Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy∗

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Abstract

We study the effect of monetary and fiscal policy in a heterogeneous-agent model where households have present-biased time preferences and naive beliefs. The model features a liquid asset and illiquid home equity, which households can use as collateral for borrowing. Because present bias substantially increases households’ marginal propensity to consume (MPC), present bias increases the impact of fiscal policy. Present bias also amplifies the effect of monetary policy but, at the same time, slows down the speed of monetary transmission. Interest rate cuts incentivize households to conduct cash-out refinances, which become targeted liquidity-injections to high-MPC households. But present bias also introduces a motive for households to procrastinate refinancing their mortgages, which slows down the speed with which this monetary channel operates.

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1 Introduction

The idea that dynamically inconsistent preferences may alter individuals’ dynamic choices has a long tradition going back to seminal work by Strotz (1956). A particular form of dynamic inconsistency, present bias, has received empirical support in both laboratory and field studies (e.g., Ashraf et al., 2006; Augenblick et al., 2015; Laibson et al., 2023; and the review by Cohen et al., 2020). Present bias implies that the current self draws a sharp distinction between a util that is experienced now versus a util experienced one time unit in the future, but draws relatively little distinction between a util consumed at any other two successive future dates (Phelps and Pollak, 1968; Laibson, 1997).\(^1\)

The two most commonly used tools of macroeconomic stabilization policy – monetary and fiscal policy – operate in large part by affecting household consumption and investment decisions, two leading examples of the types of dynamic choices that are affected by present bias. It is therefore natural to ask whether and to what extent present bias alters the potency of these policy tools. To answer these questions, we develop and calibrate a heterogeneous-agent consumption model in order to evaluate the impact of present bias on policy outcomes.

Our modeling framework is motivated in part by Campbell’s (2006) concept of positive household finance: households face a complex financial planning problem, and household behavior is influenced by a range of psychological factors. Our model aims to capture the complexities of household balance sheets that are important for the transmission of monetary and fiscal policy, as well as the channels through which present bias interacts with these balance sheet features.

We set our model in partial equilibrium in order to focus on the details of the household problem. Time is continuous, and we compare the exponential-discounting benchmark to a tractable, and empirically realistic, continuous-time limit of present-biased discounting. In addition to present bias, we assume that households have naive beliefs, meaning that households do not foresee their own future present bias (Strotz, 1956; Akerlof, 1991; O’Donoghue and Rabin, 1999). The modeling of present bias in continuous time builds on the foundational work of Barro (1999) and Luttmer and Mariotti (2003), and our specific approach follows Harris and Laibson (2013).

The household budget constraint includes stochastic auto-correlated labor income and interest rates. On the asset side of the household balance sheet, the model features a liquid savings account and an illiquid home. Households can build a buffer stock of liquid wealth to insure against income fluctuations, and can accumulate home equity by paying down their mortgage. On the liabilities side of the balance sheet, households have access to two forms of debt: credit cards and mortgages. Households can borrow on credit cards up to a

\(^1\)We use “present bias” to refer to quasi-hyperbolic discounting. Other common terms are: “$\beta$-$\delta$ preferences” in discrete-time models; and “Instantaneous Gratification preferences” in continuous-time models.
calibrated limit. If they have enough home equity, they can also borrow against their home by refinancing their mortgage. We calibrate our heterogeneous-agent model to reproduce two empirical regularities on household balance sheets: the average quantity of credit card debt and the average loan-to-value ratio in the housing market.\(^2\)

In order to focus on the effects of mortgage refinancing, we study homeowners, a large fraction of the population: two-thirds of U.S. housing units are owner-occupied. Homeowners represent an even larger fraction of aggregate income and consumption, and homeowners are an important channel for fiscal (Cloyne and Surico, 2017) and monetary policy (Wong, 2019; Cloyne et al., 2020).

Our main result is that relative to exponential discounting, present bias amplifies the balance-sheet channels of both fiscal and monetary policy, but with some important added subtlety in the case of monetary policy due to refinancing procrastination.

Fiscal policy is powerfully enhanced by present bias, because present bias sharply raises households’ average marginal propensity to consume (MPC) (Angeletos et al., 2001). In our Exponential Benchmark model the quarterly MPC is predicted to be 4\% and the quarterly marginal propensity for expenditure (MPX), which includes spending on both nondurables and durables, is predicted to be 13\%. In our Present-Bias Benchmark, the MPC rises from 4\% to 14\% and the MPX rises from 13\% to 30\%. These higher propensities to consume and spend are more consistent with the empirical literature: estimates of the quarterly response of nondurable expenditure are on the order of 15-25\%, and estimates of the quarterly response of total expenditure are typically two- to three-times larger.\(^3\)

Present bias also amplifies the overall effect of expansionary monetary policy, but slows down the speed of monetary transmission (an offsetting effect). Interest rate cuts incentivize households to conduct cash-out refinesances, which serve as targeted liquidity-injections to households with especially high MPCs (because they are near their liquidity constraint). But present bias with naive beliefs also introduces a motivation for households to procrastinate on refinancing their mortgage, which substantially slows down the speed at which this channel operates. Naive present bias implies that households will delay completing immediate-cost, delayed-reward tasks such as mortgage refinancing, which tends to take weeks and requires the borrower to go through the effortful process of negotiating with lenders, gathering documents, and filling out paperwork. Naive households will keep delaying refinancing, all the

\(^2\)The version of our model with exponential discounting is similar to Guerrieri et al. (2020) with two main differences: we assume that housing is fixed while they model a costly housing adjustment decision, and our model features credit card debt while theirs does not. Also see Mitman (2016), Berger et al. (2018), Berger et al. (2021), Wong (2019), Kaplan et al. (2020), Kinnerud (2021), and Eichenbaum et al. (2022) for related models of housing and mortgage refinancing decisions. Like us, Guerrieri et al. (2020) and McKay and Wieland (2021) use the continuous-time methods of Achdou et al. (2021) to solve their models.

\(^3\)For nondurable spending estimates, see e.g. Johnson et al. (2006), Parker et al. (2013), and the discussions in Kaplan et al. (2018) and Kaplan and Violante (2014). For total spending estimates, see e.g. Parker et al. (2013), Di Maggio et al. (2020), Fagereng et al. (2021), and the discussion in Laibson et al. (2021).
while (counterfactually) believing that the task will get done in the near future.

A noteworthy feature of our model is that present bias amplifies the direct effect of monetary policy on household consumption while, at the same time, also delivering larger MPCs. This is in contrast to standard heterogeneous-agent models, where modeling choices that amplify MPCs typically deliver smaller consumption responses to interest rate changes (Werning, 2015; Olivi, 2017; Kaplan et al., 2018; Auclert, 2019; Slacalek et al., 2020).4 Our model instead delivers a larger responsiveness to monetary policy precisely because of the higher MPCs: interest rate cuts incentivize households to conduct cash-out refinances, which become targeted liquidity-injections to high-MPC households.

Though our model is stylized, the steady state of the present-biased economy replicates a variety of empirical patterns from the household finance literature that have, collectively, proven difficult to replicate in models with exponential discounting. The present-biased economy generates empirically-plausible levels of high-cost credit card borrowing by homeowners (Zinman, 2015), cash-out behavior, and loan-to-value (LTV) ratios. It also features a buildup of liquidity-constrained households that is consistent with empirical estimates of households’ propensity to spend out of increases in credit card limits (Gross and Souleles, 2002; Agarwal et al., 2018). Present-biased households struggle to smooth consumption, resulting in a consumption function with discontinuities at the borrowing constraint (Ganong and Noel, 2019). Present bias delivers larger MPCs and MPXs, as well as MPCs and MPXs that remain elevated for large shocks (Fagereng et al., 2021).5 The time-profile of consumer spending is consistent with the intertemporal MPC evidence in Auclert et al. (2018) (using data from Fagereng et al. (2021)). The present-biased economy also generates differential MPCs out of liquid cash transfers versus illiquid home equity increases, a pattern shown empirically by Ganong and Noel (2020). Finally, there is a large literature documenting refinancing inertia: the proclivity for households to delay refinancing when it is financially optimal to do so (e.g., Keys et al., 2016; Johnson et al., 2019; Andersen et al., 2020). We show that present bias with naive beliefs provides a natural motivation for this behavior.

One may wonder: what is specific to present bias? Why not just calibrate a model with exponential discounting that generates empirically realistic MPCs and use that for our policy experiments? The answer is twofold. First, such a model would not generate the procrastination behavior just described. Second, in two-asset models like ours, high-MPC calibrations with exponential discounting often make assumptions about interest rates that

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4This statement is made precise by Auclert (2019) and Olivi (2017) who show that, in a standard one-asset consumption-saving problem, a household’s MPC is a “sufficient statistic” to determine both income and substitution effects of interest rate changes and, in particular, enters the substitution effect with a negative sign. See also the exposition in Slacalek et al. (2020).

5Besides present bias, other deviations from standard exponential discounting also have the potential to help match empirical MPCs. For example, Attanasio et al. (2020) show that temptation preferences can help in this regard, and Lian (2021) shows that the anticipation of future mistakes can increase MPCs.
are difficult to reconcile with the data. In contrast, our calibration with present bias delivers high MPCs with interest rates “taken from the data.” We view this reconciliation as a step forward for the heterogeneous-agent literature.

Section 2 lays out our model of the household balance sheet. Section 3 characterizes the effect of present bias on household consumption and refinancing decisions. Section 4 discusses our calibration and what it implies for the model’s steady state. Section 5 presents our main results about macroeconomic stabilization policy. Section 6 concludes.

2 A Model of Household Finances with Present Bias

Our model is set in partial equilibrium. The goal of the model is to capture household balance-sheet channels through which present bias can impact fiscal and monetary policy. Abstracting from general equilibrium considerations simplifies the analysis and allows for a richer investigation of the institutional factors that affect the household problem. Our partial equilibrium results should be interpreted as just one part of the overall macroeconomic analysis, providing inputs for a full general equilibrium analysis.

2.1 The Household Balance Sheet

Our model focuses on homeowners. There is a unit mass of households that are heterogeneous in their wealth and income. Here we outline the evolution of each household’s balance sheet.

Budget Constraints. Each household faces idiosyncratic income risk. The household’s income is denoted $y_t$, and $y_t$ follows a finite state Poisson process. We normalize the average income flow to 1.

Households hold three types of assets: liquid wealth $b_t$, illiquid housing $h$, and a fixed-rate mortgage $m_t$. For simplicity we assume that each household is endowed with a home of fixed value $h$. The remainder of the household balance sheet evolves as follows:

$$\dot{b}_t = y_t + r_t b_t + \omega c b_t^\gamma - (r^m_t + \xi) m_t - c_t,$$

$$\dot{m}_t = -\xi m_t,$$  

6Specifically, such models often calibrate relatively low interest rates on credit card debt and/or relatively high returns on illiquid assets in order to generate the low levels of liquid wealth accumulation and high levels of credit card borrowing that are observed empirically.

7We return to this theme in Appendix G, which briefly discusses how present bias may alter the transmission of macroeconomic policy in general equilibrium.

