occasionally binding borrowing constraint; or (ii) convexity of marginal utility of consumption (see Ljungqvist-Sargent textbook)

- Precautionary motive leads agents to continue to save even when inter-temporal motive is shutdown, i.e. $r = \rho$. For total assets to remain bounded, we require $r < \rho$
  - for proof sketch see Ethan's note 3, slide 15 (the argument using the super-martingale convergence theorem)

Shape of Aggregate Savings Function, $A(r)$

- See graphs on whiteboard
- How would you compute these graphs on a computer?
- $A(r)$ is continuous if no discontinuity in underlying consumption-savings problem when varying r
  - claim based on results in Stokey-Lucas-Prescott
- If IES≥ 1, then $A(r)$ is strictly increasing (Achdou et al, 2017). But this is not a necessary condition. In general $A(r)$ need not be strictly increasing but in most applications it is.
  - see Ethan's notes for definition of IES = intertemporal elasticity of substitution
  - e.g. with CRRA utility $\frac{c^{1-\sigma}}{1-\sigma}$, IES = $1/\sigma$ so IES≥ 1 means $\sigma \leq 1$ so log utility or less concave
  - what's the intuition for the condition IES≥ 1?

Stationary Equilibrium Interest Rate

- Stationary equilibrium interest rate r determined by equating demand and supply in the market for assets in the ergodic distribution of households

- Since $A(r) \in [\underline{a}, \infty)$ and continuous, an equilibrium will exist if the demand for assets is either constant or decreasing in the interest rate.

- Different GE HA models: different assumptions about how to interpret assets and how they are supplied:

  1. Huggett model: private IOUs in zero net supply
  2. Bewley model: money or bonds in positive net supply
  3. Aiyagari model: capital in positive net supply

- Compare rep agent model: $A(r)$ perfectly elastic at $r = \rho$

Stationary equilibrium: some general remarks

- Conceptually = steady state: "if you start there you stay there"
  - difference to before: now looking for entire distribution such that this is true!
- Importantly: aggregates constant (like st. st. in growth model)...
- but rich dynamics at individual level
  - individuals/cohorts "churning around" in stationary distribution
- Typically, no analytic solutions for stationary equilibrium
- $\Rightarrow$ solve for stationary equilibrium numerically
  - challenge: have to find stationary wealth distribution
  - much easier than time-varying equilibrium because prices (e.g. w^*, r^*) are just scalars

Huggett Model: Assets in Zero Net Supply

- Equilibrium interest rate determined by market clearing condition

$$A(r) = 0$$

- Important that households are allowed to borrow, i.e. $\underline{a} < 0$
- Compute by iterating on interest rate until convergence or using a one-dimensional equation solver

Huggett Model: Definition of Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is

1. Value and policy functions: $V(a,y), c(a,y), s(a,y)$
2. Distribution of households: $g(a,y)$
3. Interest rate: r

such that

1. Given r, the function $V(a,y)$ solves the household problem, i.e. satisfies the Bellman eqn:

$$V(a,y_j) = \max_{c,a' \geq \underline{a}} u(c) + \beta \sum_{j'} p_{jj'} V(a', y_{j'}) \quad \text{s.t.} \quad a' = (1+r)a + y - c$$

   The implied policy functions are $c(a,y)$ and $a'(a,y) = (1+r)a + y - c(a,y)$.

2. Given the saving policy function $a'(a,y)$ and transition probabilities $p_{jj'}$, the distribution $g(a,y)$ is the corresponding stationary distribution

3. Given the distribution $g(a,y)$, the market for asset clears:

$$\sum_j \int_{\underline{a}}^{\infty} a g(a,y_j) da = 0$$

Bewley Model: Assets in Positive Supply

- Government issues bonds B, finances interest payments and govt spending G by collecting taxes according to tax function $\tau(a, y)$
- Total tax revenues are

$$T(r) = \sum_j \int_a \tau\left(a, y_j\right) g\left(a, y_j; r\right) da$$

- Government budget constraint: $G + rB = T(r)$
- Market clearing condition

$$A(r) = B$$

- Computation with exogenous B: As in Huggett economy, determine $G(r) = T(r) - rB$ as residual, provided $G(r) \geq 0$
- Computation with exogenous G: Solve $A(r) = \frac{T(r)-G}{r}$ and determined equilibrium B endogenously

Aiyagari Model: Add Production Side

- Representative firm with CRS production technology

$$Y = K^{\alpha} L^{1-\alpha}$$

- Firm rents capital from households at rate r and hires efficiency units of labor at wage rate w:

$$r + \delta = \alpha \left(\frac{K}{L} \right)^{\alpha - 1}$$

$$w = (1 - \alpha) \left(\frac{K}{L} \right)^{\alpha}$$

- Note that this implies a one-to-one mapping between w and r

$$w(r) = (1 - \alpha) \left(\frac{\alpha}{r + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- HH's supply "efficiency units of labor" y_j, budget constraint is

$$c + a' = w y_j + (1 + r) a$$

Market Clearing

- Labor market clearing: exogenous labor supply

$$
\begin{aligned}
L &= \sum_j \int_a y_j g\left(a, y_j; r\right) da \\
&= \sum_j y_j \pi_j
\end{aligned}
$$

where $\pi_j :=$ stationary dist of income process = $\int_a g\left(a, y_j; r\right) da$

- Capital market clearing

$$
\begin{aligned}
A(r) &= K(r) \\
&= L\left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

Aiyagari Model: Definition of Equilibrium

A stationary Recursive Competitive Equilibrium (RCE) is

1. Value and policy functions: $V(a, y), c(a, y), s(a, y)$
2. Factor Demands: K, L
3. Distribution of households: $g(a, y)$
4. Prices: r, w

such that

1. Given r, w, the function $V(a, y)$ solves the hh problem, i.e. satisfies the Bellman eqn:

$$V(a, y_j) = \max_{c, a' \geq \underline{a}} u(c) + \beta \sum_{j'} p_{jj'} V(a', y_{j'}) \quad \text{s.t.} \quad a' = (1 + r)a + wy - c$$

   The implied policy functions are $c(a, y)$ and $a'(a, y) = (1 + r)a + wy - c(a, y)$.

