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A Details and Proofs for Theoretical Benchmarks

A.1 Constant Asset Prices (Section 2.1)

Derivation of Saving Policy Function in (2). Households maximize (1) subject to $\dot{a}_t = w + ra_t - c_t$. The corresponding HJB equation is

$$\rho v(a) = \max_c \; u(c) + v'(a)(w + ra - c) \tag{A1}$$

We solve this equation by using a guess-and-verify strategy: guess

$$v(a) = B\frac{(a + w/r)^{1-\gamma}}{1-\gamma}$$

which implies $v'(a) = B(a + w/r)^{-\gamma}$ and

$$c(a) = v'(a)^{-1/\gamma} = B^{-1/\gamma}(a + w/r) \tag{A2}$$

Substituting into (A1) and dividing by $(a + w/r)^{1-\gamma}$

$$\rho B\frac{1}{1-\gamma} = \frac{1}{1-\gamma}B^{-(1-\gamma)/\gamma} + Br - BB^{-1/\gamma}$$

Dividing by B and collecting terms we have $B^{-1/\gamma} = \frac{\rho - r}{\gamma} + r$ and hence from (A2) we have

$$c(a) = \left(\frac{\rho - r}{\gamma} + r\right)\left(a + \frac{w}{r}\right) \tag{A3}$$

Since the saving policy function is given by $s(a) = w + ra - c$, this yields (2).

A.2 Changing Asset Prices (Section 2.2) – Proof of Proposition 1

Our goal is to prove that optimal consumption and net saving satisfy (9) and (10) with

$$\begin{aligned} \xi_t\left(\frac{1}{\gamma}, \rho, \{r_s\}_{s\geq t}\right) &:= \frac{1}{\int_t^\infty e^{-\int_t^s \left(r_\tau - \frac{1}{\gamma}\left(r_\tau - \rho\right)\right)d\tau} ds}, \\ \phi_t\left(\frac{1}{\gamma}, \rho, \{r_s\}_{s\geq t}\right) &:= 1 - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds. \end{aligned} \tag{A4}$$

Lemma A1. *Consumption satisfies the usual Euler equation*

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\gamma}(r_t - \rho) \tag{A5}$$

Proof of Lemma A1: Consider the formulation in terms of net worth $a_t := p_t k_t$ so that the budget constraint is $\dot{a}_t = w + r_t a_t - c_t$ with r_t defined in (5). The current-value Hamiltonian is

$$\mathcal{H} = u(c) + \lambda[w + ra - c].$$

The first-order condition and law of motion of the co-state are

$$c_t^{-\gamma} = \lambda_t, \qquad \dot{\lambda}_t = (\rho - r_t)\lambda_t.$$

Combining yields (A5).□

Proof of Proposition 1: Integrating (4) forward in time, we have the following present-value budget constraint

$$\int_t^\infty e^{-\int_t^s r_\tau d\tau} c_s ds = p_t k_t + \int_t^\infty e^{-\int_t^s r_\tau ds\tau} w ds. \tag{A6}$$

From the Euler equation (A5)

$$c_s = c_t e^{\frac{1}{\gamma}\int_t^s (r_\tau - \rho) d\tau}.$$

Plugging into (A6) and rearranging:

$$c_t = \xi_t \left(p_t k_t + \int_t^\infty e^{-\int_t^s r_\tau d\tau} w ds\right) \tag{A7}$$

with ξ_t defined in (A4). Further using expression (6) we obtain

$$c_t = \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + D_s k_t)ds,$$

which is (9) in the Proposition. Next, net saving is

$$p_t \dot{k}_t = w + D_t k_t - c_t = w + D_t k_t - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + D_s k_t)ds.$$

Adding and subtracting $\xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + D_t k_t)ds$ from the right-hand side we have

$$\begin{aligned} p_t \dot{k}_t &= \left(1 - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds\right)(w + D_t k_t) - \xi_t k_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(D_s - D_t)ds \\ &= \phi_t (w + D_t k_t) - \xi_t k_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(D_s - D_t)ds \end{aligned}$$

with ϕ_t defined in (A4), which is (10) in the Proposition. Finally, it is easy to see that when the intertemporal elasticity of substitution, $1/\gamma = 0$, then ξ_t and ϕ_t defined in (A4) become

$$\xi_t\left(0, \rho, \{r_s\}_{s\geq t}\right) := \frac{1}{\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds}, \qquad \phi_t\left(0, \rho, \{r_s\}_{s\geq t}\right) := 1 - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds = 0,$$

as stated in the Proposition. This concludes the main part of the proof.

Expressions for consumption and gross saving in footnote 13: Using $a_t = p_t k_t$ in (A7) gives

$$c_t = \xi_t \left(a_t + \int_t^\infty e^{-\int_t^s r_\tau d\tau} w ds\right) \tag{A8}$$

which is the expression for c_t in footnote 13.

To derive the expression for $\dot{a}_t$ in footnote 13 note first that $\int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds = 1$ for all t. To see this define $v_t := \int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds$ and time-differentiate $r_t v_t = r_t + \dot{v}_t$ for all t which is solved by $v_t = 1$ for all t (alternatively, integrate the expression for v_t by parts). Therefore

$$c_t = \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + r_s a_t)ds$$

Gross saving is

$$\dot{a}_t = w + r_t a_t - c_t = w + r_t a_t - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + r_s a_t)ds.$$

Adding and subtracting $\xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + r_t a_t)ds$ from the right-hand side we have

$$\begin{aligned}\dot{a}_t &= \left(1 - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds\right)(w + r_t a_t) - \xi_t a_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(r_s - r_t)ds \\ &= \phi_t(w + r_t a_t) - \xi_t a_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(r_s - r_t)ds\end{aligned}$$

with ϕ_t defined in (A4), which is the expression for $\dot{a}_t$ in footnote 13.

Special Cases in Proposition 1: finally we derive special cases 1 through 3.

1. **Constant asset price, dividend and return:** When $r_t = r$ for all t, ξ_t and ϕ_t defined in (A4) become
$$\xi_t = r - \frac{r-\rho}{\gamma}, \qquad \phi_t = 1 - \xi_t \frac{1}{r} = \frac{r-\rho}{\gamma r}. \tag{A9}$$
Plugging into (10) and using the definition of the asset return $r = \frac{D}{p}$ we obtain $p\dot{k}_t = \frac{r-\rho}{\gamma r}(w + rpk_t)$.

2. **Price growth with dividend growth (constant return):** Just like in special case 1, a constant return $r_t = r$ for all t implies that ξ_t and ϕ_t simplify as in (A9). Furthermore
$$\begin{aligned}\int_t^\infty e^{-\int_t^s r_\tau d\tau}(D_s - D_t)ds &= \int_t^\infty e^{-r(s-t)} D_s ds - D_t \int_t^\infty e^{-r(s-t)} ds \\ &= p_t - \frac{D_t}{r} \\ &= \frac{rp_t - D_t}{r} \\ &= \frac{\dot{p}_t}{r},\end{aligned}$$
where the last equality uses (5). Plugging into (10) we obtain (11) and (12).

3. **Price growth without dividend growth (falling return):** the expression for net saving (13) follows straight from (10) and that constant dividends imply that
$$\int_t^\infty e^{-\int_t^s r_\tau d\tau}(D_s - D_t)ds = 0.$$
Next gross saving is given by $p_t\dot{k}_t + \dot{p}_t k_t = \phi_t(w + D_t k_t) + \dot{p}_t k_t$ or equivalently
$$p_t\dot{k}_t + \dot{p}_t k_t = \phi_t(w + (D_t + \dot{p}_t)k_t) + (1-\phi)\dot{p}_t k_t,$$

which is (14).

This concludes the proof of Proposition 1.□

B Extensions (Section 2.3)

B.1 Housing as a Consumption Good

We build on Berger et al. (2018) and construct a parsimonious model in which housing is a durable consumption good but counterfactually assume that it is fully divisible and freely adjustable. Households receive a constant labor income w and invest in two assets: housing and bonds. House prices change deterministically over time. Households maximize

$$\int_0^\infty e^{-\rho t} u(c_t, s_t) dt, \tag{A10}$$

where c is non-durable consumption, and s is housing services which can be generated either by housing owned by the household h or rental housing. We assume the following homothetic utility function

$$u(c,s) = \frac{C^{1-\gamma}}{1-\gamma} \quad \text{with} \quad C = \left(\alpha^{1/\eta} c^{\frac{\eta-1}{\eta}} + (1-\alpha)^{1/\eta} s^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A11}$$

where γ is the inverse of the intertemporal elasticity of substitution, α is the utility weight on non-durable consumption, and η is the elasticity of substitution between consumption and housing. The budget constraint is

$$c_t + \dot{b}_t + p_t \dot{h}_t = w + r_t b_t + R_t (h_t - s_t), \tag{A12}$$

where b_t are bonds, h_t is housing owned by the household, r_t is the return to bonds (interest rate), p_t is the house price, and R_t is the rental price per unit of housing. Households maximize (A10) subject to (A12). Our goal is to understand how households treat housing capital gains in this environment.

To analyze this problem, it is useful to rewrite the budget constraint (A12) in terms of total wealth, $a_t := b_t + p_t h_t$ as

$$c_t + R_t s_t + \dot{a}_t = w + r_t b_t + \frac{R_t + \dot{p}_t}{p_t} p_t h_t \quad \text{with} \quad a_t = b_t + p_t h_t,$$

and where $(R_t + \dot{p}_t)/p_t$ is the return to housing. In our model without indivisibilities or transaction costs, absence of arbitrage opportunities requires the return to housing to equal the return on bonds

$$r_t = \frac{R_t + \dot{p}_t}{p_t} \tag{A13}$$

so that

$$c_t + R_t s_t + \dot{a}_t = w + r_t a_t. \tag{A14}$$

An equivalent but simpler formulation of the problem is therefore that households maximize (A10) subject to (A14).

Expression (A13) for the return to housing is the exact analogue of (5) in Section 2.2. Integrating (A13) forward in time and imposing a no-bubble condition, we have

$$p_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} R_s ds, \tag{A15}$$

which is the analogue of (6). Comparing this setup, with the one with a financial asset in Section 2.2, we can see that the only difference between housing and a financial asset is that the dividend of housing is the rental payment R_t. Households optimally choose housing services such that $u_s(c_t, s_t)/u_c(c_t, s_t) = R_t$. Therefore housing equivalently pays a "utility dividend" $u_s(c_t, s_t)/u_c(c_t, s_t)$. As in Section 2.2, a rising house price can only be due to one of two factors: growing rents (dividends) or a declining interest rate (returns).

Also note that the assumption of a rental market is made only for simplicity: the results in this Section equivalently go through in a version of the model without such a rental market. In particular, consider the budget constraint (A12) but impose that $s_t = h_t$ for all t, meaning households can only consume the housing they own themselves. One can then still derive the budget constraint (A14) by defining the "implicit rental cost" or "user cost" of housing $R_t := r_t p_t - \dot{p}_t$.[44]

Just like the model with a financial asset in Section 2.2 in the main text, the model with housing as a consumption good can be solved in closed form. In particular, we obtain the following Proposition which is analogous to Proposition 1 in the main text (the proof of Proposition A2 is at the end of this Appendix).

