Behavioral SIR Models

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Recall SIR Model

- Susceptibles S_t
- Infectious I_t
- Recovered or dead R_t

$$\dot{S} = -\beta S I \tag{S}$$
$$\dot{I} = \beta S I - \gamma I \tag{I}$$
$$\dot{R} = \gamma I \tag{R}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

- Death: constant death probability out of I state π

$$\dot{D} = \pi \gamma I \quad \Leftrightarrow \quad D = \pi R$$

- Mass preservation: $\dot{S}_t + \dot{I}_t + \dot{R}_t = 0 \Rightarrow S_t + I_t + R_t = 1$, all $t \geq 0$
- For analysis, see e.g. `https://benjaminmoll.com/SIR_notes/`

Simplest Behavioral SIR Model

Simplified version of models in econ literature, e.g.

- Lukasz Rachel "An Analytical Model of Covid-19 Lockdowns"
- Bognanni-Hanley-Kolliner-Mitman "Economics and Epidemics: Evidence from an Estimated Spatial Econ-SIR Model"
- Joshua Gans "The Economic Consequences of $R = 1$: Towards a Workable Behavioural Epidemiological Model of Pandemics"
- Flavio Toxvaerd "Equilibrium Social Distancing"
- Farboodi-Jarosch-Shimer "Internal and External Effects of Social Distancing"
- Garibaldi-Moen-Pissarides "Modelling contacts and transitions in the SIR epidemics model"
- Engle-Keppo-Kudlyak-Quercioli-Smith-Wilson "The Behavioral SIR Model, with Applications to the Swine Flu and COVID-19 Pandemics"
- Kaplan-Moll-Violante "The Great Lockdown and the Big Stimulus: Tracing the Pandemic Possibility Frontier for the U.S."
- ... plus too many others to list here (apologies for omissions)...
- Version in these notes due to Gianluca Violante
  `https://conference.nber.org/confer/2020/EFGs20/Violante.pdf`

Simplest Behavioral SIR Model

- Susceptibles S_t
- Infectious I_t
- Recovered or dead R_t
- Economic activity Y_t, normalize pre-pandemic $\bar{Y} = 1$ so $Y_t \leq 1$

$$\dot{S} = -\beta(Y)SI \tag{S}$$
$$\dot{I} = \beta(Y)SI - \gamma I \tag{I}$$
$$\dot{R} = \gamma I \tag{R}$$
$$Y = \mathcal{Y}(I) \tag{Y}$$

  with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

- Death as on previous slide
- Assumptions
  1. Infections $\nearrow$ in econ activity: $\beta' > 0$, e.g. $\beta(Y) = \bar{\beta}Y^{\alpha}, \alpha > 0$
  2. Econ activity $\searrow$ in infections: $\mathcal{Y}' < 0$, e.g. $\mathcal{Y}(I) = e^{-\sigma I}, \sigma > 0$

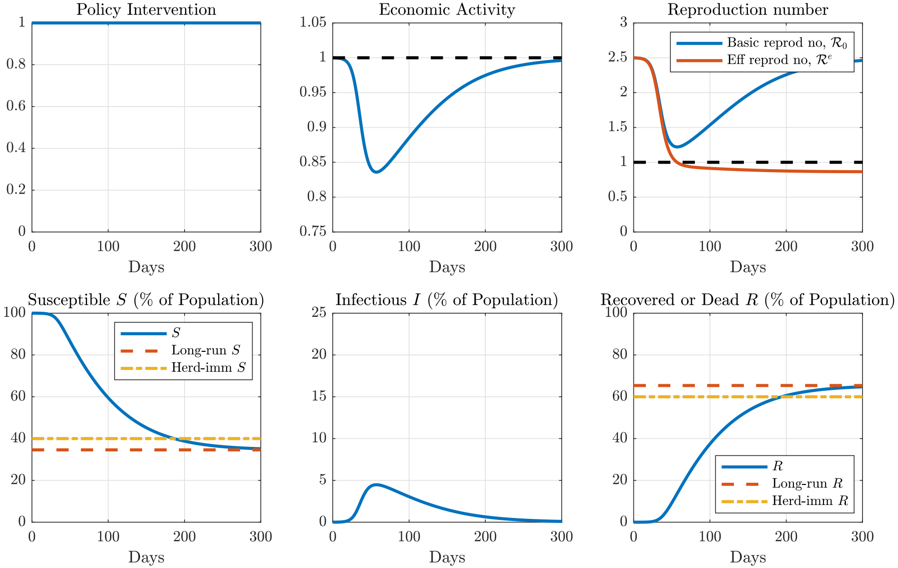
Simplest Behavioral SIR Model: Reduced Form

- Define
$$\tilde{\beta}(I) := \beta(\mathcal{Y}(I))$$
- Clearly $\beta' > 0$ and $\mathcal{Y}' < 0 \Rightarrow \tilde{\beta}' < 0$
- Example: $\beta(Y) = \bar{\beta}Y^{\alpha}$ and $\mathcal{Y}(I) = e^{-\sigma I} \Rightarrow \tilde{\beta}(I) = \bar{\beta}e^{-\alpha\sigma I}$
- Reduced form behavioral SIR model:

$$\dot{S} = -\tilde{\beta}(I)SI \quad \text{(S)}$$
$$\dot{I} = \tilde{\beta}(I)SI - \gamma I \quad \text{(I)}$$
$$\dot{R} = \gamma I \quad \text{(R)}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