2. Given r, w, the factor demands K, L solve the firm FOC
3. Given the saving policy function $a'(a, y)$ and transition probabilities $p_{jj'}$, the distribution $g(a, y)$ is the corresponding stationary distribution
4. Given the distribution $g(a, y)$, the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^{\infty} a g(a, y_j) da = K \qquad \sum_j y_j \pi_j = L$$

Main Graph of Aiyagari (1994)

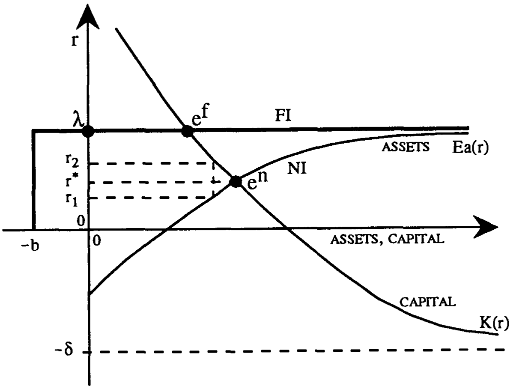
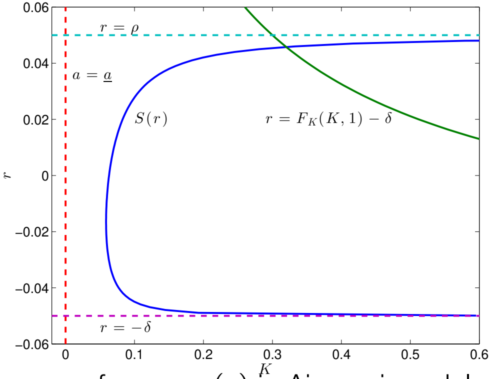


FIGURE IIb
Steady-State Determination

- Aiyagari's λ is our ρ, and his E$a(r)$ curve is our $A(r)$
- To get E$a(r)$, feed r & $w(r) = (1-\alpha)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}$ into hh problem

More precise version of this graph



- That is, presence of wage $w(r)$ in Aiyagari model already means that $A(r)$ function will typically be non-monotonic
- Nevertheless this is typically not a source of multiplicity
- Reason: like in perpetual youth model in Lecture 2, CRRA + C-D $\Rightarrow$ can write things such that everything scales with w and w drops

Aside: If assets = capital, how can they be < 0?

- Capital $K \geq 0$. If $\underline{a} < 0$, hh's can have negative assets $a < 0$
- At same time, agg assets = capital, $A(r) = K$. Or indexing individual hh's by $i \in [0, 1]$ (alternative notation = useful below)
$$\int_0^1 a_i di = K$$
- Question: So how does this make sense? If assets = capital, how can assets be negative?
- Answer: key is that assets = capital only in aggregate. There can still be borrowing and lending among different households.
- Easiest way to operationalize this:
  - households hold two assets capital $k_i \geq 0$ and bonds $b_i \gtrless 0$
  - household wealth $a_i = k_i + b_i \gtrless 0$
  - capital & bond market clearing (bonds in zero net supply)
$$K = \int_0^1 k_i di, \quad 0 = \int_0^1 b_i di \quad \Rightarrow \quad \int_0^1 a_i di = K$$

Computation of Equilibrium

- Any non-linear equation solver can be used to solve: $A(r) = K(r)$
- Often useful to iterate on $\kappa := \frac{K}{L}$. Using $r = \alpha\kappa^{\alpha-1} - \delta$:

$$\kappa = \frac{A\left(\alpha\kappa^{\alpha-1} - \delta\right)}{L}$$

- Suggests updating rule

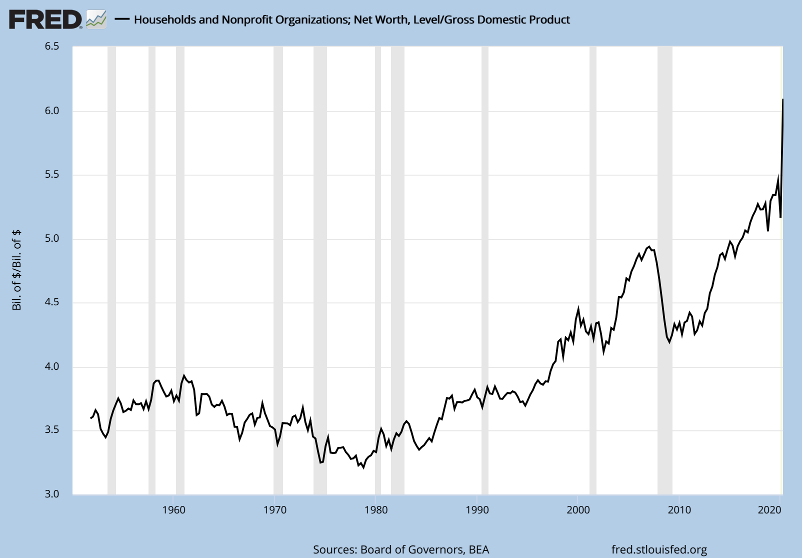
$$\kappa_{\ell+1} = \omega\frac{A\left(\alpha\kappa_{\ell}^{\alpha-1} - \delta\right)}{L} + (1-\omega)\,\kappa_{\ell}$$

where $\omega \in [0, 1]$ is dampening parameter

Aiyagari model: some aspects of the calibration

- **Discount rate:** choose discount rate β so as to match aggregate or average wealth-income ratio
  - option 1 = macro target: agg wealth/GDP from national accounts, e.g. for U.S. $\approx 3-5$ depending on time period and whether include residential capital
  - option 2 = micro target: e.g. average wealth from households survey, say SCF (next slides)
- **Labor income process:** estimate from micro data on individual income dynamics
- **Borrowing constraint:** calibrate the borrowing constraint in order to match, say, the fraction of agents with negative net worth which is around 10% in the U.S. economy.

