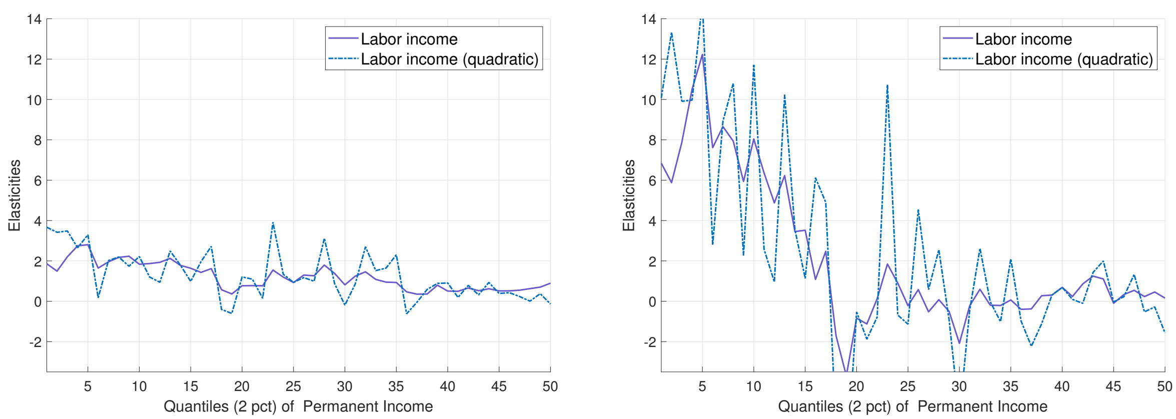
Online Appendix

A Additional material on the estimation of incidence functions

This appendix presents some additional material for our empirical exercise discussed in Section 2.2. Namely, we repeat the estimation of elasticities $\gamma(z)$ allowing for a quadratic (instead of linear) trend:

$$\log y_{it}^{z} = \beta_0(z) + \beta_1(z)t + \beta_2(z)t^2 + \gamma(z)\log Y_t + \epsilon_{it}, \qquad \forall z \in \{1, \ldots, 50\}$$
$$\text{s.t. } \sum \bar{s}(z)\gamma(z) = 1$$

Figure A1 shows the results of this specification together with our baseline estimates which assume a linear trend. Albeit noisier, the overall shape of elasticities across permanent income is robust to the specification of the trend.



(a) Elasticity using logarithm (b) Elasticity using inverse hyperbolic sine

Figure A1: Estimated elasticities of individual earnings to aggregate earnings as a function of permanent income quantile. Dotted lines are the 95% confidence bands. Source: ASEC 1967-2017.

B.1 Proof of Lemma 1

Lemma 1. *The optimal investment rate ι_t satisfies $1 + \Phi'(\iota_t) = q_t^k$ where $q_t^k := \frac{dA_t}{dK_t}$ is the fund's shadow value of capital. The value of the fund is given by $A_t = q_t^k K_t + q_t^x X_t$. And* *the return to illiquid assets r_t^a satisfies*

$$r_t^a = \frac{r_t^k - \iota_t - \Phi(\iota_t) + q_t^k(\iota_t - \delta) + \dot{q}_t^k}{q_t^k} = \frac{\psi\Pi_t + \dot{q}_t^x}{q_t^x}. \tag{B1}$$

Proof. Let $A_t(K, X)$ be the value of the fund at time t with capital stock K and X shares in the intermediate producer. It must satisfy the following Hamilton-Jacobi-Bellman equation

$$\begin{aligned} r_t^a A_t(K,X) = \max_{\iota,x} \left[r_t^k - \iota - \Phi(\iota)\right]K + \psi\Pi X - q_t^x x + \\ + \partial_K A_t(K,X)(\iota - \delta)K + \partial_X A_t(K,X)x + \partial_t A_t(K,X) \end{aligned} \tag{B2}$$

together with *first order conditions* on investment rate ι and the drift $\dot{X} = x$ of its shareholdings

$$\begin{aligned} [\iota]: &\qquad 1 + \Phi'(\iota)\big] = \partial_K A_t(K,X) \\ [x]: &\qquad -q_t^x + \partial_X A_t(K,X) = 0. \end{aligned}$$

Our proof follows the guess-and-verify approach. We guess that the value of the fund is linear in capital and shareholdings $A_t(K,X) = q_t^k K + q_t^x X$, which implies that first order conditions can be written as

$$\begin{aligned} [\iota]: &\qquad 1 + \Phi'(\iota) = q_t^k \\ [x]: &\qquad -q_t^x + q_t^x = 0. \end{aligned}$$

