Behavioral SIR Models

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Recall SIR Model

- Susceptibles S_t
- Infectious I_t
- Recovered or dead R_t

$$\dot{S} = -\beta SI \tag{S}$$
$$\dot{I} = \beta SI - \gamma I \tag{I}$$
$$\dot{R} = \gamma I \tag{R}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

- Death: constant death probability out of I state π

$$\dot{D} = \pi\gamma I \quad \Leftrightarrow \quad D = \pi R$$

- Mass preservation: $\dot{S}_t + \dot{I}_t + \dot{R}_t = 0 \Rightarrow S_t + I_t + R_t = 1$, all $t \geq 0$
- For analysis, see e.g. `https://benjaminmoll.com/SIR_notes/`

Simplest Behavioral SIR Model

Simplified version of models in econ literature, e.g.

- Lukasz Rachel "An Analytical Model of Covid-19 Lockdowns"
- Bognanni-Hanley-Kolliner-Mitman "Economics and Epidemics: Evidence from an Estimated Spatial Econ-SIR Model"
- Joshua Gans "The Economic Consequences of $R = 1$: Towards a Workable Behavioural Epidemiological Model of Pandemics"
- Flavio Toxvaerd "Equilibrium Social Distancing"
- Farboodi-Jarosch-Shimer "Internal and External Effects of Social Distancing"
- Garibaldi-Moen-Pissarides "Modelling contacts and transitions in the SIR epidemics model"
- Engle-Keppo-Kudlyak-Quercioli-Smith-Wilson "The Behavioral SIR Model, with Applications to the Swine Flu and COVID-19 Pandemics"
- Kaplan-Moll-Violante "The Great Lockdown and the Big Stimulus: Tracing the Pandemic Possibility Frontier for the U.S."
- ... plus too many others to list here (apologies for omissions)...
- Version in these notes due to Gianluca Violante
  `https://conference.nber.org/confer/2020/EFGs20/Violante.pdf`

Simplest Behavioral SIR Model

- Susceptibles S_t
- Infectious I_t
- Recovered or dead R_t
- Economic activity Y_t, normalize pre-pandemic $\bar{Y} = 1$ so $Y_t \leq 1$

$$\dot{S} = -\beta(Y)SI \tag{S}$$

$$\dot{I} = \beta(Y)SI - \gamma I \tag{I}$$

$$\dot{R} = \gamma I \tag{R}$$

$$Y = \mathcal{Y}(I) \tag{Y}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

- Deaths as two slides ago $D = \pi R$
- Assumptions
  1. Infections $\nearrow$ in econ activity: $\beta' > 0$, e.g. $\beta(Y) = \bar{\beta} Y^{\alpha}, \alpha > 0$
  2. Econ activity $\searrow$ in infections: $\mathcal{Y}' < 0$, e.g. $\mathcal{Y}(I) = e^{-\sigma I}, \sigma > 0$

Two positions in popular debate

1. **"Tradeoff between lives & livelihoods"**
   - Focuses on dynamics of infections

$$\dot{I} = \beta(Y)SI - \gamma I \quad \text{with} \quad \beta'(Y) > 0$$

   - Policy that reduces $Y \downarrow$ implies $I \downarrow$ and ultimately $D \downarrow$

2. **"To save the economy, save people first"**
   - Focuses on behavioral response

$$Y = \mathcal{Y}(I) \quad \text{with} \quad \mathcal{Y}'(I) < 0$$

   - Policy that reduces $I \downarrow$ (and hence $D \downarrow$) implies $Y \uparrow$

In standard epi-econ models, both polar positions are present

- Which is ultimately right? Depends, e.g. on available policy tools

Simplest Behavioral SIR Model: Reduced Form

- Define

$$\tilde{\beta}(I) := \beta(\mathcal{Y}(I))$$

- Clearly $\beta' > 0$ and $\mathcal{Y}' < 0 \Rightarrow \tilde{\beta}' < 0$
- Example: $\beta(Y) = \bar{\beta}Y^{\alpha}$ and $\mathcal{Y}(I) = e^{-\sigma I} \Rightarrow \tilde{\beta}(I) = \bar{\beta}e^{-\alpha\sigma I}$
- Reduced form behavioral SIR model:

$$\dot{S} = -\tilde{\beta}(I)SI \tag{S}$$

$$\dot{I} = \tilde{\beta}(I)SI - \gamma I \tag{I}$$

$$\dot{R} = \gamma I \tag{R}$$

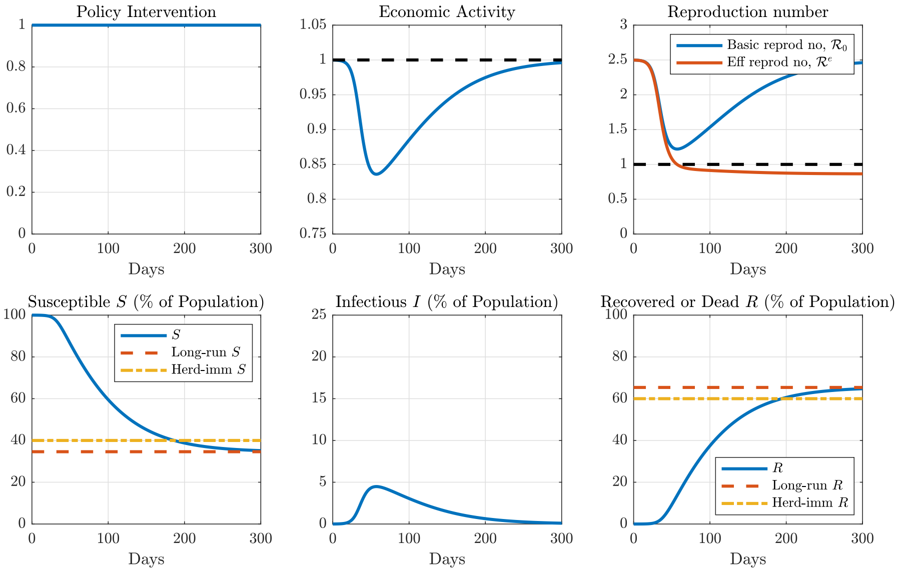
with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

- Matlab codes for all simulations, diagrams here:

`https://benjaminmoll.com/wp-content/uploads/2022/02/SIR_behavioral.m`

`https://benjaminmoll.com/wp-content/uploads/2022/02/SIR_behavioral_intervention.m`

Dynamics of epidemic in behavioral SIR model



In (S, I) Space: Phase Diagram

Recall (dropping equation for R)

$$\dot{S} = -\tilde{\beta}(I)SI \tag{S}$$

$$\dot{I} = \tilde{\beta}(I)SI - \gamma I \tag{I}$$

From (I) we have:

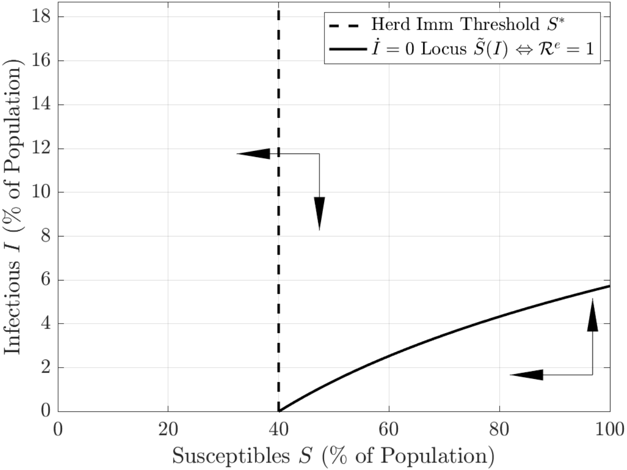
$$\dot{I} = \left(\tilde{\beta}(I)/\gamma \times S - 1\right)\gamma I = (\mathcal{R}^e - 1)\,\gamma I$$

$$\dot{I} = \left(\frac{S}{\tilde{S}(I)} - 1\right)\gamma I \quad \text{where} \quad \tilde{S}(I) := \frac{1}{\tilde{\beta}(I)/\gamma}$$

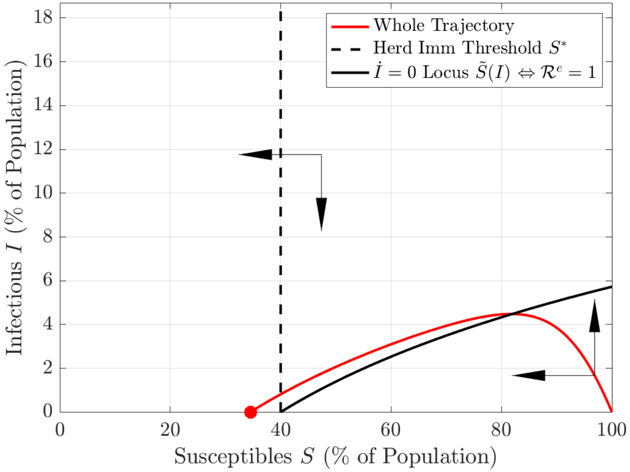
Therefore

1. $\dot{I} > 0$ if $\mathcal{R}^e > 1$ and < 0 otherwise
2. $\dot{I} > 0$ if $S > \tilde{S}(I)$ and < 0 otherwise (similarly $\mathcal{R}^e > 1 \Leftrightarrow S > \tilde{S}(I)$)
3. Whether $\dot{I} > 0$ or $\mathcal{R}^e > 1$ depends not only on S but also on I (in contrast to non-behavioral SIR model where $\dot{I} > 0 \Leftrightarrow S > S^*$)
4. $\dot{I} = 0$ locus $\tilde{S}(I)$ is increasing in (S, I) space

In (S, I) Space: Phase Diagram



In (S, I) Space: Phase Diagram



Note: trajectory "hugs" $\mathcal{R}^e = 1$ locus. In fact, $\mathcal{R}^e$ just < 1 for long time.

