416-1 Advanced Macro

Topic 2: Deleveraging and Housing

Guido Lorenzoni
Northwestern University

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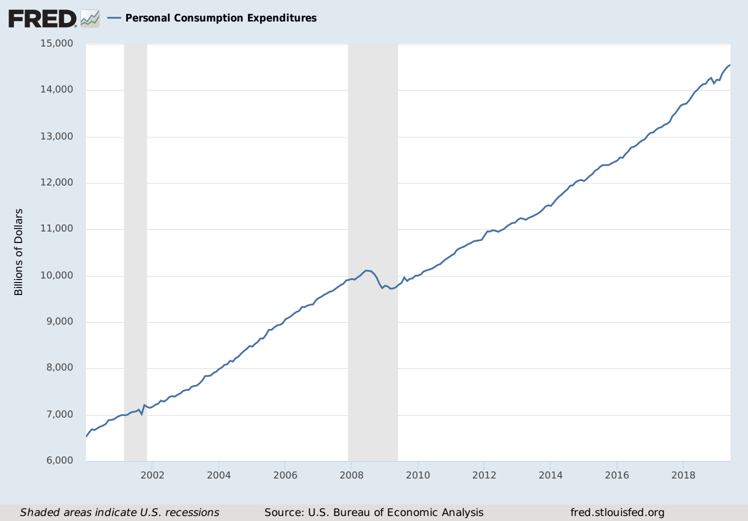
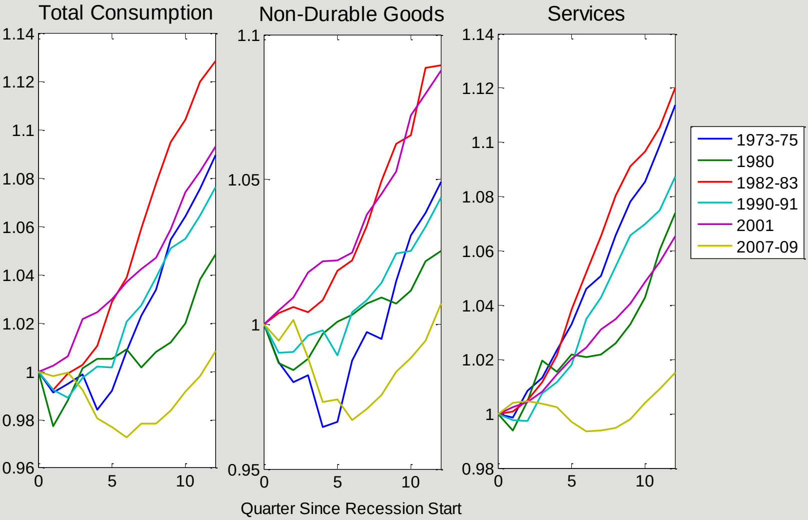
Deleveraging

