

# Distributional Macroeconomics

---

Benjamin Moll  
Princeton

Asian Meeting of the Econometric Society  
June 21, 2018

# What do I mean by “Distributional Macroeconomics”?

---

- Study of **macroeconomic questions in terms of distributions** rather than just aggregates
  - typical example: distributions of income and wealth
- More technically: macroeconomic theories in which **relevant state variable is a distribution**
- Or “heterogeneous agent models” but I want to emphasize distributions
- What’s attractive about this approach?
  - conceptually: unified approach to macro and distribution
  - empirically: unified approach to micro and macro data

# Main Message

---

- Hard to coherently think about macro if ignore distribution
- Instead, rich interaction:

distribution  $\iff$  macroeconomy

- Or perhaps more precisely:

macroeconomy **is** a distribution

# Plan

---

1. Distribution in macroeconomics: a **history of thought**
2. **Methods** for “distributional macro” models: continuous time
3. An **application** of “distributional macro” from my own work: “Monetary Policy According to HANK”
  - based on joint work with Yves Achdou, SeHyouon Ahn, Paco Buera, Andreas Fagereng, Jiequn Han, Martin Holm, Greg Kaplan, Jean-Michel Lasry, Pierre-Louis Lions, Gisle Natvik, Galo Nuño, Gianluca Violante, Tom Winberry, Christian Wolf

# Distribution in Macro: A History of Thought

---

I find it useful to categorize macroeconomic theories as follows:

- **before modern macro**: 1930 to 1970
- **1st generation** modern macro: 1970 to 1990
- **2nd generation** modern macro: 1990 to financial crisis
- **3rd generation** modern macro: after the financial crisis

Main drivers of evolution in modern macro era

1. better data
2. better computers & algorithms
3. current events (rising inequality, financial crisis)

(Warning: narrative won't be perfect – will point out failures)

# Before Modern Macro: 1930 to 1970

---

## 1. Keynesian IS/LM

- about aggregates, **no role for inequality/distribution by design**

## 2. Distribution does play role in growth theory

- mostly **factor** income distribution – capital vs labor  
Kaldor, Pasinetti, other Cambridge UK theorists
- rarely **personal** income or wealth distribution  
exceptions: Tobin, Stiglitz, Blinder

## 3. Disconnected empirical work on inequality (Kuznets)

# First Generation Macro Theories: 1970 to 1990

---

Representative agent models, e.g. RBC model

- again **no role for inequality/distribution** by design
- advertised as “microfounded” but rep agent assumption cuts 1st generation theories from much of micro research

What’s wrong with that?

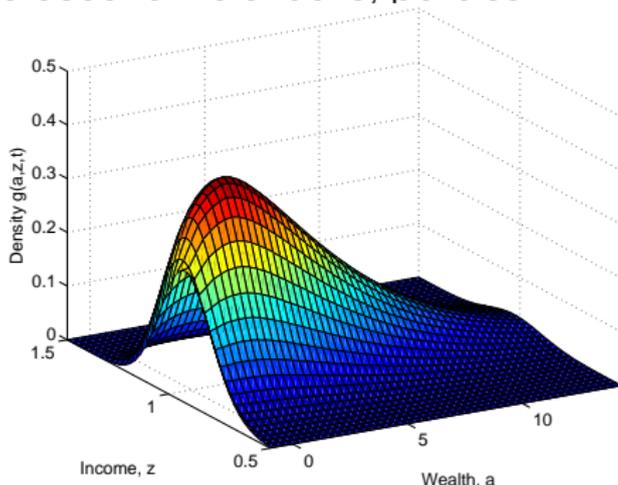
1. cannot speak to a number of important empirical facts, e.g.
  - unequally distributed growth
  - poorest hit hardest in recessions
2. cannot think coherently about **welfare** – “who gains, who loses?”

## Second Generation Macro Theories: 1990 to 2008

Incorporate micro **heterogeneity**, particularly **in income and wealth** – early “heterogeneous agent models”

Aiyagari, Bewley, Huggett, Imrohoroğlu, Krusell-Smith, Den Haan,...