Proposition A2. *Consider a household who owns housing h_t and bonds b_t and who maximizes*

[44]To see this, assume that there is no rental market and households maximize $\int_0^\infty e^{-\rho t} u(c_t, h_t) dt$ subject to $c_t + \dot{b}_t + p_t \dot{h}_t = w + r_t b_t$. Defining $a_t := b_t + p_t h_t$ we have $\dot{a}_t = \dot{p}_t + p_t \dot{h}_t + \dot{p}_t h_t$ and so

$$\dot{a}_t = w + r_t a_t - c_t - R_t h_t, \qquad R_t := r_t p_t - \dot{p}_t,$$

which is (A14) with the restriction that households can only consume the housing they own themselves, $s_t = h_t$. The only difference is that R_t is a purely fictious, implied rent rather than a market price.

the homothetic utility function (A10) *subject to the budget constraint* (A14). *Her optimal total consumption* $C_t = c_t + R_t s_t$*, non-durable consumption* c_t*, consumption of housing services* s_t*, gross saving* $\dot{a}_t = \dot{b}_t + p_t\dot{h}_t + \dot{p}_t h_t$*, and net saving* $\dot{b}_t + p_t\dot{h}_t$ *are given by*

$$C_t = \frac{\xi_t}{P_t}\left(\int_t^\infty e^{-\int_t^s r_\tau d\tau} w ds + a_t\right) \tag{A16}$$

$$c_t = \frac{\alpha}{R_t^{1-\eta}(1-\alpha)+\alpha} P_t C_t, \tag{A17}$$

$$s_t = \frac{R_t^{-\eta}(1-\alpha)}{R_t^{1-\eta}(1-\alpha)+\alpha} P_t C_t, \tag{A18}$$

$$\dot{a}_t = w + r_t a_t - \xi_t\left(\int_t^\infty e^{-\int_t^s r_\tau d\tau} w ds + a_t\right), \tag{A19}$$

$$\dot{b}_t + p_t\dot{h}_t = \phi_t(w + r_t b_t + R_t h_t) - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}[(r_s - r_t)b_t + (R_s - R_t)h_t] ds, \tag{A20}$$

where $P_t := \left(\alpha + (1-\alpha)R_t^{1-\eta}\right)^{\frac{1}{1-\eta}}$*, and where*

$$\xi_t\left(\frac{1}{\gamma}, \rho, \{r_s\}_{s\geq t}, \{P_s\}_{s\geq t}\right) := \left(\int_t^\infty (P_s/P_t)^{1-\frac{1}{\gamma}} e^{-\int_t^s \left(r_\tau - \frac{1}{\gamma}(r_\tau - \rho)\right) d\tau}\right)^{-1} \tag{A21}$$

$$\phi_t\left(\frac{1}{\gamma}, \rho, \{r_s\}_{s\geq t}, \{P_s\}_{s\geq t}\right) := 1 - \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \tag{A22}$$

are independent of h_t, b_t *and* w*. We further have the following special cases:*

1. ***Constant house price, rent and return:*** *When* $p_t = p$ *and* $R_t = R$*, all* t *so that* $r_t = \frac{R}{p} =: r$ *and* $P_t := P$ *for all* t*, then* $\xi_t = r - \frac{r-\rho}{\gamma}$ *and* $\phi_t = \frac{r-\rho}{\gamma r}$ *so that total consumption expenditure in* (A16) *and gross saving in* (A19) *equal*

$$PC_t = c_t + Rs_t = \left(r - \frac{r-\rho}{\gamma}\right)\left(a_t + \frac{w}{r}\right), \qquad \dot{a}_t = \frac{r-\rho}{\gamma}\left(a_t + \frac{w}{r}\right). \tag{A23}$$

   Gross saving (which equals net saving) therefore equals (2)*, meaning that saving behavior in the model with housing collapses to that in the one-asset model in Section* 2.1*. Both net and gross saving rates are independent of wealth.*

2. ***House price growth with rental growth (constant return), unit elasticity of intertemporal substitution (IES):*** *Assume that* p_t *and* R_t *increase over time while the asset return is constant,* $r_t = r$ *for all* t *and that the IES,* $1/\gamma = 1$ *(log utility). Then* $\xi_t = r - \frac{r-\rho}{\gamma} = \rho$ *and* $\phi_t = \frac{r-\rho}{r}$ *so that total consumption expenditure in* (A16)*, gross saving in* (A19)*, and*

net saving in (A20) *equal*

$$P_t C_t = c_t + R_t s_t = \rho \left(a_t + \frac{w}{r} \right), \tag{A24}$$

$$\dot{a}_t = \frac{r - \rho}{r} \left(w + r a_t \right), \tag{A25}$$

$$\dot{b}_t + p_t \dot{h}_t = \frac{r - \rho}{r} (w + r b_t + R_t h_t) - \frac{\rho}{r} \dot{p}_t h_t \tag{A26}$$

with $a_t = b_t + p_t h_t$ *and* p_t *given by* (A15)*. Gross saving* $\dot{a}_t$ *therefore equals* (2) *with* $\gamma = 1$*, meaning that gross saving in the model with housing collapses to that in the one-asset model in Section* 2.1*. The gross saving rate is independent of wealth and equal to* $\frac{r-\rho}{r}$ *and the net saving rate decreases with wealth as in Figure* 2*(a). When the IES* $1/\gamma$ *is close to one, this is approximately true.*

3. ***House price growth without cashflow growth (falling return):*** *When* $R_t = R$*, all* t *and* $b_t = 0$ *(so that the changing interest rate leaves cashflows unchanged), then net saving in* (A20) *is*

$$\dot{b}_t + p_t \dot{h}_t = \phi_t (w + r_t b_t + R_t h_t)$$

Therefore the net saving rate is independent of wealth and equal to ϕ_t *and the gross saving rate increases with wealth as in Figure* 2*(b). Wealthier households "save by holding."*

Somewhat surprisingly, in both special cases 1 and 2 with a constant return, saving behavior in the model with housing collapses to the one-asset model with a constant return in Section 2.1.

This result is particularly surprising in special case 2 with a growing house price due to growing (implied) rents. In this case, intuition might suggest that the consumption aspect of housing would be enough to make households hold on to their residential wealth in the face of rising asset prices and that this could explain the flat net saving rate and increasing gross saving rate observed in our data. Special case 2 shows that is not the case: the fact that housing is not only an asset but also a consumption good does not, by itself, generate households holding on to their residential wealth in the face of rising house prices. The model instead predicts that households should optimally increase their total consumption expenditure in response to housing capital gains. Interestingly, this is true regardless of the value of the elasticity of substitution η between housing and non-durable consumption in (A11).

To take an extreme case, even when $\eta = 0$ so that the utility function $u(c, s)$ is Leontief, i.e., housing and non-durables are perfect complements, the model still generates households dissaving in the face of housing capital gains. To see this, set $\eta = 0$ and $\gamma = 1$ in the expressions in Proposition (A2): the price index becomes $P_t := \alpha + (1-\alpha)R_t$, non-durable consumption and housing in (A17) and (A18) become

$$c_t = \alpha C_t, \qquad s_t = (1-\alpha)C_t, \tag{A27}$$

while total consumption $P_t C_t = c_t + R_t s_t$, gross saving and net saving continue to satisfy (A24) to (A26) but with the price index $P_t = \alpha + (1-\alpha)R_t$. The key observation is that Leontief preferences do *not* restrict the household to keep its consumption unchanged; instead they only restrict c_t and s_t to move proportionately to each other. Rising rents R_t have two effects. First the house price p_t appreciates – see (A15) – and therefore so does household wealth $a_t = b_t + p_t h_t$; second the price index of the household's total consumption basket $P_t = \alpha + (1-\alpha)R_t$ increases. From (A24), the effect on C_t is ambiguous and therefore, from (A27), so is the effect on c_t and s_t: on the one hand, the household is wealthier and consumes more; on the other hand, its consumption basket has become more expensive and so it consumes less. However, total consumption expenditures including (implied) rents $P_t C_t = c_t + R_t s_t$ unambiguously increase, and more so the larger is the household's residential wealth $p_t h_t$. Finally, consider net saving $\dot{b}_t + p_t \dot{h}_t = w + rb_t + R_t(h_t - s_t) - c_t$ which specializes to (A26): because total consumption expenditures $c_t + R_t s_t$ increase with residential wealth, net saving becomes more and more negative the larger are the housing capital gains, hence the term $-\frac{\rho}{r}\dot{p}_t h_t$ in (A26). In summary, even when the utility function between non-durables and housing is Leontief ($\eta = 0$), the housing model with rising house prices driven by growing (implied) rents, predicts that the net saving rate should be decreasing with wealth, in contrast to our empirical findings.

Finally, special case 3 of Proposition A2 shows that when house price growth is accompanied by falling returns rather than rising dividends (rents or implied rents), then the housing model generates a net saving rate that is flat across the wealth distribution and a gross saving rate that increases with wealth.

Proof of Proposition A2 Recall that households maximize (A10) subject to (A12). Equivalently, they maximize (A10) subject to (A14). It is easy to see that this problem splits into separate inter- and intratemporal problems and can be solved as a two-stage budget problem. The intertemporal problem is

$$\begin{aligned} \max_{\{C_t\}_{t\ge 0}} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \quad \text{s.t.} \\ \dot{a}_t = w + r_t a_t - P_t C_t \end{aligned} \tag{A28}$$

where

$$P_t = \left(\alpha + (1-\alpha)R_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{A29}$$

is a CES price index. Given the solution to this intertemporal problem, non-durable consumption and housing can then be found from the intratemporal problem. This problem is just a standard static utility maximization problem with CES utility. The solution is given by

$$c = \frac{\alpha}{R^{1-\eta}(1-\alpha)+\alpha} PC, \qquad s = \frac{R^{-\eta}(1-\alpha)}{R^{1-\eta}(1-\alpha)+\alpha} PC. \tag{A30}$$

Problem (A28) is just a consumption-saving problem with a time-varying price level. The solution satisfies

$$C^{-\gamma} = \lambda P, \tag{A31}$$

$$\dot{\lambda} = (\rho - r)\lambda, \tag{A32}$$

$$\dot{a} = w + ra - PC. \tag{A33}$$

The budget constraint can be integrated to yield a lifetime budget constraint

$$\int_0^\infty e^{-\int_0^t r_s ds} P_t C_t dt = \int_0^\infty e^{-\int_0^t r_s ds} w dt + a_0. \tag{A34}$$

Integrating the Euler equation (A32) forward in time, we have

$$\lambda_t = \lambda_0 e^{-\int_0^t (\rho - r_s) ds}.$$

Hence from (A31)

$$C_t^{-\gamma}/P_t = C_0^{-\gamma}/P_0 e^{-\int_0^t (\rho - r_s) ds}.$$

And in particular

$$P_t C_t / P_0 C_0 = (P_t/P_0)^{1-\frac{1}{\gamma}} e^{-\int_0^t \frac{\rho - r_s}{\gamma} ds}.$$

Substituting into the lifetime budget constraint (A34)

$$P_0 C_0 \int_0^\infty (P_t/P_0)^{-\frac{1}{\gamma}} e^{-\int_0^t \left(r_s - \frac{1}{\gamma}(r_s - \rho)\right) ds} dt = \int_0^\infty e^{-\int_0^t r_s ds} w dt + a_0,$$

and hence consumption expenditure is

$$P_0C_0 = \xi_0\left(\int_0^\infty e^{-\int_0^t r_s ds}w dt + a_0\right), \qquad \xi_0 := \left(\int_0^\infty (P_t/P_0)^{1-\frac{1}{\gamma}} e^{-\int_0^t\left(r_s - \frac{1}{\gamma}(r_s-\rho)\right)ds}dt\right)^{-1}.$$

Similarly, and using that the household's problem is recursive and purely forward-looking, consumption at time t rather than time 0 is

$$P_tC_t = \xi_t\left(\int_0^\infty e^{-\int_t^s r_\tau d\tau}w ds + a_t\right)$$

with ξ_t in (A21) which is the expression for C_t in (A16). Since $\dot{a}_t = w + ra_t - P_tC_t$, we also get the saving response, namely

$$\dot{a}_t = w + ra_t - \xi_t\left(\int_0^\infty e^{-\int_t^s r_\tau d\tau}w ds + a_t\right),$$

which is (A19).

