Uneven Growth
Automation’s Impact on Income and Wealth Inequality

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Uneven Growth in the United States:
Stagnant incomes at bottom, rising incomes at top

Candidate Cause: Technology

- Huge literature: technology affects wage inequality
- Examples: SBTC and polarization of wages
- But what about capital income and wealth?
What We Do

- Theory that links tech to income & wealth distribn, not just wages
- Use it to examine distributional effects of automation technologies = technologies that substitute labor for capital in production
- **Tractable framework** to study dynamics of
  1. macro aggregates
  2. factor income distribution: capital vs labor
  3. personal income, wealth distribution
- Key modeling difference to growth model: perpetual youth ⇒
  - nondegenerate wealth distribution
  - long-run capital supply elasticity < ∞
Our Main Point

Technology ⇒ returns ⇒ distributional consequences

Analytic version in our theory:

\[ \text{return to wealth} = \rho + \sigma g + \text{premium}(\alpha) \]

where \( \alpha = \text{capital share} = \text{average automation} \)

1. **New mechanism:** technology increases inequality via return to wealth
   - income/wealth distributions have **Pareto tail** with fatness = \( \alpha \)

2. Automation may lead to **stagnant wages** and **lackluster investment**
   - productivity gains partly accrue to capital owners
   - \( \alpha := R \times \frac{K}{Y} \) and part of \( \alpha \uparrow \) shows up in \( R \) not \( K/Y \)

Paraphrasing these results

- if “robots” increasingly outperform labor, this benefits people owning lots of robots rather than “workers”
How does this square with trends in returns?

Just told you that

\[
\text{return to wealth} = \rho + \sigma g + \text{premium}(\alpha)
\]

But haven’t treasury rates decreased over time?

1. Treasury rates = return on specific asset, ave return on US capital ↑
2. Model w risky & safe assets: relevant \( r = \sum_{j=1}^{J} r_j \times \text{portfolio share}_j \)
3. Inequality depends \( r - \rho - \sigma g \). Even if \( r \downarrow \), arguably \( r - \rho - \sigma g \uparrow \).
Model Meets Data

- Calibrate incidence of automation using exposure to routine jobs
  - accounts for changes in wage inequality 1980-2014
- Conservative (i.e. high) value for long-run capital supply elasticity
- Examine consequences of automation for
  - aggregates? Small expansions in $I, Y$
  - income, wealth inequality? Sizable increase, uneven growth
  - wages? Stagnation except for top of distribution
- Small shock (3% inc in TFP) can have large distributional effects
Small productivity gains but large distributional effects

Change in income by percentile of the income distribution

- Total income growth in our model
- Total income growth in rep. household model

Percent change vs. income percentile
Literature and Contribution

**Automation and inequality** (Acemoglu-Restrepo, Caselli-Manning, Hémous-Olsen, ...)

- capital income & wealth, not just wages
- capital supply elasticity $< \infty$ very different from $= \infty$

**Technology and wealth distribution** (Kaymak-Poschke, Hubmer-Krusell-Smith, Straub,...)

- new mechanism: technology $\Rightarrow$ return $\Rightarrow$ wealth inequality
  (in addition to: technology $\Rightarrow$ wage dispersion $\Rightarrow$ wealth inequality)

**Returns as driver of top wealth inequality** (Piketty, Benhabib-Bisin, Jones,...)

- tractable form of capital income risk, integrated in macro model
- Piketty: $r - g \uparrow$ due to lower taxes, lower $g$. This paper: technology.

**Tractable** theory of macro aggregates, factor and personal income dist

**Perpetual youth literature** (Blanchard): closed form for wealth distribution
Plan

1. Framework and model of automation
2. Steady state
3. Transition dynamics – skip today
4. Model meets data
1. Framework: Households and Technology
Model has two key building blocks

Long-run capital supply elasticity \( < \infty \) (Aiyagari, ...)

\[ r \\
\] \[ K \]

\( + \)

returns \( \Rightarrow \)
wealth inequality

(Benhabib-Bisin, Piketty, Jones, ...)