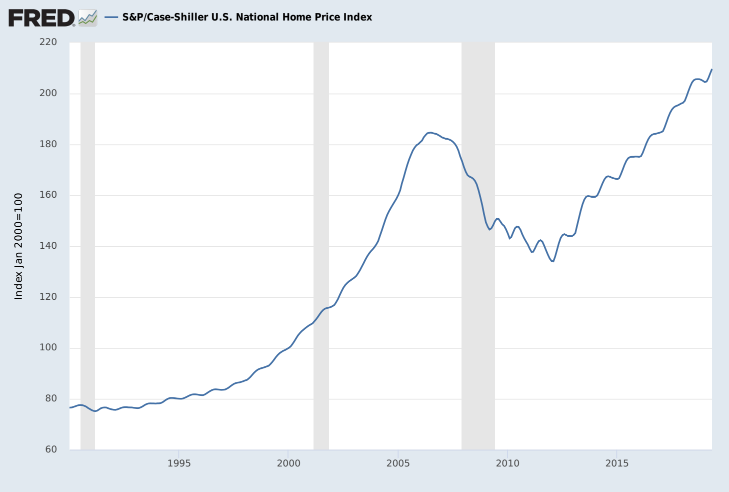
The Great Recession: Consumption



Comparison with Previous Recessions

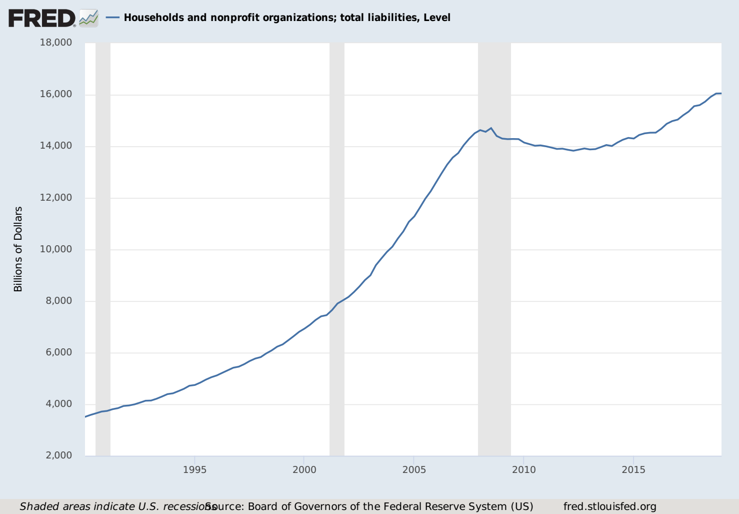


House Prices



Shaded areas indicate U.S. recessions Source: S&P Dow Jones Indices LLC fred.stlouisfed.org

Household Debt



Credit crunch and households

- Guerrieri and Lorenzoni (2017) study a credit crunch in a Bewley-Aiyagari-Huggett model with nominal rigidities

- Borrowing limit

$$b_{it} \geq -\phi$$

- What are effects of decreasing ϕ?

  1. drop in demand $\Rightarrow$ recession

  2. drop in interest rate $\Rightarrow$ liquidity trap

Precautionary effects, distribution effects

- Two interesting issues, absent in RA economies:
  - Precautionary effects: agents not at debt limit are affected
  - Distribution effects: as distribution adjusts effects change
- Analysis of
  - Partial equilibrium: effects on consumption and household debt
  - GE effects: response of output/inflation, response of monetary policy r

Household Problem

- Preferences

$$\mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t U\left(c_{it}, n_{it}\right)\right]$$

- Budget constraint

$$c_{it} + b_{it+1} \leq \frac{W_t}{P_t}\theta_{it} n_{it} + (1 + r_t)b_{it} - \tau_t$$

- θ_{it} follows Markov chain on $\{\theta^1, \ldots, \theta^S\}$

- Exogenous borrowing limit

$$b_{it+1} \geq -\phi$$

Closing the Model

- For given $\bar{B}$, government chooses τ_t to satisfy

$$\tau_t = r_t \bar{B}$$

- Bond market clearing

$$\int \int_{b,\theta} b d\Psi_t(b,\theta) = \bar{B}$$

$\Psi_t(b,\theta)$ = equilibrium wealth-productivity distribution

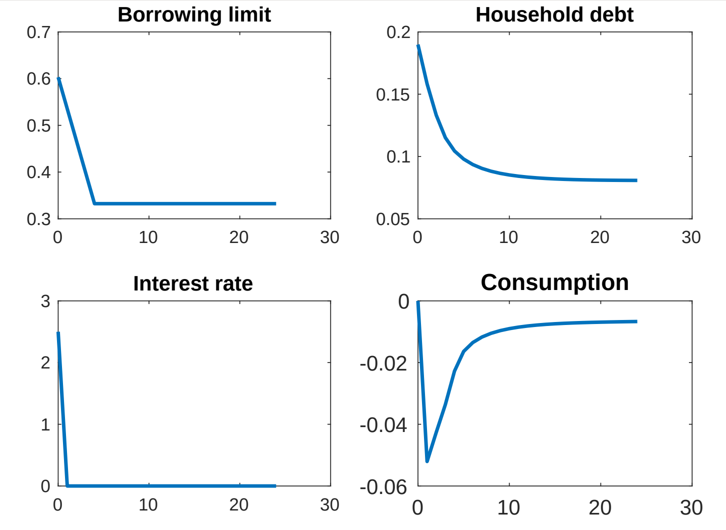
Nominal rigidities

- nominal rigidity: downward rigid nominal wages $\rightarrow W_t$ and $P_t = W_t$ stay constant
- real rate $r_t =$ nominal rate $i_t \geq 0$
- wedge ω_t in labor supply condition

$$(1 - \omega_t) U_c(c_{it}, n_{it}) \theta_{it} = -U_n(c_{it}, n_{it})$$

- assume central bank aims to set $\omega_t = 0$ as long as ZLB not binding
- Equilibrium definition: look for sequence r_t, ω_t such that goods market, labor market (with wedge), bond market clear
- ...and such that $r_t \geq 0$ and $\omega_t \geq 0$ with at least one equality