Finally we derive (A20). From (A16) we have

$$P_tC_t = \xi_t\left(\int_t^\infty e^{-\int_t^s r_\tau d\tau}w ds + b_t + p_th_t\right) \tag{A35}$$

Substituting (A15) into (A35) and using that $\int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds = 1$, we have

$$P_tC_t = \xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + r_sb_t + R_sh_t)ds. \tag{A36}$$

Net saving can be written as

$$\dot{b}_t + p_t\dot{h}_t = w + r_tb_t + R_th_t - P_tC_t$$

Substituting in from (A36), adding and subtracting $\xi_t \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w + r_tb_t + R_th_t)ds$ and rearranging yields (A20).■

B.2 Asset-Price Risk

We extend the model in Section 2.2 to feature a risky asset price. Apart from households facing this uncertainty, everything is analogous to Section 2.2. Individuals maximize

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt, \tag{A37}$$

subject to (4). Writing the budget constraint (4) in terms of market wealth $a_t := p_t k_t$ we have $da_t = p_t dk_t + k_t dp_t$ and so

$$da_t = (w - c_t)\, dt + a_t dR_t \quad \text{where} \quad dR_t := \frac{D_t dt + dp_t}{p_t} \tag{A38}$$

is the asset's instantaneous return over a time interval of length dt. As in Section 2.2, it is again useful to take the perspective of the asset-pricing literature to treat the required asset return as a primitive and the price as an outcome. To this end, define the expected return

$$r_t dt := \mathbb{E}_t[dR_t] = \frac{D_t dt + \mathbb{E}[dp_t]}{p_t}. \tag{A39}$$

Note that, in general, the expected return r_t itself follows a stochastic process. Rearranging (A39) as $r_t p_t = D_t + \mathbb{E}_t[dp_t]/dt$ and integrating forward in time and assuming a no-bubble condition, the asset price is given by the analogue of (6):

$$p_t = \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} D_s \right] ds, \tag{A40}$$

where the expectation $\mathbb{E}_t$ is taken over future realizations of returns and dividends $\{r_s, D_s\}_{s \geq t}$, conditional on information available at time t. Because the return and dividend are stochastic, so is the asset price p_t.

From (A40), and again analogous to Section 2.2, there are also still two relevant polar cases to consider: either the asset price fluctuates and so do cashflows (dividends); or the asset price changes even though cashflows do not, i.e., the return changes.

We now analyze households' optimal saving decisions in these two polar cases. We first consider the case in which all asset-price movements are due to changing cashflows, and then turn to the opposite case of asset-price fluctuations that are due to discount rate changes. In analogy to special cases 2 and 3 in Proposition 1 we refer to them as cases 2′ and 3′. (Recall that Proposition 1 also featured a special case 1 with a constant asset price – clearly there is no need to redo that special case.)

B.2.1 Case 2′: asset-price changes accounted for by changing dividends

In this polar case we assume that the expected return is constant over time, $r_t = r$ for all t, and that asset-price fluctuations are driven by dividend fluctuations. In particular, assume that dividends follow a geometric Brownian motion with drift μ and variance σ^2 (the logarithm follows a random walk):

$$dD_t = \mu D_t dt + \sigma D_t dW_t. \tag{A41}$$

With a constant discount rate, $r_t = r$ for all t, (A40) simplifies to

$$p_t = \mathbb{E}_t \int_t^\infty e^{-r(s-t)} D_s ds. \tag{A42}$$

If the dividend follows a geometric Brownian motion (A41), then $\mathbb{E}_t[D_s] = D_t e^{\mu(s-t)}$ for $s \geq t$. Therefore

$$p_t = \int_t^\infty e^{-r(s-t)} \mathbb{E}_t[D_s] ds = D_t \int_t^\infty e^{-(r-\mu)(s-t)} ds = \frac{D_t}{r-\mu}.$$

Hence also the asset price p_t evolves according to a geometric Brownian motion

$$dp_t = \mu p_t dt + \sigma p_t dW_t \tag{A43}$$

with the same drift and variance as dividends. Furthermore, the dividend yield $D_t/p_t = r - \mu$ is constant over time. Therefore the instantaneous return dR_t defined in (A38) is given by

$$dR_t = \frac{dp_t + D_t dt}{p_t} = r dt + \sigma dW_t.$$

Substituting into (A38) the budget constraint becomes

$$da_t = (w + r a_t - c_t)\, dt + a_t \sigma dW_t. \tag{A44}$$

Households therefore maximize (1) subject to (A44). When returns feature a stochastic component $\sigma > 0$ and labor income is positive $w > 0$, it is no longer possible to solve the consumption-saving problem analytically. However, Proposition A3 at the end of this Appendix derives a useful approximation to the saving policy function under the assumption that labor income is small, $w \approx 0$:

$$da_t \approx \bar{s}\left(a_t + \frac{w}{r}\right) dt + a_t dW_t \tag{A45}$$

and $\bar{s} := \frac{r-\rho}{\gamma} + (\gamma - 1)\frac{\sigma^2}{2}$. The approximation is exact if either $\sigma^2 = 0$ – so that we are back in the case with a deterministic return – or $w = 0$ – in which case the problem is a simplified version of Merton (1969) without portfolio choice. Examination of (A45) yields a key result: the optimal response to capital gains depends crucially on whether these are transitory or persistent. The logic is similar to that of the permanent income hypothesis regarding the optimal consumption response to transitory and persistent labor income shocks. When experiencing a transitory capital gain $dW_t > 0$, the household optimally saves all of it (see the additive term adW_t in (A45)). Symmetrically, transitory capital losses $dW_t < 0$ translate one-for-one into lower wealth. In contrast, when experiencing a persistent capital gain $\mu > 0$, the household optimally consumes part of the resulting income flow.

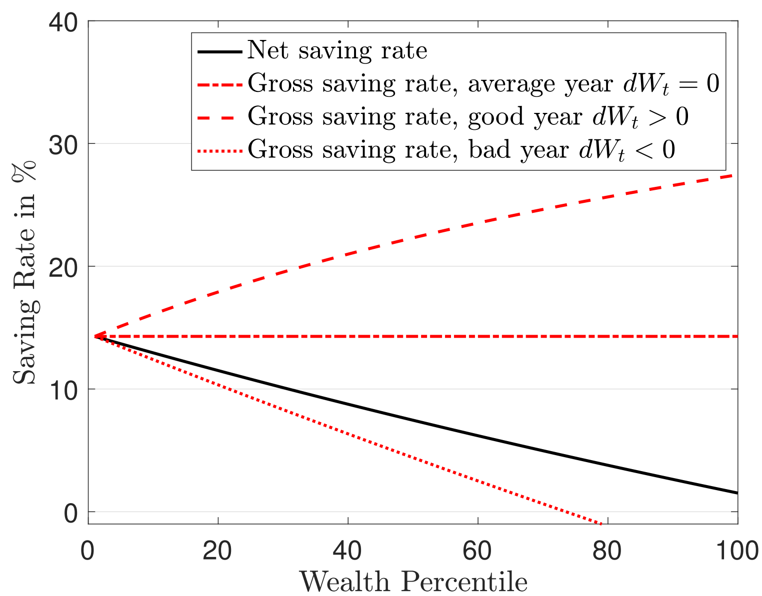


Figure A13: Saving rates across wealth distribution with risky asset price.

Figure A13 displays saving rates across the wealth distribution from the model with asset-price risk. When $dW_t = 0$, meaning that the stock market had an "average year," the figure is identical to Figure 2(a). In contrast, when the stock market does well, $dW_t > 0$, gross saving is increasing with wealth. When the stock market does badly, $dW_t < 0$, the gross saving rate is instead decreasing with wealth.[45] The reason is that households save all transitory capital gains, so these impact wealth one-for-one. Finally, the net saving rate

[45]Equation (A45) shows that $da_t \approx \hat{s}(w + (\theta + \mu)a_t)dt + a_t dW_t$ where $\hat{s} = \bar{s}/\bar{r}$. Therefore the gross saving rate is

$$\frac{\hat{s}(w + (\theta + \mu)a_t) + a_t dW_t}{w + (\theta + \mu)a_t dt + a_t dW_t}$$

is independent of stock market performance and predicted to be decreasing with wealth.[46]

B.2.2 Case 3′: asset-price changes accounted for by changing returns

In this polar case we assume that the dividend is constant over time, $D_t = D$ for all t, and that asset-price fluctuations are entirely driven by expected-return fluctuations. In particular, assume that expected returns evolve according to a diffusion process

$$dr_t = \mu_r(r_t)dt + \sigma_r(r_t)dW_t. \tag{A46}$$

With a constant dividend, $D_t = D$ for all t, (A40) simplifies to

$$p_t = D\mathbb{E}_t\left[\int_t^\infty e^{-\int_t^s r_\tau d\tau}\right]ds. \tag{A47}$$

and a particular choice of the diffusion process (A46) implies a stochastic process for the asset price p_t. Given this price process, we then compute the instantaneous return dR_t defined in (A38) and solve the household's problem. For any expected-return process, we already know from (A39) that $\mathbb{E}_t[dR_t] = r_t dt$ or, equivalently,

$$dR_t = rdt + \tilde{\sigma}(r_t)dW_t$$

for some function $\tilde{\sigma}$. Substituting into (A38) the budget constraint becomes

$$da_t = (w + r_t a_t - c_t)\,dt + a_t\tilde{\sigma}(r_t)dW_t. \tag{A48}$$

Households therefore maximize (1) subject to (A48).

Following the same steps as in the proof of Proposition 1, one can show that optimal consumption satisfies the analogue of (A7):

$$c_t = \xi_t\left(a_t + \mathbb{E}_t\left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} w ds\right]\right) \quad \text{with} \quad a_t = p_t k_t, \tag{A49}$$

for some $\xi_t > 0$, i.e., households optimally consume a fraction ξ_t of their lifetime income.

which equals $\hat{s}$ and is independent of wealth when $dW_t = 0$, is increasing in wealth a_t whenever $dW_t > 0$ and decreasing whenever $dW_t < 0$.

[46] From (A45) consumption c_t is independent of transitory capital gains dW_t. Then from (4) so is net saving $p_t\dot{k}_t$.

Using the expression for the asset price (A40)

$$c_t = \xi_t \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} (w + D_s k_t) ds \right]$$

Finally, with a constant dividend, $D_t = D$ for all t, optimal consumption and net saving are given

$$c_t = \xi_t \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \right] (w + Dk_t)$$

$$p_t \dot{k}_t = \phi(w + Dk_t), \qquad \phi_t := 1 - \xi_t \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \right]$$

Therefore, when asset price movements are entirely accounted for by expected-return movements, the net saving rate is flat across the wealth distribution, just like in special case 2 of Proposition 1 in the main text.

B.2.3 Derivation of Equation (A45)

Households maximize (1) subject to (A44). The corresponding HJB equation is

$$\rho v(a) = \max_c \; u(c) + v'(a)(w + ra - c) + \frac{\sigma^2 a^2}{2} v''(a). \tag{A50}$$

This problem can no longer be solved in closed form. However, we can use a perturbation method around small w. We first solve (A50) analytically when $w = 0$. We then perturb v that solves (A50) for $w > 0$ around the solution for the case $w = 0$, thus obtaining (A45). Our argument makes use of a standard perturbation method as in Fleming (1971), Judd (1996), Anderson, Hansen and Sargent (2012) and Kasa and Lei (2018).