8We study only the short-run response to fiscal and monetary policy, and house prices are slow-moving over short horizons (Case and Shiller, 1989). Appendix D.2.1 presents an extension with house price shocks.
subject to the borrowing constraint \( b_t \geq b \) and the loan-to-value (LTV) constraint \( m_t \in [0, \theta h] \). Equation (1) characterizes the evolution of liquid wealth \( b_t \). Equation (2) describes the evolution of mortgage balances. Explaining equation (1), households earn income \( y_t \) and have a consumption outflow of \( c_t \). The return to liquid wealth is given by \( r_t b_t + \omega^c b_t^- \), where \( b_t^- = \min\{b_t, 0\} \). \( \omega^c > 0 \) is a credit card borrowing wedge, which generates a "soft constraint" at \( b = 0 \) (Kaplan and Violante, 2014; Achdou et al., 2021). The household’s total mortgage payment is captured by \( (r_t^m + \xi)m_t \). This is composed of a mortgage interest payment \( (r_t^m \times m_t) \) and a principal repayment \( (\xi \times m_t) \). To economize on state variables, we make the slightly non-standard assumption that the household pays down its mortgage at a constant proportional rate \( \xi \) (Agarwal et al., 2013). The more realistic assumption of a constant flow payment would require an additional state variable.

The borrowing constraint is important for our results, so we emphasize its effects here. In continuous time, the consumption rate \( c_t \) is unconstrained for all \( b_t > b \): any finite rate of consumption can be adopted without violating the borrowing constraint, so long as that rate of consumption persists for a short enough period of time (Achdou et al., 2021). However, at the liquid-wealth constraint of \( b_t = b \) the household is restricted to consume at a rate

\[
c_t \leq y_t + (r_t + \omega^c)b - (r_t^m + \xi)m_t.
\]

That is, consumption features an occasionally-binding constraint when \( b_t = b \).

Equations (1) and (2) characterize how the household’s balance sheet evolves continuously. Households also have the option of paying a fixed cost to discretely adjust their mortgage. We provide details of this discrete adjustment process, which includes the option to refinance, further below after outlining the interest rate process.

**Interest Rates and Stimulus Payments.** The real interest rate is denoted \( r_t \). We assume that \( r_t \) follows a finite state Poisson process. Since our goal is to study the effect of changes in mortgage rates we treat \( r_t \) as a long rate (e.g., 10-year TIPS). When discussing monetary policy in Section 5, we implicitly assume that the Federal Reserve is implementing the necessary short rate adjustments to generate the corresponding changes in long rate \( r_t \). Because this paper studies the refinancing channel of monetary policy, it is important that households have reasonable expectations about how mortgage rates evolve over time.\(^9\)

Our calibration of households’ interest-rate expectations is discussed in Section 4.1. Each household pays a mortgage interest rate \( r_t^m \). To capture features of the U.S. mortgage market, this mortgage rate is fixed until the household decides to refinance. At the time of refinancing, the household switches to a new mortgage rate of \( r_t^m = r_t + \omega^m \), where \( \omega^m \)

\(^9\)Importantly, this means that interest rate shocks in our model are not “MIT shocks.” This feature differentiates our model from many other heterogeneous-agent models that study monetary policy.
represents the mortgage borrowing wedge over the current interest rate $r_t$.

When discussing fiscal policy in Section 5, we consider an unexpected one-time stimulus payment to each household. While our analysis is set in partial equilibrium, we do impose a government budget constraint by levying a flow income tax on all households to finance the initial stimulus payment.

**Discrete Adjustments and Mortgage Refinancing.** Equations (1) and (2) describe how liquid wealth and mortgage balances evolve continuously. Households also have two methods for discretely adjusting their balance sheet position between liquid wealth and illiquid home equity. First, households can pay a small fixed cost to prepay their mortgage. Second, households can pay a larger fixed cost to refinance. We detail these options below.

A household’s first adjustment option is to prepay its mortgage. We introduce prepayment because mortgage contracts typically allow for households to pay down their mortgage faster than contractually required. Prepayment requires a small fixed cost of $\kappa^{\text{prepay}}$, and the household chooses a new liquid wealth value $b'$ and a new mortgage value $m'$ such that

$$b' - m' = b_t - m_t - \kappa^{\text{prepay}}, \quad \text{subject to } m' \in [0, m_t) \text{ and } b' \geq b.$$  

(4)

Prepayment does not affect the mortgage interest rate, which remains the same as before the adjustment decision. By using part of their liquid wealth to prepay their mortgage, households are effectively shifting their portfolio from a low-return liquid asset to a high-return illiquid asset. Home equity is illiquid because it can only be accessed through a cash-out refinance. But, the benefit of accumulating home equity is that wedge $\omega^m$ makes mortgage debt costly. Households will therefore first build a buffer-stock of liquid wealth and then use additional liquidity to prepay their mortgage.

A household’s second adjustment option is to refinance its mortgage (or take out a new one if $m_t = 0$). Refinancing requires payment of a fixed cost of $\kappa^{\text{refi}}$. When refinancing, the household chooses a new liquid wealth value $b'$ and a new mortgage value $m'$ such that

$$b' - m' = b_t - m_t - \kappa^{\text{refi}}, \quad \text{subject to } m' \in [0, \theta h] \text{ and } b' \geq b.$$  

(5)

By refinancing, the household also resets the interest rate on its mortgage to $r_t^m = r_t + \omega^m$.

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10We impose a small fixed cost for numerical stability. This small fixed cost can be thought of as capturing, for example, the time cost of the additional budgeting required to make a mortgage prepayment.

11Kaplan and Violante (2014) highlight how high-return illiquid assets can prevent households from building sizable liquid buffer stocks. Laibson et al. (2023) demonstrate that asset illiquidity combined with present bias allows lifecycle consumption-saving models to match the joint accumulation of credit card debt and illiquid savings that characterizes the balance sheets of many U.S. households.

12In our model, some households will have fully paid off their mortgage ($m_t = 0$) prior to taking cash out. For simplicity we refer to adjustments starting from both $m_t > 0$ and $m_t = 0$ as refinancing.
Though refinancing requires up-front costs, there are two reasons why households may choose to refinance. First, if the market interest rate falls then refinancing “locks in” lower mortgage interest payments. Second, refinancing allows households to rebalance their asset allocation across liquid wealth and illiquid home equity. For example, a cash-out refinance lets households convert illiquid home equity into liquid wealth. Accessing home equity is useful during spells of low income (consumption smoothing),\textsuperscript{13} and also as a means of converting costly credit card debt into cheaper mortgage debt. Consistent with the empirical evidence in Berger et al. (2021), we find that these two motives for refinancing are not mutually exclusive in our model. Indeed, one of our main results is that interest rate cuts can be highly stimulative precisely because they induce a wave of home-equity extractions.

We assume that both types of discrete adjustments require a small effort cost $\bar{\varepsilon}$ (in addition to the monetary costs $\kappa^{\text{refi}}$ and $\kappa^{\text{prepay}}$). This cost $\bar{\varepsilon}$ is intended to capture the effort associated with filling out paperwork, negotiating with mortgage brokers, etc. We will show in Section 2.3 that this effort cost provides a natural mechanism for producing refinancing procrastination. In that section we also slightly generalize the setup by making the effort cost stochastic to capture the idea that households face occasional windows of time in which the marginal cost of making effortful budgeting adjustments is lower than normal, such as a free weekend or a cancelled afternoon meeting.

Finally, we note that our model does not allow for home equity lines of credit, second mortgages, or reverse mortgages. These alternate products are much more likely to be used when interest rates are rising, in order to extract home equity without resetting the entire mortgage balance to a higher interest rate (Bhutta and Keys, 2016). We abstract from these alternate products because our paper focuses on the stimulative effect of rate cuts.

Other Structural Assumptions. To capture exogenous mortgage adjustment dynamics such as moving for a new job, we introduce an exogenous hazard rate $\lambda^F$ at which households are forced to adjust their mortgage (and pay the cost to either refinance or prepay). We assume that households adjust their mortgage optimally when they are forced to do so.

To capture lifecycle dynamics, we assume that households retire at rate $\lambda^R$ and are replaced by “first-time homeowners.” To avoid needing an additional state variable we model retirement using a “perpetual youth” framework (Blanchard, 1985). A household who retires at time $t$ receives a constant consumption flow of $y^R + \bar{r}(h - m_t + b_t)$ in perpetuity, where $y^R$ is a fixed retirement income flow and $\bar{r}$ is the average interest rate. We denote the exponentially discounted value of the retirement consumption flow by $v^R(b_t, m_t) = \frac{u(y^R + \bar{r}(h - m_t + b_t))}{\rho}$, where $\rho$ is an exponential discount rate. This parameterization captures a retirement pension of size $y^R$ plus the annuity value of a household’s assets at retirement.

\textsuperscript{13}See also Hurst and Stafford (2004) and Chen et al. (2020) for related insights.
Summary. The goal of our model is to provide a simple characterization of the household balance sheet features that are important for the conduct of macroeconomic stabilization policy. Our partial equilibrium model has five state variables: \((b, m, y, r^m, r)\). Liquid wealth \(b\) and stochastic income \(y\) introduce uninsurable income risk and wealth heterogeneity. Mortgage \(m\) introduces a realistic role for housing, which is the primary illiquid asset held by most American households (Campbell, 2006). Time-varying interest rate \(r\) provides a role for monetary policy. Mortgage interest rate \(r^m\) introduces a refinancing motive, and allows us to study the refinancing channel of monetary policy.

To simplify notation, let \(x = (b, m, y, r^m, r)\) denote the vector of state variables that characterize the household problem. Households can be heterogeneous in dimensions \(b, m, y\) and \(r^m\). All households face the same time-varying market interest rate \(r_t\).

2.2 Utility and Value

Utility. Households have CRRA utility over consumption:\(^{14}\)

\[
u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.
\]

Time Preferences: Instantaneous Gratification. This paper’s key departure from rationality is the household’s discount function. Households have naive Instantaneous Gratification (IG) time preferences. IG time preferences were first derived by Harris and Laibson (2013), and are extended in Laibson and Maxted (2022) and Maxted (2023).

In discrete time, the quasi-hyperbolic discount function is given by \(1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots\). IG preferences are the continuous-time limit of this discount function, where each self lives for a vanishingly small length of time.\(^{15}\) For \(t \geq 0\), the limiting IG discount function \(D(t)\) is:

\[
D(t) = \begin{cases} 
1 & \text{if } t = 0 \\
\beta e^{-\rho t} & \text{if } t > 0
\end{cases}
\]

Since the current instantaneous self discounts all future selves by factor \(\beta\), discount function \(D(t)\) features a discontinuity at \(t = 0\) whenever \(\beta < 1\). The IG household values instantaneous utility flows, and all later utility is discounted by \(\beta\). Note that \(\beta = 1\) recovers the standard, time-consistent, exponential discount function.

We assume that households are naive about their present bias. This means that the

\(^{14}\)We could also let households earn utility from housing \(h\). We ignore this element since housing \(h\) is constant. This assumption is isomorphic to households having separable utility over consumption and housing, or CES utility with a unitary elasticity of substitution.

\(^{15}\)See Appendix B for a heuristic derivation, and Harris and Laibson (2013) for a rigorous derivation.
current self is unaware of the self-control problems of future selves. Under naive present bias the current self discounts the utility flows of all future selves by \( \beta < 1 \), while expecting that all future selves will be exponential discounters (\( \beta = 1 \)). We assume naiveté because we need (at least partial) naiveté to generate procrastination from small effort costs. We extend our analysis to partial and full sophistication in Appendix D.5, and briefly discuss the key takeaways from that extension at the end of Section 3.

As detailed in Laibson and Maxted (2022), IG preferences are a mathematically tractable limit case. They are not a psychologically realistic model of time preferences. The temporal division between “now” and “later” is certainly longer than a single instant \( dt \). Nonetheless, Laibson and Maxted (2022) show that discrete-time models with psychologically appropriate time-steps (e.g., each period lasts for one day or one week) are closely approximated by the continuous-time IG model. The current paper leverages the tractability of the IG approximation to study the effect of present bias on macroeconomic stabilization policy.