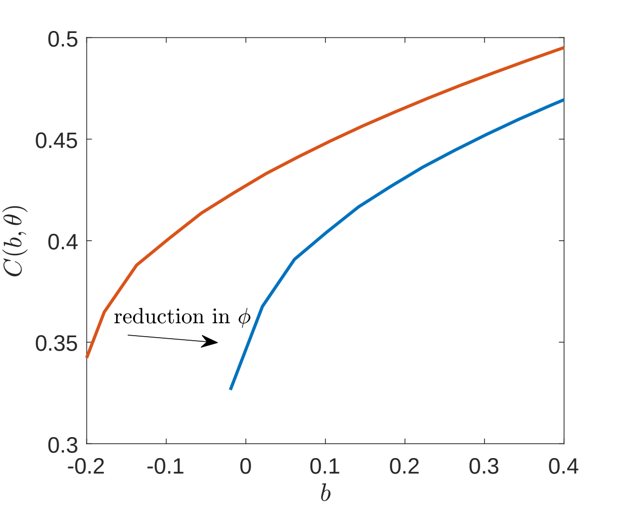
Unexpected Contraction in Borrowing Limit



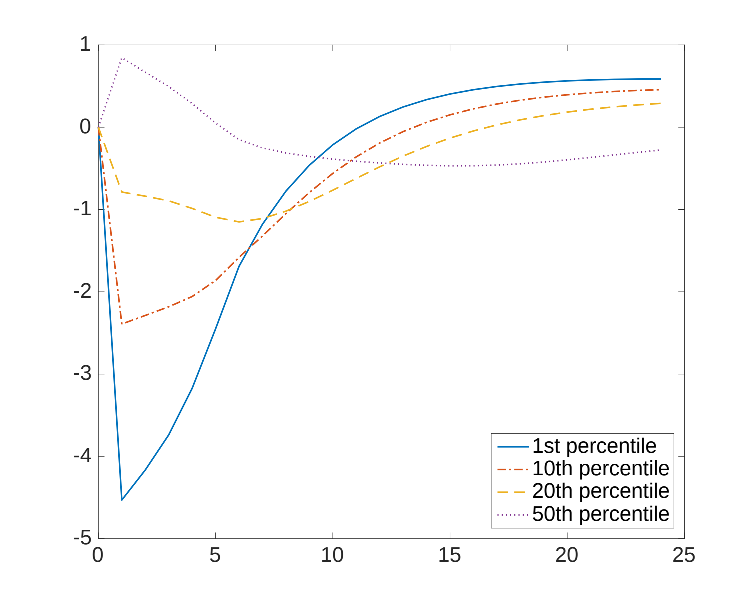
Distributional Effects

- crucial: delevereging/consumption responses depend on asset position
- who is mostly affected by tightening in the borrowing limit?
  1. households at the constraint: forced to delver
  2. households close to constraint: precautionary savings
- these tend to be poor households with large marginal propensity to consume
- richer households may even increase consumption in response to the drop of the interest rate (but low MPC)

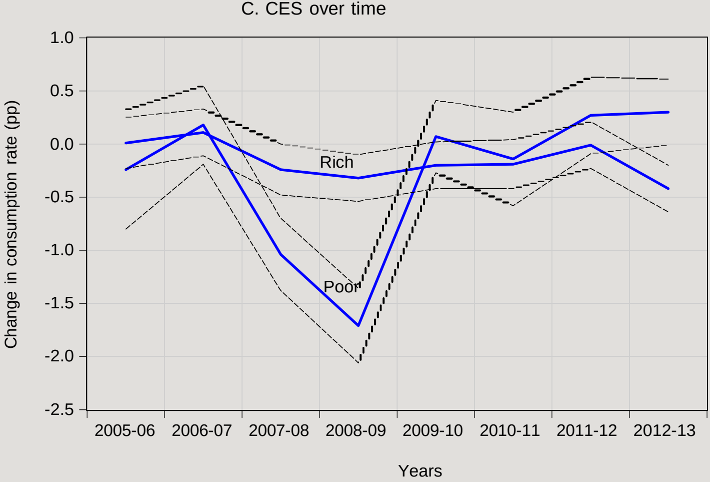
Precautionary effect and MPCs



Consumption Responses by Wealth



Consumption Data by Wealth



Note: the thin dashed lines delimit 2 (bootstrapped) standard error bands.