Step 1: Closed form with $w = 0$. For the first step, consider the value function for $w = 0$ which we denote by $v_0(a)$. It solves:

$$\rho v_0(a) = \max_c \; u(c) + v_0'(a)(ra - c) + \frac{\sigma^2 a^2}{2} v_0''(a) \tag{A51}$$

This is the HJB equation for a simplified version without portfolio choice of the problem analyzed by Merton (1969) and it is well-known to have a closed-form solution.

Lemma A2. *The value function and consumption policy function with $w = 0$ are*

$$v_0(a) = B_0^{-\gamma} \frac{a^{1-\gamma}}{1-\gamma}, \quad c_0(a) = B_0 a, \quad B_0 := \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \tag{A52}$$

Proof of Lemma A2: the proof uses a guess-and-verify strategy. Start by guessing that $v_0(a) = B_0^{-\gamma} \frac{a^{1-\gamma}}{1-\gamma}$ for a constant B_0 to be determined. Then $v'(a) = (B_0 a)^{-\gamma}, c(a) = B_0 a$ and $v''(a) = -\gamma B_0^{-\gamma} a^{-\gamma-1}$. Substituting into (A51), we have

$$\rho B_0^{-\gamma} \frac{1}{1-\gamma} = B_0^{1-\gamma}/(1-\gamma) + B_0^{-\gamma} r - B_0^{1-\gamma} - \frac{\sigma^2}{2}\gamma B_0^{-\gamma}$$

Rearranging

$$\rho \frac{1}{1-\gamma} = \gamma B_0/(1-\gamma) + r - \frac{\sigma^2}{2}\gamma.$$

Rearranging again we obtain the expression for B_0 in (A52).■

Step 2: Perturbation around $w = 0$. As already mentioned, it is no longer possible to solve (A50) in closed form. However, we can look for approximate solutions of the form

$$v(a) = v_0(a) + w v_1(a) + O(w^2), \quad c(a) = c_0(a) + w c_1(a) + O(w^2) \tag{A53}$$

where v_0 and c_0 are the value and consumption policy functions from Lemma A2 and where v_1 and c_1 are to be determined.

Proposition A3. *The value and consumption policy functions solving* (A50) *satisfy*

$$\begin{aligned} v(a) &= \bar{c}^{-\gamma}\left(\frac{a^{1-\gamma}}{1-\gamma} + \frac{w a^{-\gamma}}{r - \gamma\sigma^2}\right) + O(w^2) \\ c(a) &= \bar{c}\left(a + \frac{w}{r - \gamma\sigma^2}\right) + O(w^2) \\ \bar{c} &:= \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \end{aligned} \tag{A54}$$

Before proving the Proposition, we first note that it immediately implies the approximate saving policy function (A45) in the main text. To see this simply substitute $c(a)$ in (A54) into (A38) to get

$$da_t \approx \left[w + r a_t - \bar{c}\left(a + \frac{w}{r - \gamma\sigma^2}\right)\right] dt + \sigma a_t dW_t.$$

which is equation (A45).

Proof of Proposition A3: In the proof it is convenient to use slightly different notation than in the statement of the Proposition: for reasons that will become apparent momentarily, we use B_0 in place of the variable $\bar{c}$ used in the statement of the Proposition. As already mentioned we look for solutions of the form (A53). Further, we restrict our attention to solutions such that $v_1'(a) > 0$ for all a which ensures that $v'(a) \approx v_0'(a) + wv_1'(a) > 0$ for all $w > 0$ and therefore consumption $c(a) = (v'(a))^{-1/\gamma}$ is positive. However, we do not make any assumptions about the sign of $v_1(a)$. Substituting (A53) into (A50)

$$\rho(v_0(a) + wv_1(a)) = \max_c \ u(c) + (v_0'(a) + wv_1'(a))(w + ra - c) + (v_0''(a) + wv_1''(a))\frac{\sigma^2}{2}a^2$$

and the first-order condition is $(c_0(a) + wc_1(a))^{-\gamma} = v_0'(a) + wv_1'(a)$. Differentiating both the HJB equation and the first-order condition with respect to w and evaluating at $w = 0$:

$$\rho v_1(a) = v_1'(a)(ra - c_0(a)) + v_1''(a)\frac{\sigma^2}{2}a^2 + v_0'(a), \tag{A55}$$

$$-\gamma c_0(a)^{-\gamma-1}c_1(a) = v_1'(a). \tag{A56}$$

Substituting the expressions for v_0 and c_0 from Lemma A2 into (A55)

$$\rho v_1(a) = v_1'(a)\left(\frac{r-\rho}{\gamma} + (\gamma - 1)\frac{\sigma^2}{2}\right)a + v_1''(a)\frac{\sigma^2}{2}a^2 + B_0^{-\gamma}a^{-\gamma} \tag{A57}$$

It remains to find a solution $v_1(a)$ that solves the ODE (A57). We solve it using a guess-and-verify strategy. Guess $v_1(a) = B_1a^{-\gamma}$. Then $v_1'(a) = -\gamma B_1 a^{-\gamma-1}$ and $v_1''(a) = \gamma(1+\gamma)B_1a^{-\gamma-2}$. Substituting into (A57)

$$\rho B_1 a^{-\gamma} = -\gamma B_1 a^{-\gamma}\left(\frac{r-\rho}{\gamma} + (\gamma - 1)\frac{\sigma^2}{2}\right) + \gamma(1+\gamma)B_1a^{-\gamma}\frac{\sigma^2}{2} + B_0^{-\gamma}a^{-\gamma}$$

Rearranging we find that $B_1 = \frac{1}{r-\gamma\sigma^2}B_0^{-\gamma}$. Therefore

$$v_1(a) = \frac{1}{r - \gamma\sigma^2}B_0^{-\gamma}a^{-\gamma}$$

Similarly, substituting the expressions for $v_1'(a)$ and $c_0(a) = B_0 a$ into (A56) we find

$$c_1(a) = \frac{1}{r-\gamma\sigma^2}B_0$$

Substituting v_1 and c_1 as well as v_0 and c_0 from Lemma A2 into (A53), we obtain (A54) (recall again that the statement of the Proposition uses $\bar{c}$ to denote B_0).■

B.3 Labor Income Risk

Another extension is to allow for labor income risk and borrowing constraints, as in Aiyagari (1994) and Huggett (1993). A continuum of ex-ante identical and infinitely-lived households maximize the discounted utility flow from consumption,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt,$$

subject to a budget constraint

$$c_t + \dot{a}_t = w_t + r a_t,$$

where w_t is labor income that evolves stochastically according to an Ornstein-Uhlenbeck process (continuous-time analogue of an AR(1) process) in logs

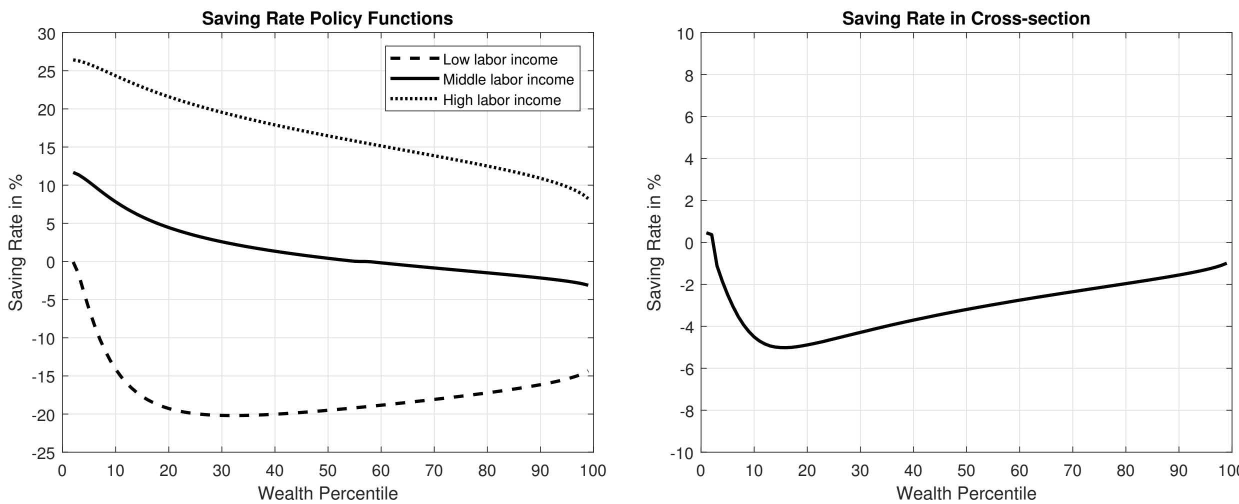
$$d \log w_t = -\nu \log w_t + \sigma_w dW_t.$$

We impose a no-borrowing constraint, $a_t \geq 0$. Markets are incomplete and households self-insure by accumulating wealth a. Conditional on their earnings history, households differ in their level of wealth and income. Table A4 presents our calibration.

	Value	
γ	2	Relative risk aversion / inverse IES
ρ	0.05	Discount rate
r	0.045	Dividend yield
ν	0.03	Persistence of income innovations (annual autocorrelation = 0.97)
σ_w	0.14	Standard deviation of income innovations

Table A4: Calibration of Huggett model

The model generates a saving policy function $\dot{a} = s(a, w)$ where w is labor income. Figure A14 plots the resultant saving rate out of total income against wealth in this environment. The left panel displays the "saving rate policy function" $s(a, w)/(w + ra)$ for three different levels of labor income w. Conditional on labor income, the saving rate is declining in wealth, and more steeply so the closer a household is to the borrowing constraint. For households with different labor income realizations, the policy function simply shifts up or down.[47] The right panel displays saving rates without conditioning



(a) Policy function

(b) Saving across the wealth distribution

Figure A14: Saving rates with income risk and borrowing constraints.

on income. It simply plots the cross-sectional relationship between saving rates and wealth in the model's stationary distribution. The relationship is first decreasing, then increasing. This reflects two opposing forces. On the one hand, conditional on income, saving rates are decreasing with wealth. On the other hand, saving rates increase with labor income (see panel (a)), and labor income and wealth are positively correlated in the stationary distribution. Summarizing, while saving rates conditional on income should decline slightly with wealth, they may be increasing with wealth when not conditioning on income. But this increasing relationship is driven by a cross-sectional correlation between labor income and wealth.

[47] It is straightforward to show that the flat-saving-rate result from Section 2.1 now applies as wealth becomes large. More precisely, for all w, $s(a,w)/(w+ra) \rightarrow (r-\rho)/(\gamma r)$ as $a \rightarrow \infty$ meaning that the saving rate policy function even converges to the same value as in Section 2.1. The steep decline close to the borrowing constraint reflects two familiar effects. First, precautionary saving of high-income, low-wealth households. Second, low-income households rapidly decumulating wealth and then hitting the constraint (i.e., their saving rate is zero at the constraint but strongly negative above the constraint).

