Lectures 8: Policy Analysis in the Growth Model (Capital Taxation)

ECO 503: Macroeconomic Theory I

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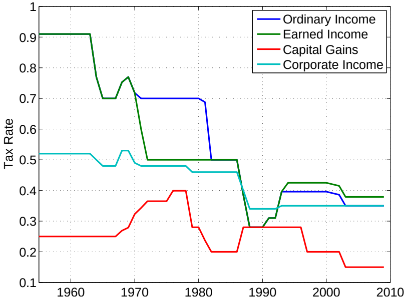
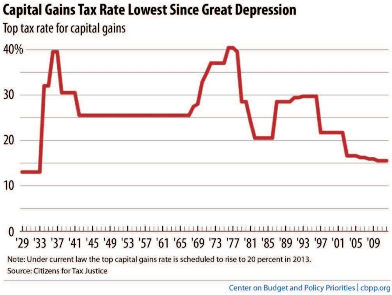
Policy Analysis in the Growth Model

- Classic question: what are the consequences for allocations and welfare of policy x?
- Today: $x =$ capital income taxation
- but approach works more generally

Capital Taxes in the U.S.

- U.S. top marginal tax rates (from Saez, Slemrod and Giertz, 2012, Table A1)





Capital Taxation in Theory

- Most influential: Chamley and Judd's **zero capital tax** result
  - somewhat more precisely: in the **long-run**, the optimal **linear** capital income tax should be zero
  - perhaps even reflected in observed policy (see previous slide)

Plan

1. Capital income taxation and redistribution
   - a growth model with capitalists and workers
   - “Ramsey taxation” (Judd, 1985)
   - critique by Straub and Werning (2014)
2. Capital income taxation without redistribution
   - “Ramsey taxation” (Chamley, 1986)
   - only quick overview
3. Summary: takeaway on capital taxation

Growth Model with Capitalists & Workers

- Consider a variant of the growth model with two types of individuals:
  - **capitalists**: rep. capitalist derives all income from returns to capital
  - **workers**: rep. worker derives all income from labor income
- Originally due to Judd (1985), use discrete-time formulation from Straub and Werning (2014)
- Two reasons why variant is better model for thinking about capital income taxation than standard growth model
  - some distributional conflict (as opposed to rep. agent)
  - math turns out to be easier
- End of lecture: capital taxation in **representative agent** model (Chamley, 1986)

Growth Model with Capitalists & Workers

- **Preferences**
  - capitalist

$$\sum_{t=0}^{\infty} \beta^t U(C_t), \quad U(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

  - workers

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

- **Technology**

$$c_t + C_t + k_{t+1} = F(k_t, h_t) + (1-\delta)k_t$$

- **Endowments**: capitalists own $k_0 = \hat{k}_0$ units of capital

Competitive Equilibrium without Taxes

- **Definition**: A SOMCE for the growth model with capitalists and workers are sequences $\{c_t, h_t, k_t, a_t, w_t, r_t\}_{t=0}^{\infty}$ s.t.

  1. (Capitalist max) Taking $\{r_t\}$ as given, $\{C_t, a_t\}$ solves

  $$\max_{\{C_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.}$$

  $$C_t + a_{t+1} = (1 + r_t)a_t, \quad \lim_{T\to\infty} \left(\prod_{s=0}^{T} \frac{1}{1+r_s}\right) a_{T+1} \geq 0, \quad a_0 = \hat{k}_0.$$

  2. (Worker max) Taking $\{w_t\}$ as given, $\{c_t, h_t\}$ solves

  $$\max_{\{c_t, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad c_t = w_t h_t$$

  3. (Firm max) Taking $\{w_t, r_t\}$ as given $\{k_t, h_t\}$ solves

  $$\max_{\{k_t, h_t\}} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \frac{1}{1 + r_s}\right) (F(k_t, h_t) - w_t h_t - i_t), \quad k_{t+1} = i_t + (1-\delta)k_t$$