Predictions of simple behavioral SIR model

1. Relative to standard (non-behavioral) SIR model, epidemic "overshoots" herd immunity threshold by less

2. Effective reproduction number $\mathcal{R}^e$ just < 1 for long time

   - Gans uses $\mathcal{R}^e \approx 1$ to construct approximate solutions that can be analyzed graphically

Policy Interventions

Policy Interventions

Consider two types of policies

1. ℓ = lockdowns: reduce transmissions via reducing econ activity Y
2. m = "masks": reduce transmissions without affecting activity Y

$$\dot{S} = -\beta(Y, m)SI \tag{S}$$

$$\dot{I} = \beta(Y, m)SI - \gamma I \tag{I}$$

$$\dot{R} = \gamma I \tag{R}$$

$$Y = \mathcal{Y}(I, \ell) \tag{Y}$$

with initial conditions S_0, I_0, R_0 satisfying $S_0 + I_0 + R_0 = 1$

Potential functional forms

- ℓ = lockdowns

$$\mathcal{Y}(I, \ell) = \min\left\{(1-\ell), e^{-\sigma I}\right\}$$

- m = "masks"

$$\beta(Y, m) = \bar{\beta}(1-m)Y^{\alpha}$$

Measures of cumulative economic activity

Recall daily economic activity $Y_t \leq 1$

Different options for cumulating these over time. Two examples

1. present discounted value

$$V_0 = \rho \int_0^\infty e^{-\rho t} Y_t dt$$

   satisfying $V_0 \leq 1$

2. Alternatively, use GDP over the first year of the pandemic

$$\text{GDP} = \int_0^T Y_t dt, \quad T = 365$$

   satisfying GDP $\leq T$

Effects of Policy Interventions

Some questions:

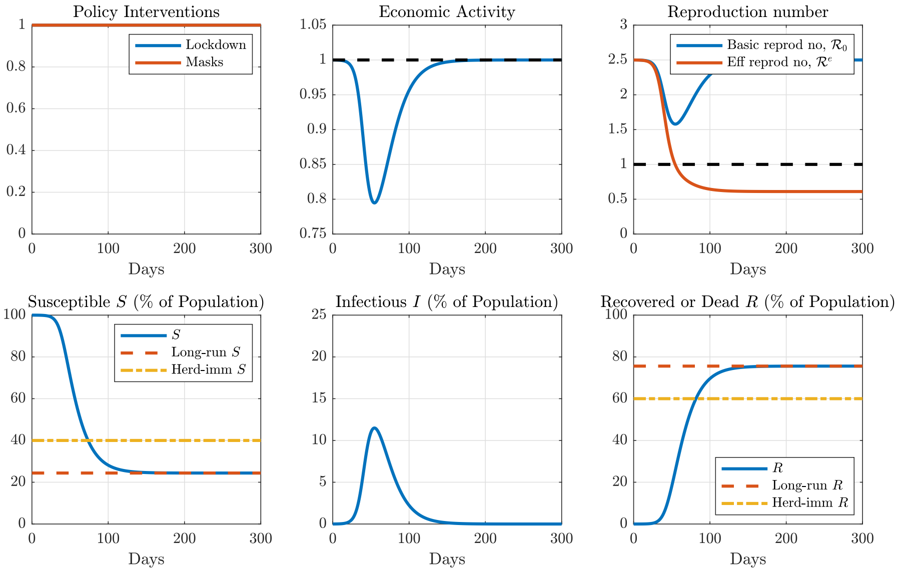
1. Do policies lead to tradeoff between reducing infections (saving lives) and econ activity?