C Appendix for Section 3

C.1 Data Sources and Variables

Source:	Variables:
Income and wealth from tax returns	Labor income
Annual, 1993 -	Business income
	Capital income
	Transfers received
	Taxes paid
	Asset holdings (e.g deposits, mutual funds, bonds, real estate)
	Debt (total debt)
	Pensionable income (since 1967)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/inntekt	
Housing wealth database	Value of housing (including cabins and secondary homes)
Annual, 1993 -	(as in Fagereng, Holm and Torstensen, 2019)
Norwegian educational database	Highest completed education (length and type)
Annual, 1964 -	
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/utdanning	
Stockholder registry	ISIN / firm ID
Annual, 2004 -	Owner ID
	Quantity owned of stock
https://www.ssb.no/383782/utlan-av-data-om-aksjonaerer-aksjeselskaper-og-allmennaksjeselskaper	
Firm balance sheet and tax return data data	Balance sheet information (e.g., book value of equity, retained earnings)
Annual, 1995 -	Assessed value of private companies
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/regnskap	
The central population register	Region of residence at the end of the year
Annual, 1964 -	Date (month) of birth
	Gender, indicator variable for gender
	Marital status indicator variable
	Spousal id (unique identifier of spouse)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/befolkning	
Ambita / Norwegian mapping authority	Buyer/seller ID
Event data, 1993 -	Price
https://www.ambita.com/tjenester/eiendomsinformasjon/	
Other public data sources:	
Consumer price index, Statistics Norway	
https://www.ssb.no/en/priser-og-prisindekser/statistikker/kpi	
Flow of funds, Statistics Norway	
https://www.ssb.no/en/nasjonalregnskap-og-konjunkturer/statistikker/finsekv	
Stock price index and general stock prices, Oslo Børs	
https://www.oslobors.no/ob_eng/markedsaktivitet/#/details/OBX.OSE	
House price indices, Eitrheim and Erlandsen (2004)	
https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/	
Exchange rate data, Norges Bank	
https://www.norges-bank.no/en/topics/Statistics/exchange_rates/	
MSCI stock index	
https://finance.yahoo.com/quote/MSCI/history/	

C.2 Net and Gross Saving with Multiple Assets

Section 2.2 defined net and gross saving with one asset. The data feature multiple assets and in this section, we generalize the saving definitions.

A household receives annual income w_t (labor income and transfers) and pays taxes τ_t. There are J assets indexed by $j = 1, ..., J$. Let $k_{j,t-1}$ denote the household's holdings of asset j at the end of period $t-1$. To simplify notation, assume that the household holds each asset $k_{j,t-1}$ throughout the year and only makes transactions at the end of the year. Throughout the year, asset holdings $k_{j,t-1}$ earn capital income $\theta_{j,t} p_{j,t} k_{j,t-1}$. The general versions of (7) and (8) are then

$$c_t + \underbrace{\sum_{j=1}^{J} p_{j,t}(k_{j,t} - k_{j,t-1})}_{\text{net saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J} \theta_{j,t} p_{j,t} k_{j,t-1}}_{\text{disposable income}} \tag{A58}$$

$$c_t + \underbrace{\sum_{j=1}^{J}(p_{j,t}k_{j,t} - p_{j,t-1}k_{j,t-1})}_{\text{gross saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J}\left(\theta_{j,t} + \frac{p_{j,t} - p_{j,t-1}}{p_{j,t-1}}\right) p_{j,t-1}k_{j,t-1}}_{\text{Haig-Simons income}}. \tag{A59}$$

C.3 Separating Gross Saving into Net Saving and Capital Gains

C.3.1 Housing: Using Transaction Data

To explain our approach, it is helpful to introduce some notation. Time is continuous and we consider a household that makes housing transactions at discrete time intervals. We denote by $h(t), p(t)$ and $a_h(t) = p(t)h(t)$ the household's physical number of housing units, the price of housing and the value of the house at the beginning of year t. Throughout a year, i.e., between dates t and $t+1$, the household makes $N \geq 0$ transactions at ordered dates τ_n: $t \leq \tau_1 < \tau_2 < ... < \tau_N < t+1$. We decompose gross saving, i.e., the change over the year in housing wealth $a_h(t) = p(t)h(t)$, into net saving and capital gains using the following decomposition

$$\underbrace{a_h(t+1) - a_h(t)}_{\text{gross saving}} = \underbrace{\sum_{n=1}^{N+1}(p(\tau_n) - p(\tau_{n-1}))h(\tau_{n-1})}_{\text{capital gains}} + \underbrace{\sum_{n=1}^{N} p(\tau_n)(h(\tau_n) - h(\tau_{n-1}))}_{\text{net saving}}. \tag{A60}$$

For our purpose, the main implication is that net saving can only be non-zero for households with housing transactions. Hence, for households without housing transactions, net saving in housing is zero and all changes in housing wealth are due to capital gains. In the case with transactions, on the other hand, (A60) implies that net saving is equal to net transactions at market value during the year. Capital gains in housing is then the change in housing wealth minus net transactions at market value.

C.3.2 Stocks: Using Ownership Data

For most asset classes, we do not know the individual transactions within the year. We therefore approximate capital gains and net saving based only on available information at time t and $t+1$. For example, for stocks we know the number of shares q in each stock and its price p at the beginning and end of the year. We make three simplifying assumptions to compute capital gains and net saving:

1. All transactions are of the same size and direction: $dq_{\tau_n} = \frac{q_{t+1}-q_t}{N}$.
2. All prices move monotonically and with same step size within a year: $p_{\tau_n} = (\tau_n - t)(p_{t+1} - p_t) + p_t = \frac{n}{N+1}p_{t+1} + \frac{N+1-n}{N+1}p_t$.
3. All transactions are distributed uniformly across the year: $\tau_n - \tau_{n-1} = d\tau\ \forall n$.

Under these assumptions, we derive an expression for net saving from observables

$$
\begin{aligned}
\sum_{n=1}^{N} p_{\tau_n} dq_{\tau_n} &= \sum_{n=1}^{N} \left(\frac{n}{N+1} p_{t+1} + \frac{N+1-n}{N+1} p_t \right) \frac{q_{t+1}-q_t}{N} \\
&= \frac{q_{t+1}-q_t}{N} \sum_{n=1}^{N} \left(\frac{n p_{t+1} + (N+1-n) p_t}{N+1} \right) \\
&= \frac{q_{t+1}-q_t}{N} \sum_{n=1}^{N} \left(\frac{n(p_{t+1}-p_t)}{N+1} + p_t \right) \\
&= \frac{q_{t+1}-q_t}{N} (N p_t + \frac{1}{2} N (p_{t+1}-p_t)) \\
&= \frac{1}{2}(p_t + p_{t+1})(q_{t+1}-q_t)
\end{aligned}
$$

Capital gains is next defined as the change in total value of an asset not accounted for by net saving.

C.4 Private Businesses

The portfolio shares in Figure 3 show that many of the wealthiest households hold a substantial share of their wealth in private businesses. Since these firms are not publicly traded, there is no available market price. In this appendix, we describe how we account for private businesses.

A private business is a company that is not listed on a stock exchange and owned by a small number of shareholders. Control of the firm is therefore limited to a few persons. These firms are typically small to medium sized businesses or holding companies. In 2006, Norway introduced a dividend tax at the individual level. One response to this tax reform was that the number of holding companies grew such that individuals could retain earnings in firms to avoid paying the dividend tax. These holding companies are therefore common, especially at the top of the wealth distribution.

Our aim is to find the ultimate owners of private businesses to be able to allocate retained earnings, public stock ownership, debt, and capital gains onto the ultimate owner's balance sheet. The approach is similar to other papers using Norwegian data (Alstadsæter et al., 2016; Fagereng et al., 2020).

Ultimate owners of private businesses. We use the stock holder registry to find the ultimate owners of private businesses. The stock holder registry contains information of individuals' and firms' ownership of stocks in all companies in Norway. Some companies are held directly. In this case, the ownership share is the fraction of total shares owned by the individual. However, many companies are owned by other firms. To fix ideas, assume an individual owns shares in company A and company A owns shares in company B. In this case, the individual holds an ownership share in company B equal to that individual's ownership share in company A multiplied with company A's ownership share in company B. We compute indirect ownership through up to 7 layers.

Retained earnings. Retained earnings is the profit of the firm that is withheld in the firm by not paying dividends. These are profits that accrue to the company but will not be accounted for on the income statement of individuals. Alstadsæter et al. (2016) show that retained earnings in private businesses have grown sharply after the dividend tax reform in 2006. By not accounting for retained earnings properly, we underestimate earnings of (wealthy) individuals.

To compute retained earnings, we follow the method in Alstadsæter et al. (2016) and exploit the Norwegian accounting concept of earned equity, defined as accumulated retained earnings. Retained earnings in a private business in year t is therefore the difference between earned equity at the end of year t and the beginning of year t. After obtaining retained earnings at the level of the private business, we allocate it to the ultimate owners' income using the ownership register.

Balance sheets. To more precisely measure individuals' portfolio shares and exposure to risky assets and debt, we allocate all publicly-traded stocks and debt onto the ultimate owners' balance sheets. At the end of each year, the private business reports its balance sheet to the tax authorities. Both publicly-traded stocks and debt are directly observed on these firm balance sheets and we allocate these to the ultimate owners' balance sheet using the ownership register.

Capital gains. Private businesses hold publicly traded stocks that accumulate capital gains. From the stockholder registry, we can see which stocks a private business hold. We are therefore able to compute capital gains in a private business in the same way as for publicly traded stocks held by individuals in Appendix C.3.2. Once we obtain a measure of capital gains at the level of the private business, we allocate these capital gains to ultimate owners' capital gains using the ownership register.

C.5 Stock Ownership in Norway

The portfolio shares in Figure 3 show that ownership of publicly-traded stocks, held either directly by the individual or indirectly though stock funds or private businesses, is relatively low in Norway compared with other OECD countries. For example, the mean portfolio share in publicly-traded stocks is about 1.5% for all individuals and less than 5% for the top 1% of the wealth distribution. In contrast, the top 1% in the US hold more than 40% of their assets in public equity (Campbell, 2006). There are two main reasons why the portfolio share of publicly-traded stocks is lower in Norway than in the US. First, Norway has a public pension system that holds a substantial position of Oslo Stock Exchange (OSE) on behalf of the Norwegian population. This indirect ownership of publicly-traded stocks does not enter individuals' balance sheets in the way for example 401k accounts enter the balance sheets of US citizens. Second, Oslo Stock Exchange is smaller as a share of GDP than in other similar countries. For example, the market capitalization of listed domestic companies relative to GDP is about twice as large in Sweden as in Norway.[48] In

[48]See https://data.worldbank.org/indicator/CM.MKT.LCAP.GD.ZS?locations=NO-SE.

this appendix we document the ownership structure of Oslo Stock Exchange and to what extent we are able to account for aggregate stock ownership at the individual level.

Ownership structure of Oslo Stock Exchange. Table A5 presents the ownership structure in aggregate data from the Oslo Stock Exchange and in the ownership registry in 2015. In 2015, 33.6% of the market capitalization of Oslo Stock Exchange was held by the government sector. There are two main reasons why the government sector has such a large ownership share. First, the government sector includes pension funds both at the state and municipality level.[49] Norway has a public pension system where all citizens are enrolled. A part of the pension funds is invested in public stocks in Norway. Second, it includes the government's direct ownership of firms. The Norwegian government owns substantial fractions of many publicly-traded companies in Norway, both for historic and strategic reasons. For example, the Norwegian government still holds large positions in many Norwegian banks after the re-capitalization of the banking system in the early 1990s.

In 2015, foreign investors held 36.8% of Oslo Stock Exchange. This ownership share has also been stable between 33% and 41% in our sample period. Next, 8.8% of the stock exchange is held by the financial sector. This is mainly stock funds (6.8%), while the rest (2.0%) is held by banks and mortgage companies, private pension funds or life insurance companies, and general insurance companies.