Source: Heathcote and Perri (2017)

Precautionary savings in continuous time

Bewley model in continuous time

- We will do exercises analogous to GL (2017), but in a continuous time framework
- Why? It will introduce tools that will be useful later when we introduce housing and adjustment costs
- A consumer with an income process $\{y_t\}_{t\geq 0}$ chooses the consumption process $\{c_t\}_{t\geq 0}$ to maximize:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right]$$

subject to the budget constraint

$$\dot{a}_t = r a_t + y_t - c_t$$

and the borrowing constraint

$$a_t \geq -\phi.$$

Deterministic case

- The income process is a Poisson process that switches between J values $\{y_1, ..., y_J\}$ with intensities $\lambda_{j'j}$
- Based on Achdou, Han, Lasry, Moll, Lions (2019)
- To set ideas let's consider first the case with no shocks, so $y_t = y$ constant
- Recursive formulation: $V(a)$ is the maximum expected utility for an agent starting at wealth a
- Claim: if u CRRA, $\rho \geq r$, and $\underline{a} > -y/r$ the problem is well defined and the value function V exists and is differentiable (right differentiable at $-\phi$)
- Claim: V satisfies the Hamilton-Jacobi-Bellman equation

$$\rho V(a) = \max_c u(c) + V'(a)(ra + y - c)$$

Deterministic case (continued)

- Going from discrete to continuous time (to help interpretation of HJB)
- Let $u_t = u(c(a_t))$ and $V_t = V(a_t)$, in discrete time with time period Δt we have

$$V_t = u_t + e^{-\rho \Delta t} V_{t+\Delta t}$$

so

$$(1 - e^{-\rho \Delta t}) V_t = u_t \Delta t + e^{-\rho \Delta t} (V_{t+\Delta t} - V_t)$$

divide by Δt and take $\Delta t \to 0$

$$\rho V_t = u_t + \frac{dV}{dt}$$

and since problem is stationary $V_t = V(a_t)$ we have

$$\frac{dV}{dt} = V'(a) \dot{a}_t$$

Deterministic case (continued)

- First order condition from HJB

$$u'(c) = V'(a)$$

- How do we deal with the constraint $a \geq -\phi$?
- We impose that either $u'(c) = V'(-\phi)$ gives $c < -r\phi + y$ or with $c = -r\phi + y$
- This is summarized by boundary inequality for V'

$$V'(-\phi) \geq u'(-r\phi + y)$$

Deterministic case (continued)

- Summarizing, V must satisfy:

$$\rho V(a) = \max_c u(c) + V'(a)(ra + y - c)$$

$$V'(-\phi) \geq u'(-r\phi + y)$$

- Or solving the optimization problem the HJB becomes

$$\rho V(a) = u(u'^{-1}(V'(a))) + V'(a)(ra + y - u'^{-1}(V'(a)))$$

- So we have an ODE for V, with a terminal inequality condition

Deterministic case (continued)

- Suppose we find a V that solves the ODE, have we found the value function?
- There are verification theorems to ensure that the answer is yes
- But we are also ok if ODE has a unique solution
- It seems that we don't have enough initial conditions, but ODEs that come from HJB have special structure and uniqueness actually emerges in many cases

Numerical approach

- How to find a solution
- A finite difference approach
- Transform the problem into a discrete time problem with a small time interval Δt
- Crucial advantage: a discrete time problem where you can only move to points immediately to the left or right on the grid

Numerical approach (continued)

- Grid $A = \{a_i\} = \{a_1, a_2, ..., a_I\}$
- Maximization problem at point i on the grid:

$$\max_s u(ra_i + y - s) + s^+ \frac{V(a_{i+1}) - V(a_i)}{\Delta a} - s^- \frac{V(a_i) - V(a_{i-1})}{\Delta a}$$

  where $s^+ = \max\{s, 0\}$ and $s^- = -\min\{s, 0\}$
- Think of s as the (Poisson) intensity with which you try to move left or right, it's like choosing a lottery that moves you up or down