Aggregate Wealth-GDP Ratio for U.S. Economy



- Source `https://fred.stlouisfed.org/graph/?g=dGy`

Average Wealth (SCF 2016)

Wealth Definition		All	Exclude top 1%
Total	Mean	11.2	7.8
	Median	1.6	1.7
Non-housing	Mean	8.5	5.3
	Median	0.5	0.5
Financial	Mean	5.3	4.0
	Median	0.3	0.3
Mean earnings		$61,600	$54,900
Mean income		$102,200	$ 84,300

- Wealth numbers expressed as ratios to mean household earnings
- Average wealth = $11.2 \times \$61,600 = \$689,920$!
- Average wealth/average income = $\$689,920/\$102,200 = 6.75$
  - surprisingly hard to square with agg numbers on previous slide

Low Wealth Households (SCF 2016)

Wealth Definition		
Total	$<= 0$	11%
	$<= \$2,000$	17%
Non-housing	$<= 0$	15%
	$<= \$2,000$	22%
Financial	$<= 0$	11%
	$<= \$2,000$	31%

Comparing Model and Data

Baseline Model Wealth Statistics – `egp_AR1_IID_tax.m`

Discount factor	0.945	0.95	**0.955**	0.96	0.97
Var log gross labor inc	0.982	0.982	**0.982**	0.982	0.982
Gini gross labor inc	0.505	0.505	**0.505**	0.505	0.505
Var log net labor inc	0.982	0.982	**0.982**	0.982	0.982
Gini net labor inc	0.505	0.505	**0.505**	0.505	0.505
Var log consumption	0.987	0.980	**0.966**	0.941	0.833
Gini consumption	0.497	0.493	**0.486**	0.476	0.443
Mean wealth	1.414	2.067	**3.053**	4.599	12.003
Median wealth	0.017	0.130	**0.379**	0.930	4.935
Gini wealth	0.858	0.831	**0.799**	0.762	0.662
P90-P50 wealth	220	44	**23**	14	7
P99-P50 wealth	1217	209	**94**	50	17
Frac wealth<=0	47%	30%	**25%**	20%	6%
Frac wealth<= 5% $E[y]$	53%	46%	**39%**	26%	11%
Top 10% wealth share	75%	70%	**64%**	59%	47%
Top 1% wealth share	22%	19%	**17%**	14%	9%
Top 0.1% wealth share	4%	3%	**3%**	2%	1%

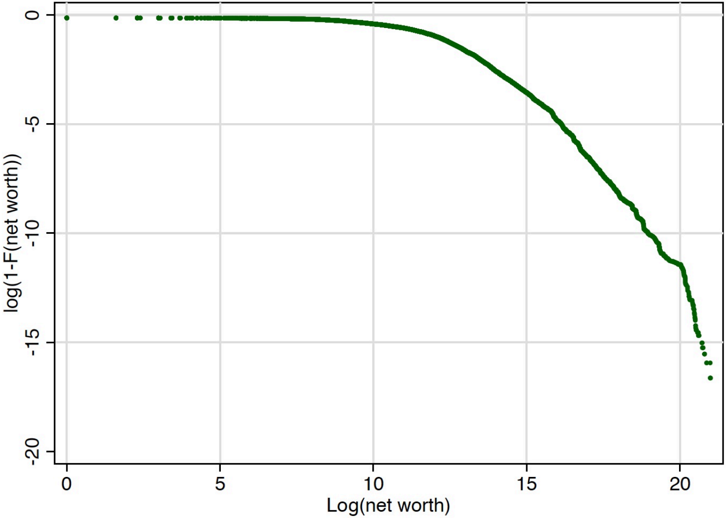
Wealth Statistics in SCF 2016

Full Distribution	Total	Non-Housing	Financial
Mean	11.19	8.48	5.34
Median	1.58	0.49	0.31
P90	19.3	12.6	9.6
P99	168.0	141.4	87.0
P99.9	700.5	634.6	358.9
P90-P50 Ratio	12	26	31
P99-P50 Ratio	106	288	284
Top 10% Share	77%	84%	81%
Top 1% Share	39%	45%	40%
Top 0.1% Share	15%	18%	14%
Gini	0.86	0.91	0.89
Frac <=0	11%	15%	11%
Frac <= $2000	17%	22%	31%
Frac <= 5% Av Earns	18%	23%	34%