  4. (Market clearing) For each t:

  $$c_t + C_t + k_{t+1} = F(k_t, h_t) + (1 - \delta)k_t, \quad a_t = k_t$$

Comments

- Only capitalist can save
- Worker cannot save, lives "hand to mouth"
- Work with decentralization in which
  - firms own capital
  - capitalists save in riskless bond
  - in contrast, in last lecture: households owned capital, rented it to firms
- Relative to Straub and Werning
  - make notation as similar as possible to last lecture
  - impose no-Ponzi condition rather than borrowing limit $a_{t+1} \geq 0$ (doesn't matter)

Necessary Conditions

- Necessary conditions for **capitalist** problem

$$U'(C_t) = \beta(1 + r_{t+1})U'(C_{t+1}) \tag{1}$$
$$0 = \lim_{T\to\infty} \beta^T U'(C_T)a_{T+1}$$

- Solution to **worker** problem

$$h_t = 1, \quad c_t = w_t$$

- Necessary conditions for **firm** problem

$$F_h(k_t, h_t) = w_t$$
$$F_k(k_t, h_t) + 1 - \delta = 1 + r_t \tag{2}$$

- **Market Clearing**

$$c_t + C_t + k_{t+1} = F(k_t, h_t) + (1 - \delta)k_t$$

Necessary Conditions

- (6) is same no-arbitrage condition we had in last lecture, but now coming directly from firm's problem
- Combining (1) and (6) and defining $F(k_t, 1) = f(k_t)$ we get

$$U'(C_t) = \beta U'(C_t)(f'(k_{t+1}) + 1 - \delta)$$

- Same condition as usual, except that C_t is consumption of capitalists
- In steady state $C_t = C^*, c_t = c^*, k_t = k^*$

$$f'(k^*) + 1 - \delta = \frac{1}{\beta}$$

$\Rightarrow$ same steady state as standard growth model.

Analytic Solution in Special Case: $\sigma = 1$

- **Lemma**: with $\sigma = 1$ capitalists save a constant fraction β

$$a_{t+1} = \beta(1 + r_t)a_t, \quad C_t = (1 - \beta)(1 + r_t)a_t$$

- **Proof**: "guess and verify". Consider nec. cond's w/ $\sigma = 1$

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_{t+1}) \qquad (*)$$

$$0 = \lim_{T \to \infty} \beta^T \frac{a_{T+1}}{C_T}$$

$$C_t + a_{t+1} = R_t a_t$$

- Guess $C_t = (1 - s)(1 + r_t)a_t$. From $(*)$

$$\frac{(1 - s)(1 + r_{t+1})a_{t+1}}{(1 - s)(1 + r_t)a_t} = \beta(1 + r_{t+1}) \quad \Rightarrow \quad \frac{a_{t+1}}{a_t} = \beta(1 + r_t)$$

i.e. $s = \beta$.$\square$

$\sigma = 1$: Intuition for Constant Saving Rate

- Log utility $\Rightarrow$ offsetting income and substitution effects
  - (a_{t+1}, C_t) do **not** depend on r_{t+1}
- $1/\sigma =$ "intertemporal elasticity of substitution (IES)"
  - low $\sigma \Rightarrow U$ close to linear ...
  - ... capitalists like to substitute intertemporally ("high IES")
- To understand, consider effect of unexpected increase of r_{t+1}
  - $\sigma > 1$: income effect dominates $\Rightarrow C_t \uparrow, a_{t+1} \downarrow$
  - $\sigma < 1$: substitution effect dominates $\Rightarrow C_t \downarrow, a_{t+1} \uparrow$
  - $\sigma = 1$: income & subst. effects cancel $\Rightarrow C_t, a_{t+1}$ constant
- Same logic as in Lecture 4
  - there condition was $\sigma \gtrless \alpha$ where $\alpha =$ curvature of prod. fn.
  - reason for difference: planner in Lecture 4 faced concave saving technology, εk_t^α
  - ... here instead, capitalists face linear saving technology $((1 + r_t)a_t)$. In effect, $\alpha = 1$.