Numerical approach (continued)

- V concave in the application we are considering now
- Therefore only 3 cases possible
  - First order condition

    $$u'(ra_i + y - s) = \frac{V(a_{i+1}) - V(a_i)}{\Delta a}$$

    with $s > 0$
  - First order condition

    $$u'(ra_i + y - s) = \frac{V(a_i) - V(a_{i-1})}{\Delta a}$$

    with $s < 0$
  - Or

    $$\frac{V(a_{i+1}) - V(a_i)}{\Delta a} < u'(ra_i + y) < \frac{V(a_i) - V(a_{i-1})}{\Delta a}$$

Value function iterations

- The step described above was the optimization step
- We combine it with a step to update V and we get iterative procedure:
  - Start with guess V
  - Optimize
  - Update V
  - Continue until convergence

Updating the value function

- To update, useful to think of general non-stationary value function $V(a,t)$, so HJB takes the form of PDE

$$0 = \max_c u(c) + \frac{\partial V(a,t)}{\partial a}(ra + y - c) + \frac{\partial V(a,t)}{\partial t} - \rho V(a,t)$$

- Discretizing we get

$$V(a_i, t - \Delta t) = (h - \rho V(a_i, t))\Delta t + V(a_i, t)$$

where

$$h = \max_s u(ra_i + y - s) + s^+ \frac{V(a_{i+1}, t) - V(a_i, t)}{\Delta a} - s^- \frac{V(a_i, t) - V(a_{i-1}, t)}{\Delta a}$$

Adding back income shocks

- Now we can add back the income shocks
- So value function is $V(a,j)$
- Just changes definition of h

$$
\begin{aligned}
h &= \max_c u(c) + \frac{\partial V}{\partial a}(ra + y_j - c) \\
&\quad + \sum_{j'} \lambda_{j',j}(V(a_i, j', t) - V(a_i, j, t))
\end{aligned}
$$

- And in numerical version

$$
\begin{aligned}
h &= \max_s u(ra_i + y - s) + s^+ \frac{V(a_{i+1}, j) - V(a_i, j)}{\Delta a} - s^- \frac{V(a_i, j) - V(a_{i-1}, j)}{\Delta a} + \\
&+ \sum_{j'} \lambda_{j',j}(V(a_i, j') - V(a_i, j))
\end{aligned}
$$

In matrix form

- Stack $v_{ij} \equiv V(a_i, j)$ in a vector v (first by i then by j) the HJB can be written as

$$\rho \mathrm{v} = \mathrm{u} + \mathrm{A}(\mathrm{v})\mathrm{v}$$

  where in matrix A we put savings s^+ and s^- and transition probabilities $\lambda_{j'j}$

- Value function iteration described above then looks like this

$$\mathrm{v}' = \mathrm{v} + [\mathrm{u} + \mathrm{A}(\mathrm{v})\mathrm{v} - \rho \mathrm{v}]\,\Delta t$$

Improvement step

- Faster algorithm
- Do the optimization step and find optimal saving policy for each point on grid, find u and A
- Then iterate several times on value function without re-optimizing
- Then optimize again

The “implicit method”

- A very fast approach, similar in spirit to improvement step
- Approximate $\partial V/\partial t$ forward so the value update looks like this

$$\mathrm{v}' = \mathrm{v} + \left(\mathrm{u}' + \mathrm{A}\left(\mathrm{v}'\right)\mathrm{v}' - \rho\mathrm{v}'\right)\Delta t$$

- However instead of solving this implicit equation, use old v to compute u and A so solve

$$\mathrm{v}' = \mathrm{v} + \left(\mathrm{u} + \mathrm{A}\left(\mathrm{v}\right)\mathrm{v}' - \rho\mathrm{v}'\right)\Delta t$$

- System of linear equations that becomes

$$\left[\left(\frac{1}{\Delta t} + \rho\right)\mathrm{I} - \mathrm{A}\left(\mathrm{v}\right)\right]\mathrm{v}' = \mathrm{u} + \frac{1}{\Delta t}\mathrm{v}$$