... represent economy with a **distribution** that moves over time, responding to macroeconomic shocks, policies



Can speak to facts on previous slide, useful for welfare analysis

## Second Generation Theories: Inequality $\nrightarrow$ Macro

---

- Typical finding: **heterogeneity doesn't matter much for macro agg's**  
Krusell-Smith (1998) "approximate aggregation"
- Reason: linearity – **rich = scaled version of poor**  
Hence "inequality  $\nrightarrow$  macro", but also a **knife-edge result**
- Interestingly, some more nuanced, cautionary results in literature:
  - **even in KS98, extension where heterogeneity does matter (§4)**
  - but gets lost, economists' perception = "inequality  $\nrightarrow$  macro"
- Either way: in data, rich  $\neq$  scaled version of poor, e.g. rich have
  - e.g. lower MPCs out of transitory income changes
- Note: some important contributions from 90s don't fit my narrative
  - **Banerjee-Newman, Benabou, Galor-Zeira, Persson-Tabellini, ...**

## Third Generation Theories: after the Crisis

---

- 3rd generation theories **take micro data more seriously**
- Leads them to emphasize things like
  - household balance sheets
  - credit constraints
  - MPCs that are high on average but heterogeneous
  - non-homotheticities, non-convexities
- ⇒ **move away from knife-edge case**
- Typical finding: **distribution matters for macro**
- Momentarily: an application from my own work (HANK)

# Distribution in Macro: Summary

---

- **Before modern macro:** 1930 to 1970
  - it's complicated
- **1st generation:** 1970 to 1990
  - representative agent models (RBC, New Keynesian etc)
  - no role for inequality by design
- **2nd generation:** 1990 to financial crisis
  - early heterogeneous agent models
  - “macro  $\Rightarrow$  inequality” but “macro  $\nRightarrow$  inequality” (perception)
- **3rd generation:** after the financial crisis
  - current heterogeneous agent models
  - rich interaction: “inequality  $\iff$  macro”

# Distribution in Modern Macro: Summary

---

Janet Yellen speech “Macroeconomic Research After the Crisis”

<http://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm>

- “Prior to the financial crisis, representative-agent models were the dominant paradigm for analyzing many macroeconomic questions [= 1st generation].”
- “However, a disaggregated approach seems needed to understand some key aspects of the Great Recession...”
- “While the economics profession has long been aware that these issues matter, their effects had been incorporated into macro models only to a very limited extent prior to the financial crisis [= 2nd generation].”
- “I am glad to now see a greater emphasis on the possible macroeconomic consequences of heterogeneity [= 3rd generation].”

# Methods for Solving 3rd Generation Models: Continuous Time

# Solving heterogeneous agent model as PDEs

---

- 3rd generation theories: richer economics, distribution matters
  - $\Rightarrow$  standard numerical solution methods may not work
  - need to carry around distribution – “can’t do Krusell-Smith”
- One approach to make progress: solve het. agent model as PDEs
  1. Hamilton-Jacobi-Bellman equation for individual choices
  2. Kolmogorov Forward equation for evolution of distribution= application of “Mean Field Games” framework (Lasry-Lions)
- Apparatus is very general: applies to any heterogeneous agent model with continuum of atomistic agents
  1. heterogeneous households (Aiyagari, Bewley, Huggett,...)
  2. heterogeneous producers (Hopenhayn,...)

# References

---

1. “Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach” (with Achdou, Han, Lasry & Lions)
  - discussion of **computational advantages** over discrete time
  - Codes: <http://www.princeton.edu/~moll/HACTproject.htm>
2. With aggregate shocks: “When Inequality Matters for Macro and Macro Matters for Inequality” (with Ahn, Kaplan, Winberry & Wolf)
  - Matlab toolbox: <https://github.com/gregkaplan/phact>

# A nerdy version of my main message

---

Question: What is the **central equation in macro**?