**Remark 1.** IG time preferences are a generalization of standard time-consistent preferences. Exponential discounting is recovered by setting \( \beta = 1 \).

**Value Function (\( \beta = 1 \)).** We start by presenting the value function for an exponential (\( \beta = 1 \)) household. We present the value function in steps, suppressing notation at first in order to clarify the structure of the household’s decision-making problem. As a first step, the value function of a \( \beta = 1 \) household can be defined as the solution to the sequence problem:

\[
v(x_0) = \max_{\{c_t\}, \tau} \mathbb{E}_0 \left[ \int_0^\tau e^{-\rho t} u(c_t) dt + e^{-\rho \tau}(v^*(x_\tau) - \bar{\varepsilon}) \right] \quad \text{s.t. (1) and (2) hold and with}
\]

\[
v^*(x) = \max \{ v_{\text{prepay}}(x), v_{\text{refi}}(x) \}
\]

\[
v_{\text{prepay}}(x) = \max_{b', m'} v(b', m', y, r, m, r) \quad \text{s.t. prepayment constraint (4) holds}
\]

\[
v_{\text{refi}}(x) = \max_{b', m'} v(b', m', y, r + \omega, m, r) \quad \text{s.t. refinancing constraint (5) holds}
\]

Equation (7) subsumes all Poisson shocks inside the expectation operator.

The integral \( \mathbb{E}_0 \left[ \int_0^\tau e^{-\rho t} u(c_t) dt \right] \) captures utility from consumption, which the household chooses continuously. The term \( \mathbb{E}_0 [e^{-\rho \tau}(v^*(x_\tau) - \bar{\varepsilon})] \) captures discrete adjustment, which the household chooses at time \( \tau \) (a stopping time). These discrete adjustments form an optimal stopping (option value) problem. Function \( v^* \) denotes the optimal value function conditional on adjusting, which also requires effort cost \( \bar{\varepsilon} \). Adjustment takes the form of either mortgage prepayment or refinancing. Note that the mortgage interest rate remains

\[^{16}\text{Augenblick (2018) finds that the division between “now” and “later” is roughly 2 hours. Using fMRI data, McClure et al. (2007) find a one-hour discount rate of 50% for food rewards. More generally, Augenblick (2018) and Augenblick and Rabin (2019) show that almost all discounting occurs within one week.} \]
constant when the household chooses a mortgage prepayment, while refinancing resets the mortgage interest rate to \( r_t + \omega^m \).

Equation (7) highlights that the household faces a simultaneous optimal control problem plus an optimal stopping problem. The household continuously chooses consumption \( c_t \), and also possesses the option to discretely rebalance its asset allocation across liquid wealth and illiquid home equity. To capture these dual decisions, the value function in equation (7) can also be expressed as a Hamilton-Jacobi-Bellman Quasi-Variational Inequality (HJBQVI).\(^{17}\)

Starting with compact notation to highlight the general structure of variational inequalities:

\[
\rho v(x) = \max \left\{ \max_c \left( u(c) + (\mathcal{A}v)(x), \rho(v^*(x) - \varepsilon) \right) \right\}. \tag{8}
\]

Operator \( \mathcal{A} \) is an infinitesimal generator, which we will define momentarily by writing out equation (8) in a less compact fashion. The left branch of equation (8) captures the optimal control problem while the right branch captures the optimal stopping problem. If it is not optimal to adjust, the left branch imposes that value function \( v(x) \) satisfies a standard HJB equation \( \rho v(x) = \max_c \left( u(c) + (\mathcal{A}v)(x) \right) \), and the right branch imposes that \( v(x) \) is larger than the value of adjusting: \( v(x) \geq v^*(x) - \varepsilon \). If it is optimal to adjust, the right branch imposes that the value function equals the value of adjusting: \( v(x) = v^*(x) - \varepsilon \).

Expanding the operator \( \mathcal{A} \) to explicitly show the Poisson risks faced by the household, the HJBQVI can be written out fully as follows:

\[
\rho v(x) = \max \left\{ \max_c \left\{ u(c) + \frac{\partial v(x)}{\partial b} \left( y + rb + \omega \sigma b - (\sigma^m + \xi)m - c \right) \right\} \right\} \tag{8'}
\]

\[
\begin{align*}
&- \frac{\partial v(x)}{\partial m} (\xi m) \\
&+ \sum_{y' \neq y} \lambda^{y \rightarrow y'} \left[ v(b, m, y', r, r, r) - v(b, m, y, r, r, r) \right] \\
&+ \sum_{r' \neq r} \lambda^{r \rightarrow r'} \left[ v(b, m, y, r, r, r') - v(b, m, y, r, r, r) \right] \\
&+ \lambda^R \left[ v_R(x) - v(x) \right] \\
&+ \lambda^F \left[ v^*(x) - (v(x) - \varepsilon) \right],
\end{align*}
\]

\[
\rho(v^*(x) - \varepsilon).
\]

\(^{17}\)See Bensoussan and Lions (1982, 1984) and Bardi and Capuzzo-Dolcetta (1997). For additional details and a discussion of more economic applications, see http://benjaminmoll.com/liquid_illiquid_numerical/, and Guerrieri et al. (2020) and McKay and Wieland (2021). Relative to our formulation in equation (8), the mathematics literature studying HJBQVIs typically uses somewhat different but equivalent notation, for example: \( 0 = \min \{ \rho v(x) - \max_c u(c) + (\mathcal{A}v)(x), v(x) - (v^*(x) - \varepsilon) \} \). We use the formulation in (8) with the max operator because it is more economically intuitive.
Each of the seven rows of equation \((8')\) reflects the value function’s dependence on, respectively, liquid wealth \(b\) (row one), mortgage level \(m\) (row two), income state \(y\) (row three), interest rate state \(r\) (row four), retirement (row five), forced adjustment (row six), and discrete adjustment (row seven). In rows three and four of the equation, which correspond respectively to the income process and the interest rate process, we use notation \(\lambda^{s\rightarrow s'}\) to denote the transition rate from state \(s\) to state \(s'\). \(\lambda^R\) is the transition rate into retirement, and \(\lambda^F\) is the rate at which households are forced to adjust their mortgage.

**Value Functions** (\(\beta < 1\)). We now introduce naive present bias. Naifs incorrectly perceive that all future selves will discount exponentially (\(\beta = 1\)). Thus, the value function \(v(x)\) that solves equation (7) – or equivalently (8) – characterizes the naive IG household’s perceived value function starting in the next instant. For this reason, we will refer to \(v(x)\) as the *continuation-value function*. The current-value function characterizes the household’s perceived value of future utility flows in the current period. Since the current self discounts all future selves by \(\beta\), the current-value function of the naive IG household is given by:

\[
\begin{align*}
\text{w}(x) & = \max \left\{ \beta v(x), \ w^*(x) - \bar{\varepsilon} \right\} \quad \text{with} \\
\text{w}^*(x) & = \max \left\{ \text{w}^{\text{prepay}}(x), \text{w}^{\text{refi}}(x) \right\} \\
\text{w}^{\text{prepay}}(x) & = \max_{b',m'} \text{w}(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds} \\
\text{w}^{\text{refi}}(x) & = \max_{b',m'} \text{w}(b',m',y,r + \omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}
\end{align*}
\] (9)

In the first line of equation (9), the left branch captures the current-value function if the household does not adjust its mortgage, and the right branch captures the current-value function if the household chooses to adjust its mortgage. Importantly, the utility flow of the current self does not show up in equation (9). Because each self lives for a single instant of \(dt\), no individual self’s utility flow has a measurable effect on the overall value function. Further discussion is provided in Appendix B, which derives the current-value function of the naive IG household as the continuous-time limit of a discrete-time model.

It is worth emphasizing again that, with naive present bias, the perceived value function \(v\) on the right-hand side of (9) is the value function of a \(\beta = 1\) household, i.e. the one that solves (8). This property of naïveté is the key reason for its tractability. In particular, it implies that both theoretical and computational approaches can use the following two-step procedure: first, solve the value function \(v\) of an exponential \(\beta = 1\) household from (8); second, find the value function of a present-biased \(\beta < 1\) household immediately from (9).
Policy Functions. Households make two choices in the model: they choose consumption continuously, and they have the option to adjust their mortgage discretely. Introducing notation for these policy functions, let $c : x \to [0, \infty)$ denote a household’s consumption policy in state $x = (b, m, y, r^m, r)$. Mortgage adjustment consists of two nested decisions. First, conditional on adjustment the household chooses its new mortgage level $m'$ and liquid wealth $b'$. Let $m' : x \to [0, \theta h]$ and $b' : x \to [b, \infty)$ denote the household’s optimal mortgage and liquid wealth choice, conditional on adjustment. Second, the household chooses whether or not to adjust. Let $R : x \to \{0, 1, 2\}$ denote whether a household finds it optimal to not adjust ($R = 0$), prepay ($R = 1$), or refinance ($R = 2$).

2.3 Procrastination

There is a large literature documenting that households are slow to refinance after interest rate declines (e.g., Keys et al., 2016; Johnson et al., 2019; Andersen et al., 2020). Refinancing involves a series of up-front effort costs, such as negotiating with mortgage brokers and filling out paperwork, in exchange for long-run financial benefits. Households with naive present bias will delay completing these sorts of immediate-benefit delayed-reward tasks, instead deferring them for future selves (O’Donoghue and Rabin, 1999, 2001; DellaVigna and Malmendier, 2004). In this way, naive present bias provides one natural motivation for refinancing inertia: procrastination. From a theoretical standpoint, our incorporation of present-bias-driven procrastination also differentiates the analysis here from other papers using IG preferences, such as Harris and Laibson (2013) and Maxted (2023).

Keys et al. (2016) provide direct evidence of procrastination as an important channel through which refinancing inertia arises. The financial calculations involved in refinancing are complex (Agarwal et al., 2013), and refinancing generates a range of non-pecuniary short-term costs in exchange for uncertain long-term benefits. This creates an environment where a variety of psychological factors – such as trust in the financial system, financial illiteracy, sticky information, attention costs, and bounded rationality – underlie the effort costs that drive procrastination. Our goal here is to provide a simple and transparent model that captures the intuition for how such cognitive costs can interact with present bias to produce refinancing inertia.

The key model ingredient that generates procrastination for $\beta < 1$ households is the effort cost that households face to discretely adjust their mortgage. While the setup with a constant effort cost that we spelled out in Sections 2.1 and 2.2 already gives rise to procrastination, we here generalize this setup slightly by assuming that the effort cost is stochastic. This stochasticity captures the idea that households face occasional windows of time in which the

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18 For references to some of these factors, see respectively Johnson et al. (2019); Agarwal et al. (2017); Mankiw and Reis (2002); Sims (2003); Woodford (2003); Gabaix (2019).
marginal cost of making effortful financial adjustments is lower than normal. As we explain below, a stochastic effort cost will generate a particular form of procrastination, namely “Calvo-style” procrastination, that is not only tractable but also a useful approximation of household refinancing behavior (Andersen et al., 2020).

**Assumption 1** The effort cost \( \varepsilon_t \) takes two values \( \bar{\varepsilon} \) and \( \breve{\varepsilon} \) satisfying
\[
\bar{\varepsilon} > \frac{1}{\beta} \varepsilon > 0.
\]

Effort cost \( \varepsilon_t \) usually equals the high value of \( \bar{\varepsilon} \). But, at Poisson arrival rate \( \phi \), \( \varepsilon_t \) falls to the low value of \( \varepsilon \) for a single instant before immediately reverting back to the high value of \( \bar{\varepsilon} \).