- Our paper: model this in very **stylized** fashion – perpetual youth
  - cost: some unrealistic implications
  - payoff: **analytic solution** for everything incl distributions

- Same mechanisms would be present in richer, less tractable models
Framework: Perpetual Youth Households

Households: age $s$, skills $z$, solve

$$\max \{c_z(s), a_z(s)\}_{s \geq 0} \int_0^\infty e^{- (\rho p) s} \frac{c_z(s)^{1-\sigma}}{1 - \sigma} ds \quad \text{s.t.} \quad \dot{a}_z(s) = r a_z(s) + w_z - c_z(s)$$

- $w_z$: wage for skill $z$, $\ell_z$ households
- $r$: return to wealth
- $\rho$: discount rate
- $p$: probability of dying ($p = 0 \Rightarrow$ rep agent)
- $\rho = \rho + p$: effective discount rate

Key assumption: “imperfect dynasties”

- average wealth of newborn $<$ average wealth of living
- stark implementation: eat wealth when die $\Rightarrow$ no bequests, $a_z(0) = 0$
- other mechanisms: annuities, pop growth, estate taxation
- perpetual youth = just tractable stand-in for other sources of churn
Framework: Technology (Zeira, Acemoglu-Restrepo)

Task-based model: machines/software substitute for tasks, not jobs

First: “reduced form” production side, next slide: where this comes from

1. Each skill type $z$ works in different sector that produces $Y_z$

$$Y = A \prod_{z} Y_z^{\gamma_z} \quad \text{with} \quad \sum_{z} \gamma_z = 1$$

2. $Y_z$ produced using Cobb-Douglas tech with skill-specific exponent $\alpha_z$

$$Y_z = \left( \frac{k_z}{\alpha_z} \right)^{\alpha_z} \left( \frac{\psi_z \ell_z}{1 - \alpha_z} \right)^{1-\alpha_z}$$

$\alpha_z = \text{share of tasks technologically automated. Automation: } \alpha_z(t) \uparrow$

3. Capital mobile across sectors, labor immobile
Derivation from Task-based Model (Zeira, Acemoglu-Restrepo)

For simplicity, derivation with only one skill type. Reduced form:

\[ Y = \left( \frac{K}{\alpha} \right)^{\alpha} \left( \frac{\psi L}{1 - \alpha} \right)^{1-\alpha} \]  

(*)

Comes out of following task-based model:

1. Final good produced combining unit continuum of tasks \( u \)

\[ \ln Y = \int_{0}^{1} \ln \mathcal{Y}(u) du \]

2. Tasks produced using capital \( k(u) \) or labor \( \ell(u) \) at prices \( R \) and \( w \)

\[ \mathcal{Y}(u) = \begin{cases} \psi \ell(u) + k(u) & \text{if } u \in [0, \alpha] \\ \psi \ell(u) & \text{if } u \in (\alpha, 1] \end{cases} \]

- \( \alpha = \) share of tasks technologically automated. Automation: \( \alpha(t) \uparrow \)
- Example: HR manager, tasks = screen CVs, interview applicants,…
- Displacement vs productivity effects
Derivation from Task-based Model (Zeira, Acemoglu-Restrepo)

For simplicity, derivation with only one skill type. Reduced form:

\[ Y = \left( \frac{K}{\alpha} \right)^\alpha \left( \frac{\psi L}{1 - \alpha} \right)^{1 - \alpha} \]

Comes out of following task-based model:

1. Final good produced combining unit continuum of tasks \( u \)

\[ \ln Y = \int_0^1 \ln \mathcal{Y}(u) du \]

2. Assumption 1 (full adoption): \( w/\psi > R \) (sufficient to have \( L < \bar{L} \))

\[ \mathcal{Y}(u) = \begin{cases} 
\psi l(u) + k(u) & \text{if } u \in [0, \alpha] \\
\psi l(u) & \text{if } u \in (\alpha, 1]
\end{cases} \]
Derivation from Task-based Model (Zeira, Acemoglu-Restrepo)

For simplicity, derivation with only one skill type. Reduced form:

\[ Y = \left(\frac{K}{\alpha}\right)^{\alpha} \left(\frac{\psi L}{1 - \alpha}\right)^{1-\alpha} \]  

(\text{(*)})

Comes out of following task-based model:

1. Final good produced combining unit continuum of tasks \( u \)

\[ \ln Y = \int_{0}^{1} \ln \gamma(u) du \]

2. Assumption 1 (full adoption): \( w/\psi > R \) (sufficient to have \( L < \bar{L} \))

\[ \gamma(u) = \begin{cases} 
  k(u) & \text{if } u \in [0, \alpha] \\
  \psi \ell(u) & \text{if } u \in (\alpha, 1] 
\end{cases} \]

- 1. and 2. with \( k(u) = K/\alpha, \ell(u) = L/(1 - \alpha) \) imply (\text{(*)}). □
2. Characterizing Steady State
Output, Factor Payments and Capital Demand

- Aggregate output:

\[ Y = AK\sum_z \gamma_z \alpha_z \prod_z (\psi_z \ell_z)^{\gamma_z(1-\alpha_z)} \]

\[ \alpha = \sum_z \gamma_z \alpha_z : \text{aggregate capital-intensity}, \ A = \text{constant}(\alpha_z, \gamma_z) \]