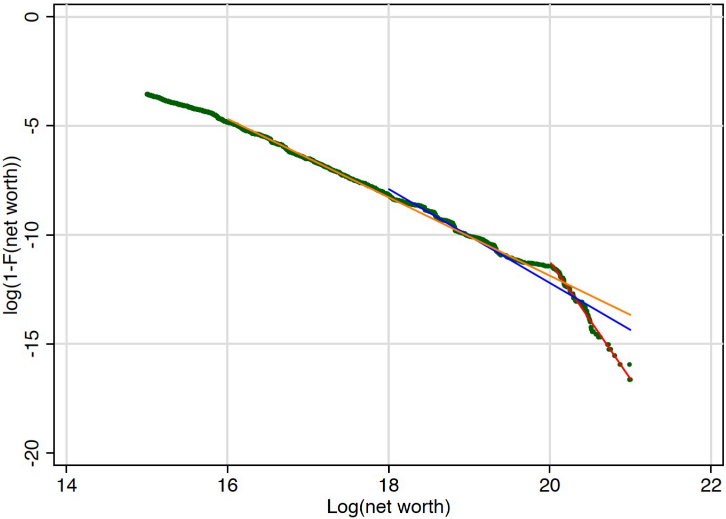
Wealth Statistics in SCF 2016: Exclude Top 1%

Excluding top 1%	Total	Non-Housing	Financial
Mean	7.79	5.32	3.95
Median	1.71	0.53	0.33
P90	19.8	12.6	9.7
P99	106.0	85.1	56.9
P99.9	179.1	163.1	133.5
P90-P50 Ratio	12	24	30
P99-P50 Ratio	62	160	174
Top 10% Share	65%	74%	73%
Top 1% Share	18%	23%	23%
Top 0.1% Share	2%	3%	4%
Gini	0.79	0.86	0.84
Frac <=0	12%	15%	11%
Frac <= $2000	17%	22%	31%
Frac <= 5% Av Earns	18%	23%	33%

Pareto Tail SCF 2013



Pareto Tail SCF 2013



Model Wealth Statistics with Transitory Shocks

Transitory shock size	0.05	0.1	**0.2**	0.25	**0.2**
Discount factor	0.955	0.955	**0.955**	0.955	**0.9**
Var log gross labor inc	0.985	0.992	**1.021**	1.043	**1.021**
Gini gross labor inc	0.509	0.513	**0.521**	0.526	**0.521**
Var log net labor inc	0.985	0.992	**1.021**	1.043	**1.021**
Gini net labor inc	0.509	0.513	**0.521**	0.526	**0.521**
Var log consumption	0.964	0.963	**0.959**	0.957	**1.001**
Gini consumption	0.486	0.486	**0.486**	0.486	**0.513**
Mean wealth	3.071	3.105	**3.212**	3.285	**0.178**
Median wealth	0.399	0.430	0.543	0.617	**0.026**
Gini wealth	0.795	0.790	**0.776**	0.767	**0.814**
P90-P50 wealth	22	21	**17**	15	**16**
P99-P50 wealth	90	83	**66**	59	**90**
Frac wealth<=0	6%	5%	**4%**	3%	**24%**
Frac wealth<= 5% $E[y]$	34%	29%	**20%**	16%	**59%**
Top 10% wealth share	64%	64%	**62%**	61%	**69%**
Top 1% wealth share	16%	16%	**16%**	16%	**25%**
Top 0.1% wealth share	3%	3%	**3%**	3%	**6%**

Modifications I: Discount Factor Heterogeneity

Discount factor spread	$\pm 5\%$	$\pm 6\%$	$\mathbf{\pm 6.5\%}$	$\pm 7\%$	$\mathbf{\pm 6.5\%}$
Mean discount factor	0.9	0.9	**0.9**	0.9	**0.9**
Switching probability	0	0	**0**	0	$\frac{1}{40}$
Var log consumption	0.992	0.977	**0.963**	0.946	**0.987**
Gini consumption	0.505	0.497	**0.491**	0.483	**0.507**
Mean wealth	1.152	2.400	**3.678**	6.073	**1.184**
Median wealth	0.046	0.064	**0.083**	0.120	**0.060**
Gini wealth	0.883	0.866	**0.850**	0.826	**0.864**
P90-P50 wealth	57	103	**133**	162	**48**
P99-P50 wealth	423	555	**594**	582	**306**
Frac wealth<=0	25%	24%	**24%**	24%	**22%**
Frac wealth<= 5% $E[y]$	51%	48%	**46%**	44%	**48%**
Top 10% wealth share	82%	77%	**73%**	67%	**78%**
Top 1% wealth share	28%	22%	**19%**	16%	**25%**
Top 0.1% wealth share	5%	4%	**3%**	3%	**5%**

Modifications II: Bequests

Warm-glow B	0	0.07	0.07	0.07	0.08
Luxury parameter ζ	0	0.01	4	6	4
Discount factor	0.955	0.955	0.955	0.955	0.955
Var log consumption	0.959	0.973	0.964	0.962	0.971
Gini consumption	0.486	0.488	0.485	0.484	0.485
Mean wealth	3.212	3.245	3.421	3.483	4.400
Median wealth	0.543	0.147	0.188	0.197	0.203
Gini wealth	0.776	0.900	0.887	0.884	0.893
P90-P50 wealth	17	37	33	33	39
P99-P50 wealth	66	421	332	318	398
Frac wealth<=0	4%	9%	8%	8%	8%
Frac wealth<= 5% $E[y]$	20%	33%	30%	30%	29%
Top 10% wealth share	62%	86%	83%	83%	84%
Top 1% wealth share	16%	36%	34%	33%	34%
Top 0.1% wealth share	3%	9%	8%	8%	8%

Overview of Current Research

Some general observations

1. If you want to ask policy questions about whole economy, you will typically need a macro model (at least think through one)
    - policy question = question about counterfactuals
    - GE effects/spillovers typically key – think Keynesian cross
    - estimates identified off macro variation hard to come by
    - estimates identified off cross-sectional var in micro data (DiD, RCTs) silent on GE effects/spillovers ("missing intercept")
2. If question involves economy's household side, the current go-to model is some variant of the Aiyagari model
    - "Aiyagari on steroids" = basis for much of current research
    - will show you 2-3 from broad spectrum – there are many more
3. Also recall examples of succesful JMPs in my first lecture