Though stylized with a two-state process for tractability, Assumption 1 captures the sorts of stochastic life events, such as a free weekend or a cancelled afternoon meeting, in which the household becomes more willing to complete chores because the opportunity cost of time has temporarily fallen.

What is critical about these sorts of stochastic windows of availability is that they typically have explicit end dates. For example, a free weekend may represent a low-cost period for a household, but that window closes on Sunday night. These sorts of deadlines are forcing mechanisms that encourage present-biased households to complete effortful tasks, because even present-biased households will want to take advantage of relatively low-cost periods before they come to an end (see e.g. Carroll et al., 2009; Allcott et al., 2022).

Assumption 1 makes the simplification that these low-cost windows last for exactly one instant \( dt \). This simplification maintains the stationarity of our continuous-time model (avoiding the need to include time as a state variable), but it is not critical to the results. The essential feature of these low-cost windows is that they have a defined expiration date.

This simple generalization of our model to a stochastic effort cost requires us to append our model equations in a few places. Appendix B.3 spells out the full set of equations. For example, in addition to the current-value of a \( \beta < 1 \) household in the high-effort-cost state defined in equation (9), there is now an analogous equation for a household in the low-effort-cost state:
\[
\bar{w}(x) = \max \left\{ \beta v(x), \ w^*(x) - \varepsilon \right\}.
\]

Intuitively, since the low-cost state only lasts for an instant (Assumption 1), the household either takes advantage of adjusting its mortgage at the lower effort cost \( \varepsilon \) in which case its value is \( w^*(x) - \varepsilon \), or else the household reverts to the high-cost state and its value is \( \beta v(x) \). Equation (10) highlights the cost of not refinancing when \( \varepsilon = \varepsilon \) — the low-cost period is lost and the effort cost reverts back to \( \bar{\varepsilon} \). For future reference, we will denote by \( \bar{\mu}(x), \bar{m}'(x), \) and \( \bar{y}'(x) \) the corresponding refinancing policy function in the low-effort-cost state (i.e., the
prepayment or refinancing decisions corresponding to (10), while $\mathcal{R}(x), m'(x)$, and $b'(x)$ continue to denote the analogous policy function when the effort cost is high.

Next, we make an assumption so that the effort cost only matters for $\beta < 1$ households:

**Assumption 2** $\bar{\varepsilon}$ and $\xi$ are vanishingly small.

Assumption 2 represents the idea that households typically consider refinancing to be a nuisance, but not costly in an economically meaningful sense. By making the effort cost arbitrarily small, the effort cost is inconsequential for the behavior of $\beta = 1$ households.

However, this trivial effort cost becomes important when interacted with present bias. When $\beta < 1$, the small effort cost is sufficient to generate procrastination.\(^{19}\) This is because naive present-biased households will always choose to delay the task of refinancing (for one instant in expectation) whenever $\varepsilon_t = \bar{\varepsilon}$. The perceived benefit of procrastinating is that the effort cost of adjustment gets pushed into the future, where it is discounted by $\beta$. When $\varepsilon_t = \bar{\varepsilon}$, the perceived cost of delaying for one instant is infinitesimal. So, naive present-biased households will continually procrastinate when $\varepsilon_t = \bar{\varepsilon}$.

For $\beta < 1$ households, procrastination persists until the household stochastically enters a low-cost window and the effort cost $\varepsilon_t$ momentarily drops to $\xi$. Now, there is an explicit cost to waiting: further procrastination causes the low-cost window to expire and the effort cost to revert to $\varepsilon > \frac{1}{\beta} \bar{\varepsilon}$. Since the effort cost of completion now, $\varepsilon$, is less than the discounted effort cost of completion next instant, $\beta \bar{\varepsilon}$, this fleeting opportunity incentivizes the present-biased household to stop procrastinating. This is formalized in Proposition 2 below.\(^{20}\)

### 3 The Effect of Present Bias on Policy Functions

We can now describe the effect of present bias on the consumption and mortgage adjustment decisions. In order to characterize the policy functions of a present-biased household relative to those of a standard exponential household, we use hat-notation to denote the policy functions of an otherwise-identical household that has $\beta = 1$. Accordingly, the policy functions denoted by hats are what the naive household perceives all future selves will follow.

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\(^{19}\)Of course, when $\beta = 1$ it is possible to rationalize refinancing inertia by making other assumptions about effort cost $\varepsilon_t$, for example by making $\bar{\varepsilon}$ arbitrarily large.

\(^{20}\)As this discussion makes clear (and we confirm in Proposition 2), our baseline model in Sections 2.1 and 2.2 with a constant effort cost $\bar{\varepsilon}$ would generate indefinite procrastination by $\beta < 1$ households. Perhaps surprisingly, this is true even though this effort cost is arbitrarily small (Assumption 2). Thus, an additional rationale for extending our model to feature a stochastic effort cost, besides increased realism (low-cost windows like weekends and cancelled meetings do exist), is that it generates $\beta < 1$ households who refinance.
Consumption. Present-biased households want to bring utility into the present, which implies that present bias has a direct effect on households’ consumption decisions. Specifically, present-biased households overconsume by factor $\beta^{-\frac{1}{\gamma}}$:

Proposition 1 (Continuous Control)

1. For all $b > b$, the household sets $c(x) = \beta^{-\frac{1}{\gamma}}\hat{c}(x)$.

2. For $b = b$, the household sets $c(x) = \min\left\{\beta^{-\frac{1}{\gamma}}\hat{c}(x), \ y + (r + \omega c)b - (r^m + \xi)m\right\}$.

The proof of Proposition 1 makes use of an important intermediate step, which we state first and which is proved in Appendix B:

Lemma 1 When the borrowing constraint does not bind, $b > b$, consumption is defined implicitly by the first-order condition:

$$u'(c(x)) = \beta \frac{\partial v(x)}{\partial b},$$

where the continuation-value function $v$ is equal to the value function of an exponential $\beta = 1$ household and solves (7) or equivalently (8).

Equation (11) is a first-order condition: consume until the marginal utility of consumption equals the marginal value of liquid wealth. For $\beta = 1$, the standard continuous-time first-order condition of $u'(c(x)) = \frac{\partial v(x)}{\partial b}$ is recovered. The additional discount factor $\beta$ appears in equation (11) because the present-biased household discounts future consumption (and hence current wealth) by $\beta$.\footnote{While (11) looks like a standard discrete-time first-order condition, it is important to note that the interpretation is different. In particular, $\beta$ is not the standard exponential discrete-time discount factor; instead, $\beta$ is the discount factor between now and one instant from now, see (6), with $\beta = 1$ corresponding to exponential discounting, i.e. no present bias.} It is worth reemphasizing that naiveté implies that the continuation-value function $v$ in (11) is the value function of an exponential $\beta = 1$ household. This means that, similarly to the value function, one can recover the consumption policy function with a two-step procedure: first, solve the value function $v$ of an exponential $\beta = 1$ household from (8); second, find the consumption policy function of a present-biased $\beta < 1$ household from (11) (with an additional condition when $b = b$).

Proof of Proposition 1. Let $\hat{c}(x)$ denote the consumption function that the naive household expects all future selves to adopt ($\beta = 1$). Expanding (11) under CRRA utility:

$$c(x)^{-\gamma} = \beta \frac{\partial v(x)}{\partial b} \quad \text{and} \quad \hat{c}(x)^{-\gamma} = \frac{\partial v(x)}{\partial b}.$$
Rearranging gives \( c(x) = \beta^{-\frac{1}{\gamma}} \hat{c}(x) \). This holds as long as \( b > b \). For \( b = b \), overconsumption will be restricted if the borrowing constraint binds (see equation (3)). ■

Proposition 1 provides a tractable formula that relates the IG household’s consumption to that of a standard exponential household. This can be used to construct an Euler equation for the IG household:

**Corollary 1 (Maxted (2023))** Let \( r_t(b_t) \) denote the household’s effective borrowing cost: \( r_t(b_t) = r_t \) if \( b_t \geq 0 \), and \( r_t(b_t) = r_t + \omega \hat{c}\) if \( b_t < 0 \). Whenever \( c(x_t) \) is locally differentiable in \( b \), consumption obeys the following Euler equation:

\[
E_t \frac{du'(c(x_t))/dt}{u'(c(x_t))} = \left[ \rho + \gamma \left( 1 - \beta^{\frac{1}{\gamma}} \right) \frac{\partial c(x_t)}{\partial b} \right] - r_t(b_t).
\] (12)

**Proof.** Appendix C extends the proof of Maxted (2023) to our environment. ■

Equation (12) is the naive, continuous-time, analogue of the Hyperbolic Euler Relation for sophisticates of Harris and Laibson (2001). The growth rate of marginal utility is given by \( E_t \frac{du'(c(x_t))/dt}{u'(c(x_t))} \), and the term in brackets can be interpreted as the household’s effective discount rate at time \( t \). When \( \beta = 1 \), equation (12) reduces to a standard Euler equation: \( E_t \frac{du'(c(x_t))/dt}{u'(c(x_t))} = \rho - r_t(b_t) \). When \( \beta < 1 \), the household’s effective discount rate is increasing in its instantaneous MPC, \( \frac{\partial c(x_t)}{\partial b} \). Intuitively, overconsumption will have a larger effect on the growth rate of marginal utility when consumption itself is sensitive to \( b \).

For \( \beta < 1 \), an important consequence of equation (12) is that the household’s effective discount rate varies over the state space. In particular, since households with a higher instantaneous MPC will consume more impatiently, households near \( b = b \) and \( b = 0 \) will act more impatiently, while households with plentiful liquidity will act more patiently.

**Mortgage Adjustment.** Next we characterize the effect of present bias on the mortgage adjustment decision. To this end, recall that \( \mathfrak{R}(x) \in \{0, 1, 2\} \) and \( \mathfrak{R}(x) \in \{0, 1, 2\} \) denote the household’s decision to not adjust, prepay, or refinance in the high- and low-effort-cost states, and \( m'(x), b'(x), \hat{m}'(x), \) and \( \hat{b}'(x) \) denote its adjustment targets conditional on adjusting. Unlike consumption, present bias has a muted impact on these adjustment decisions. In particular, the only way that \( \beta < 1 \) affects the mortgage adjustment decision is through procrastination.\(^{22}\) This is formalized in the following proposition:

**Proposition 2 (Optimal Stopping)**

1. Adjustment targets \( m' \) and \( b' \) are independent of \( \beta \). Thus, \( m'(x) = \hat{m}'(x) \), \( b'(x) = \hat{b}'(x) \), \( \hat{m}'(x) = \hat{m}'(x) \), and \( \hat{b}'(x) = \hat{b}'(x) \) for all \( x \).

\(^{22}\) Accordingly, setting \( \varepsilon_t \equiv 0 \) would remove the effect of present bias on the mortgage adjustment decision.
2. (a) For $\beta = 1$, the refinancing policy function $R(x)$ converges pointwise to $\hat{R}(x)$ as the effort cost vanishes. This effectively means that the $\beta = 1$ household’s mortgage adjustment behavior does not depend on the state of the effort cost.

(b) For $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$, $R(x) = 0$ for all $x$. This means that the present-biased household procrastinates and will not adjust its mortgage when $\varepsilon = \bar{\varepsilon}$.

(c) For $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$, $\hat{R}(x)$ converges pointwise to $\hat{R}(x)$ as the effort cost vanishes. This effectively means that the present-biased household does not procrastinate when $\varepsilon = \bar{\varepsilon}$.