Analytic Solution in Special Case: $\sigma = 1$

- Necessary conditions reduce to

$$k_{t+1} = \beta(f'(k_t) + 1 - \delta)k_t \qquad (*)$$

$$C_t = (1-\beta)(f'(k_t) + 1 - \delta)k_t$$

$$c_t = f(k_t) - f'(k_t)k_t$$

(used $F = F_k k + F_h h$ and so $F_h(k_t, 1) = f(k_t) - f'(k_t)k_t$)

- Model basically boils down to **Solow model**
  - e.g. with $f(k) = Ak^\alpha$

$$k_{t+1} = \alpha\beta A k_t^\alpha + \beta(1-\delta)k_t$$

  - effective saving rate $\alpha\beta$ and depreciation term $\beta(1-\delta)$
- **Extremely convenient:** compute entire transition by hand
  - no need for phase diagram etc, simply do Solow zig-zag graph
  - but still same steady state at standard growth model

$$f'(k^*) = 1/\beta + 1 - \delta$$

Policy in GE Models

- Next: policy in growth model with capitalists and workers
- Questions about policy need to be **well posed**
  - example of question that is not well-posed: "What happens if we introduce a proportional tax τ on capital?"
  - reason: if a policy raises revenue (or requires expenditure), then one must specify what is done with the revenue (where the revenue comes from)
- There are many possible uses of revenue $\Rightarrow$ many possible exercises
- Here, ask: What are the consequences of introducing
  - a proportional (linear) tax on capital income of τ_t

  when the revenues are used to fund
  - constant government consumption $g \geq 0$ and
  - a lump-sum transfer to workers T_t

  with period-by-period budget balance?

Competitive Equilibrium with Taxes

- **Definition**: A SOMCE **with taxes** for the growth model with capitalists and workers are sequences $\{c_t, h_t, k_t, a_t, w_t, r_t, \tau_t T_t\}_{t=0}^{\infty}$ s.t.

  1. (Capitalist max) Taking $\{r_t, \tau_t\}$ as given, $\{C_t, a_t\}$ solves

     $$\max_{\{C_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.}$$

     $$C_t + a_{t+1} = (1 - \tau_t)(1 + r_t)a_t, \ \lim_{T \to \infty} \left(\prod_{s=0}^{T} \frac{1}{1+r_s} \right) a_{T+1} \geq 0, a_0 = \hat{k}_0.$$

  2. (Worker max) Taking $\{w_t\}$ as given, $\{c_t, h_t\}$ solves

     $$\max_{\{c_t, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad c_t = w_t h_t + T_t$$

  3. (Firm max) Taking $\{w_t, r_t\}$ as given $\{k_t, h_t\}$ solves

     $$\max_{\{k_t, h_t\}} \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t} \frac{1}{1 + r_s} \right) (F(k_t, h_t) - w_t h_t - i_t), \quad k_{t+1} = i_t + (1 - \delta) k_t$$

Competitive Equilibrium with Taxes

- **Definition**: A SOMCE **with taxes** for the growth model with capitalists and workers are sequences $\{c_t, h_t, k_t, a_t, w_t, r_t, \tau_t T_t\}_{t=0}^{\infty}$ s.t.

  4. (Government) For each t

$$g + T_t = \tau_t k_t$$

  5. (Market clearing) For each t:

$$c_t + C_t + k_{t+1} = F(k_t, h_t) + (1 - \delta)k_t, \quad a_t = k_t$$

Comments

- Tax is **linear** as opposed to **non-linear** tax function $\tilde{\tau}$

$$C_t + a_{t+1} = (1 + r_t)a_t - \tilde{\tau}((1 + r_t)a_t)$$

with $\tilde{\tau}'' \neq 0$ (e.g. $\tilde{\tau}'' > 0 =$ progressive)

Characterizing CE with Taxes

- Necessary conditions unchanged except for

$$U'(C_t) = \beta(1 - \tau_{t+1})(1 + r_{t+1})U'(C_{t+1})$$

  and resource constraint

- Therefore

$$U'(C_t) = \beta U'(C_{t+1})(1 - \tau_{t+1})(f'(k_{t+1}) + 1 - \delta)$$

- For any $\{\tau_t\}_{t=0}^{\infty}$ can use shooting algorithm to solve for eqm
  - natural initial condition: steady state without taxes
- What about steady state with taxes? Suppose $\tau_t = \tau$. Then

$$(1 - \tau)(f'(k^*) + 1 - \delta) = \frac{1}{\beta}$$

  Hence higher $\tau \uparrow \Rightarrow k^* \downarrow$, e.g. if $f(k) = Ak^{\alpha}$

$$k^* = \left(\frac{\alpha A}{\frac{1}{\beta(1-\tau)} + 1 - \delta}\right)^{\frac{1}{1-\alpha}}$$