The next two ownership categories, "other companies" and "private investors," are the most interesting for our purpose because a large share of these categories are controlled by Norwegian individuals. Other companies includes all stocks that are held by Norwegian companies. Many individuals hold stocks through private businesses and these are included in this sector. The private investors sector includes all stocks that are directly held by Norwegian individuals. The sum of other companies and private investors is therefore an upper bound the share of the stock exchange that is controlled by Norwegian individuals and that we may be able to find using the available data.[50]

Although the sum of "other companies" and "private investors" is the upper bound for the share of stocks that are directly controlled by Norwegian individuals in the annual report, it is not the upper bound that we can unravel from the registry data for two

[49]Note that the Norwegian Sovereign Wealth Fund does not hold stocks in Norway and is therefore not included in this ownership share.

[50]Note that the sum of "other company" and "private investors" is not the strict upper bound of ownership that is held by Norwegian individuals since they can hold stocks on Oslo Stock Exchange indirectly through foreign companies. For example, Alstadsæter, Johannesen and Zucman (2019) document that almost 30% of taxes among the top 0.01% are evaded in Norway.

Owner sector	OSE Annual report	Ownership registry	Potentially controlled by individuals	Allocated to individuals
Government[1]	33.6%	33.0%		
Foreign investors	36.8%	32.4%		
Financial sector[2]	8.8%	8.4%		
Other companies	16.7%	11.2%	9.7%	6.0%
Private investors	3.9%	3.7%	3.7%	3.5%
Others	0.1%	0.0%		
Sum	100.0%	88.6%	13.4%	9.4%

Notes: "OSE Annual report" refers to the annual report of Oslo Stock Exchange and "Ownership registry" refers to the ownership registry that is available with individual owners. "Potentially held by individuals" refers to the share of Oslo Stock Exchange that is potentially controlled directly by individuals. The difference between "ownership registry" and "potentially controlled by individuals" is the stocks that are held by companies that are listed on the stock exchange. "Allocated to individuals" is the share of stocks at Oslo Stock Exchange that we ultimately allocate to individuals, either via indirect ownership through private businesses or directly held stocks.

[1]Government includes the categories "government and municipalities" and "companies with government ownership."
[2]Financial sector includes the categories "banks and mortgage companies," "private pension funds/life insurance," "stock funds," and "general insurance."

Table A5: Ownership structure of Oslo Stock Exchange (OSE), 2015

reasons. First, there is a discrepancy between the ownership registry in the micro data and the official data from Oslo Stock Exchange. This discrepancy brings the ownership share of other companies down from 16.7% to 11.2%. Second, a share of the sector other companies are stocks that are either held by the company itself, held by other publicly traded companies or held by foreign companies. By excluding the share that is owned by other publicly-traded companies, the share of the sector "other companies" declines further from 11.2% to 9.7%. 9.7% is therefore the upper bound on what share of stocks on Oslo Stock Exchange that we potentially can allocate to individuals. At the end of the day, we are able to allocate 6.0% of the stock exchange to the ultimate owners held through private businesses.

C.6 Public Pension Wealth

This appendix describes how we compute public pension wealth. We define public pension wealth as the net present value of future pension income, discounted at the risk free interest rate and accounting for the probability of living to the retirement age. The main complication is that there are currently four different pension systems depending on birth cohorts. We describe each system in detail before we define public pension wealth, savings, and income. A common feature of all public pension systems is that an individual accumulates claims pen_t^c in units of the basis-amount in the social security system G_t.

C.6.1 The Four Pension Systems

Cohorts born prior to 1944. For these cohorts, we observe pension transfers in our data. Pension wealth is therefore computed as the net present value of these pension earnings.

Cohorts born between 1944 and 1953. For these cohorts, there are two parts of the pension system: the social security and the service pension.

1. **Social Security.** The social security system is based on a point system. Each year, individuals accumulate pension points based on the following formula

$$point_t = \begin{cases} \frac{y_t}{G_t} - 1 & \text{if } G_t \leq y_t \leq 6G_t \\ 5 + \frac{y_t - 6G_t}{3G_t} & \text{if } 6G_t \leq y_t \leq 12G_t \\ 7 & \text{if } y_t \geq 12G_t \end{cases}$$

   where y_t is gross earnings in year t and G_t is the basis-amount in the social security system.

   An individual's pension number P is then defined as the average of the 20 years with the highest pension points, or the average of the n years that person has earned pension points. The payouts from the social security pension is approximately

$$pen^{SC} = \alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}$$

   where $\alpha = 1$ for singles and 0.85 for couples, and κ is a proportionality factor equal to 0.45 if income was accumulated prior to 1992 and 0.42 after 1992.

2. **Service Pension.** All public sector employees and about 50 percent of private sector employees have an additional service pension on top of their social security pension. This service pension guarantees an individual a fraction ψ of their final income. This fraction depends on how long the individual has worked for the company/government, but the maximum is $\psi = 0.66$.

The service pension pays the difference between the sum implied by the fraction rule and the level received from the social security pension. We can therefore approximate the pensions in units of G as

$$pen_t^c = \max\left\{\alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}, \frac{0.66 y^{final}}{G^{final}}\right\}$$

where n is the number of working years, and y^{final} and G^{final} are income and basis-unit in social security in your final working year, respectively.

Cohorts born between 1954 and 1962. Pensions in units of G is a linear combination of the system for those born prior to 1954 outlined above and those born after 1963 outlined below:

$$pen_t^c = \frac{63 - c}{10} pen_t^{44 \leq c \leq 53} + \frac{c - 53}{10} pen_t^{c \geq 63}.$$

Cohorts born after 1963. In 2010, the government simplified the pension system for all earners born after 1964. The new system implies that every year, 18.1 % of your gross income below 7.1 G is added to your pension holdings ("pensjonsbeholdning," P_t).

$$P_t^c = \left(\sum_{\tau=c+13}^{t} \max\left\{0.181 \frac{y_\tau}{G_\tau}, 0.181 \cdot 7.1\right\}\right)$$

P_t^c is, as before, defined in units of G, the basis-amount in the Norwegian pension system.

One complication is that you can start taking out pensions from age 62. However, to simplify the exposition, we assume that all households value their pension as if they would start taking out pensions from age 67. From age 67, your income from pensions is the value of your pension holdings P_t^c in units of G divided by your expected remaining years (e.g., 16.02 for the cohort born in 1964).

$$pen_t^c = \frac{P_t^c}{d_c}$$

C.6.2 Pension Wealth, Saving, and Income

We define pension wealth as the net present value of pension income from age 67. That income is $pen_t^c G_t$ in each year t when t is greater than 67, where pen_t^c is defined by one of the four systems above depending on your birth cohort. In order to calculate the net present value, we discount the pension contributions net of taxes by the risk free real interest rate and the survival probability[51]

$$V_t^c = (1-\tau) pen_t^c G_t \mathcal{M}_{t,c+67} \left\{ \sum_{\tau=\max\{c+67,t\}}^{\max\{c+67+d_c,t\}} \frac{\prod_{s=t}^{\tau}(1+\pi_{w,s})}{\prod_{s=t}^{\tau}(1+r_s)} \right\} \tag{A61}$$

where τ is the median tax rate on pensions (17 %), $\mathcal{M}_{t,c+67}$ is the probability of surviving from year t to year $c+67$ when the household is born in year c, and $\pi_{w,s}$ is the real growth rate in G in year s and r_s is the real interest rate in year s. The max operator is there because an individual start withdrawing from the pension account after age 67. We have to make two assumptions to calculate pension wealth in the data. First, we assume perfect foresight in the years where we observe r_s and $\pi_{w,s}$. Second, we assume that after 2015, the expected real interest rate and growth rate of G are the observed geometric mean in the years from 1993 to 2015. For example, in order to calculate pension wealth in 2006, we discount by the observed real interest rate from 2006 to 2015, and with the mean real interest rate after 2015.

We define pension saving as the change in pension wealth. Arguably, changes in pension wealth may be due one of the following three reasons: (i) net withdrawals or contributions, (ii) revaluation due to discounting by the real interest rate, or (iii) revaluation because the probability of surviving to age 67 increases. We count all changes in pension wealth as net saving. Furthermore, we define pension income in such a way to ensure that the budget constraint adds up. This implies that pension income always equals pension saving.

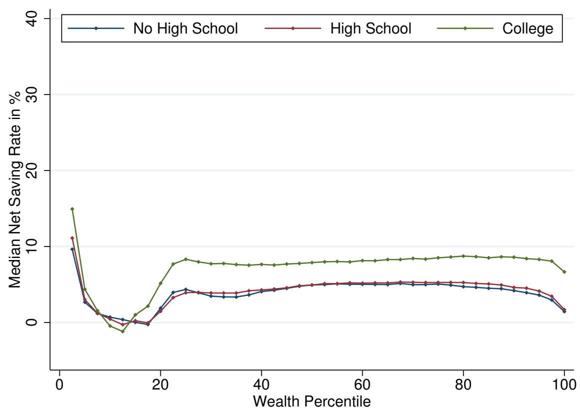
D Appendix for Section 4: Additional Exercises

Saving Rates by Education and within Deciles of Historical Saving Rates. These results and the approach behind them are discussed in the main text. Figure A15 plots the results.

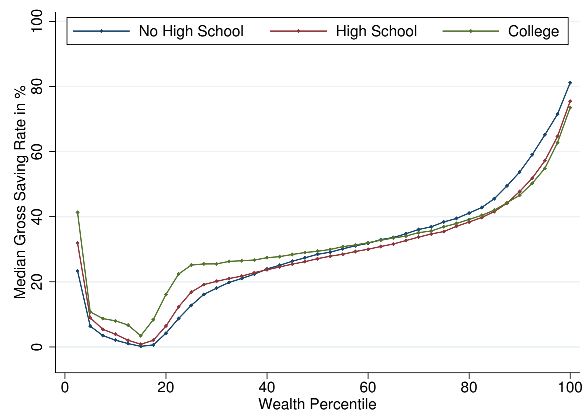
Dispersion in Saving Rates. In our main exercise, we compute medians within wealth percentiles. While interesting and informative for models, our approach ignores the dispersion in saving rates that exists within wealth percentiles. Figure A16(a) and A16(b) present the net saving rate and the gross saving rate together with their respective 25th and 75th percentile within the wealth percentile. The additional lines are computed in the same way as the main graph. For example, the 25th percentile line is computed by

[51] We calculate the survival probability of living from age t to $c+67$ from the Norwegian mortality tables. It is about 90 % for a 20 year old in our sample and increases toward 1 as the individual ages.

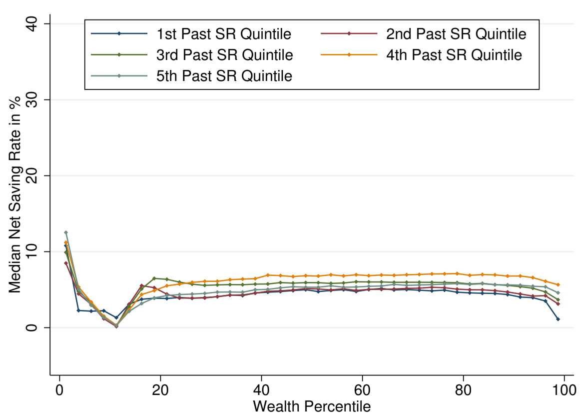
first computing the 25th percentile within the wealth percentile in each year, and then averaging across all years (2005-2015). The most striking feature of the dispersion graphs is that, dispersion is relatively stable across the wealth distribution except for the tails. For the net saving rate, also the 25th and 75th percentiles are flat across the wealth distribution for households with positive net worth. Similarly, for the gross saving rate, the 25th and 75th percentiles are increasing in wealth for households with positive net worth.