- Likely answer of most macroeconomists: the **Euler equation**
- My answer: the **Kolmogorov Forward equation**
  - (closely followed by an Euler/Bellman equation for het agents)
  - again, macroeconomy **is** joint distribution of micro variables
  - not special to continuous time, analogous eq'n in discrete time

# An Application of Distributional Macro Monetary Policy According to HANK

(with Greg Kaplan and Gianluca Violante)

(HANK = “Heterogeneous Agent New Keynesian” model)

# How monetary policy works in RANK

---

- Total **consumption response** to a drop in real rates

$$C \text{ response} = \underbrace{\text{direct response to } r}_{>95\%} + \underbrace{\text{indirect effects due to } Y}_{<5\%}$$

- **Direct response is everything**, pure intertemporal substitution
- However, data suggest:
  1. **Low** sensitivity of  $C$  to  $r$
  2. **Sizable** sensitivity of  $C$  to  $Y$
  3. Micro sensitivity vastly **heterogeneous**, depends crucially on household **balance sheets**

# How monetary policy works in HANK

---

- HANK delivers realistic distributions of household wealth and MPCs

$$C \text{ response} = \underbrace{\text{direct response to } r}_{\text{RANK: } >95\%} + \underbrace{\text{indirect effects due to } Y}_{\text{RANK: } <5\%}$$

RANK: >95%

RANK: <5%

HANK: <1/3

HANK: >2/3

- Overall effect depends crucially on fiscal response, unlike in RANK where Ricardian equivalence holds

# HANK: a framework for monetary policy analysis

---

## Households

- Face uninsured idiosyncratic labor income risk
- Consume and supply labor
- Hold two assets: liquid and illiquid
- Budget constraints (simplified version)

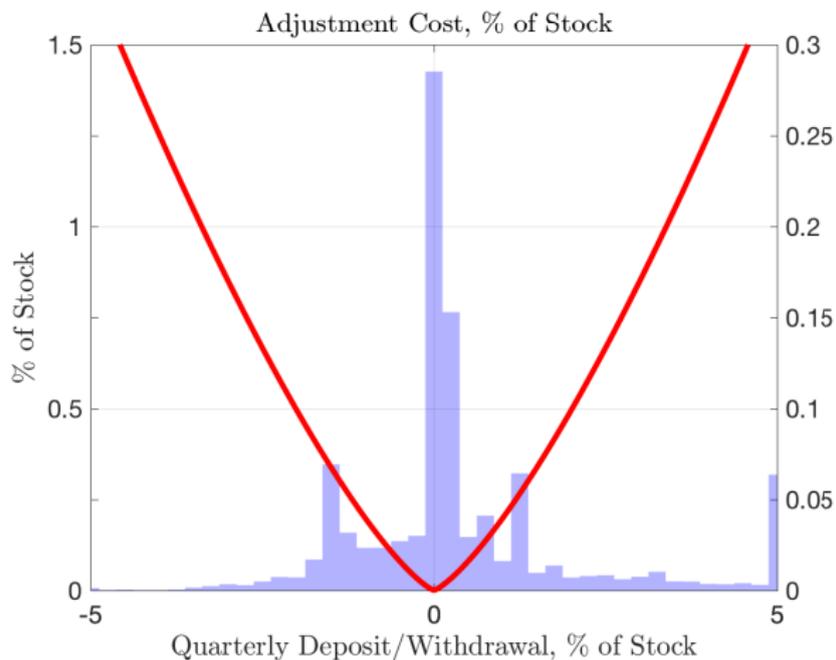
$$\dot{b}_t = r^b b_t + w z_t \ell_t - c_t - d_t - \chi(d_t, a_t)$$

$$\dot{a}_t = r^a a_t + d_t$$

- $b_t$ : liquid assets
- $d_t$ : illiquid deposits ( $\geq 0$ )
- $a_t$ : illiquid assets
- $\chi$ : transaction cost function
- In equilibrium:  $r^a > r^b$
- Full model: borrowing/saving rate wedge, taxes/transfers

# Kinked adjustment cost function $\chi(d, a)$

---



# Remaining model ingredients

---

## Firms

- monopolistically competitive intermediate-good producers
- quadratic price adjustment costs à la Rotemberg (1982)

## Illiquid assets

- consist of both productive capital and equity = claim to profits
- pins down illiquid return