- Exploiting sparsity (using only neighbors) can be solved very fast

Solve for the distribution

- Distribution is $g(a, j)$
- Evolution of distribution (adding time t)

$$\frac{\partial g(a,j,t)}{\partial t} = -\frac{d}{da}(s(a,j,t)g(a,j,t)) - \sum_{j'} \lambda_{j'j} g(a,j,t) + \sum_{j'} \lambda_{jj} g(a,j',t)$$

- Intuition for first term
- Suppose $s > 0$, then lose mass at a at rate $s(a,j,t)g(a,j,t)$ and gain mass at rate $s(a-\epsilon,j,t)g(a-\epsilon,j,t)$
- If $s < 0$ gain from $a+\epsilon$ but because of different sign of s end up with same expression

Solve for the distribution (continued)

- Computing the first term on the grid
- Simple example ignoring income shocks for a moment, 5 points on grid
- Suppose $s_1, s_2, s_3 > 0$ and $s_4, s_5 < 0$
- Remember that $\frac{\partial V}{\partial a}\dot{a}$ is approximated by

$$\begin{bmatrix} -s_1 & s_1 & 0 & 0 & 0 \\ 0 & -s_2 & s_2 & 0 & 0 \\ 0 & 0 & -s_3 & s_3 & 0 \\ 0 & 0 & -s_4 & s_4 & 0 \\ 0 & 0 & 0 & -s_5 & s_5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

Solve for the distribution (continued)

- How do we update the discrete distribution g_i?
- Take same matrix of savings and compute:

$$\begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \end{bmatrix} \begin{bmatrix} -s_1 & s_1 & 0 & 0 & 0 \\ 0 & -s_2 & s_2 & 0 & 0 \\ 0 & 0 & -s_3 & s_3 & 0 \\ 0 & 0 & -s_4 & s_4 & 0 \\ 0 & 0 & 0 & -s_5 & s_5 \end{bmatrix}$$

- Why?

Solve for the distribution (continued)

- The matrix expression

$$\begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \end{bmatrix} \begin{bmatrix} -s_1 & s_1 & 0 & 0 & 0 \\ 0 & -s_2 & s_2 & 0 & 0 \\ 0 & 0 & -s_3 & s_3 & 0 \\ 0 & 0 & -s_4 & s_4 & 0 \\ 0 & 0 & 0 & -s_5 & s_5 \end{bmatrix}$$

  approximates

$$-\frac{d}{da}(s(a)g(a))$$

- For the second term this is clear:

$$\Delta g_2 = (s_1 g_1 - s_2 g_2)\Delta t$$

- For the other terms things are less transparent, but make sense once we realize that: (i) g is zero outside the grid; (ii) $s = 0$ cannot cross 0 continuosly on a discrete grid (so sometimes 3 terms appear)

Two methods

- Again, two methods to find g in steady state
- Direct method, iterate on difference equation

$$\mathrm{g}' - \mathrm{g} = -\mathrm{A}^{\mathrm{T}}\mathrm{g}\Delta t$$

- Much faster to ask matlab to look for solution to

$$\mathrm{A}^{\mathrm{T}}\mathrm{g} = 0$$

Soft borrowing constraint

- Consider the case where there are two interest rates, one for positive, one for positive a balances, with

$$r_+ < \rho < r_-$$

- No income shocks
- ra is replaced by

$$f(a) = r_- \min\{a, 0\} + r_+ \max\{a, 0\}$$

- So the HJB can be written as

$$\rho V(a) = \max_c u(c) + V'(a)(f(a) + y - c)$$

Dynamics near steady state

- This model has a nice steady state at $a = 0$
- Differentiate the HJB for $a \neq 0$

$$\rho V'(a) = V'(a)f'(a) + V''(a)s(a)$$

$$(f'(a) - \rho)V'(a) = -V''(a)s(a)$$

(using an envelope argument)