Proof. See Appendix C. The intuition for clause 1 is that the current self composes only an infinitesimal part of the household’s overall value function. This implies that $w_{\text{prepay}}(x)$ and $w_{\text{refi}}(x)$ in (9) can be rewritten as:

$$w_{\text{prepay}}(x) = \max_{b', m'} \beta v(b', m', y, r_m, r) \quad \text{s.t. prepayment constraint (4) holds}$$

$$w_{\text{refi}}(x) = \max_{b', m'} \beta v(b', m', y, r + \omega_m, r) \quad \text{s.t. refinancing constraint (5) holds}$$

Since maximizing $\beta v$ is equivalent to maximizing $v$, the same $(b', m')$ will be optimal for the $\beta < 1$ household and the $\beta = 1$ household. The intuition for clause 2 was discussed above in Section 2.3.

Clauses 2b and 2c of Proposition 2 state that present-biased households refinance only when they are in a refinancing region of the state space and they experience a low-cost window (weekends, cancelled meetings, etc). Recall from Assumption 1 that these low-cost windows occur at Poisson rate $\phi$. Therefore the model with naive present bias reproduces the state- plus time-dependent refinancing behavior documented in Andersen et al. (2020).

Cash-Out Refinances as Targeted Liquidity Injections. The combination of Propositions 1 and 2 yields one of our key results: when households are present-biased, interest rate cuts incentivize households to conduct cash-out refinances, which become targeted liquidity-injections to high-MPC households. The explanation proceeds in two steps. First, Proposition 2 states that present bias ($\beta < 1$) does not affect refinancing behavior except through procrastination: conditional on refinancing, the mortgage adjustment and hence the size of any potential cash-out is the same. Second, Proposition 1 implies that present-biased households overconsume: hence they spend down any given cash-out amount faster than exponential discounters. This is precisely what we find when we conduct our monetary policy experiments in Section 5.2 further below.
A Comparison: Consumption versus Mortgage Adjustment. The juxtaposition of Propositions 1 and 2 also highlights the subtleties of present bias’ impact on household balance-sheet decisions. In our model, present bias directly affects the consumption decision, whereas present bias only affects the mortgage adjustment decision through procrastination.

Present bias has a differential impact on these decisions because consumption and procrastination are “small” decisions (flow decisions) while discrete adjustment is a “large” decision (a stock decision). The current self wants to overconsume in the moment, but this overconsumption has only an infinitesimal effect on the household’s balance sheet. Similarly, procrastination is expected to last only for an instant. The same is not true of the mortgage adjustment decision. This decision discretely adjusts the household’s asset allocation between liquid wealth and illiquid home equity. Since each self only lasts for an instant, any short-term benefits from myopia-driven refinancing are dominated by the accumulation of costs borne across all future selves. In short, naive present bias causes a persistent accumulation of small mistakes, not the intermittent occurrence of large mistakes.

Though the proofs of Propositions 1 and 2 utilize IG preferences, the assumption that each self lasts for a single instant is mathematically convenient but not quantitatively necessary (Laibson and Maxted, 2022). Propositions 1 and 2 will be robust so long as the temporal division between the present and the future is relatively short (e.g., one week), meaning that each self composes a negligible part of the overall value function.23

The Effect of Present Bias on the Propensity to Refinance. There is an informal intuition in the literature that present bias increases the propensity for households to extract home equity in order to finance near-term consumption (see e.g. Mian and Sufi, 2011; Campbell, 2013). Our model illustrates that the effect of present bias on refinancing is more complex. When effort cost \( \varepsilon = \bar{\varepsilon} \), Proposition 2 says that present bias has no effect on the refinancing decision at any given point \( x \). When \( \varepsilon = \bar{\varepsilon} \), procrastination slows down the rate at which present-biased households refinance. Both of these effects counter the intuition that present bias increases cash-out refinancing. However, present bias causes a build-up of credit card debt due to overconsumption. This incentivizes home-equity extractions as a means of converting costly credit card debt to cheaper mortgage debt. In summary, present bias inhibits refinancing at any given point \( x \). But, present bias changes the distribution of households over the state space in a way that encourages cash-out refinancing.

\[ \text{For intuition, consider a discrete-time model where each self lives for one week. Let } \delta = \exp \left( \frac{\rho}{\beta \Delta} \right). \text{ Assume that each self consumes a constant amount, } \bar{c}, \text{ in each period. Each self has a current value of } u(\bar{c}) + \frac{\beta \delta}{1 - \delta} u(\bar{c}), \text{ meaning that the utility of each self composes a share } 1/ \left( 1 + \frac{\beta \delta}{1 - \delta} \right) \text{ of the total value function. Under our benchmark calibration with } \beta = 0.83 \text{ and } \rho = 0.88\% \text{ (Section 4.2) so that } \delta = \exp \left( \frac{\rho}{\beta \Delta} \right) = 0.9998, \text{ this share equals } 0.02\% \text{ of the total value function.} \]
Generalization to (Partial) Sophistication. We extend our analysis to partial and full sophistication in Appendix D.5. Letting $\beta^E$ denote the short-run discount factor that the current self expects all future selves to have, full naiveté means that $\beta^E = 1$. Partial sophistication instead sets $\beta^E \in (\beta, 1)$, such that the current self is partially aware of future selves’ self-control problems. Full sophistication is the limiting case of $\beta^E = \beta$.

There are two key takeaways from the extension in Appendix D.5. First, our main-text analysis (which assumes full naiveté) is robust to all but the limiting case of full sophistication. In particular, all that is needed to generate refinancing procrastination is for the current self to think that future selves will be less present biased than the current self is, meaning that any amount of naiveté is sufficient (i.e., $\beta^E > \beta$). This is because the current self then perceives that the next self will be more willing to refinance, and hence pushes refinancing into the future. Second, for the limiting case of full sophistication, households do not procrastinate. This second result follows from Assumption 2 that effort costs are vanishingly small. Intuitively, without at least some scope for incorrect expectations, we cannot generate non-vanishing bouts of procrastination from vanishingly small effort costs.\footnote{Even with full sophistication, procrastination can still arise with non-vanishing effort costs. See the discussion in Appendix D.5.4.}

4 Calibration and Steady State Household Behavior

4.1 Externally Calibrated Parameters

We begin by describing the model’s externally calibrated parameters. These parameter choices are summarized in Appendix Table 4.

Home Owners in the 2016 SCF. For many of our calibration targets we use the 2016 Survey of Consumer Finances (SCF) wave. We often express these targets relative to a measure of households’ permanent income for which – following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) – we use the SCF’s question about “normal income.” To align the SCF with the households in our model, we restrict our SCF sample to households that own a home, possess a credit card, have a head aged 25-61, and have a head or spouse in the labor force. The average after-tax permanent income for households in our SCF sample is roughly $100,000. See Appendix A.2 for details.

Income. We assume that the Poisson income process takes one of three values $y_t \in \{y_L, y_M, y_H\}$. We limit the model to three income states – low, middle, and high – so
that we can more fully illustrate the resulting model solution (see e.g. Figures 1 and 2).\textsuperscript{25} Our calibration of the income process follows Guerrieri and Lorenzoni (2017), who assume that the logarithm of income follows an AR(1) process at a quarterly frequency. We first convert this quarterly AR(1) process to a continuous-time Ornstein-Uhlenbeck (OU) process and then discretize the OU process with a three-state Poisson process using a finite difference method. See Appendix A.3 for details.

**Interest Rates.** We view $r_t$ as a longer-term interest rate, and assume that the monetary authority adjusts short rates in the background to generate these movements in the long rate. Our model aims to capture larger movements in the federal funds rate, such as those in 2001, 2008, and 2019-20, which have economically significant effects on long rates and hence on mortgage refinancing. The focus of our paper is not on smaller, fine-tuning, movements in the federal funds rate that do not significantly affect mortgage rates.

Rather than treating all shocks to $r_t$ as unexpected “MIT shocks,” households in our model have calibrated expectations about the data-generating process for $r_t$. In order to calibrate household interest rate expectations we estimate an AR(1) process via maximum likelihood estimation using weekly data on the yield of 10-year TIPS from 2003 – 2019. We first convert this weekly AR(1) process to a continuous-time OU process (with rate of mean reversion 0.29 and volatility 0.63%), and then discretize it into a four-state Poisson process with states $r_t \in \{-1\%, 0\%, 1\%, 2\%\}$ using a finite difference method.

We set the credit card wedge $\omega^{cc}$ to 10.3% to capture the difference between the commercial bank interest rate charged on credit cards and the 10-year treasury yield from 2015 – 2017. We set $\omega^{m} = 1.7\%$ to capture the difference between the average 30-year fixed rate mortgage and the 10-year treasury yield from 2015 – 2017.

**Assets and Liabilities.** Using the sample of home owners in the 2016 SCF, we estimate an average home value to permanent income ratio of 3.29 (roughly $329,000) and therefore set $h = 3.29$. We set $\theta$, the maximum LTV ratio, equal to 0.8. Although this is a tight restriction on the maximum LTV allowed for first-time homebuyers, it is consistent with the maximum LTV available to households conducting a cash-out refinance.\textsuperscript{26}

Mortgages are paid down at rate $\xi = 3.5\%$, which generates a 20-year half-life for mortgages.\textsuperscript{27} We set the fixed cost of refinancing to $\kappa^{refi} = 0.05$ (approximately $5,000), which

\textsuperscript{25}For $\beta = 1$ in consumption-saving models such as this, three income states are the minimum number of states that are needed to generate a mass of households who borrow, a mass of households at the soft constraint, and a mass of households who save (Achdou et al., 2021).

\textsuperscript{26}See Greenwald (2018) for data on realized LTVs for first-time homebuyers and for cash-out refinances.

\textsuperscript{27}Recall that mortgage payments are not constant in our model. We choose a 20-year half-life for mortgage paydowns so that the mortgage payment required by large mortgages is not exceedingly onerous.
is 3% of the average outstanding mortgage principal.\textsuperscript{28} For numerical tractability, we also impose a small cost to prepaying mortgages of $\kappa_{\text{prepay}} = 0.002$ (or approximately $\$200$).

We set credit card borrowing limit $b$ to one third of permanent income. This is consistent with reported credit limits in the SCF, and in line with typical choices in the literature.\textsuperscript{29}

**Preferences.** Discount function parameters $\rho$ and $\beta$ are calibrated internally to match home equity and credit card debt levels in the 2016 SCF — see Section 4.2 below. We set the intertemporal elasticity of substitution $\frac{1}{\gamma}$ equal to $\frac{1}{2}$, which is a standard calibration in the consumption-saving literature. We choose a Poisson rate governing procrastination of $\phi = -\ln(0.5)$. This implies that there is a 50% probability that a household for whom it is optimal to refinance will do so within one year, consistent with Andersen et al. (2020).\textsuperscript{30}

**Other Structural Parameters.** We set the rate of forced adjustment to $\lambda^F = \frac{1}{15}$, which approximates the moving rate of homeowners reported in the Current Population Survey’s Annual Social and Economic Supplement for 2016. The retirement rate $\lambda^R = \frac{1}{30}$ so that households exist in our model for 30 years on average. We set the retirement income flow $y^R$ equal to the minimum income level $y_L$, since $y_L = 0.75$ and a retirement replacement rate of 70-80% is a common benchmark. Retired households are dropped from the model, and are replaced by new households with mortgage $m_t = \theta h$ and liquid wealth $b_t$ drawn from a uniform distribution with support $[0, \frac{y_L}{2}]$.