Let ι_t be the solution to the *foc* with respect to capital – notice that the *foc* for shares is satisfied for any value of the controls. Substituting these and our guess back into (B2) we get

$$r_t^a \left(q_t^k K + q_t^x X\right) = \left[r_t^k - \iota_t - \Phi(\iota_t)\right]K + \psi\Pi X + q_t^k(\iota_t - \delta)K + \left(\dot{q}_t^k K + \dot{q}_t^x X\right),$$

which must hold for any value of the capital stock K and shares X in order to verify our guess. Matching coefficients on capital K and shares X we arrive at the pricing conditions in the lemma

$$r_t^a = \frac{r_t^k - \iota_t - \Phi(\iota_t) + q_t^k(\iota_t - \delta) + \dot{q}_t^k}{q_t^k} \tag{B3}$$

$$r_t^a = \frac{\psi\Pi + \dot{q}_t^x}{q_t^x}. \tag{B4}$$

□

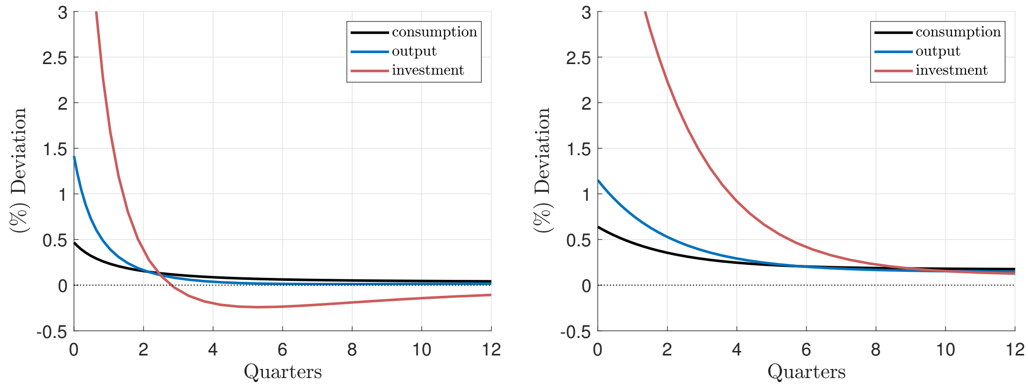
B.2 Representative Agent Model

In this section we compare the response of our baseline HANK model to a properly calibrated RANK economy. Our calibration of the RANK economy targets the same aggregates as in HANK. Our strategy for choosing the two transaction cost parameters (scale, curvature) in RANK is necessarily different though, because in HANK we choose them to replicate moments of the cross-sectional distribution of liquid and illiquid assets, for which the RA model does not make predictions. We choose the scale parameter of the adjustment cost function so that total transaction costs as a fraction of output in steady-state are the same as in HANK. The curvature parameter determines the responsiveness of aggregate deposits to the gap in rates of return between the two assets. Hence, we choose the curvature parameter so that the elasticity of aggregate deposits to a change in the real liquid rate r_t^b is the same as in HANK (keeping all other prices, including equity prices, fixed at their steady-state values). This ensures that, in partial equilibrium, in the two models investment has the same sensitivity to the liquid interest rate.

Figure B1 plots the impulse response functions (IRFs) to a monetary shock for output, consumption, investment for the two economies. Consumption reacts by more in RANK, but most striking is the difference in the investment response—the RANK features a much more persistent rise in investment.

The Representative Agent setting also differs in its sensitivity to the different channels studied in Section 4. To explore this, Table B1 repeats the exercises of Table 3 for RANK. Let us first focus on the relative impact of capital adjustment costs and partial adjustment rule for the baseline (the differences across the columns). While adjustment costs had only a small impact on aggregate consumption in HANK, we observe a 20% reduction in first quarter consumption in RANK.

Changing our focus from columns to rows, we see that consumption is invariant for most channels studied. In some cases this is by construction (there is no notion



(a) HANK (b) RANK

Figure B1: Aggregate responses to a monetary policy shock in HANK and RANK.

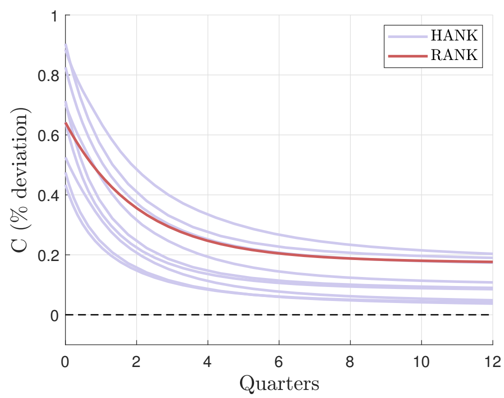


Figure B2: Consumption IRF to a monetary shock under RANK and a variety of HANK specifications.

of unequal incidence of income in the representative agent setup), but in others it is a result (the timing of transfers is irrelevant because of Ricardian equivalence). As for the channels that still have an effect, namely profit distribution and adjustment of government expenditures, the results are in line with our discussion for HANK. Distributing profits in the liquid account amplifies the consumption response, while using government expenditures to adjust the government budget leads to a crowding out effect.