Ramsey Taxation

- So far: **positive** analysis
  - what is the effect of τ_t ...?
- Now: **normative**
  - what is the **optimal** τ_t
- **Ramsey problem**: find $\{\tau_t\}$ that produces a CE with taxes with highest utility for agents (capitalists and workers).
- that is, find optimal $\{\tau_t\}$ subject to the fact that agents behave competitively for those taxes
- Important assumption

Ramsey Problem

- Need to take stand on objective of policy
- Here use

$$\sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t))$$

  for a "Pareto weight" $\gamma \geq 0$

  - $\gamma = 0$: only care about workers
  - $\gamma \to \infty$: only care about capitalists

Ramsey Problem

- Recall necessary conditions for CE with taxes

$$U'(C_t) = \beta(1 + r_{t+1})(1 - \tau_{t+1})U'(C_{t+1}) \quad (1)$$

$$0 = \lim_{T\to\infty} \beta^T U'(C_T)a_{T+1} \quad (2)$$

$$C_t + a_{t+1} = (1 - \tau_t)(1 + r_t)a_t \quad (3)$$

$$c_t = w_t + T_t \quad (4)$$

$$F_h(k_t, 1) = w_t \quad (5)$$

$$F_k(k_t, 1) + 1 - \delta = 1 + r_t \quad (6)$$

$$c_t + C_t + g + k_{t+1} = F(k_t, 1) + (1 - \delta)k_t \quad (7)$$

$$k_t = a_t \quad (8)$$

$$a_0 = k_0 = \hat{k}_0 \quad (9)$$

- **Ramsey problem** is

$$\max_{\{\tau_t, c_t, C_t, k_{t+1}, a_{t+1}, w_t, r_t\}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)) \quad \text{s.t.} \quad (1)\text{-}(9)$$

Ramsey Problem

- Can simplify by combining/eliminating some of the constraints
- From (3) and (8)

$$(1-\tau_t)(1+r_t) = \frac{C_t}{k_t} + \frac{k_{t+1}}{k_t}$$

- Substituting into (1)

$$U'(C_{t-1})k_t = \beta U'(C_t)(C_t + k_{t+1})$$

- Write $F(k_t, 1) = f(k_t)$ as usual
- Walras' Law: can drop one budget constraint or resource constraint. Drop (4).
- Also drop (5) and (6) since $\{r_t, w_t\}_{t=0}^{\infty}$ only show up in equations we already dropped.

Ramsey Problem

- After simplifications:

$$\max_{\{c_t, C_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)) \quad \text{s.t.}$$

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1-\delta)k_t$$

$$\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t$$

$$\lim_{T\to\infty} \beta^T U'(C_T)k_{T+1} = 0$$

Comments

- Note: problem only in terms of **allocation**
- Given optimal $\{c_t, C_t, k_{t+1}\}_{t=0}^{\infty}$, can always back out taxes and prices

$$w_t = F_h(k_t, 1) = f(k_t) - f'(k_t)k_t$$
$$r_t = F_k(k_t, 1) - \delta = f'(k_t) - \delta$$
$$1 - \tau_t = \frac{1}{f'(k_t) + 1 - \delta} \frac{U'(C_t)}{\beta U'(C_{t+1})}$$

- In other applications, typically combine constraints in different way, leading to so-called "implementability" condition.
  - same outcome: Ramsey problem in terms of allocations only
- But here follow Judd (1985) and Straub and Werning (2014). Easier to work with.