Dynamics of epidemic in behavioral SIR model



In (S, I) Space: Phase Diagram

Recall (dropping equation for R)

$$\dot{S} = -\tilde{\beta}(I)SI \tag{S}$$

$$\dot{I} = \tilde{\beta}(I)SI - \gamma I \tag{I}$$

From (I) we have:

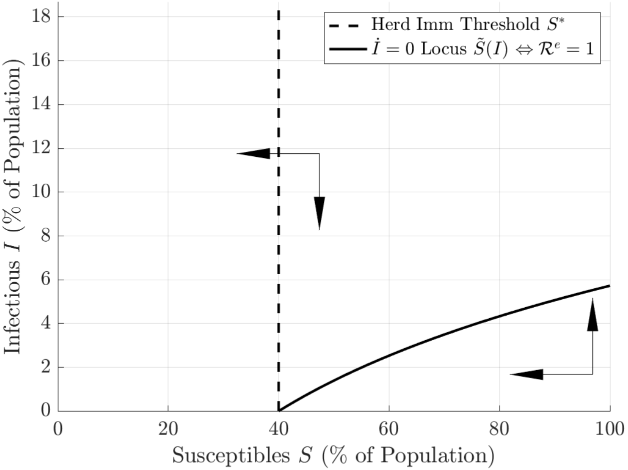
$$\dot{I} = \left(\tilde{\beta}(I)/\gamma \times S - 1\right)\gamma I = (\mathcal{R}^e - 1)\,\gamma I$$

$$\dot{I} = \left(\frac{S}{\tilde{S}(I)} - 1\right)\gamma I \quad \text{where} \quad \tilde{S}(I) := \frac{1}{\tilde{\beta}(I)/\gamma}$$

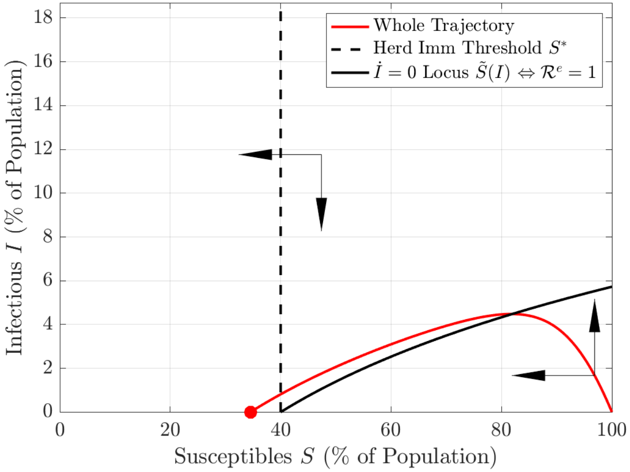
Therefore

1. $\dot{I} > 0$ if $\mathcal{R}^e > 1$ and < 0 otherwise
2. $\dot{I} > 0$ if $S > \tilde{S}(I)$ and < 0 otherwise (similarly $\mathcal{R}^e > 1 \Leftrightarrow S > \tilde{S}(I)$)
3. Whether $\dot{I} > 0$ or $\mathcal{R}^e > 1$ depends not only on S but also on I (in contrast to non-behavioral SIR model where $\dot{I} > 0 \Leftrightarrow S > S^*$)
4. $\dot{I} = 0$ locus $\tilde{S}(I)$ is increasing in (S, I) space

In (S, I) Space: Phase Diagram



In (S, I) Space: Phase Diagram



Note: trajectory “hugs” $\mathcal{R}^e = 1$ locus. In fact, $\mathcal{R}^e$ just < 1 for long time.

Predictions of simple behavioral SIR model

1. Relative to standard (non-behavioral) SIR model, epidemic "overshoots" herd immunity threshold by less

2. Effective reproduction number $\mathcal{R}^e$ just < 1 for long time

    - Gans uses $\mathcal{R}^e \approx 1$ to construct approximate solutions that can be analyzed graphically

Policy Interventions

Policy Interventions

Consider two types of policies

1. ℓ = lockdowns: reduce transmissions via reducing econ activity Y
2. m = "masks": reduce transmissions without affecting activity Y

$$\dot{S} = -\beta(Y, m)SI \tag{S}$$

$$\dot{I} = \beta(Y, m)SI - \gamma I \tag{I}$$

$$\dot{R} = \gamma I \tag{R}$$

$$Y = \mathcal{Y}(I, \ell) \tag{Y}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

Effects of Policy Interventions

- Some questions:

    1. Do policies lead to tradeoff between reducing infections (saving lives) and econ activity?

    2. How much can policies reduce cumulative infections (disease burden) and how does this depend on size of overshoot?

- Haven't had time to write this down yet, but you should be able to work this out yourselves!

- See discussion in papers on slide 2