### 4.2 Internally Calibrated Parameters and Steady State

**“Steady State.”** We start all policy counterfactuals from a “steady state” in which the interest rate has been permanently fixed at $r^* = 1\%$. This implies that all households in our steady state will have a mortgage interest rate of $r_t^m = r^* + \omega^m = 2.7\%$. The assumption that the interest rate has been fixed at 1\% is formally just one possible sample path of the interest rate process. This assumption is helpful for pedagogy because it reduces the dimensionality of our steady state by eliminating heterogeneity in $r_t^m$.\textsuperscript{31}

\textsuperscript{28}The Federal Reserve’s website on refinancing suggests that refinancing costs roughly 3\% of outstanding principle. The average LTV in our model is 0.51, suggesting $\kappa_{\text{refi}} = 0.03 \times (0.51 h)$. This yields roughly $\kappa_{\text{refi}} = 0.05$. For details, see https://www.federalreserve.gov/pubs/refinancings/. While we assume a uniform fixed cost for simplicity, refinancing costs vary from state to state and also over time (e.g., the 2017 Tax Cuts and Jobs Act lowered the mortgage interest deduction threshold from $\$1$ million to $\$750,000 and thus altered cash-out refinancing incentives for some households). Though not explored in depth in the current paper, our results suggest that variation in refinancing costs can affect both the steady-state distribution of households and their subsequent responses to macroeconomic stabilization policy.

\textsuperscript{29}For example, Kaplan et al. (2018) calibrate a borrowing limit of one times quarterly labor income.

\textsuperscript{30}Andersen et al. (2020) estimate that 84\% of households are “asleep” each quarter, and $0.84^4 \approx 0.5$.

\textsuperscript{31}This fixed-$r$ assumption implies that once rates fall, all households can lower their mortgage rate by refinancing. This broadly reflects the rate cuts that followed the Early 2000s, Great, and COVID-19 Recessions,
**Calibration Cases.** We present two benchmark calibration cases for the “steady state”: an *Exponential Benchmark* with $\beta = 1$ and a *Present-Bias Benchmark* with $\beta < 1$. To reproduce the home equity that we observe in our 2016 SCF sample, we calibrate the long-run discount rate $\rho$ in both cases to match an estimated average LTV ratio of 0.51. To capture the fragility of household balance sheets, we also calibrate $\beta$ in the Present-Bias Benchmark to fit an estimated average credit card debt to permanent income ratio of 0.09 (the Exponential Benchmark restricts $\beta = 1$). Note that SCF-measured credit card debt is designed to capture *revolving* debt that accrues interest. Moreover, to exclude balances on promotional low-interest products such as balance-transfer cards, we follow Lee and Maxted (2023) and restrict our focus to *high-interest credit card debt* on which households report paying an interest rate of greater than 5%. Finally, because credit card debt appears to be underreported in the SCF (Zinman, 2009), we adjust reported credit card borrowing upward by a factor of 1.5 following the methodology of Beshears et al. (2018, Appendix C).

Table 1 presents the discount function calibration. In the Exponential Benchmark, $\rho = 1.25\%$ matches the average LTV moment of 0.51. However, the Exponential Benchmark fails to generate the level of credit card borrowing observed in the data. In the Present-Bias Benchmark, $\beta = 0.83$ matches the average credit card debt to income ratio of 0.09,\(^{32}\) while lowering $\rho$ to 0.88% ensures that households still accumulate sufficient home equity to match the LTV moment. Importantly, Table 1 highlights that in a calibrated model, introducing present bias does not simply mean that households are more impatient. Rather, present-biased households are more impatient in the short run since $\beta < 1$, but are simultaneously more patient in the long run due to their lower $\rho$. This differential patience allows the Present-Bias Benchmark to fit high-cost credit card borrowing jointly with illiquid home equity accumulation (relatedly, see also Laibson et al., 2023).

For readers who are uncomfortable with our 1.5-times adjustment to SCF credit card borrowing, we also estimate that 53% of households in our SCF sample report carrying a high-interest credit card balance from one month to the next. Table 1 shows that this feature is qualitatively matched by our Present-Bias Benchmark, but not by our Exponential Benchmark. Indeed, we would calibrate an even lower $\beta$ value if we were to set $\beta$ to reproduce the share of households with revolving (high-interest) credit card debt.

Compared to heterogeneous-agent macro models with exponential time preferences, one key difference with our calibration is that we take both the credit card wedge ($\omega^{cc} = 10.3\%$) and the effective illiquid return spread ($\omega^{m} = 1.7\%$) “from the data.” Despite the large

\(^{32}\)Meier and Sprenger (2010) combine surveyed time preferences with administrative credit card data to provide direct evidence of the relationship between present bias and credit card debt.
borrowing wedge and the low illiquidity wedge, our Present-Bias Benchmark is nonetheless able to match the credit card debt levels observed empirically. In contrast, models with exponential time preferences such as Kaplan and Violante (2014) and Kaplan et al. (2018) typically internally calibrate at least one of the model’s interest rates in order to generate similar levels of low-liquidity households. Doing so typically results in a lower borrowing wedge and higher illiquidity spread than in the data. This also implies that our own Exponential Benchmark – which sets $\beta = 1$ without recalibrating interest rates to generate constrained households – has too few low-liquidity households and hence low MPCs.

### Supplement: Intermediate Cases.

As shown by Proposition 2, the Present-Bias Benchmark features procrastination while the Exponential Benchmark does not. Because our Present-Bias Benchmark therefore introduces both present bias and procrastination, for conceptual clarity we also discuss various “intermediate cases” in Appendix D.4. These intermediate cases allow us to bridge the gap between the Exponential Benchmark and the Present-Bias Benchmark by studying present bias without refinancing inertia, and exponential discounting with refinancing inertia. These cases also allow us to explore sophistication, which effectively shows up as present bias without refinancing inertia.

---

For example, Kaplan et al. (2018) internally calibrate a borrowing wedge of 6% to fit the share of households with negative net liquidity (a borrowing rate of 8% minus a liquid return of 2%), which is lower than our empirical credit card wedge of 10.3%. Kaplan et al. (2018) relatedly calibrate an illiquidity wedge of 3.7% (5.7% minus 2%), which is higher than our number of 1.7%. Additionally, our approach of taking interest rate wedges “from the data” also differentiates our approach from models with unsecured borrowing rates determined endogenously based on default risk (e.g., Mitman, 2016), though such models have also had success in producing realistic levels of unsecured debt and home equity accumulation.

Another difference is that our model includes mortgages, which allows households to borrow against their illiquid (housing) wealth. Alternatively, in models such as Kaplan et al. (2018) that only have net illiquid wealth, households must sell their illiquid wealth in order to access it. Since illiquid assets offer high returns and hence are costly to sell, we conjecture that allowing households to borrow against their illiquid wealth effectively makes that wealth more liquid. This will also lead to fewer constrained households and hence lower MPCs in our model. A similar effect is also reported in McKay and Wieland (2021).

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### Table 1: Internally Calibrated Parameters.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Exponential Benchmark</th>
<th>Present-Bias Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discount Function</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>-</td>
<td>1</td>
<td>0.83</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-</td>
<td>1.25%</td>
<td>0.88%</td>
</tr>
<tr>
<td><strong>Calibration Targets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV</td>
<td>0.51</td>
<td><strong>0.51</strong></td>
<td>0.51</td>
</tr>
<tr>
<td>Avg. C.C. Debt</td>
<td>0.09</td>
<td>0.03</td>
<td><strong>0.09</strong></td>
</tr>
<tr>
<td>Share C.C. Debt $&gt; 0$</td>
<td>53%</td>
<td>27%</td>
<td>47%</td>
</tr>
</tbody>
</table>

Notes: This table presents the calibration of the discount function for the two benchmark cases we study.
4.3 Steady State Household Behavior

Households solve an optimal control problem augmented with an optimal stopping problem, and the steady state features cross-sectional heterogeneity in four variables: $b_t, m_t, y_t,$ and $\varepsilon_t$. When characterizing the steady state we focus on the features that will be important for the macroeconomic policy results to follow. As we detail below, many of the equilibrium behaviors that differentiate the Present-Bias Benchmark from the Exponential Benchmark are consistent with an emerging set of empirical findings in the household finance literature that have, collectively, proven challenging for models with exponential discounting to replicate.

Phase Diagrams of the Household Balance Sheet. Figure 1 uses phase diagrams to describe the evolution of households over the state space. From left to right, each panel represents a different income state. The top row shows the Exponential Benchmark, and the bottom row shows the Present-Bias Benchmark. The horizontal axis of each panel is liquid wealth $b$ and the vertical axis is the household’s mortgage balance, expressed as the LTV ratio $m/h$. The red and blue shaded areas indicate discrete adjustment regions: red indicates cash-out refinancing, and blue indicates discrete mortgage prepayment. In areas of non-adjustment, the arrows indicate how the household balance sheet evolves over time.

Looking first at the red regions, households choose to take a cash-out refinance when they have relatively low income (either $y_L$ or $y_M$) and are near the credit limit of $b$. During spells of lower income, the first margin on which households adjust is to decumulate liquid wealth. The second margin is credit card debt: even with wedge $\omega_{cc}$, the fixed cost of refinancing implies that temporarily taking on credit card debt can be prudent. Cash-outs are the final margin that households turn to, but only after accumulating sizable credit card debt.

Looking next at the blue regions, households choose to prepay their mortgage once they have built a buffer stock of liquid wealth. Having some liquid wealth is useful because it allows households to avoid taking on costly credit card debt during low-income spells. However, it is suboptimal to hold too much liquidity because the mortgage wedge $\omega_{m}$ implies that mortgage debt is more costly than the household’s return on liquid wealth. Thus, high-liquidity households will use some of their accumulated liquidity to pay down their mortgage.

As shown in Proposition 2, differences in adjustment regions across the two calibrations are driven by variation in $\rho$. The top row (Exponential Benchmark) features a higher calibration of $\rho = 1.25\%$, while the bottom row (Present-Bias Benchmark) calibrates $\rho = 0.88\%$. Households who perceive themselves to be less patient (higher $\rho$) will be more willing to take a cash-out refinance and less willing to prepay their mortgage. The variation in the red cash-out region and blue prepayment region across the two calibrations follows accordingly.

35In the steady state with constant interest rates, the only reason to refinance is to withdraw home equity.
Figure 1: Steady State Phase Diagrams.

Notes: Arrows display the evolution of household balance sheets. Red regions mark where the household chooses to conduct a cash-out refinance, and blue regions mark mortgage prepayment. See text for details.

The blue arrows indicate how household balance sheets evolve over the non-adjustment regions. The arrows always point downward due to mortgage principal repayment, as specified by equation (2). The arrows point either right or left to indicate liquid saving or dissaving, and the length of the arrow corresponds to the speed of evolution. Arrows point strongly leftward when $y_t = y_L$, indicating liquid dissaving. Arrows point strongly rightward when $y_t = y_H$, indicating liquid saving. When $y_t = y_M$ the arrows typically point to the left but are small, indicating slight dissaving by households.

The bottom row features gray arrows in the adjustment regions, while the top row does not. In the top row there is no procrastination. Households will “jump” as soon as they move into an adjustment region, and therefore households will never find themselves in the shaded regions. The bottom row features procrastination. This means that households will find themselves in the adjustment regions, and the gray arrows indicate how household balance sheets evolve when households procrastinate.
Steady State Consumption Dynamics. Figure 2 plots the steady state consumption function. Liquid wealth is on the horizontal axis. Each panel plots the consumption function for an LTV \( \in \{0, 0.4, 0.8\} \).