First order conditions

- Lagrangean

$$\mathcal{L} = \sum_{t=0}^{\infty} \{ \beta^t (u(c_t) + \gamma U(C_t)) \\ + \beta^t \lambda_t (f(k_t) + (1-\delta)k_t - c_t - C_t - g - k_{t+1}) \\ + \beta^t \mu_t (\beta U'(C_t)(C_t + k_{t+1}) - U'(C_{t-1})k_t) \}$$

- First order conditions (use that $U'(C_t)C_t = C_t^{1-\sigma}$)

$$c_t: \quad 0 = u'(c_t) - \lambda_t \tag{1}$$

$$C_t: \quad 0 = \gamma U'(C_t) - \lambda_t - \beta \mu_{t+1} U''(C_t) k_{t+1} \\ + \beta \mu_t ((1-\sigma) U'(C_t) + U''(C_t) k_{t+1}) \tag{2}$$

$$k_{t+1}: \quad 0 = -\lambda_t + \mu_t \beta U'(C_t) \\ + \beta \lambda_{t+1} (f'(k_{t+1}) + 1 - \delta) - \beta \mu_{t+1} U'(C_t) \tag{3}$$

Tricky Detail: C_{-1}

- Treated C_t as a state variable, even though it's a jump var
  - C_{-1} is not-predetermined
- Can show: multiplier μ_t corresponding to $\{C_t\}$ has to satisfy

$$\mu_0 = 0$$

- Heuristic derivation: for any (k_0, C_{-1}) define $V(k_0, C_{-1})$ by

$$V(k_0, C_{-1}) = \max_{\{c_t, C_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma U(C_t)) \quad \text{s.t.}$$

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1-\delta)k_t$$

$$\beta U'(C_t)(C_t + k_{t+1}) = U'(C_{t-1})k_t$$

$$\lim_{T\to\infty} \beta^T U'(C_T) k_{T+1} = 0$$

- C_{-1} pinned down from $V_C(k_0, C_{-1}) = 0$. Envelope condition

$$V_C(k_0, C_{-1}) = \frac{\partial \mathcal{L}}{\partial C_{-1}} = -\mu_0 U''(C_{-1}) k_0 \quad \Rightarrow \quad \mu_0 = 0$$

First order conditions

- Manipulate (2) as follows

  $-\beta\mu_{t+1}U''(C_t)k_{t+1} = -\gamma U'(C_t)+\lambda_t-\beta\mu_t((1-\sigma)U'(C_t)+U''(C_t)k_{t+1})$

  Use that $U''(C_t)k_{t+1} = -\sigma U'(C_t)\kappa_{t+1}, \kappa_{t+1} = k_{t+1}/C_t$

  $\mu_{t+1}\beta\sigma U'(C_t)\kappa_{t+1} = \beta\mu_t((\sigma-1)U'(C_t)+U'(C_t)\kappa_{t+1}\sigma)-\gamma U'(C_t)+\lambda_t$

$$\mu_{t+1} = \mu_t\left(\frac{\sigma-1}{\sigma\kappa_{t+1}}+1\right)+\frac{\lambda_t/U'(C_t)-\gamma}{\beta\sigma\kappa_{t+1}}$$

$$\mu_{t+1} = \mu_t\left(\frac{\sigma-1}{\sigma\kappa_{t+1}}+1\right)+\frac{1-\gamma v_t}{\beta\sigma\kappa_{t+1}v_t}, \quad v_t = \frac{U'(C_t)}{u'(c_t)}$$

- Manipulate (3) as follows

$$\beta\lambda_{t+1}(f'(k_{t+1})+1-\delta) = \lambda_t-\mu_t\beta U'(C_t)+\beta\mu_{t+1}U'(C_t)$$

  Divididing by $\beta\lambda_t$ and using $\lambda_t = u'(c_t), v_t = U'(C_t)/u'(c_t)$

$$\frac{u'(c_{t+1})}{u'(c_t)}(f'(k_{t+1})+1-\delta) = \frac{1}{\beta}+v_t(\mu_{t+1}-\mu_t) \qquad (4)$$