## Government

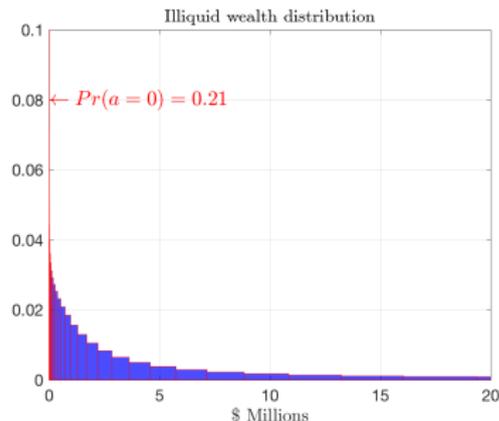
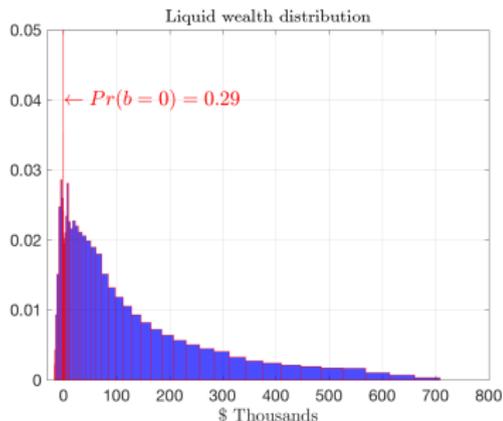
- issues liquid debt, spends, taxes

## Monetary Authority

- sets nominal rate on liquid assets based on a Taylor rule

# Model matches key feature of U.S. wealth distribution

---



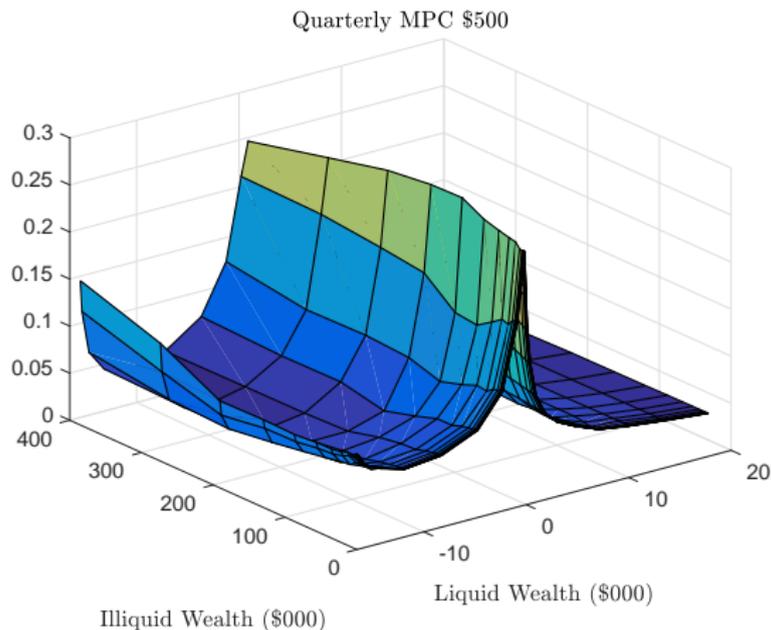
---

	Data	Model
Mean illiquid assets (rel to GDP)	2.920	2.920
Mean liquid assets (rel to GDP)	0.260	0.263
Poor hand-to-mouth	10%	10%
Wealthy hand-to-mouth	20%	19%

---

# Model generates high and heterogeneous MPCs

---

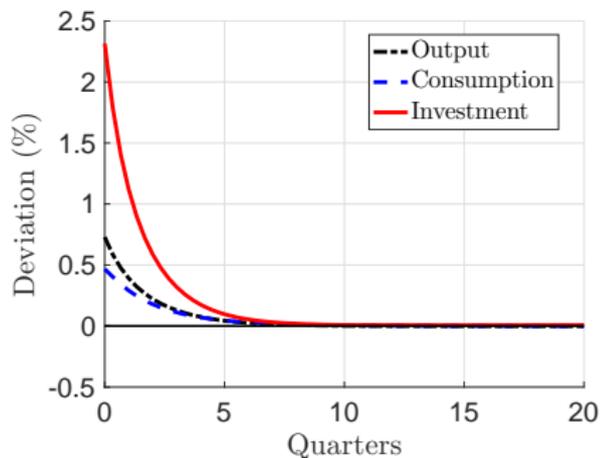
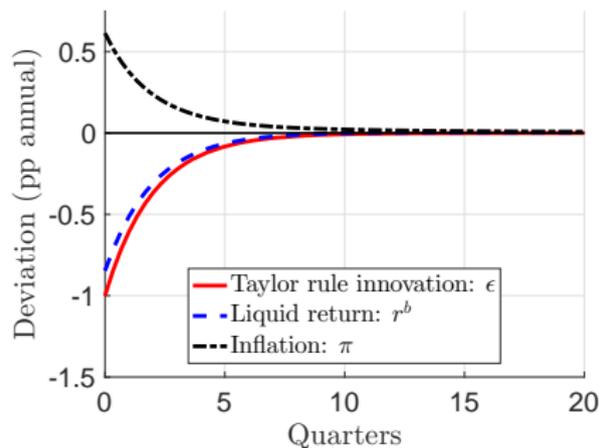