- Substitute $u'(C(a)) = V'(a)$ and (differentiating) $u''(C(a))C'(a) = V''(a)$

$$(f'(a) - \rho)u'(C(a)) = -u''(C(a))C'(a)s(a)$$

- Notice that this is an Euler equation (e.g. for $a > 0$)

$$(r_+ - \rho)\frac{u'}{-u''c} = \frac{C'(a)}{C(a)}s(a) = \frac{\dot{c}}{c}$$

Limit properties of $s(a)$

- Limit behavior

$$(r_+ - \rho)\frac{u'(C(a))}{-u''(C(a))} = C'(a)s(a) = (f'(a) - s'(a))s(a)$$

- As $a \to 0$ $s(a) \to 0$ and $C(a) \to y$
- So we obtain

$$\lim_{a \downarrow 0} s'(a)s(a) = (\rho - r_+)\frac{u'(y)}{-u''(y)} > 0$$

where $s(a) < 0$ and $s'(a) < 0$

- And

$$\lim_{a \uparrow 0} s'(a)s(a) = (\rho - r_-)\frac{u'(y)}{-u''(y)} < 0$$

where $s(a) > 0$ and $s'(a) < 0$

Local characterization of $s(a)$

- In the limit behavior comes from ODE

$$s'(a)s(a) = const.$$

- Solution for $a > 0$ is

$$s(a) = -\zeta a^{\frac{1}{2}}$$

$$s'(a) = -\zeta \frac{1}{2} a^{\frac{1}{2}-1}$$

- So

$$s'(a)s(a) = \zeta^2 \frac{1}{2}$$

Illiquid assets

Motivation

- Three observations:
  - A lot of the leveraging/deleveraging of US households has to do with mortgages
  - Credit crunch is relatively short lived
  - The house price cycle was a slower process
  - The debt dynamic was even slower
  - The increase in household saving rate was very persistent (permanent?)
- Broad question: how do we put all these pieces together?

Specific critiques to deleveraging story

- Critique 1: mortgages are not short term debt, when credit tightens households do not need to repay old mortgages
- Critique 2: houses are not traded often, and when you sell a house you buy a new one, so house prices should not matter for consumption
- So overall maybe stories based on household balance sheet adjustment story do not work well
- Use a model to dig deeper in these issues and see if they are well founded

- (Critique 3: "deleveraging " is bad terminology as debt/asset values went up for households. Just semantic, but connected to #1)

Illiquid assets and spending

- Here we'll study some preliminary steps to think about these issues
- The first question is the following
- In simulations in Berger et al (2017) we find large house price effects, even when houses are illiquid
- Why?
- Work on basic question: how does the value of an illiquid asset affect consumption?
- Preliminary question to understand housing/mortgages

Simple model with illiquid asset

- Illiquid asset x
- Price of the asset is p when you buy, $p - \phi$ when you sell
- Asset pays dividend d
- ϕ is a fixed transaction cost
- Agents as in Bewley continuous time model, with Poisson shocks to income

Simple model with illiquid asset (continued)

- Present value of holding asset for interval $[0, T]$

$$\int_0^T de^{-rt}dt + e^{-rT}(p - \phi)$$

so holding profitable if

$$\frac{d}{r}\left(1 - e^{-rT}\right) + e^{-rT}(p - \phi) > p$$

- Assume $d > rp$ so holding asset profitable for T large enough
- Never profitable to hold for short T because of fixed cost ϕ

Simple model with illiquid asset (continued)

- HJB for W value if no adjustment to illiquid asset

$$\rho W\left(a,x,y\right) = \max_{c} u\left(c\right) + V_a\left(a,x,y\right)\left(ra + dx + y_j - c\right) + \\ + \sum_{y'} \lambda_{y',y}\left(V\left(a,x,y'\right) - V\left(a,x,y\right)\right)$$

- Optimal adjustment

$$V(a,x,y) = \max\left\{W(a,x,y), \max_{x'} W(a+(p-\phi)x - px', x', y)\right\}$$

Simple model with illiquid asset (continued)

- Numerical example with a one time decision
- $x \in \{0, \bar{x}\}$
- Start at $x = \bar{x}$, can only switch to $x = 0$
- Solve for

$$\begin{aligned}\rho V(a,0,y) = \quad & \max_c u(c) + V_a(a,0,y)(ra + y_j - c) + \\ & + \sum_{y'} \lambda_{y',y} \left(V\left(a,0,y'\right) - V(a,0,y) \right)\end{aligned}$$

- Then solve for $x = \bar{x}$ with optimal stopping

A simple scrapping problem (aside on smooth pasting conditions)