2. How much can policies reduce cumulative infections (disease burden) and how does this depend on size of overshoot?

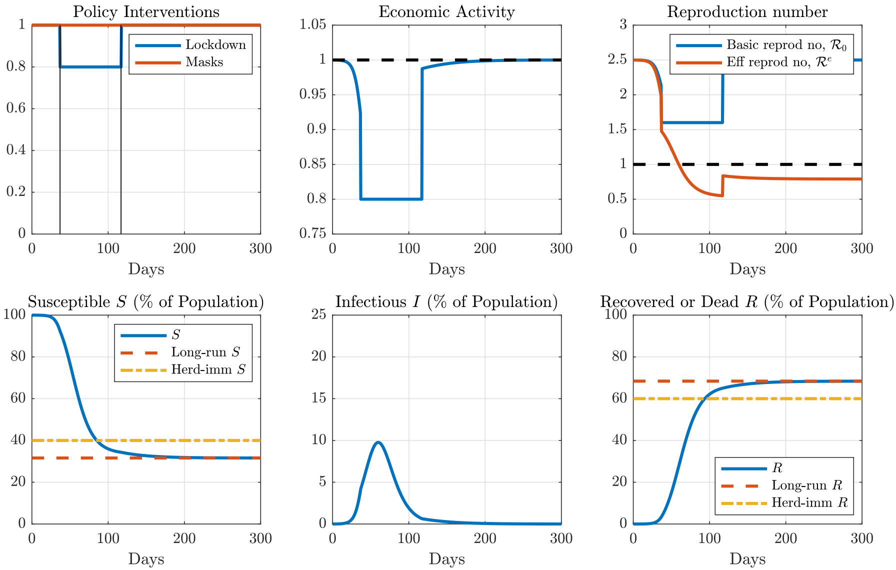
Next slides:

- some simulations of policy interventions
- `https://benjaminmoll.com/wp-content/uploads/2022/02/SIR_behavioral_intervention.m`

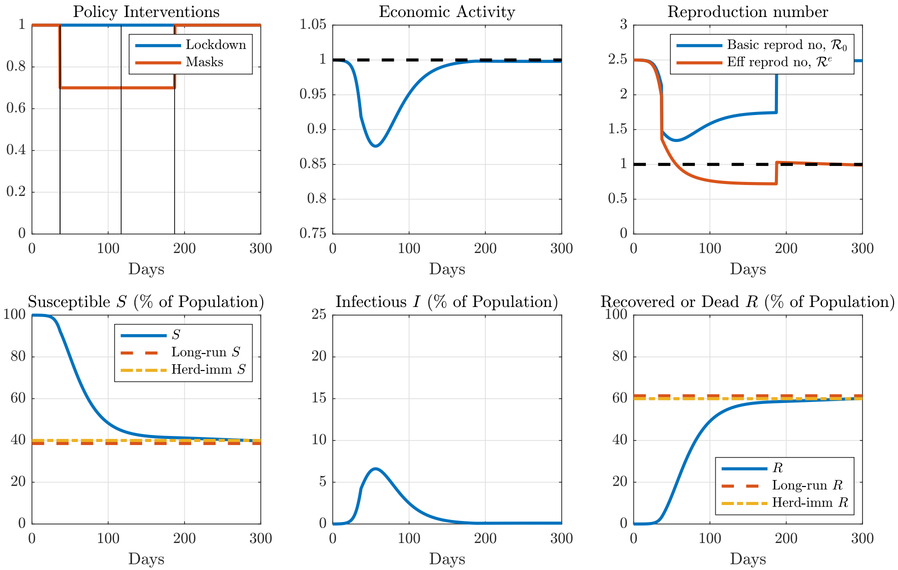
Scenario 1: do nothing



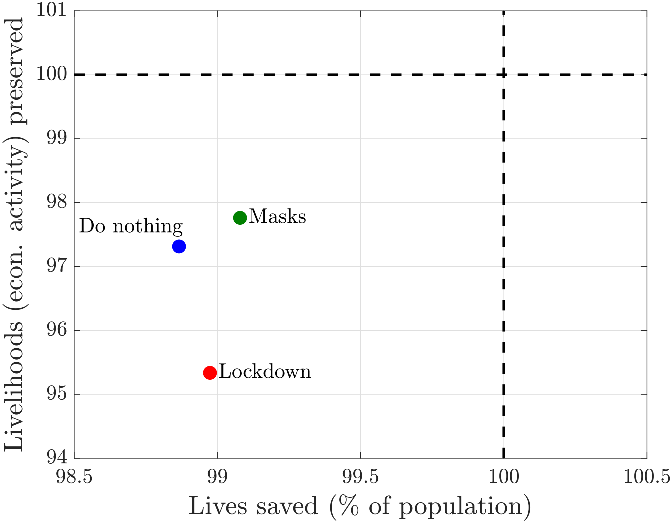
Scenario 2: lockdown



Scenario 3: “masks”



Pandemic Possibility Frontier



"Lives saved" = $1 - D_\infty$, "Livelihoods preserved" = $\frac{1}{T}\int_0^T Y_t\,dt, T = 365$