- Factor payments:

\[ w_z \ell_z = (1 - \alpha_z) \gamma_z Y, \quad RK = \alpha Y, \quad \bar{w} = (1 - \alpha) Y \]

\( \alpha_z \)'s \(\Rightarrow\) relative wages, factor shares. But effect on levels unclear

- Aggregate capital demand

\[ \frac{K}{\bar{w}} = \frac{\alpha}{1 - \alpha} \frac{1}{R} \]

- Expositional assumption for presentation: \( g = 0, \delta = 0 \Rightarrow R = r \)
Steady State Capital Supply

Households’ consumption and saving decisions:

\[ c_z(s) = \left( \frac{\rho - r}{\sigma} + r \right) \left( a_z(s) + \frac{w_z}{r} \right) \]

\[ \dot{a}_z(s) = \frac{1}{\sigma} (r - \rho) \left( a_z(s) + \frac{w_z}{r} \right) \]  

Useful later: relevant state = effective wealth = assets + human capital

\[ x_z(s) := a_z(s) + \frac{w_z}{r} \]

Find aggregate capital supply by integrating (*) with \( \bar{w} := \sum_z w_z \ell_z \):

\[ 0 = \dot{K} = \frac{1}{\sigma} (r - \rho) \left( K + \frac{\bar{w}}{r} \right) - pK \quad \Rightarrow \quad \frac{K}{\bar{w}} = \frac{1 - \rho/r}{\rho + p\sigma - r} \]

Wealth accumulated by surviving households

Imperfect dynasties
Steady-State Equilibrium: Return to Wealth

Demand and Supply of Capital

\[ r^* = \rho + p\sigma \alpha \]

Supply
\[ \frac{K}{w} = \frac{1 - \rho/r}{\rho + p\sigma - r} \]

Demand
\[ \frac{K}{\bar{w}} = \frac{\alpha}{1 - \alpha r} \]

Graph showing the relationship between \( r \), \( \rho + p\sigma \), \( \rho \), and \( K/w \) with the equilibrium point labeled as \( r^* = \rho + p\sigma \alpha \).
Same diagram as in richer theories (Aiyagari, Benhabib-Bisin,...)

Demand and Supply of Capital

Supply \( \frac{K}{w} = \frac{1 - \rho/r}{\rho + p\sigma - r} \)

Demand \( \frac{K}{\bar{w}} = \frac{\alpha}{1 - \alpha r} \)

\( r^* = \rho + p\sigma \alpha \)
Automation ⇒ higher $r$ and modest expansion in $K$
Steady State Income and Wealth Distributions

Recall wealth dynamics: \( \dot{a}_z(s) = \frac{1}{\sigma}(r - \rho) \left( a_z(s) + \frac{w_z}{r} \right) \)

**Proposition:** stationary distribution of effective wealth by skill type is

\[
g_z(x) = \left( \frac{w_z}{r} \right)^\zeta \zeta x^{-\zeta - 1}, \quad \frac{1}{\zeta} = \frac{1}{p} \frac{r - \rho}{\sigma}
\]

Pareto distribution with scale \( w_z/r \) and inverse tail parameter \( \zeta \).
Steady State Income and Wealth Distributions

\[ \dot{x}_z(s) = \frac{1}{\sigma} (r - \rho) x_z(s), \quad x_z(s) := a_z(s) + \frac{w_z}{r} \]

**Proposition:** stationary distribution of effective wealth by skill type is

\[ g_z(x) = \left( \frac{w_z}{r} \right)^{-\zeta} \zeta \cdot x^{-\zeta-1}, \quad \frac{1}{\zeta} = \frac{1}{p} \frac{r - \rho}{\sigma} \]

Pareto distribution with scale \( w_z/r \) and inverse tail parameter \( \zeta \).
Steady State Income and Wealth Distributions

\[ \dot{x}_z(s) = \frac{1}{\sigma}(r - \rho)x_z(s), \quad x_z(s) := a_z(s) + \frac{w_z}{r} \]

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Pareto distribution with scale \( w_z/r \) and inverse tail parameter \( \frac{1}{p} \frac{r-\rho}{\sigma} \)
Steady State Income and Wealth Distributions

\[ \dot{x}_z(s) = \frac{1}{\sigma} (r - \rho)x_z(s), \quad x_z(s) := a_z(s) + \frac{w_z}{r} \]

**Proposition:** stationary distribution of effective wealth by skill type is

\[ g_z(x) = \left( \frac{w_z}{r} \right)^\zeta x^{-\zeta - 1}, \quad \frac{1}{\zeta} = \frac{1}{p} \frac{r - \rho}{\sigma} = \alpha \quad \text{(recall} \ r = \rho + p\sigma\alpha) \]