(a) Exponential Benchmark

(b) Present-Bias Benchmark

Notes: This figure shows the steady state consumption function over income and liquid wealth for LTV \( \in \{0, 0.4, 0.8\} \). See text for details.

Starting with the Exponential Benchmark in the top row, note that the curves do not always span the entire liquid-wealth axis (e.g., the blue and orange curves do not extend to \( \bar{b} \) in the Low Income panel). These missing areas reflect the adjustment regions. Households will never reach these parts of the state space.

The bottom row shows consumption in the Present-Bias Benchmark. There are two key differences under present bias. First, we now use dashed lines and open circles to mark consumption in the adjustment regions, since procrastination implies that households can enter these regions. Second, the consumption function occasionally features a discontinuity.
at $b$. This discontinuity arises because the borrowing constraint restricts overconsumption (see constraint (3)). The consumption discontinuity is consistent with the evidence presented in Ganong and Noel (2019), who use high-frequency data to document that consumption drops sharply at the expiration of unemployment insurance.

This discontinuity can be particularly large when households procrastinate on refinancing. For example, households with $y_t = y_L$ and $LTV = 0.4$ experience a drop in consumption of approximately 30% if they fail to refinance before hitting $b$. The reason for this large discontinuity is that households are naive about their procrastination. Once households are in an adjustment region they expect their next self to refinance. As a result, households choose consumption today in order to smooth consumption relative to where they expect to be following a cash-out refinance. Households do not anticipate hitting the borrowing constraint $b$, creating a large downward discontinuity if the constraint does eventually bind.

**Marginal Propensities to Consume (MPCs).** Consumption behavior can also be investigated through MPCs. The marginal propensity to consume (MPC) over $\tau$ years is defined as follows (Achdou et al., 2021):

$$MPC_{\tau}(x) = \frac{\partial}{\partial b} \mathbb{E}\left[ \int_0^\tau c(x_t) dt \mid x_0 = x \right].$$ (13)

The left panel of Figure 3 plots the quarterly MPC out of $1,000 as a function of liquid wealth $b$, averaging over the income and mortgage dimensions. Consistent with the typical behavior in these sorts of models, the Exponential Benchmark features elevated MPCs at $b = 0$ where the interest rate jumps and at the borrowing constraint $b$ (e.g., Kaplan and Violante, 2014; Kaplan et al., 2018). The key effect of present bias is that it drastically increases MPCs near the borrowing constraint. This follows directly from the consumption function discontinuities at $b$ detailed above: present-biased households do not smooth consumption into the borrowing constraint, instead choosing a high consumption rate all the way into $b$.37

**Steady State Wealth Distribution.** The right panel of Figure 3 presents the steady state distribution of liquid wealth for the Exponential Benchmark and the Present-Bias Benchmark. There are two key differences. First, present bias generates a leftward shift in the

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36Equation (13) defines the MPC out of an infinitesimal increase in liquid wealth. However, tax rebates and fiscal stimulus payments increase liquid wealth discretely. The definition of the MPC can easily be extended to discrete liquidity shocks (see Appendix E.1 for details).

37The left panel of Figure 3 also illustrates that in both calibrations, MPCs are driven by borrowing-constrained households at either $b = 0$ or $b = b$ and decline quickly with liquid wealth. Accordingly, present bias alone is not capable of matching the empirical evidence that some high-liquidity households also have high MPCs (see for example Figure 4 in Ganong et al., 2020, and the studies referenced therein).
liquid wealth distribution because present-biased households overconsume out of liquidity.\footnote{In particular, the overconsumption shown in Proposition 1 implies that the soft constraint does not typically bind for $\beta < 1$ households. See Maxted (2023, Prop. 7) and Lee and Maxted (2023) for details.} Second, present bias produces a larger share of households at the borrowing constraint $b$.

There are three factors that contribute to this large mass of constrained households at $b$ when $\beta < 1$. First, households fail to maintain liquidity buffers. Second, the $\beta < 1$ calibration features a lower value of $\rho$, which reduces households’ willingness to refinance at $b$ (see the Middle Income panels in Figure 1). Third, procrastination means that households can be slow to refinance when they hit the constraint.

The buildup of households at $b$ is consistent with the responsiveness of debt accumulation to borrowing limits documented empirically. Gross and Souleles (2002) study how borrowing responds to credit limit increases, and estimate that credit card debt increases by 10-14\% of the increased limit after one year. In our model, the first-order effect of a small increase in the borrowing limit $b$ is that constrained households accumulate additional debt by exactly the same amount, so that the overall marginal borrowing propensity simply equals the share of constrained households. Consistent with this evidence, the share of constrained households is 12\% in the Present-Bias Benchmark, compared to only 0.1\% in the Exponential Benchmark. Moreover, we find in our 2016 SCF sample of homeowners that roughly 14\% of households have less than two-weeks’ pay of available credit remaining on their credit card, again consistent with the buildup of households near $b$ in the Present-Bias Benchmark.
Table 2: Steady State Summary Statistics.
Notes: This table summarizes household consumption, expenditure, and saving behavior in the steady state.

Summary Statistics. Table 2 summarizes the model’s steady state. The Present-Bias Benchmark features an average quarterly MPC of 14.4% out of a $1,000 transfer. For the Exponential Benchmark, this MPC is 4.2%. Only the Present-Bias Benchmark comes close to empirical MPC estimates, which range from 15-25% for fiscal transfers of $500 – $1,000.\(^{39}\) The Present-Bias Benchmark also features much larger variation in MPCs based on transitory income (second row). Low- and middle-income households have high MPCs because they compose a larger share of households on or near the borrowing constraint. Though this MPC heterogeneity still exists in the Exponential Benchmark, it is much less drastic. Time-consistent households maintain liquidity buffers optimally, so the Exponential Benchmark features far fewer constrained households. We also find that the elevated MPC exhibited in the Present-Bias Benchmark is robust to the size of the wealth shock. The average quarterly MPC remains at 8.3% for a transfer of $10,000 (for more, Appendix Figure 12 reports quarterly MPCs out of transfers ranging from $1,000 to $50,000).\(^{40}\)

When comparing the model to the data, it is important to delineate between the marginal propensity to consume (MPC) versus the marginal propensity for expenditure (MPX) (Laibson et al., 2021).\(^{41}\) Durables drive a wedge between expenditure and model-based “notional” consumption, because spending on durables does not translate into immediate consumption. The difference can be substantial empirically. For example, Parker et al. (2013) document that over 75% of spending from the 2008 fiscal stimulus was on durable goods.

To bridge the gap between notional consumption and expenditure, we follow the method

\(^{39}\)See footnote 3 for references reviewing the empirical literature on MPCs. While our model focuses on homeowners, the empirical evidence on large consumption responses to liquidity injections has also been found to extend to homeowners specifically (see e.g. Parker et al., 2013; Lewis et al., 2022).

\(^{40}\)See Kueng (2018) and Hamilton et al. (2023) for empirical evidence of high MPCs from large transfers. Hamilton et al. (2023) argue that a two-asset model with naive present bias is consistent with the evidence they document whereas the version with exponential discounting is not. Attanasio et al. (2020) show that temptation preferences can also generate high MPCs out of large transfers.

\(^{41}\)The MPX is also likely to be the more relevant concept for general equilibrium analyses, since the MPX captures changes in overall consumption expenditure on both durables and nondurables which is what matters for aggregate demand.
of Laibson et al. (2021) for converting our model’s predictions about MPCs into predictions about MPXs. Table 2 reports the model’s MPX predictions. As with MPCs, the Present-Bias Benchmark features elevated quarterly MPXs that remain large even for large wealth transfers. Elevated MPXs out of large transfers is consistent with Fagereng et al. (2021), who use Norwegian administrative data to estimate that lottery winners of amounts ranging from $8,300 – $150,000 spend 50% of their prize within the year of winning.

The final two rows of Table 2 summarize credit card borrowing. In the Exponential Benchmark, only 0.1% of households are constrained, and only 27% of households hold credit card debt (compared to 53% in the SCF). The Present-Bias Benchmark is more in line with the data: 12% of households are constrained and 47% hold credit card debt.

5 Results: Macro Stabilization Policy with Present Bias

We now present our results for fiscal and monetary policy under present-biased time preferences. We start all policy counterfactuals from the pre-shock “steady state” with $r^* = 1\%$.

5.1 Fiscal Policy

We first study the efficacy of fiscal policy. Starting from the “steady state” at time $t = 0$, the government (unexpectedly) makes a one-time stimulus payment of $1,000 to each household. This stimulus payment is financed by a flow income tax levied on all households in perpetuity, which is chosen to satisfy the government budget constraint in our environment with a stochastic interest rate on government debt. Specifically, the government follows a simple fiscal rule: at each point in time $t > 0$, levy a (stochastic) flow income tax on all households that is “just a little bit higher” than the additional (stochastic) interest payments resulting from the initial stimulus. As we explain in Appendix F.1, this simple fiscal rule ensures that the government budget constraint is satisfied and that government debt eventually reverts to its initial steady state level. This simple modeling trick could also prove useful in other environments with stochastic interest rates.

Given that a substantial fraction of households are financially constrained and that households have finite working lives (without transfers across generations), Ricardian equivalence does not hold in our model (Barro, 1974). In fact, the initial short-run consumption response under this tax scheme is similar to that without imposing a government budget constraint. Intuitively, the costs of the initial fiscal stimulus are spread over a long horizon (resulting in a small per-period tax) and borne largely by future generations (Blanchard, 1985).

Figure 4 plots the impulse response function (IRF) of aggregate consumption to this $1,000 fiscal transfer. To make this IRF easier to connect to MPCs, we scale the consumption
response at time $t$ by the size of the initial fiscal transfer. We provide the cumulative (scaled) consumption responses over different time horizons in the corresponding table. The model’s predicted consumption responses can also be interpreted using the “intertemporal MPC” framework of Auclert et al. (2018), who show that this dynamic spending response is important for characterizing the general equilibrium propagation of fiscal policy shocks.

![Figure 4: Consumption Response to Fiscal Policy.](image)

<table>
<thead>
<tr>
<th></th>
<th>Exponential</th>
<th>Present Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year Consumption Response</td>
<td>14%</td>
<td>28%</td>
</tr>
<tr>
<td>2 Year Consumption Response</td>
<td>23%</td>
<td>39%</td>
</tr>
<tr>
<td>3 Year Consumption Response</td>
<td>30%</td>
<td>46%</td>
</tr>
</tbody>
</table>

Table 2 and Appendix Figure 12 show that MPCs remain elevated even for large transfers, suggesting that present bias also allows fiscal policy to remain potent when implemented at larger scales.

Fiscal Policy Implementation: Liquidity. In response to the 2007-08 Financial Crisis, policymakers utilized a mixture of liquid and illiquid fiscal transfers (e.g., stimulus checks versus mortgage principal reductions). To study these different transfer designs in our model, Appendix Figure 15 compares the consumption response following a liquid fiscal transfer to...
an illiquid mortgage reduction of the same magnitude. In the Present-Bias Benchmark, we find that the large consumption response shown in Figure 4 relies critically on the transfer being liquid. This is because the liquidity-constrained households who drive that consumption response are unable to immediately consume out of illiquid home equity. Such sensitivity to transfer liquidity is consistent with Ganong and Noel (2020), who use variation in mortgage modification programs following the financial crisis to document that liquidity is the critical driver of short-term household consumption decisions. Alternatively, the consumption response in the Exponential Benchmark is far less sensitive to the liquidity of the transfer.

5.2 Monetary Policy

Next we study the impact of stimulative monetary policy. Starting from the “steady state” where $r^* = 1\%$ and all households have a mortgage interest rate of $r^m_t = r^* + \omega^m$, interest rates are cut from 1% to 0% and held at 0% for 3 years. Figure 5 illustrates the consumption response to this 1% rate cut in our Exponential and Present-Bias Benchmarks compared to the “steady state.” We also provide the present value of these consumption responses in the corresponding table (discounted at the market interest rate).

In both cases, approximately 70% of households find themselves in a refinancing region following the rate cut. By encouraging households to refinance, an important feature of this interest rate cut is that it also provides households with an opportunity to extract home equity. Indeed, roughly 75% of households who refinance engage in a cash-out refinance. Details of the refinancing decision are presented in Appendix F.3.

Starting with the Exponential Benchmark, Figure 5 shows that this case features a 3.7% increase in consumption on impact, which then decays slowly over longer horizons. Intuitively, the rate cut induces a wave of cash-out refinances on impact, and this extracted home equity is steadily spent down over time. Turning to the Present-Bias Benchmark, the solid red line shows a similar on-impact consumption response to the rate cut. But, in contrast to the Exponential Benchmark, the Present-Bias Benchmark also exhibits essentially no decay in potency over the three-year period that we study.

To provide intuition for the Present-Bias Benchmark, the transparent red line plots a counterfactual consumption response where we start from the Present-Bias Benchmark, but shut off refinancing procrastination for one quarter after the rate cut. Without procrastination, we see that monetary policy is roughly twice as effective on impact, but also burns out quickly over time. The intuition is the same as for fiscal policy: the rate cut incentivizes households – especially those who are constrained – to extract equity from their home.

---

43Since we only study the three-year response, we do not need to specify the interest rate path beyond that horizon. While our monetary policy experiment examines just one sample path of interest rates for simplicity, households continue to have calibrated interest-rate expectations along that sample path.
### Figure 5: Consumption Response to Monetary Policy.

Notes: This figure plots the percent change in aggregate consumption following an interest rate cut from 1% to 0%. The corresponding table provides the cumulative consumption response in present value terms. The transparent line labeled 1Q No Proc. plots a counterfactual consumption response that starts from the Present-Bias Benchmark, but shuts off refinancing procrastination for one quarter after the rate cut.

This way, the refinancing channel of monetary policy imitates the liquidity-injection features of fiscal policy. Since present-biased households overconsume out of liquid wealth, this wave of home-equity extractions produces a large consumption boom.

Connecting this counterfactual experiment to our Present-Bias Benchmark, the key difference is that households procrastinate on refinancing. Though procrastination lowers the consumption response to monetary policy on impact, it also helps to sustain that response over time as households slowly get around to refinancing.

A noteworthy feature of the Present-Bias Benchmark is that the consumption response to monetary policy in Figure 5 is mildly hump-shaped, reaching its peak after around 2 years. Appendix Figure 18 shows that this hump shape can also be more pronounced if the procrastination duration is shorter.\(^\text{44}\) A hump-shaped response of aggregate consumption

\[^{44}\text{Intuitively, even short bouts of procrastination will limit the on-impact consumption response, since many households need to extract home equity before they can increase consumption. But when procrastination is only short-lived, a wave of home-equity extractions still follows quickly after the rate cut. Thus, short bouts of procrastination generate a muted on-impact, but large subsequent, response to monetary policy, which then burns out as households consume their cash-out — i.e., a hump-shaped consumption response.}\]

<table>
<thead>
<tr>
<th></th>
<th>Exponential</th>
<th>Present Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year Present Value</td>
<td>3.45%</td>
<td>3.53%</td>
</tr>
<tr>
<td>2 Year Present Value</td>
<td>6.40%</td>
<td>7.14%</td>
</tr>
<tr>
<td>3 Year Present Value</td>
<td>9.03%</td>
<td>10.72%</td>
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</tbody>
</table>
to monetary policy shocks is a common finding in the literature estimating such IRFs using
time-series data (e.g., Rotemberg and Woodford, 1997; Christiano et al., 2005). Present bias
with procrastination thus has the potential to qualitatively generate this empirical finding.\textsuperscript{45}

There is also an emerging literature showing that the refinancing channel of monetary
policy is sensitive to the time-path of mortgage interest rates (Berger et al., 2021; Eichen-
baum et al., 2022). We highlight a different form of sensitivity — sensitivity to procrastina-
tion. Procrastination means that households are slow to adjust on their refinancing margin,
prolonging the pass-through of monetary policy to household consumption.

**Consumption Response Decomposition: Three Channels.** Monetary policy affects
household consumption through three channels. First, there is the standard direct effect on
liquid wealth — the change in interest rate \( r_t \) affects the household’s return on \( b_t \).
\textsuperscript{46} Second, the interest rate cut gives households the option to refinance into a lower-rate mortgage.
Third, households can extract home equity when refinancing their mortgage.

We decompose the initial consumption response to monetary policy into its three com-
ponents. First, we isolate the direct effect on liquid wealth by shutting down households’
ability to adjust their mortgage. Second, we reintroduce the ability for households to con-
duct a rate refinance, but keep the cash-out channel shut down.\textsuperscript{47} Third, we reintroduce the
cash-out channel to get back to the benchmark results shown in Figure 5.

Table 3 presents this decomposition. Each cell gives the on-impact consumption elasticity
in the modified model.\textsuperscript{48} In both calibration cases, the liquid wealth channel and the rate
refinancing channel each drive roughly one quarter of the total consumption response, with
the bulk of the response coming from the cash-out channel of monetary policy.\textsuperscript{49}

### 5.3 Summary

Present bias creates both high MPCs and a large share of constrained households. Because
these households are quick to spend down liquidity, present bias makes fiscal policy a powerful
and robust tool for generating a short-run consumption boom. Present bias also increases

\textsuperscript{45}See also Auclert et al. (2020), who propose “sticky expectations” as a source of such hump-shaped IRFs.

\textsuperscript{46}In particular, the direct effect on liquid wealth includes the usual income and substitution effects, which
are the focus of most single-asset models of monetary policy.

\textsuperscript{47}To do this, we modify the refinancing budget constraint in equation (5) so that it instead becomes:
\( b' - m' = b_t - m_t - \kappa \text{refi} \), subject to \( m' \in [0, m_t + \kappa \text{refi}] \) and \( b' \geq b_t \).
This modified budget constraint means that households cannot increase their liquid wealth by refinancing; i.e., \( b' \leq b_t \).

\textsuperscript{48}In all three steps we use the pre-shock distribution of households from the full model. This prevents the
distribution of households from changing as we change households’ access to refinancing technology.

\textsuperscript{49}One important caveat here is that our model provides an incomplete picture of the spending response to
cash-outs. In particular, while households often report using extracted home equity for residential investment
(e.g., Greenspan and Kennedy, 2008), this channel is broadly missing from our model with fixed housing \( h \).
### Table 3: Consumption Response Decomposition.

<table>
<thead>
<tr>
<th>Step</th>
<th>Exponential</th>
<th>Present Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1. No Adjustment</td>
<td>0.81%</td>
<td>0.84%</td>
</tr>
<tr>
<td>Step 2. No Cash-Outs</td>
<td>1.78%</td>
<td>1.89%</td>
</tr>
<tr>
<td>Step 3. Full Response</td>
<td>3.74%</td>
<td>3.60%</td>
</tr>
</tbody>
</table>

Notes: This table decomposes the channels through which monetary policy produces a consumption response (on impact). The first row presents the consumption elasticity when households are not allowed to adjust their mortgage. The second row allows for rate refinances only. The third row presents the full response.

the potency of monetary policy for a similar reason: rate cuts produce cash-out refinances, which mirror the liquidity-injection features of fiscal policy. Though powerful, the refinancing channel of monetary policy is sensitive to refinancing inertia. Procrastination increases the lag between rate cuts and refinancing, generating a milder but longer-lived stimulus.

### 5.4 Extensions

**Distributional Effects of Fiscal and Monetary Policy.** Appendix D.1 leverages the heterogeneous-agent structure of the model to explore how present bias affects the distributional consequences of policy. The key takeaway from this analysis is that present bias reverses the distributional consequences of fiscal versus monetary policy. In the Exponential Benchmark, monetary policy is an effective way to stimulate the consumption of low-consumption households: a cut to interest rates allows low-consumption households to refinance. In the Present-Bias Benchmark, procrastination implies that monetary policy no longer stimulates the short-run consumption of constrained households. Instead, fiscal policy is highly effective at increasing the short-run consumption of low-consumption households.

**Shocks to House Prices and Income.** Appendix D.2 examines the effect of house price shocks and recessionary income shocks on our results. House price shocks are of particular interest for our analysis, since house price shocks can quickly create or destroy the home equity that is central to the refinancing channel of monetary policy (e.g., Beraja et al., 2019). Though the magnitude of the consumption response to monetary policy is sensitive to house price shocks, our main result that present bias amplifies this response continues to hold.

**A Call to ARMs?** Thus far we have assumed that households have fixed-rate mortgages (FRMs) in order to reflect the typical features of the U.S. mortgage market. However, adjustable-rate mortgages (ARMs) are often the modal mortgage contract outside of the U.S. (Badarinza et al., 2016). Moreover, since the 2007-08 Financial Crisis, many economists...

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50See also Wolf (2021) for a comparison of the distributional effects of monetary and fiscal policy.
have argued that downwardly flexible mortgages may improve macroeconomic stability by creating a direct transmission of rate cuts to household mortgage payments (e.g., Eberly and Krishnamurthy, 2014; Andersen et al., 2020; Campbell et al., 2021; Guren et al., 2021).

In Appendix D.3 we explore how present bias interacts with ARMs, and highlight a novel tradeoff between FRMs and ARMs that policymakers should be aware of when considering different mortgage contract designs. On the one hand, the benefit of ARMs is that the direct pass-through of monetary policy offsets refinancing procrastination. On the other hand, ARMs also reduce the cash-out channel of monetary policy – which is particularly potent when households are present biased – because ARMs imply that households no longer need to refinance when rates fall. Though our model is too stylized to make rigorous quantitative claims, we find that these two effects roughly offset in our benchmark calibration.

Discussion of General Equilibrium. Appendix G discusses how present bias could affect the transmission of monetary and fiscal policy in a full general equilibrium analysis. We provide only a brief discussion through the lens of the literature on Heterogeneous Agent New Keynesian (HANK) models. Fully evaluating the impact of present bias in a general equilibrium model is an important task for future work.

6 Conclusion

This paper’s main messages are twofold. First, present bias improves the model’s ability to replicate a variety of empirical patterns exhibited in household consumption-saving behavior. Second, present bias amplifies the balance-sheet channels of both fiscal and monetary policy but, at the same time, slows down the transmission of monetary policy due to refinancing procrastination.

We conclude by repeating a number of limitations of our analysis. First, we do not model general equilibrium forces and touch upon this issue only briefly in Appendix G. Second, our model abstracts from many important macroeconomic dimensions. We focus on a subset of the population, homeowners. We do not model the endogenous responses of the financial sector nor do we model businesses, both of which are affected by macroeconomic stabilization policy. Third, even in partial equilibrium, the household side of our model is highly stylized. Fourth, our discussion on the timing of fiscal and monetary policy abstracts from policy lags which, in practice, are a critical difference between the speed of fiscal versus monetary policy. Fifth, we do not study the welfare consequences of fiscal and monetary policy. All of these considerations are likely fruitful areas for future research.
References


Reis, Ricardo, “The Constraint on Public Debt when r < g but g < m,” Mimeo, 2021.


