Lecture 2: New Keynesian Model in Continuous Time

ECO 521: Advanced Macroeconomics I

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New Keynesian Model

- New Keynesian model = RBC model with sticky prices
- References:
  - Gali (2008): most accessible intro
  - Woodford (2003): New Keynesian bible
  - Clarida, Gali and Gertler (1999): most influential article
  - Gali and Monacelli (2005): small open economy version

Why Should You Care?

- Simple framework to think about relationship between monetary policy, inflation and the business cycle.
- RBC model: cannot even think about these issues! Real variables are completely separate from nominal variables ("monetary neutrality", "classical dichotomy").
- Corollary: monetary policy has **no effect** on any real variables.
- Sticky prices break "monetary neutrality"
- Workhorse model at central banks (see Fed presentation `/DB_ECO521_2012_2013/LectureNotes/MacroModelsAtTheFed.pdf`)
- Makes some sense of newspaper statements like: "a boom leads the economy to overheat and creates inflationary pressure"
- Some reason to believe that "demand shocks" (e.g. consumer confidence, animal spirits) may drive business cycle. Sticky prices = one way to get this story off the ground.

Outline

(1) Model with flexible prices

(2) Model with sticky prices

Setup: Flexible Prices

- Households maximize

$$\int_0^\infty e^{-\rho t} \left\{ \log C(t) - \frac{N(t)^{1+\varphi}}{1+\varphi} \right\} dt$$

subject to

$$PC + \dot{B} = iB + WN$$

- C: consumption
- N: labor
- P: price level
- B: bonds
- i: nominal interest rate
- W: nominal wage
- Note: no capital

Households

- Hamiltonian

$$\mathcal{H}(B, C, N, \lambda) = \log C - \frac{N^{1+\varphi}}{1+\varphi} + \lambda[iB + WN - PC]$$

- Conditions for optimum

$$\dot{\lambda} = \rho\lambda - \lambda i$$

$$\frac{1}{C} = \lambda P \quad \Rightarrow \quad \frac{\dot{C}}{C} = -\frac{\dot{\lambda}}{\lambda} - \frac{\dot{P}}{P}$$

$$N^{\varphi} = \lambda W$$

- Defining the inflation rate $\pi = \dot{P}/P$

$$\frac{\dot{C}}{C} = i - \pi - \rho$$

$$CN^{\varphi} = \frac{W}{P}$$

Firms – Final Goods Producer

- A competitive final goods producer aggregates a continuum of intermediate inputs

$$Y = \left(\int_0^1 y_j^{\frac{\varepsilon - 1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}$$

- Cost minimization $\Rightarrow$ demand for intermediate good j

$$y_j(p_j) = \left(\frac{p_j}{P} \right)^{-\varepsilon} Y$$

where

$$P = \left(\int_0^1 p_j^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$$

Firms – Intermediate Goods Producers

- Continuum of monopolistically competitive intermediate goods producers $j \in [0, 1]$.
- Production uses labor only

$$y_j(t) = A(t) n_j(t).$$

- Solve (drop j subscripts for simplicity)

$$\max_p \; p \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{W(t)}{A(t)} \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t)$$

- Solution

$$p(t) = P(t) = \frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{A(t)}$$

where $P = p_j$ follows because all producers are identical.

Equilibrium with Flexible Prices

- Market clearing:

$$C = AN$$

- Combining with household FOC $CN^{\varphi} = W/P$ and $P = \frac{\varepsilon}{\varepsilon - 1} W/A$

$$C = Y = A \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{-1}{1+\varphi}}$$

- Note: distortion from monopolistic competition
- Back out real interest rate from

$$r = i - \pi = \rho - \frac{\dot{C}}{C} = \rho + \frac{\dot{A}}{A} = \rho + g$$

Some Notable Features

- Like an RBC model, this model features "monetary neutrality"

  `http://lmgtfy.com/?q=monetary+neutrality`

- Equivalently: there is a "classical dichotomy"

  `http://lmgtfy.com/?q=classical+dichotomy`

- Real variables $(C(t), Y(t), N(t), W(t)/P(t), r(t))$ are determined completely separately from nominal variables $(P(t), W(t), \pi(t), i(t))$.