First order conditions

- Using these manipulations we obtain

$$\mu_0 = 0 \tag{1}$$

$$u'(c_t) = \lambda_t \tag{2}$$

$$\mu_{t+1} = \mu_t \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} v_t} (1 - \gamma v_t) \tag{3}$$

$$\frac{u'(c_{t+1})}{u'(c_t)} (f'(k_{t+1}) + 1 - \delta) = \frac{1}{\beta} + v_t (\mu_{t+1} - \mu_t) \tag{4}$$

where $\kappa_t = k_t / C_{t-1}$, $v_t = U'(C_t)/u'(c_t)$

- Straub and Werning find it convenient to denote (note $R_t \neq$ rental rate)

$$R_t^e = f'(k_t) + 1 - \delta$$

$$R_t = (1 - \tau_t)(f'(k_t) + 1 - \delta) = \frac{U'(C_t)}{\beta U'(C_{t+1})} \tag{5}$$

$$\tau = 0 \quad \Leftrightarrow \quad R_t^e / R_t = 1$$

First order conditions

Theorem (Judd, 1985)

Suppose quantities and multipliers converge to an interior steady state, i.e. c_t, C_t, k_{t+1} *converge to positive values, and* μ_t *converges. Then the tax on capital is zero in the limit:* $R^e_t / R_t \to 1$.

- **Proof:** Theorem assumes $(c_t, C_t, k_t, \mu_t) \to (c^*, C^*, k^*, \mu^*)$. Hence also $(v_t, \kappa_t) \to (v^*, \kappa^*)$.
- From (4) with $c_t = c_{t+1} = c^*$

$$R^e_t \to R^{e*} = \frac{1}{\beta}$$

- Similarly, from (5) with $C^*_t = C^*_{t+1} = C^*$

$$R_t \to R^* = \frac{1}{\beta}$$

- Hence $R^*_t / R_t \to 1$ or equivalently $\tau_t \to 0.\square$

Comments

- **Theorem** seems to prove: capital taxes converge to zero in the long-run
- Really striking: this is true **even if** $\gamma = 0$, i.e. Ramsey planner only cares about workers!
- Is this really true? Let's consider again the tractable case with log utility, $\sigma = 1$

Ramsey Problem for $\sigma = 1, \gamma = 0$

- Recall analytic solution for capitalists's saving decision

$$a_{t+1} = s(1-\tau_t)(1+r_t)a_t, \quad C_t = (1-s)(1-\tau_t)(1+r_t)a_t$$

with $s = \beta$. Follow Straub-Werning in writing s, could come from somewhere else than $\sigma = 1$ assumption

- Using $C_t = \frac{1-s}{s}k_{t+1}$, resource constraint becomes

$$c_t + \frac{1}{s}k_{t+1} + g = f(k_{t+1}) + (1-\delta)k_t$$

- Also assume $\gamma = 0$ (planner only cares about workers)
- Ramsey problem with $\sigma = 1, \gamma = 0$:

$$\max_{\{c_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

$$c_t + \frac{1}{s}k_{t+1} + g = f(k_{t+1}) + (1-\delta)k_t$$

- Mathematically equivalent to standard growth model

Ramsey Problem for $\sigma = 1, \gamma = 0$

- Euler equation is

$$u'(c_t) = s\beta u'(c_{t+1})(f'(k_{t+1}) + 1 - \delta) \qquad (*)$$

- Because this is equivalent to growth model
  - unique interior steady state

$$1 = s\beta(f'(k^*) + 1 - \delta)$$

  - globally stable
- With $R^* = 1/s$ and $R^{e*} = f'(k^*) + 1 - \delta$ have

$$\frac{R^e}{R} = \frac{1}{\beta} \quad \Rightarrow \quad \tau^* = 1 - \beta > 0$$

- **Counterexample** to zero long-run capital taxes.

What Went Wrong?