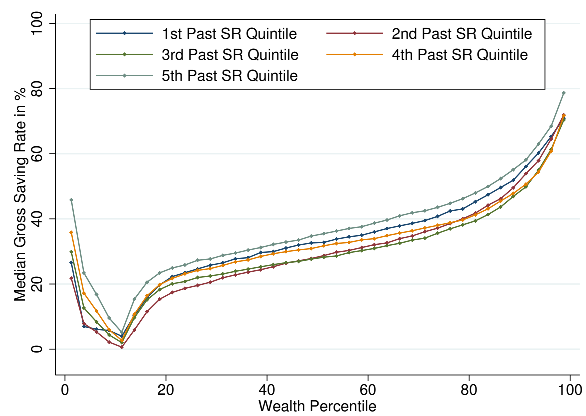
(a) Net saving rate by education group



(b) Gross saving rate by education group



(c) Net saving rate by past saving rate group

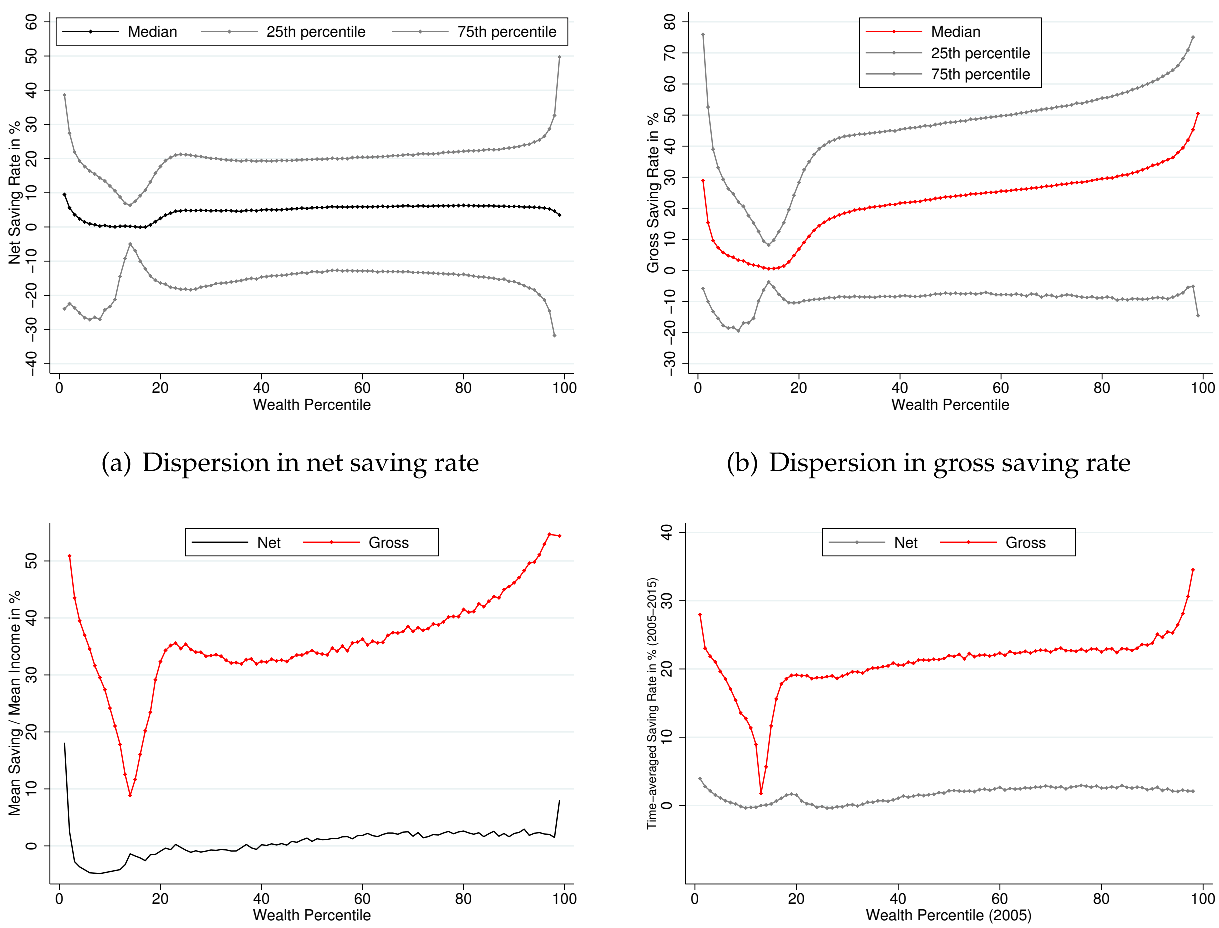


(d) Gross saving rate by past saving rate group

Notes: The figures display the median net saving rates (left) and median gross saving rates (right) within percentiles of past saving rates. Conditional upon observing a household for at least 4 prior years, we compute each household's past gross saving rate for every year, and thereafter stratify each household-year observation by average past gross saving rate. All variables are computed as the median within wealth percentile and year, averaged across all years (2005-2015).

Figure A15: Saving rates across the wealth distribution by education and within deciles of past saving rates

Saving Rate as Ratio of Means. An alternative approach to what we do in the main text is to compute the saving rate in a percentile as the ratio of average saving and average income. For example, Krueger, Mitman and Perri (2016, Table 2) show that expenditures as a share of disposable income in the U.S. is declining in wealth. The comparable prediction in our data is to say that the net saving rate, as measured by the ratio of average saving and average income within wealth percentiles, should be increasing in wealth. In Figure A16(c), we do find that the net saving rate measured as ratios of means is approximately flat across the wealth distribution, while the gross saving rate is increasing in wealth, similar to our main results.



(a) Dispersion in net saving rate

(b) Dispersion in gross saving rate

(c) Mean(saving)/mean(income)

(d) Alternative Time-averaging

Figure A16: Additional exercises.

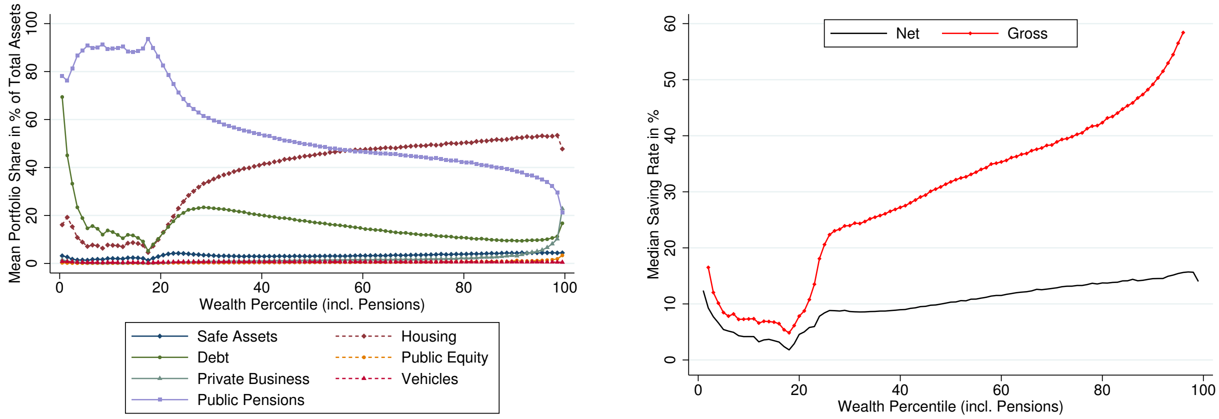
Alternative Time-Averaging. Our main graph was constructed by first computing median saving rates within each percentile for every year, and then plotting the average of these yearly median saving rates. A concern with this approach is that if households transact irregularly, for example if they buy homes every fifth year, the median households might always be non-transactors who therefore save a relatively large fraction of capital gains. To address this concern, we here present an alternative way of time-averaging saving rates. We first compute the average saving rate for each household over the entire period (2005-2015). Thereafter we stratify households by their percentile in the 2005 wealth distribution. Then we compute the within-percentile median and mean of households' time-averaged saving rate. This way, the saving rate we compute also includes years in which households transact. Figure A16(d) presents the median net and gross saving rates plotted against the 2005 wealth distribution on the horizontal axis. We see that also with this definition, our main qualitative result withstands: the net saving rate is approximately flat across the wealth distribution, while the gross saving rate increases with wealth.

Saving Rates across the Wealth Distribution Including Pensions. Figure A17 presents the portfolio shares and the saving rates when we include pensions.[52] The first thing to note is that the public pension system has complete coverage of the Norwegian population. This means that including public pensions adds wealth to every person with a Norwegian passport. Furthermore, the public pension system is relatively generous. In particular, everyone is entitled to a minimum pension equal to approximately $20,000 every year after retirement. Since the discounting approximately cancels out the real wage growth, the value of this pension claim is approximately equal to $20,000 per year multiplied by 20 years, net of taxes on pension benefits (approximately 17%) and the probability of living until age 67. This implies that every 20 year old has a pension wealth approximately equal to $300,000. Figure A17(a) reveals that almost no Norwegian has negative net wealth if we count their claims to public pensions. Furthermore, public pensions are a substantial share of net worth across the wealth distribution, ranging from almost 100% to about 20% in the top 1% wealth group.[53]

Figure A17(b) shows the saving rates across the wealth distribution when we include

[52] We exclude retirees from sample when we look on portfolios and saving rates including pensions. This is because retirees typically have approximately zero income when we include pension income (their income = pension benefits - reduction in pension wealth ≈ 0), making saving rates explosive.

[53] The 20% share at the top may seem large. However, most individuals in the top 1% wealth group has the maximum public pension, equal to about $55,000 per year and worth about $900,000. Taking into account that the threshold for entering the top 1% wealth is a little less than $2,000,000 (excluding pensions), 20% is a reasonable average.



(a) Portfolio Shares

(b) Saving Rates

Figure A17: Portfolio shares and saving rates, including pension wealth, and income and savings from pensions. The sample excludes retirees.

pensions. The first thing to note is that all saving rates shift up when we include pension saving. For almost all Norwegians, pension wealth increases from one year to the other because (i) they work that year and add to their pension wealth or (ii) they live another year such that the probability of living until age 67 increases. Hence, including pensions adds saving for almost all individuals.

The second notable change when comparing figure A17(b) with figure 4 is that by including pension saving, the saving rate becomes increasing in wealth percentiles, even for net saving rates. The minimum pension is the main reason this happens. Young individuals, typically at the lower part of the wealth distribution, have not yet worked long enough to accumulate pension wealth above the minimum pension. Hence, from year to year, their pension saving is small. Older individuals, on the other hand, have long enough earnings history to have accumulated pension wealth above the minimum pension. Hence, for every working year, old individuals add a part of their labor earnings to their pension wealth. Since old individuals typically are wealthier, this saving channel makes saving rates more increasing in wealth percentiles.

Saving Rates by Capital Gains Year by Year (Section 5.1). See Table A6.

	Dependent variable: $\frac{\text{Net saving}}{\text{Gross income}} \cdot 100$										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Constant	5.50	6.24	5.55	5.07	5.91	5.49	6.93	7.35	8.68	9.68	9.28
	(0.025)	(0.026)	(0.027)	(0.045)	(0.032)	(0.033)	(0.024)	(0.024)	(0.026)	(0.028)	(0.028)
$\frac{\text{Capital gains}}{\text{Gross income}}$	-8.29	-8.27	-7.26	-7.11	-7.82	-5.90	-9.09	-9.27	-10.43	-10.71	-10.82
	(0.042)	(0.039)	(0.039)	(0.063)	(0.050)	(0.043)	(0.038)	(0.037)	(0.040)	(0.039)	(0.041)
Observations	2,322,491	2,231,267	2,345,122	749,837	1,135,712	1,444,789	2,153,979	2,136,213	1,828,634	1,688,959	1,671,642

Notes: The table displays coefficients from quantile regressions of net saving relative to gross income on capital gains relative to gross income for 2005-2015. The sample is restricted to households with positive capital gains to avoid issues with small or negative gross income. Robust standard errors are reported in parentheses.

Table A6: Saving rates by capital gains by year.

E Appendix for Section 5: Model with Portfolio Adjustment Frictions

This section presents a model with portfolio adjustment frictions and shows how saving rates look across the wealth distribution in such a model. Our model has two key ingredients. First, motivated by empirical household balance sheets, households are assumed to hold multiple distinct assets.[54] Second, households are subject to portfolio adjustment "frictions" broadly defined. These frictions inhibit withdrawals to finance consumption and give rise to inertia in response to asset price changes. Related types of inertia are an important idea in the household finance literature (e.g., Brunnermeier and Nagel, 2008; Calvet, Campbell and Sodini, 2009).

Households hold two assets. A "consumption asset" b is used to smooth consumption. An "investment asset" k experiences capital gains. The portfolio adjustment friction is a stand-in for several alternative reasons why households move funds from the investment asset to the consumption asset only infrequently. Candidates include physical adjustment costs and capital gains taxes that generate a "lock-in-effect" because they are levied on realized rather than accrued gains. The model builds on Grossman and Laroque (1990) and, more recently, Kaplan and Violante (2014) and Kaplan, Moll and Violante (2018) who label the two assets "liquid" and "illiquid." Our framework differs from theirs in two dimensions: (i) households face an asset price that moves over time and (ii) we take a broader view of portfolio adjustment frictions rather than just physical transaction costs.

[54] In contrast, canonical models of household wealth accumulation typically collapse to one-asset models.

If the investment asset is interpreted as housing, the model is a version of what Guren et al. (2021) call the "new canonical model of housing" except that we do not model long-term mortgages.

Households have standard preferences over utility flows from consumption c_t, discounted at rate ρ. Additionally, they now face a constant hazard of death at rate ϕ. Their expected lifetime utility is $\mathbb{E}_0 \int_0^\infty e^{-(\rho+\phi)t}u(c_t)dt$. The investment asset k pays a dividend D_t that grows at a constant rate μ, $D_t = D_0 e^{\mu t}$ and its price p_t grows at the same growth rate $p_t = p_0 e^{\mu t}$ so that the dividend yield $\theta = D_t/p_t$ and the return $r_k := \theta + \mu$ are constant, i.e. the rising asset price comes with a rising cashflow as in special case 2 of Proposition 1.

We use d_t to denote a household's purchases of the investment asset (with $d_t < 0$ corresponding to sales). In order to buy or sell investment assets, households must pay a flow transaction cost $\chi(d_t, p_t k_t)$ which is zero whenever $d_t = 0$. Households can borrow in the consumption asset b up to an exogenous limit $\underline{b}$ at the real interest rate $r^b_- = r^b + \kappa$, where κ is an exogenous wedge between borrowing and lending rates. We denote by $r^b(b)$ the full interest rate schedule. A household's assets evolve according to

$$\begin{aligned} \dot{b}_t &= wz_t + r^b(b_t)b_t + \theta p_t k_t - p_t d_t - \chi(d_t, p_t k_t) - c_t, \\ \dot{k}_t &= d_t, \qquad \frac{\dot{p}_t}{p_t} = \mu, \end{aligned}$$

with the borrowing constraints $b_t \geq \underline{b}$ and $k_t \geq 0$, and where z_t denotes an individual's idiosyncratic labor productivity, the logarithm of which follows an Ornstein-Uhlenbeck process (the continuous-time analogue of an AR(1) process). Following Kaplan, Moll and Violante (2018), we assume that the adjustment cost function is symmetric and has a kink at $d = 0$ which generates an inaction region.[55]

Table A7 presents the calibration of the two-asset model. We present two calibrations: a benchmark calibration with reasonable numbers for dividends and capital gains, and an alternative calibration to illustrate that the model can generate patterns similar to those observed in the data. There are two set of parameters that matter. First, the wedge between borrowing and lending rates κ is sizable, ensuring that the borrowing rate is greater than the sum of the discount rate and the mortality rate. The wedge ensures that saving rates are high for households with debt. Second, the sum of dividends θ and capital gains μ on the investment asset is lower than the cost of debt. This ensures that households repay debt before they accumulate the investment asset. At the same time, the return on the

[55] We assume $\chi(d, a) = \chi_0|d| + \frac{\chi_1}{2}\left(\frac{d}{a}\right)^2 a$, with $\chi_0, \chi_1 > 0$. Because this function is infinite at $a = 0$, in computations we replace the term a with $\max\{a, \underline{a}\}$, where $\underline{a} > 0$ is a small value guaranteeing a finite cost.

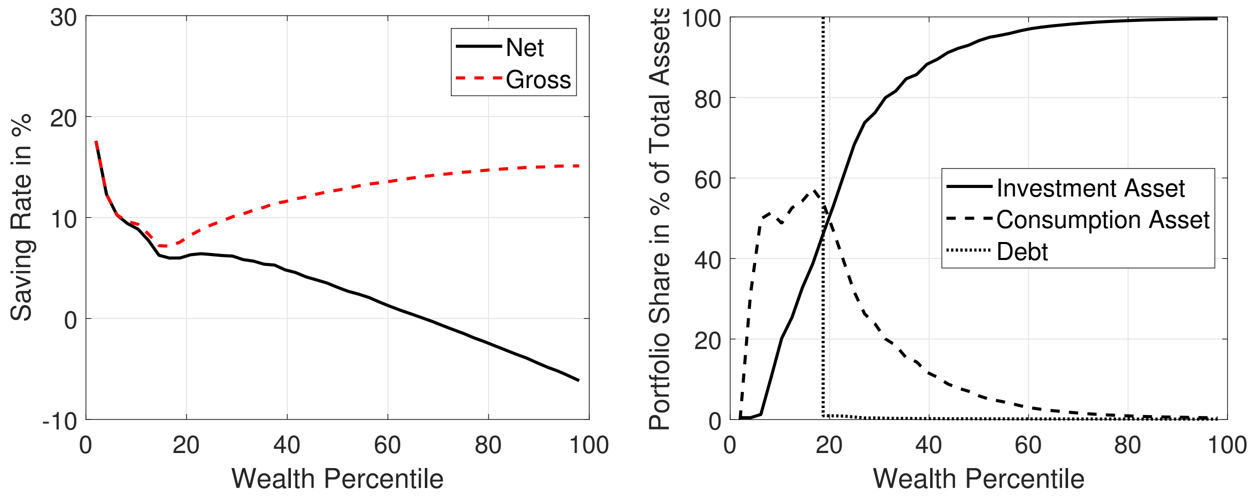
	Benchmark	Alternative	
Preferences			
γ	2.00	2.00	Relative risk aversion / inverse IES
ρ	0.03	0.03	Discount rate
η	0.01	0.01	Mortality rate
Capital markets			
θ	0.02	0.10	Dividend yield
μ	0.03	0.015	Capital gains
r^b	0.02	0.02	Interest rate on consumption asset
κ	0.10	0.10	Wedge between borrowing and lending rates
Adjustment costs			
χ_0	0.10	0.50	
χ_1	2.00	6.50	
Income			
w	5.00	5.00	Wage
ν	0.03	0.03	Persistence of income innovations
σ_z	0.14	0.14	Standard deviation of income innovations

Table A7: Calibration of two-asset model

investment asset (the sum of dividends and capital gains) is higher than both the return on the consumption asset and the sum of the discount rate and the mortality rate, making the investment asset attractive.

Figure A18 shows the saving rates and portfolio shares across the wealth distribution in our model with portfolio adjustment frictions. The model is able to reproduce the key qualitative characteristics of the gross saving rate in Figure 4: the gross saving rate first drops and then increases. In contrast to the data, net saving rates decrease with wealth for households with positive wealth. The patterns of gross and net saving rates are similar to that in Case 2 in Proposition 1 since capital gains in the model are implicitly modelled as information about future dividends. However, because adjustment frictions make it costly to realize capital gains, the presence of these twists the gross saving rate upward such that it is increasing with wealth.

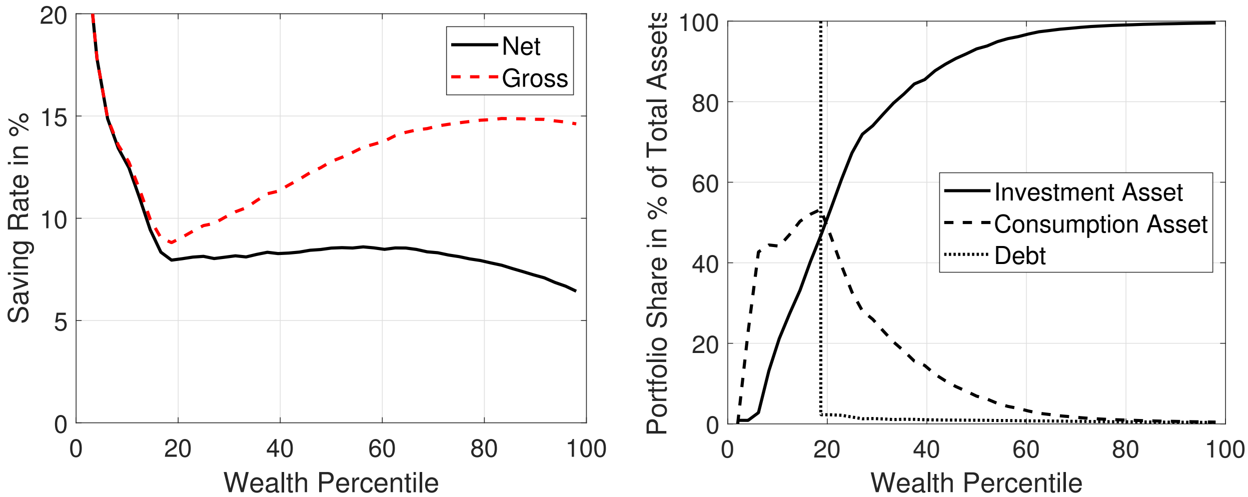
To illustrate this last point, Figure A19 presents the same set of figures from an alternative calibration where adjustment costs and returns to the illiquid asset are higher. The gross saving rate first drops and then increases, while the net saving rate first decreases and then flattens out. In contrast to the data, net saving rates decrease with wealth for households above the 70th percentile of the wealth distribution, meaning that the wealth-



(a) Saving Rates (b) Portfolio Shares

Figure A18: Saving behavior in two-asset model with portfolio adjustment friction.

iest households reduce net saving to consume some of their investment assets. Hence, a model with adjustment frictions can generate a flat net saving rate if adjustment costs are sufficiently high. However, the underlying model without adjustment frictions produces a net saving rate that is decreasing with wealth so that the flat net saving rate can only be generated in knife-edge calibrations.



(a) Saving Rates

(b) Portfolio Shares

Figure A19: Saving behavior in two-asset model with portfolio adjustment friction. Alternative calibration.