- In fact, $P(t)$ and $\pi(t)$ are not even determined in the absence of a description of a determination of the economy's money stock (e.g. through monetary policy). But this doesn't matter for real variables.

- As a corollary, monetary policy has **no effect** on real variables

Sticky Prices

- Everything same except intermediate goods producers.
- Per period profits are still

$$\Pi_t(p) = p\left(\frac{p}{P(t)}\right)^{-\varepsilon} Y(t) - \frac{W(t)}{A(t)}\left(\frac{p}{P(t)}\right)^{-\varepsilon} Y(t)$$

- But now have to pay quadratic price adjustment cost

$$\Theta_t\left(\frac{\dot{p}}{p}\right) = \frac{\theta}{2}\left(\frac{\dot{p}}{p}\right)^2 P(t)Y(t)$$

- Optimal control problem:

$$V_0(p_0) = \max_{p(t),t\geq 0} \int_0^\infty e^{-\int_0^t i(s)ds}\left\{\Pi_t(p(t)) - \Theta_t\left(\frac{\dot{p}(t)}{p(t)}\right)\right\} dt$$

- θ: degree of price stickiness

Comparison to Literature

- Note: my formulation uses quadratic price adjustment costs as in Rotemberg (1982).
- Different from standard Calvo (1983) pricing formulation: allowed to change price at Poisson rate α
- I like Rotemberg better because pricing is state dependent as opposed time dependent ("Calvo fairy").
- Closer to "menu cost" models.
- Schmitt-Grohe and Uribe (2004), Fernandez-Villaverde et al. (2011) also use Rotemberg
- I also assume that adjustment costs are paid as a transfer to consumers, $T = \Theta_t(\pi) = (\theta/2)\pi^2 PY$. Just a trick to eliminate real resource costs of inflation ($\Theta_t(\pi) \approx 0$ anyway).

Optimal Price Setting

- Hamiltonian (state: p, control: $\dot{p}$, co-state: η):

$$\mathcal{H}(p, \dot{p}, \eta) = p \left(\frac{p}{P}\right)^{-\varepsilon} Y - \frac{W}{A}\left(\frac{p}{P}\right)^{-\varepsilon} Y - \frac{\theta}{2}\left(\frac{\dot{p}}{p}\right)^{2} PY + \eta\dot{p}$$

- Conditions for optimum

$$\theta \frac{\dot{p}}{p}\frac{P}{p} Y = \eta$$

$$\dot{\eta} = i\eta - \left[(1-\varepsilon)\left(\frac{p}{P}\right)^{-\varepsilon} Y + \varepsilon \frac{W}{p}\frac{1}{A}\left(\frac{p}{P}\right)^{-\varepsilon} Y + \theta\left(\frac{\dot{p}}{p}\right)^{2}\frac{P}{p} Y\right].$$

- Symmetric equilibrium: $p = P$

$$\theta\pi Y = \eta$$

$$\dot{\eta} = i\eta - \left[(1-\varepsilon)Y + \varepsilon\frac{W}{P}\frac{1}{A}Y + \theta\pi^{2}Y\right].$$

Optimal Price Setting

- Recall the FOC: $\theta \pi Y = \eta$. Differentiate with respect to time

$$\theta \dot{\pi} Y + \theta \pi \dot{Y} = \dot{\eta}$$

- Substitute into equation for co-state and rearrange

Lemma

The price setting of firms implies that the inflation rate $\pi = \dot{P}/P$ is determined by

$$\left(i - \pi - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W}{P} \frac{1}{A} - 1 \right) + \dot{\pi}.$$

Optimal Price Setting in Equilibrium

- In equilibrium $C = Y$ and Euler equation

$$\frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = i - \pi - \rho$$

- Substitute into expression on previous slide $\Rightarrow$ Inflation determined by

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W}{P}\frac{1}{A} - 1\right) + \dot{\pi}. \qquad (*)$$

- In integral form (check that differentiating gives back above)

$$\pi(t) = \frac{\varepsilon - 1}{\theta}\int_t^\infty e^{-\rho(s-t)}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W(s)}{P(s)}\frac{1}{A(s)} - 1\right) ds$$

- Compare with equation (16) in Chapter 3.3. of Gali's book and expression just below.