- Crucial part of Judd's Theorem: "**Suppose** quantities and multipliers converge to an interior steady state ..."
- Turns out this doesn't happen: **multipliers explode!**
- Consider planner's equations (3), (4) in case $\sigma = 1, \gamma = 0$

$$\mu_{t+1} = \mu_t + \frac{1}{\beta \kappa_{t+1} v_t} \qquad (3')$$

$$\frac{u'(c_{t+1})}{u'(c_t)}(f'(k_{t+1}) + 1 - \delta) = \frac{1}{\beta} + v_t(\mu_{t+1} - \mu_t) \qquad (4')$$

- Judd: **if** $\mu_t \to \mu^*$, then $\tau_t \to 0$ (follows from (4'))
- But from (3') $\mu_{t+1} > \mu_t$ for all $t \Rightarrow \mu_t \to \infty$
- In fact, with log-utility

$$\kappa_{t+1} = \frac{k_{t+1}}{C_t} = \frac{s}{1-s} \quad \Rightarrow \quad v_t(\mu_{t+1} - \mu_t) = \frac{1}{\beta \kappa_{t+1}} = \frac{1-s}{\beta s}$$

and so (4) implies $(*)$ on previous slide and $\tau^* = 1 - \beta$

General Case $\sigma \neq 1$

- Straub and Werning (2014) analyze general case
- Not surprisingly, asymptotic behavior of τ_t different whether
  - $\sigma > 1$: positive limit tax
  - $\sigma < 1$: zero limit tax
- This is where the meat of the paper is

General Case $\sigma \neq 1$

Proposition

If $\sigma > 1$ and $\gamma = 0$ then for any initial k_0 the solution to the planning problem converges to $c_t \to 0, k_t \to k_g, C_t \to \frac{1-\beta}{\beta} k_g$, with a positive limit tax on wealth: $1 - \frac{R_t}{R_t^*} \to \tau_g > 0$. The limit tax is decreasing in spending g, with $\tau_g \to 1$ as $g \to 0$.

- Proof: see pp.34-48!
- What about $\sigma < 1$?
    - zero long-run capital tax is correct
    - **but** convergence may take many hundred years
    - to be expected for $\sigma \approx 1$ due to continuity

Optimal Time Paths for k_t and τ_t

Left panel: k_t, Right panel: τ_t

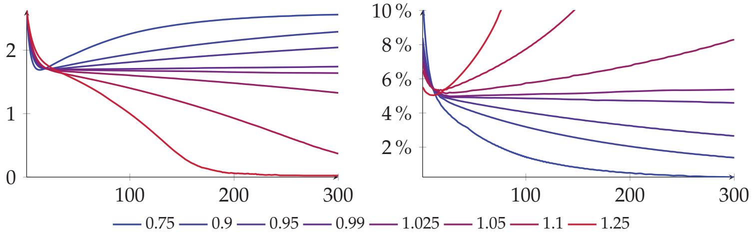
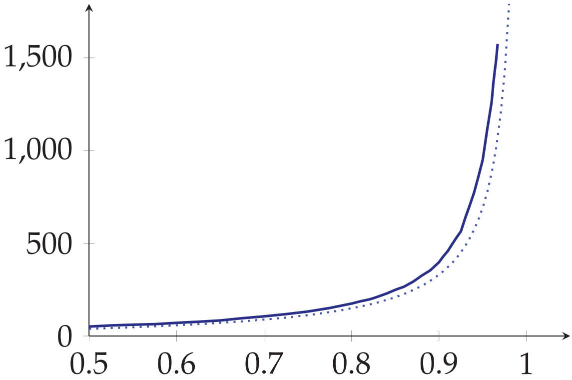


Figure 1: Optimal time paths over 300 years for capital stock (left panel) and wealth taxes (right panel) for various value of σ. Note: tax rates apply to gross returns not net returns, i.e. they represent an annual wealth tax.

$\sigma < 1$: Years until $\tau_t < 1\%$



Intuition

- In long-run, why is optimal $\{\tau_t\}$ **increasing** when $\sigma > 1$ and **decreasing** when $\sigma < 1$?
- Guess what? **Income and substitution effects!**
- Warm-up exercise: consider unexpected higher future taxation $(1 + r_{t+1})(1 - \tau_{t+1}) \downarrow$
  - $\sigma > 1$: income effect dominates $\Rightarrow C_t \downarrow, a_{t+1} \uparrow$
  - $\sigma < 1$: substitution effect dominates $\Rightarrow C_t \uparrow, a_{t+1} \downarrow$
  - $\sigma = 1$: income & subst. effects cancel $\Rightarrow C_t, a_{t+1}$ constant
- One objective of optimal tax policy: high $k_t \Rightarrow$ high output, high tax base
- $\Rightarrow$ want to encourage savings a_{t+1}
  - $\sigma > 1$: income effect dominates $\Rightarrow$ want $\tau_{t+1} \geq \tau_t$
  - $\sigma < 1$: substitution effect dominates $\Rightarrow$ want $\tau_{t+1} \leq \tau_t$
  - $\sigma = 1$: income & subst. effects cancel $\Rightarrow$ want τ_t constant