- Consider the problem of a firm with per period productivity x that follows the ODE

$$\dot{x} = \mu$$

  where $\mu < 0$, so productivity is gradually declining
- The firm per period profits are $u(x)$
- The firm can stop and sell the firm assets at the price S
- The firm payoff if it stops at T is

$$\int_0^T u(x(t))\, e^{-\rho t} dt + e^{-\rho T} S$$

Monotonicity and cutoff

- For what value of x will the firm sell?
- Let $V(x)$ be the optimal value if the initial profits are $x(0) = x$
- Easy to prove that V is non-decreasing in x, as for $x' > x$ the firm can choose the same T and increase its payoff
- This implies that there is a cutoff $\hat{x}$ such that $V(x) \geq S$ iff $x \geq \hat{x}$

HJB

- Above the cutoff V must satisfy the HJB

$$\rho V(x) = u(x) + V'(x)\dot{x}$$

- Solution is given by a V that satisfies HJB for $x > \hat{x}$ and $V(x) = S$ for $x \leq \hat{x}$
- Claim: The solution also satisfies the smooth pasting condition

$$V'(x) = 0.$$

Discrete time

- Consider the discrete time problem and consider the Bellman equation

$$V(x) = u(x)\Delta t + (1 - \rho\Delta t)\max\{V(x + \mu\Delta t), S\}$$

- Notice that if the cutoff $\hat{x}$ is optimal we must have (from the monotonicity of V, $\hat{x} + \mu\Delta t < \hat{x}$, and $V \geq S$)

$$V(\hat{x}) = u(\hat{x})\Delta t + (1 - \rho\Delta t) S = S$$

which implies

$$u(\hat{x}) = \rho S \tag{1}$$

- Observation 1: if we are indifferent between stopping or not the flow payoff from stopping must be same as the flow payoff from continuing

Discrete time (continued)

- Moreover, when we are exactly a step away from adjusting (at $\hat{x} - \mu\Delta t > \hat{x}$) we must have

$$V(\hat{x} - \mu\Delta t) = u(\hat{x} - \mu\Delta t)\,\Delta t + (1 - \rho\Delta t)\,S$$

which implies (with $V(\hat{x}) = S$)

$$V(\hat{x} - \mu\Delta t) - V(\hat{x}) = u(\hat{x} - \mu\Delta t)\,\Delta t - \rho\Delta t S$$

- So we obtain

$$\frac{V(\hat{x} - \mu\Delta t) - V(\hat{x})}{\Delta t} = u(\hat{x} - \mu\Delta t) - \rho S$$

- Taking limits for $\Delta t \to 0$ and using (1) we get the smooth pasting condition
- Observation 2: If we are close to stopping we cannot be much better off than stopping now, given Observation 1

House prices and mortgages

Housing and mortgage debt

- Growing literature
- Quantitative models with rich structure
- Question: can we make sense of recent dynamics of house prices, household debt, consumption?
- Shocks?
- Relaxation of "lending standards"
- Possibly driven by increase in securitization
- Suppose you have collateral constraint:

$$b_t \leq \theta p_t h_t$$

- You can shock θ

Shocks to θ

- In standard model small effects of θ shocks
- Favilukis, Ludvigson, Van Nieuwerburgh (2017) obtain larger effects
- Double shock: financial liberalization and foreign savings (to keep interest rates low)
- Greenwald (2019) large effects from a different shock

Housing demand

- Consumers mazimize

$$E\sum_{t=0}^{\infty}\beta^t u\left(c_t, h_t\right)$$

- Budget constraint

$$p_t h_t + a_t + c_t = p_t\left(1-\delta\right)h_{t-1} + \left(1+r\right)a_{t-1} + y_t$$

- Collateral constraint

$$\theta p_{t+1}\left(1-\delta\right)h_t + \left(1+r\right)a_t \geq 0$$

- Optimality (if constraint not binding)

$$u_c\left(c_t, h_t\right) = \beta\left(1+r\right)E_t u_c\left(c_{t+1}, h_{t+1}\right)$$

$$u_h\left(c_t, h_t\right) = p_t u_c\left(c_t, h_t\right) - \beta E_t p_{t+1}\left(1-\delta\right)u_c\left(c_{t+1}, h_{t+1}\right)$$

Housing and mortgage debt

- Housing demand depends on correlation of p_{t+1} and $u_{c,t+1}$
- Financial liberalization affects the correlation
- Demand for housing goes up

Nature of borrowing constraints

- Greenwald: interaction of payment to income (PTI) and loan to value (LTV) constraints