We end by noticing that the larger RANK consumption response for the baseline does not necessarily carry over to other exercises. In Figure B2 we show the RANK baseline consumption against a series of HANK consumption responses computed under different fiscal and labor incidences assumptions, assumptions that are inconsequential in the RANK case. The range of HANK responses is big enough that we get anything from 30% of dampening to 50% of amplification with respect to RANK. This figure illustrates an important point: whether heterogeneity amplifies or dampens the impact of monetary policy shock depends on a multitude of channels, assumptions and parameterizations, some which are absent or irrelevant in the Representative Agent framework.

Experiment	RANK	RANK capital adj. cost	RANK partial adj. Taylor
Baseline	1.000	1.000	1.000
-	(1.000)	(0.797)	(0.960)
Unequal labor incidence			
CPS (asinh)	–	–	–
CPS (log)	–	–	–
SSA	–	–	–
Profit distribution			
Profit all liq.	1.140	1.011	1.077
Profit all illiq.	0.879	0.984	0.920
Fiscal reaction			
$\rho^R = 1, \rho^N = 0$	1.000	1.000	1.000
$\rho^R = 0, \rho^N = 0$	-	-	-
(Transfers adjust)	1.000	1.000	1.000
(Labor tax adjusts)	1.000	1.000	1.000
(Government exp. adjust)	0.939	0.944	0.943

Table B1: First quarter aggregate consumption response to monetary shock relative to the baseline.

Values in each column are normalized by the consumption in the first row of the corresponding column (baselines). Values in brackets in the second row denote the relative consumption of the baseline specification relative to the baseline in RANK without capital adjustment costs and with the standard Taylor rule (first column, first row). See Section 4.3 for a description of the unequal labor incidence exercises, Section 4.4 for profit distribution, and Section 4.5 for fiscal adjustment.

C Comparison with Kaplan, Moll, and Violante (2018)

Description	AER	JMCB
Model elements		
Capital adj cost	NA	target the investment/output response to a monetary shock $\Phi\left(\frac{\dot{K}}{K}\right)K = \frac{\phi_0}{2}\left(\frac{\dot{K}}{K}-\delta\right)^2 K$
Taylor rule	$i_t = r^b + \phi_\pi \pi_t + \epsilon_t$	$i_t = r^b + \phi_\pi \pi_t + \epsilon_t$ $\frac{di}{dt} = -\rho_i(i_t - r^b - \phi_\pi \pi_t)$
Labor market	competitive individual labor supply $w_t(1-\tau)u'(c_i) = \frac{1}{\varphi} n_i$	real wage rigidities exogenous wage-setting rule $w_t = \bar{w}\left(\frac{N_t}{\bar{N}}\right)^{\epsilon_w}$
Explored channels		
Income incidence	NA	labor, transfers and profit $\Gamma(z, Y_t) = \frac{\nu(z)\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_Y(z)}}{\int \nu(z)\left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_Y(z)} d\Pi(z)} Y_t$
Profit (liquid,illiquid)	fraction accruing in each account $\big((1-\alpha)\Pi_t\,,\ \alpha\Pi_t\big)$	fraction accruing in each account $\Big((1-\alpha)\bar{\Pi} + (1-\omega)(\Pi_t - \bar{\Pi})\,,$ $\alpha\bar{\Pi} + \omega(\Pi_t - \bar{\Pi})\Big)$
Fiscal rule	3 alternative instruments: bonds, transfers, expenditures	flexible primary surplus rule parametrized by (ρ^R, ρ^N, ρ^B) $S_t = c - (1-\rho^R) r_t B_t + \rho^N w_t N_t - \rho^B B_t$
Calibration		
Borrowing limit	$1\times$ quarterly labor income	0
$(\chi_1, \chi_2, \rho, \delta)$	internally calibrated to target (i) liquid and illiquid wealth (ii) poor and wealthy HtM shares	same

Table C1: List of differences between the models in the AER and the model in this article.

References

KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2018): "Monetary Policy According to HANK," *American Economic Review*, 108(3), 697–743.