Optimal Price Setting in Equilibrium

- Inflation determined by
$$\pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W(s)}{P(s)} \frac{1}{A(s)} - 1 \right) ds$$
- Intuition: term in brackets = marginal payoff to a firm from increasing its price
$$\Pi_t'(P(t)) = (\varepsilon - 1) Y(t) \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{P(t)} \frac{1}{A(t)} - 1 \right).$$
- Positive whenever P less than optimal markup $\frac{\varepsilon}{\varepsilon-1}$ over marginal cost W/A.
- With flexible prices, $\theta = 0$: $\Pi_t'(P(t)) = 0$ for all t, $P = \frac{\varepsilon}{\varepsilon-1}\frac{W}{A}$.
- With sticky prices, $\theta > 0$: π = PDV of all future $\Pi_t'(P(t))$.
- Adjustment cost is convex. So if expect reason to adjust in the future – e.g. $W(t)/A(t) \uparrow$ – already adjust now.

IS Curve and Phillips Curve

- Call outcomes under flexible prices, $\theta = 0$, "natural" output Y^n and "natural" real interest rate. Recall

$$Y^n = A\left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\frac{-1}{1+\varphi}}, \quad \frac{\dot{Y}^n}{Y^n} = r - \rho, \quad r = \rho + \frac{\dot{A}}{A}$$

- Define output gap: $X = Y/Y^n$. Recall Euler equation under sticky prices

$$\frac{\dot{Y}}{Y} = i - \pi - \rho$$

- Euler equation in terms of output gap $\dot{X}/X = \dot{Y}/Y - \dot{Y}^n/Y^n$

$$\frac{\dot{X}}{X} = i - \pi - r$$

- This is basically an IS curve.

IS Curve and Phillips Curve

- Can obtain "Phillips Curve" in similar way. Recall

$$P^n = \frac{\varepsilon}{\varepsilon - 1}\frac{W^n}{A} \quad \Rightarrow \quad \frac{W}{P}\frac{1}{A} = \frac{W/P}{W^n/P^n}$$

- Equation for inflation $(*)$ becomes

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\frac{W/P - W^n/P^n}{W^n/P^n} + \dot{\pi}.$$

- From FOC $CN^{\varphi} = \frac{W}{P}$, and mkt clearing $C = Y, N = Y/A$

$$\frac{W/P}{W^n/P^n} = \left(\frac{Y}{Y^n}\right)^{1+\varphi} = X^{1+\varphi}.$$

IS Curve and Phillips Curve

- Relation between inflation and output gap: "New Keynesian Phillips Curve"

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\left(X^{1+\varphi} - 1\right) + \dot{\pi}.$$

- In integral form

$$\pi(t) = \frac{\varepsilon - 1}{\theta}\int_t^\infty e^{-\rho(s-t)}\left(X(s)^{1+\varphi} - 1\right) ds.$$

- Inflation high when future output gaps are high, i.e. when economy "overheats"

Three Equation Model

- Recall: IS curve and Phillips curve

$$\frac{\dot{X}}{X} = i - \pi - r \tag{IS}$$

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\left(X^{1+\varphi} - 1\right) + \dot{\pi} \tag{PC}$$

- To close model: Taylor rule

$$i = i^* + \phi\pi + \phi_x \log X \tag{TR}$$

- "Three equation model," see modern undergraduate textbooks (e.g. Carlin and Soskice)
- Substitute (TR) into (IS) $\Rightarrow$ system of two ODEs in (π, X), analyze with phase diagram.