- Average quarterly MPC out of a \$500 windfall: 16%

# Transmission of monetary policy shock to $C$

Innovation  $\epsilon < 0$  to the Taylor rule:  $i = \bar{r}^b + \phi\pi + \epsilon$

- All experiments:  $\epsilon_0 = -0.0025$ , i.e.  $-1\%$  annualized



## Transmission of monetary policy shock to $C$

---

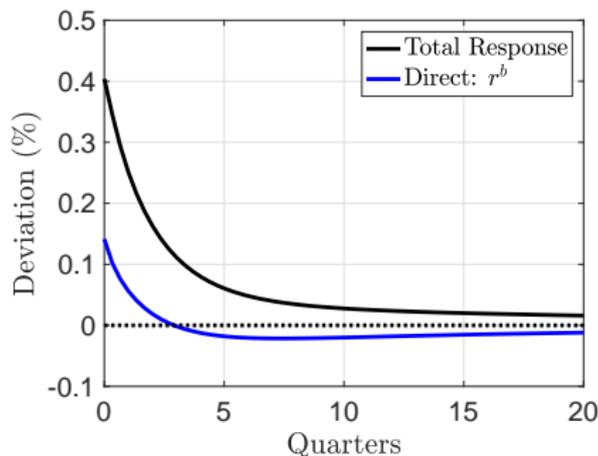
$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{\text{direct}} + \underbrace{\int_0^\infty \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt}_{\text{indirect}}$$

# Transmission of monetary policy shock to $C$

$$dC_0 = \int_0^{\infty} \frac{\partial C_0}{\partial r_t^b} dr_t^b dt + \int_0^{\infty} \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt$$

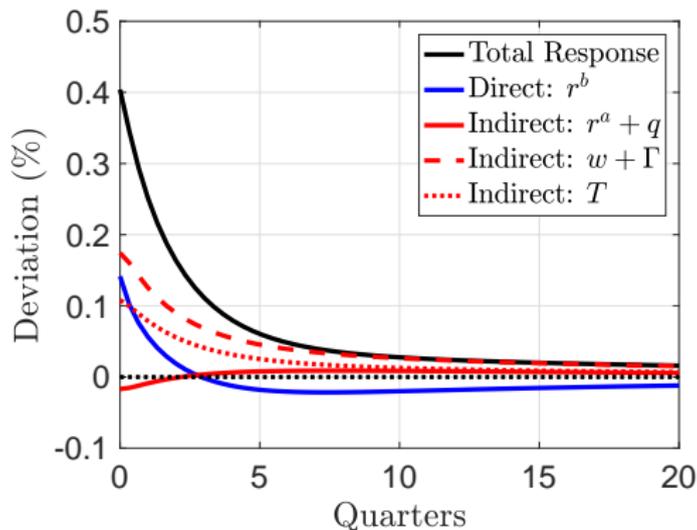
✓

Intertemporal substitution and income effects from  $r^b \downarrow$



# Transmission of monetary policy shock to $C$

$$dC_0 = \underbrace{\int_0^{\infty} \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{19\%} + \underbrace{\int_0^{\infty} \left[ \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right] dt}_{81\%}$$