Pareto distribution with scale \( \frac{w_z}{r} \) and inverse tail parameter \( \alpha \)
Distribution of Wealth

- Closed form for entire distributions:

\[
Pr(\text{wealth} \geq a | z) = \left( \frac{a + w_z/r}{w_z/r} \right)^{-\zeta}, \quad \frac{1}{\zeta} = \text{fatness}(r) = \alpha
\]

\[
Pr(\text{wealth} \geq a) = \sum_z \ell_z \left( \frac{a + w_z/r}{w_z/r} \right)^{-\zeta}.
\]

- Automation has two effects on wealth distribution
  1. via wages: determine scale of wealth distribution by type
  2. via return: determines fatness of tail
Distribution of Income

- Again, two sources of inequality: wages and return to wealth
- Again, closed form for entire distributions:

\[
\Pr(\text{income } \geq y|z) = \left( \frac{\max\{y, w_z\}}{w_z} \right)^{-1/\alpha}
\]

\[
\Pr(\text{income } \geq y) = \sum_z \ell_z \left( \frac{\max\{y, w_z\}}{w_z} \right)^{-1/\alpha}.
\]
Wage Stagnation with Upward-sloping Capital Supply

- CRS aggregate production function with technology indexed by $\theta$
  
  \[ F(K, \{l_z\}_{z \in Z}; \theta), \quad F_\theta > 0 \]

- Question: effect of technological change $d\theta > 0$ on factor prices?

\[
\begin{align*}
    d\ln TFP &= \alpha d\ln R + (1 - \alpha) d\ln \bar{w}, \\
    \text{TFP gains} &> 0 \quad \text{change in average wage} \leq 0
\end{align*}
\]

(Derivation: see e.g. Jaffe-Minton-Mulligan-Murphy (2019), uses $F = RK + \sum_z w_z l_z$)

- Bulk of literature: $d\ln R = 0$ because perfectly elastic capital supply
  - rep agent or small open economy (Acemoglu-Restrepo, Caselli-Manning, ...)
    \[ \Rightarrow \text{all productivity gains accrue to labor, wages track TFP} \]

- Our paper: $d\ln R > 0 \Rightarrow$ wages may stagnate or even decrease
  \[ \Rightarrow \text{lackluster investment response} \]
3. Transition Dynamics

Skip this today
4. Model meets Data
Consequences of automation for income inequality and aggregates?

- interpret each $z$ as percentile of wage dist; focus on 1980-2014
- use variation in routine jobs across wage percentiles $z$
  (Autor-Levy-Murnane, Autor-Dorn, Acemoglu-Autor, ...)

$$\Delta \alpha_z(t) \approx -\text{exposure}_z \times \Delta \text{Labor share}(t)$$

$\text{exposure}_z$: share of wages paid to routine jobs in $z$ (2000 Census)

scale: automation drives decline in $\text{Labor share}(t) = 1 - \alpha(t)$

- calibrate $\psi_z$ so automation yields cost-saving gains $\ln \frac{w_z}{\psi_z R} = 30\%$
- calibrate $p = 3.85\%$ to target capital-supply elasticity $\frac{d \log K}{d r} = 50$
Automation of Routine Jobs: The Shock

Panel A. Calibrated behavior of $\alpha_z(t)$

Panel B. Implied behavior of aggregate labor share

Model
BLS data
1 pp increase in return to wealth; 15% increase in $K/Y$.

$$d \ln \text{TFP} = 0.03 \alpha d \ln R + 0.04 \left(1 - \alpha\right) d \ln \overline{w}, \quad \overline{w} := \sum_z w_z \ell_z$$
Declining wages except at top

Recall

\[
w_z(t) = (1 - \alpha_z(t))\gamma_z\frac{Y(t)}{\ell_z}
\]
... and substantial uneven growth
... and substantial uneven growth
... and substantial uneven growth
... and substantial uneven growth
Empirical counterpart: uneven growth in IRS, Piketty-Saez-Zucman data

Panel A. Change in income by percentile of the income distribution, IRS data

Panel B. Change in income by percentile of the income distribution, PSZ data
Caveat: model transition too slow

Good news: know how to fix this (Gabaix-Lasry-Lions-Moll)

- heterogeneous returns or saving rates
Conclusion

- Tractable framework to think about uneven growth
  - have used it to study distributional effects of automation
  - not just on wages but also on income and wealth distributions
- Technology $\Rightarrow$ returns $\Rightarrow$ distributional effects
  - rising concentration of capital income at top
  - stagnant or declining wages at the bottom
- Framework has lots of other potential applications
  - trade: globalization’s impact on income and wealth inequality?
  - PF: optimal capital income and wealth taxation?
  - ...
- Needed: better evidence on asset returns (x-section & time-series)
Thanks for listening!