Three Equation Model in Literature

- Literature uses log-linearization all over the place.
- Obtain exact analogues by defining

$$x \equiv \log X = \log Y - \log Y^n$$

- Using that for small x (Taylor-series)

$$X^{1+\varphi} - 1 = e^{(1+\varphi)x} - 1 \approx (1+\varphi)x$$

- and defining $\kappa \equiv (\varepsilon - 1)(1+\varphi)/\theta$

$$\dot{x} = i - \pi - r \qquad \text{(IS')}$$

$$\rho\pi = \kappa x + \dot{\pi} \qquad \text{(PC')}$$

$$i = i^* + \phi\pi + \phi_x x \qquad \text{(TR')}$$

- Exact continuous time analogues of (21), (22), (25) in Chapter 3 of Gali's book, same as in Werning (2012)

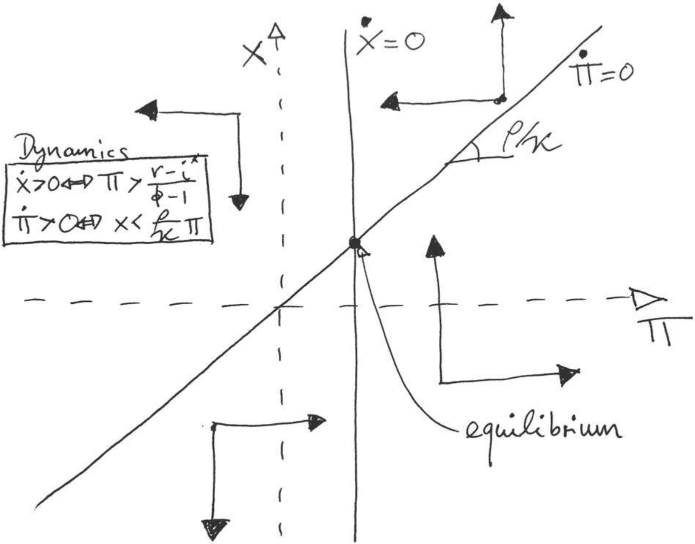
Phase Diagrams

- For simplicity, assume $\phi_x = 0$. Makes some math easier.
- Also ignore ZLB, $i \geq 0$ (next time).
- Substitute (TR') into (IS')

$$\dot{x} = i^* - r + (\phi - 1)\pi$$
$$\dot{\pi} = \rho\pi - \kappa x \tag{ODE}$$

- See phase diagrams I drew in lectures.
- Important: both π and x are jump-variables. No state variables.
- Two cases:
  - $\phi > 1$: unique equilibrium. "Taylor principle": i increases more than one-for-one with π so that also real rates increase.
  - $\phi < 1$: equilibrium indeterminacy
- From now assume $\phi > 1$

Phase Diagram with $\phi > 1$



Monetary Policy

- Can achieve $\pi = 0$ and $x = 0$ by setting $i^* = r$ (and $\phi > 1$).
- Scenario 1: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, r increases, e.g. because TFP growth increases (recall $r = \rho + \dot{A}/A$).
- Scenario 2: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, someone at the Fed goes crazy and increases i^* (e.g. because mistakenly think that TFP growth goes up).
- Draw time paths for $(x(t), \pi(t))$ for both scenarios.

Recursive Formulation

- Convenient for analyzing more complicated dynamics and also in stochastic case (later).
- Suppose $i^* = \rho$ but $A(t)$ moves around, e.g. mean reverting

$$\frac{\dot{A}}{A} = -\nu \log A$$

- Can show: implies

$$\dot{r} = \nu(\rho - r) \equiv \mu^r(r)$$

- Use r as state variable. x and π only depend on r:

$$(x(t), \pi(t)) = (x(r(t)), \pi(r(t)))$$

Recursive Formulation

- Write (ODE) recursively as

$$x'(r)\mu^r(r) = \rho - r + (\phi - 1)\pi(r)$$

$$\pi'(r)\mu^r(r) = \rho\pi(r) - \kappa x(r)$$

- Method of undetermined coefficients: guess

$$\pi(r) = \psi_\pi(r - \rho), \quad x(r) = \psi_x(r - \rho)$$

- Obtain

$$\pi(r) = \frac{\kappa}{(\phi - 1)\kappa + \nu(\rho + \nu)}(r - \rho)$$

$$x(r) = \frac{\rho + \nu}{(\phi - 1)\kappa + \nu(\rho + \nu)}(r - \rho)$$