## Role of fiscal response in determining total effect

---

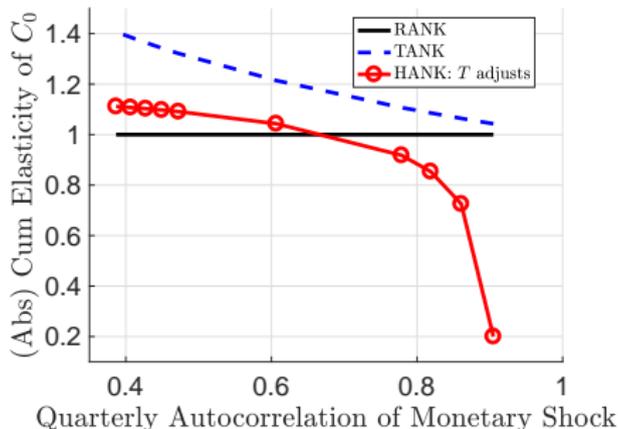
	<i>T</i> adjusts	<i>G</i> adjusts	<i>B<sup>g</sup></i> adjusts
	(1)	(2)	(3)
<b>Elasticity of <math>C_0</math> to <math>r^b</math></b>	-2.21	-2.07	-1.48
Share of Direct effects:	19%	22%	46%

- Fiscal response to lower interest payments on debt:
  - *T* adjusts: stimulates AD through MPC of HtM households
  - *G* adjusts: translates 1-1 into AD
  - *B<sup>g</sup>* adjusts: no initial stimulus to AD from fiscal side

# When is HANK $\neq$ RANK? Persistence

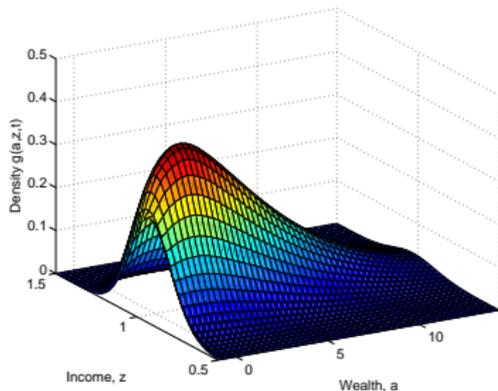
- RANK:  $\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma}(r_t - \rho) \Rightarrow C_0 = \bar{C} \exp\left(-\frac{1}{\gamma} \int_0^\infty (r_s - \rho) ds\right)$
- Cumulative  $r$ -deviation  $R_0 := \int_0^\infty (r_s - \rho) ds$  is sufficient statistic
- Persistence  $\eta$  only matters insofar as it affects  $R_0$

$$-\frac{d \log C_0}{dR_0} = \frac{1}{\gamma} = 1 \quad \text{for all } \eta$$



# Distributional Macroeconomics: Summary

- Current macro research: economy = joint distribution of micro variables, not collection of aggregates



- Often: **can't ignore distribution even if care only about aggregates**
- Not yet part of policy makers' toolkit, but starting to change:
  - various central banks, other policy institutions currently developing their own 3rd generation frameworks
- Think in terms of Kolmogorov Forward not Euler equations!



# Computational Advantages relative to Discrete Time

---

1. **Borrowing constraints** only show up in **boundary conditions**
  - FOCs always hold with “=”
2. **“Tomorrow is today”**
  - FOCs are “static”, compute by hand:  $c^{-\gamma} = v_a(a, y)$
3. **Sparsity**
  - solving Bellman, distribution = inverting matrix
  - but matrices very sparse (“tridiagonal”)
  - reason: continuous time  $\Rightarrow$  one step left or one step right
4. **Two birds with one stone**
  - (KF) for distribution is “transpose problem” of (HJB) (“adjoint”)
  - matrix in discrete (KF) is transpose of matrix in discrete (HJB)

## References: Some “Third Generation” Papers

---

- Ahn, Kaplan, Moll, Winberry & Wolf (2017) “When Inequality Matters for Macro and Macro Matters for Inequality”
- Auclert (2016) “Monetary Policy and the Redistribution Channel”
- Auclert & Rognlie (2016) “Inequality and Aggregate Demand”
- Bayer, Pham, Luetticke & Tjaden (2015) “Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk”
- Carroll, Slacalek & Tokuoka (2016) “The Distribution of Wealth and the Marginal Propensity to Consume”
- Den Haan, Rendahl & Riegler (2017) “Unemployment (fears) and Deflationary Spirals,”
- Gornemann, Kuester & Nakajima (2016) “Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy”
- Guerrieri & Lorenzoni (2017) “Credit Crises, Precautionary Savings, and the Liquidity Trap”
- Kaplan, Moll & Violante (2017) “Monetary Policy According to HANK”