Overview of some current research

1. Wealth inequality at the top
2. Monetary and fiscal policy with heterogeneous households
    - Heterogeneous Agent New Keynesian (HANK) models
3. A good model for doing “micro to macro” research
4. General equilibrium effects in development economics

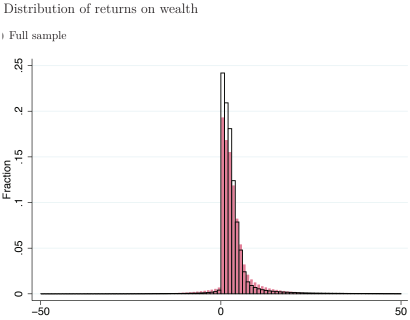
- For a nice history of earlier HA macro, see Beatrice Cherrier’s blog
  `https://beatricecherrier.wordpress.com/2018/11/28/`
  `heterogeneous-agent-macroeconomics-has-a-long-history-and-it-raises-many-questions/`
- Interesting open questions in HA macro? See Section 4 of
  `http://benjaminmoll.com/research_agenda_2020/` (= my views obviously)

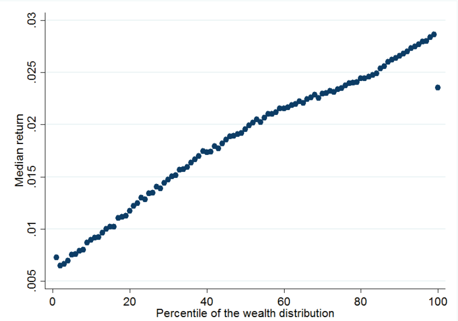
1. Wealth inequality at the top

- Standard model does not generate enough wealth at the top: e.g. top 1% wealth share in model = 15%, in data = 40%
  - Heterogeneity in discount factors: patients households (richer) save more (Krusell and Smith, 1998)
  - Non-homothetic preferences: rich save more, e.g. to bequeath (Atkinson, De Nardi, Straub)
  - Super high but transitory income realization "awesome state": rich save more for precautionary reasons (Castaneda & al, 2003)
  - Heterogeneous/stochastic rates of return (Benhabib & al, 2014)
  - Entrepreneurs with projects yielding higher, but stochastic, rate of return than r (Quadrini, 2000)
- See survey "Skewed Wealth Distribution: Theory and Empirics" by Benhabib-Bisin
- Current work: empirical evidence for these ingredients?

Example: Returns to Wealth – Fagereng et al (2019)

- Using Norwegian administrative data (Norway has wealth tax), document massive heterogeneity in returns to wealth
  - range of over 500 basis points between 10th and 90th pctile
  - returns positively correlated with wealth



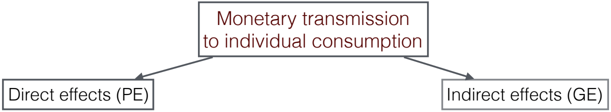


- Note: figures are from working paper version

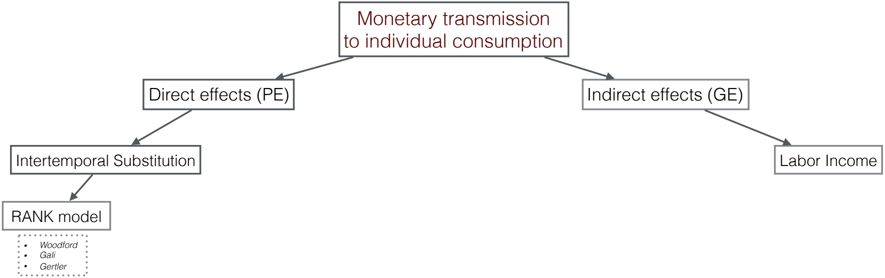
2. Monetary and fiscal policy

- HA models have potential to explain distribution of MPCs observed in data
- Long tradition: emphasis on MPCs as important for monetary and fiscal policy
  - e.g. Keynesian cross and multiplier = $\frac{1}{1-\text{MPC}}$
- Fast-growing literature studies how policy affects households and aggregate economy in models with realistic MPC distributions
  - here at LSE, Ricardo, Wouter and myself have worked on this
  - next slides: monetary policy
  - more? Section 1 of `http://benjaminmoll.com/research_agenda_2020/`

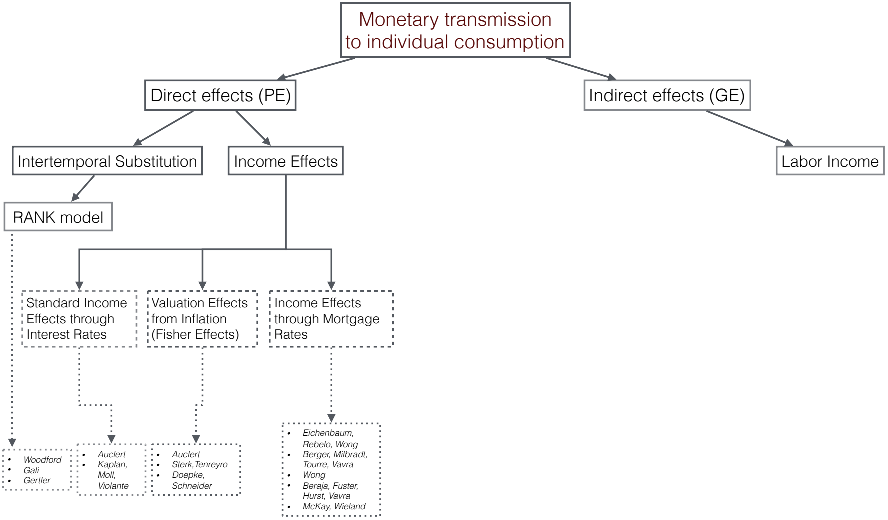
Monetary policy and consumption (RANK, HANK,...)



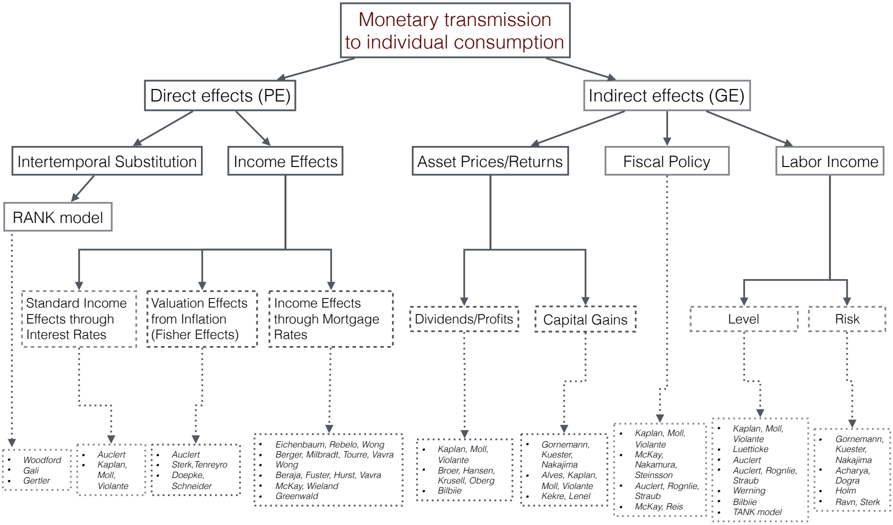
RANK: all about intertemporal substitution (Euler Eqn)



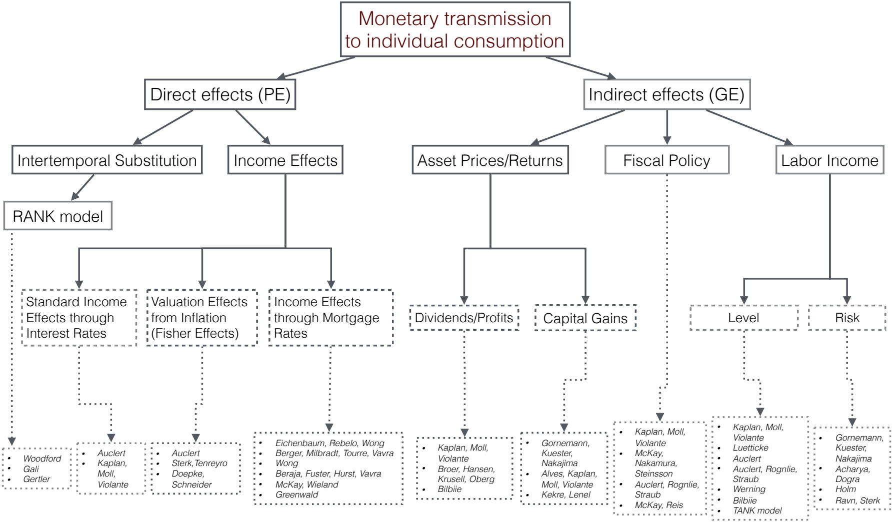
HANK: emphasizes alternative direct effects...



HANK: ... and indirect effects (given high MPCs)



Macro has come long way since RA Euler equation



Policy implications of HA models?

Nice example: Wolf (2021) “Interest Rate Cuts vs. Stimulus Payments: A Macro Equivalence Result” `https://www.christiankwolf.com/research`

For background, see Correia-Farhi-Nicolini-Teles (AER, 2013) “Unconventional Fiscal Policy at the Zero Bound”

When the zero lower bound on nominal interest rates binds, monetary policy cannot provide appropriate stimulus. We show that, in the standard New Keynesian model, tax policy can deliver such stimulus at no cost and in a time-consistent manner. There is no need to use inefficient policies such as wasteful public spending or future commitments to low interest rates. (*JEL* E12, E43, E52, E62, H20)

- Influential result, used for actual policy
- Example: explicit motivation for Germany’s 2020 VAT reduction
- e.g. `https://twitter.com/BachmannRudi/status/1268308925780242437`

Policy implications of HA models?

Nice example: Wolf (2021) "Interest Rate Cuts vs. Stimulus Payments: A Macro Equivalence Result" `https://www.christiankwolf.com/research`

1 Introduction

The prescription of standard New Keynesian theory is to conduct stabilization policy through changes in short-term nominal interest rates. Recently, however, with interest rates close to an effective lower bound (ELB), the policy space for rate-based stimulus has narrowed. A natural question, then, is how to replicate the effects of conventional monetary policy through other, feasible policy instruments. Previous work has largely focussed on fiscal purchases (Christiano et al., 2011), commitment to future monetary policy (Werning, 2011), and time variation in tax policy (Farhi & Werning, 2007; Correia et al., 2013). Among those, time-varying tax rates – labeled unconventional fiscal policy – are, at least in theory, particularly appealing: they require no wasteful public spending, are time-consistent, and can robustly replicate any desired monetary allocation. The key challenge is practical feasibility: to mimic monetary stimulus, a policymaker needs to be able to fine-tune consumption and labor taxes at business-cycle frequencies. That is a formidable task.

Policy implications of HA models?

Nice example: Wolf (2021) "Interest Rate Cuts vs. Stimulus Payments: A Macro Equivalence Result" `https://www.christiankwolf.com/research`

In this paper I focus on a different, entirely conventional fiscal policy instrument: uniform lump-sum payments ("stimulus checks") sent to households, as seen in each of the past three U.S. recessions. I show that, in a standard business-cycle model with nominal rigidities and a general departure from Ricardian equivalence, any sequence of macroeconomic aggregates that is implementable via conventional interest rate policy is also implementable by adjusting uniform lump-sum transfers. This policy equivalence fails in the limit case of Ricardian consumers, but generically holds in models with incomplete asset markets or behavioral biases in consumption. Having established that an equivalent transfer policy exists, I then proceed to explicitly characterize it. I do so under the assumption of a particular form of Ricardian non-equivalence: uninsurable household income risk. In analytically tractable models of such market incompleteness, I can prove that the mapping from interest rate policies to equivalent transfer stimulus is fully characterized by a small number of empirically measurable sufficient statistics. I then document that this sufficient statistics characterization remains almost exact even in a rich Heterogeneous Agent New Keynesian (HANK) model.

3. A Good Model for Doing "Micro to Macro" Research

- What should interplay of theory and data look like?
- What's a good model for doing macro work that takes heterogeneity and aggregation seriously?
- Disclaimer: like everything else, my personal opinion – really there is no single "right" approach here!
- I like the following model for doing research:
    - combine "causal" micro estimates with macro GE model. Examples: `http://benjaminmoll.com/micro_to_macro/`
    - idea has been around for some time but increasingly popular
    - nicely fits in with het agent / distributional macro philosophy
    - Nakamura & Steinsson (2018) call this "identified moments" (see footnote: "The term 'identified moments' may seems odd to some...")
    - typical strategy for empirically disciplining parameters of macro models: use some set of moments (calibration or GMM)
    - key idea: some moments are better than others

Example: Marginal Propensity to Consume

- Huge literature, some with arguably random variation:
  - e.g. Johnson-Parker-Souleles, Parker-Souleles-Johnson-McClelland, Fagereng-Holm-Natvik, ...
- Idea: MPCs from this literature are credibly "identified moments"
- $\Rightarrow$ if you have a macro model, and MPCs are central to what you are using it for, your model better match these MPC estimates
- Nice example: Kaplan and Violante (2014)
- In principle, could include MPC estimates as explicit calibration targets (if I recall correctly, Kaplan-Violante don't)
- End product of "identified moments" research model:
  - structural model that can be used for policy analysis
  - but at least partly satisfies "applied micro standard" for credible identification of a causal effect

4. GE effects in development economics

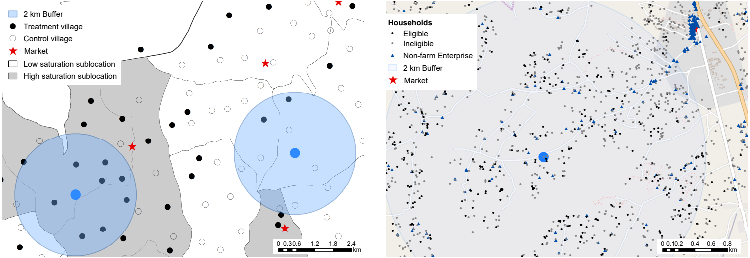
- Very interesting recent paper
- Egger-Haushofer-Miguel-Niehaus-Walker "General equilibrium effects of cash transfers: experimental evidence from Kenya"
- Collaboration with `https://www.givedirectly.org/` = NGO that makes unconditional cash transfers in east Africa (co-founded by Niehaus)
- RCT provides $\approx$ \$1000 to $> 10k$ poor households across 653 randomized villages = $> 15\%$ of local GDP
- Paper's goal: identify general equilibrium effects
    - key idea: generate spatial variation in intensity of transfers (similar to Miguel-Kremer 2004 worms paper)
    - headline result: local fiscal multiplier = 2.6 (= very large)

Key idea: spatial variation

GE effects via spatial var in share of neighboring villages that are treated

Figure B.1: Spatial variation of data and treatment

(a) Comparing 2 villages (b) Households and enterprises



Notes: This figure provides an example of the spatial variation that we use to identify spillover effects. Both panels provide zoomed-in views on a selection of villages from Figure 2. Panel A illustrates variation in the density of treatment villages around 2 treated villages. It plots village centers for treatment (filled circles) and control (open circles) villages, as well as a 2 km radius around the village center. While both villages themselves are treated, the share of treated villages around them varies considerably. Panel

- Estimate specifications like (i = households, v = village, s = sublocation)

$$y_{ivs} = \alpha_1 Treat_v + \alpha_2 HighSat_s + \delta_1 y_{ivs,t=0} + \delta_2 M_{ivs} + \varepsilon_{ivs}, \quad (1)$$

- Idea: estimate of α_2 identifies GE effects/spillovers

4. GE effects in development economics

- Identify GE effects within sublocations/2km radii (=blue circles)
  - = what "local" in "local fiscal multiplier" means
- Interesting finding: large consumption ↑ in untreated villages
  - puzzling: even though income, labor supply unchanged
- GE effects across subloc'ns (> 2-5 km) e.g. tradables demand ↑?
  - silent on those by design ("missing intercept")
- Thinking through macro model would be helpful for two purposes
  1. What's going on? Mechanism?
  2. GE effects across sublocations, e.g. regional multipliers?
- Some sort of variant of Aiyagari model, perhaps with entrepreneurs, seems like right model to think through this
- This type of cross-field work = great opportunity for grad students!

From Cross-section to Aggregates: the “Missing Intercept Problem”

The Missing Intercept Problem

- Example: Autor-Dorn-Hanson (2013) "import competition explains one-quarter of the contemporaneous aggregate decline in US manufacturing employment"
- Arrive at this number by scaling regression coefficient estimated from regional data by total Chinese import penetration
- Important: can only do this under strong assumptions
- True much more generally, whenever you want to learn about aggregates from cross-sectional variation
- Point made in many of these `http://benjaminmoll.com/micro_to_macro/`
  - some of them: strategies for recovering missing intercept
- Next slides: explain issue in context of "stock market capital gains $\Rightarrow$ consumption, employment?" (ChodorowReich-Nenov-Simsek)
  - but point much more general, see list at end of section

The Missing Intercept Problem

- Problem: regression coefficient estimated with x-sectional variation only tells you what happens in some counties relative to others...
  - what happens in counties with large capital gains relative to those with small capital gains
- ... but not the aggregate effect of stock market capital gains
- Extreme case (just to make the point):
  - employment in high-cap-gain counties unaffected
  - employment in low-cap-gain counties actually decreases
  - $\Rightarrow$ in cross-section, observe positive correlation between cap gains and employment
- Can also imagine opposite: cap gains increase employment a lot in both low- and high-cap-gain counties (just more so in the latter)
- Naively scaling up coefficient estimated with x-sectional variation gives completely wrong result – "Missing Intercept Problem"

The Missing Intercept Problem

- Notation
  - x_i: stock market wealth in county i
  - y_i: employment in county i
  - $X = \frac{1}{N}\sum_{i=1}^{N} x_i$: aggregate stock market wealth
  - $Y = \frac{1}{N}\sum_{i=1}^{N} y_i$: aggregate employment
  - ε_i: other determinants of y_i, $\frac{1}{N}\sum_{i=1}^{N} \varepsilon_i = 0$
- Assume employment in county i satisfies

$$y_i = \alpha + \beta x_i + \gamma X + \varepsilon_i \qquad (*)$$

(Other specifications similar, e.g. $y_i = \alpha + \tilde{\beta} x_i + \tilde{\gamma} X_{-i} + \varepsilon_i$, $X_{-i} := \sum_{j \neq i} x_j$)

  - $\gamma > 0$ e.g. due to tradables $\Rightarrow$ demand from j "spills over" to i
  - $\gamma < 0$ e.g. due to factor mobility $\Rightarrow$ boom in county j hurts i
- True aggregate relation

$$Y = \alpha + (\beta + \gamma) X \quad \text{or} \quad \Delta Y = (\beta + \gamma) \times \Delta X$$

- Aggregate elasticity $\beta + \gamma$ may be $\gtrless 0$ depending on β, γ

The Missing Intercept Problem

- Now suppose that estimate ($*$) using cross-sectional variation
- No variation in aggregate $X \Rightarrow$ soaked into intercept

$$y_i = \tilde{\alpha} + \beta x_i + \varepsilon_i, \qquad \tilde{\alpha} := \alpha + \gamma X$$

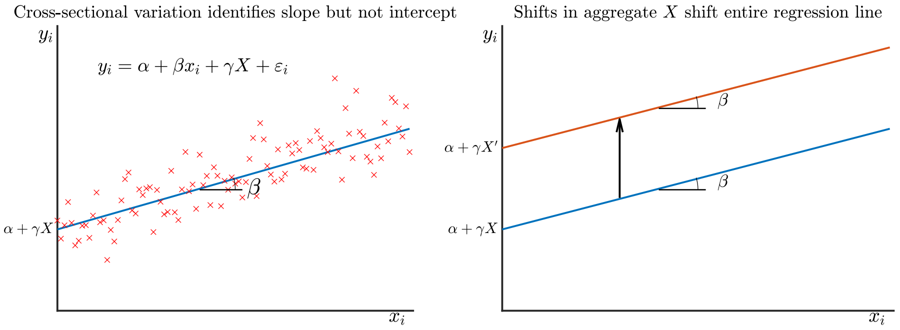
- Equivalently, estimate in changes $\Delta y_i = \beta \Delta x_i + v_i$ or w time FEs
- Naive exercise concludes that aggregate relationship is

$$\Delta Y = \beta \times \Delta X$$

  i.e. aggregate elasticity is β which is wrong!

- Logic: cross-sectional variation identifies the slope but not the intercept. But intercept is what we really care about!

Graphical Version



The Missing Intercept Problem

- More general version of same logic

$$y_i = \alpha + \beta x_i + \gamma Z + \varepsilon_i, \quad Cov(Z, X) \neq 0$$

  where Z = other aggregate factors driving employment

- Naive exercise again gets it wrong: true aggregate elasticity $\neq \beta$

- Also many other possible specifications with same logic

Other examples of missing intercept problem

1. China shock: x =import competition, y =employment (e.g. Autor-Dorn-Hanson)
2. Fiscal multipliers: x=government spending/transfers, y=GDP/consumption/employment (e.g. Nakamura-Steinsson, Wolf)
3. Household balance sheets in Great Recession: x=housing net worth, y=consumption, employment (e.g. Mian-Sufi)
4. Bank lending cuts to firms: x=bank lending, y=firm production (e.g. ChodorowReich, Herreño)
5. Unemployment benefits: x= unemployment benefits, y=unemployment (e.g. ChodorowReich-Coglianese-Karabarbounis)
6. Monetary policy and mortgage refinancing: x=housing equity, y=refinancing/consumption (e.g. Beraja-Fuster-Hurst-Vavra)
7. Consumer bankruptcy: x=debt forgiveness, y=employment (e.g. Auclert-Dobbie-GoldsmithPinkham)
8. ... and many more ...