Effect of Redistributive Preferences γ

Left panel: k_t, Right panel: τ_t

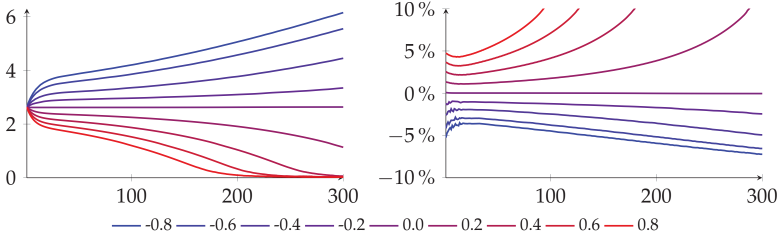


Figure 3: Optimal time paths over 300 years for capital stock (left panel) and wealth taxes (right panel) for various redistribution preferences (zero represents no desire for redistribution; see footnote 16).

Linearized Dynamics

- Straub and Werning also analyze linearized system
  - see their Proposition 4
  - linearize around zero-tax steady state (i.e. Judd's st. st.)
  - same tools as in Lecture 4 but 4-dimensional system (2 states, 2 co-states)
  - careful: they use "saddle-path stable" to refer to system of 2 states, i.e. "no. of negative eigenvalues = 1" or system is unstable except for knife-edge initial conditions (k_0, C_{-1})
- Analysis confirms numerical results

Capital Taxation without Redistribution

- So far: capital taxation in environment with **redistributive** motif (capitalists and workers as in Judd, 1985)
- Different question: if government has to finance a flow of expenditure g, how should it raise the revenue?
    - capital taxes?
    - labor taxes?
- This is the question asked in Chamley (1986)
    - $\Rightarrow$ Ramsey taxation in **representative agent** model
- Won't cover this case in detail
    - logic of Ramsey problem same: max. utility s.t. allocation = CE with taxes
    - see Chamley (1986), Atkeson et al. (1999) among others, and Straub and Werning (2014, Section 3)
    - here: brief intuitive discussion

Capital Taxation without Redistribution

- Key to results in rep. agent models is thinking about **"supply of capital"** and its elasticity (responsiveness to rate of return)
- **inelastic** in **short-run**, **elastic** in **long-run**
- In standard growth model, consider $k_t(r_t, ...)$
  - supply at $t = 0$:

$$k_0 = \hat{k}_0 \quad \Rightarrow \quad \text{elasticity} = 0$$

  - supply as $t \to \infty$:

$$r^* = 1/\beta - 1 \quad \Rightarrow \quad \text{elasticity} = \infty$$

(if **de**crease r by ε, $k_t \to 0$; if **in**crease r by ε, $k_t \to \infty$)

- "Infinite elasticity in long-run" prediction a bit extreme
  - relies on time-separability of preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$
  - but "more elastic in long-run than in short-run" is very general

Capital Taxation without Redistribution

- What does "more elastic in long-run than in short-run" imply for capital taxation?
  - motif for **"front-loading"** capital taxes: tax more today, than tomorrow
  - Chamley: no upper bounds on capital taxes $\Rightarrow$ capital tax $\Rightarrow 0$ as $t \to \infty$
  - in fact, time-separable preferences + no bounds on taxes $\Rightarrow$ all taxation at $t = 0$
- Werning and Straub point to extreme assumption: no upper bound on capital taxation
  - bounds $\Rightarrow$ less front-loading
  - bounds may even bind indefinitely, i.e. capital taxes > 0 in long-run

Takeaway on Capital Taxation

- **Robust prediction:** if possible, want to tax more today than tomorrow
- **Not robust:** this implies that capital taxes should be zero in long-run