## References: Some “Third Generation” Papers

---

- Luetticke (2017), “Transmission of Monetary Policy with Heterogeneity in Household Portfolios”
- McKay & Reis (2016), “The Role of Automatic Stabilizers in the U.S. Business Cycle”
- McKay, Nakamura & Steinsson (2016) “The Power of Forward Guidance Revisited”
- Hedlund, Karahan, Mitman & Ozkan (2017) “Monetary Policy, Heterogeneity and the Housing Channel”
- Hagedorn, Manovskii & Mitman (2017) “The Fiscal Multiplier”
- Oh & Reis (2012), “Targeted Transfers and the Fiscal Response to the Great Recession”
- Ravn & Sterk (2016), “Job Uncertainty and Deep Recessions”
- Straub (2017), “Consumption, Savings & the Distribution of Permanent Income”
- Werning (2016), “Incomplete Markets and Aggregate Demand” (depends)
- Wong (2016), “Population Aging and the Transmission of Monetary Policy to Consumption”

## References: Other Academic Articles

---

- Aiyagari (1994) “Uninsured Idiosyncratic Risk and Aggregate Saving”
- Banerjee & Newman (1993) “Occupational Choice and the Process of Development”
- Banerjee & Duflo (2003) “Inequality and Growth: What Can the Data Say?”
- Benabou (1996) “Inequality and Growth”
- Bewley (1986) “Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers”
- Carroll (2000) “Requiem for the Representative Consumer? Aggregate Implications of Microeconomic Consumption Behavior”
- De Nardi & Fella (2017) “Saving and Wealth Inequality”
- Den Haan (1996) “Heterogeneity, Aggregate Uncertainty, and the Short-Term Interest Rate”

## References: Other Academic Articles

---

- Galor & Zeira (1993) “Income Distribution and Macroeconomics”
- Heathcote, Perri & Violante (2010), “An Empirical Analysis of Economic Inequality in the United States, 1967-2006”
- Heathcote, Storesletten & Violante (2009) “Quantitative Macroeconomics with Heterogeneous Households”
- Huggett (1993) “The Risk-free Rate in Heterogeneous-Agent Incomplete-Insurance Economies”
- Jappelli & Pistaferri (2014) “Fiscal Policy and MPC Heterogeneity”
- Krueger, Mitman & Perri (2016) “Macroeconomics and Household Heterogeneity”
- Krusell & Smith (1998) “Income and wealth heterogeneity in the macroeconomy”
- Tobin (1952) “Asset Holdings and Spending Decisions”
- Persson & Tabellini (1994) “Is Inequality Harmful for Growth?”

# References: Speeches, Newspapers, Blogs

---

- Blinder (2014) “The Supply-Side Case for Government Redistribution”  
<http://economistsview.typepad.com/economistsview/2014/08/the-supply-side-case-for-government-redistribution.html>
- Coeure (2013) “The Relevance of Household-Level Data for Monetary Policy and Financial Stability Analysis”  
<https://www.ecb.europa.eu/press/key/date/2013/html/sp131017.en.html>
- Constâncio (2017) “Inequality and Macroeconomic Policies”  
<https://www.ecb.europa.eu/press/key/date/2017/html/ecb.sp170822.en.html>
- Kaplan and Violante (2016) “Wealthy ‘hand-to-mouth’ households: key to understanding the impacts of fiscal stimulus”  
<http://microeconomicinsights.org/wealthy-hand-to-mouth-households-key-to-understanding-the-impacts-of-fiscal-stimulus/>
- Kocherlakota (2009) “Some Thoughts on the State of Macro”  
<http://online.wsj.com/public/resources/documents/KOCHERLAKOTA20090930.pdf>
- Kuroda (2017) “Opening Remarks at the 2017 BOJ-IMES Conference”  
[https://www.boj.or.jp/en/announcements/press/koen\\_2017/ko170524a.htm](https://www.boj.or.jp/en/announcements/press/koen_2017/ko170524a.htm)
- Olsen (2018) “How does the key policy rate operate?”  
<https://www.bis.org/review/r181012c.htm>
- Yellen (2016) “Macroeconomic Research After the Crisis”  
<https://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm>