Appendix to "What if? The macroeconomic and distributional effects for Germany of a stop of energy imports from Russia"

April 29, 2022

for latest version, see https://benjaminmoll.com/RussianGas_Appendix/

A Appendix to Section 2 "The macroeconomic effects of a stop of energy imports from Russia on the German economy"

We pursue a two-pronged approach for assessing the macroeconomic effects. First, we use economic theory to isolate two of the key determinants of the macroeconomic effects of cutting energy imports from Russia. These are (i) the importance of Russian imports of gas, oil and coal ("brown" energy) in production and (ii) the elasticity of substitution between these energy sources and other inputs (e.g. "green" energy).

Second, we use the multi-sector model of Baqaee and Farhi (2021) to run counterfactual simulations of the macroeconomic effects of cutting energy imports from Russia. The Baqaee-Farhi model is a state-of-the-art multi-sector model with rich input-output linkages and in which energy is a critical input in production.

Our findings are as follows:

1. In appendix A.1 we summarize some statistics relating to the German economy's energy dependence that provide important signposts for assessing the effects of an import stop.

2. Standard theory predicts that the losses to the German economy of embargoing energy imports from Russia are extremely sensitive to the degree of substitutability of brown energy with other inputs as measured by the elasticity of substitution between these factors. This elasticity of substitution is hard to discipline empirically, especially for large changes in the economy's input mix of the type we are concerned with, so that any macroeconomic analysis is necessarily subject to a large degree of uncertainty.

3. This elasticity of substitution is likely low in the very short run but larger in the medium- and long-run so that the size of economic losses depends crucially on the time frame over which adjustments take place.

4. We review empirical evidence on this elasticities of substitution (which also equals the own-price elasticity of energy). The meta-analysis by Labandeira et al. (2017) provides a summary of the existing estimates on own-price elasticities for energy consumption differentiated between the short run (less than one year) and the long run (after one year). The relevant short-run average short-run elasticity for energy is -0.22, for natural gas it is -0.18, and the least elastic in the short run is heating oil with -0.02. Differences between residential and industrial consumers are small.

5. Even for elasticities of substitution below this range, the Baqaee-Farhi multi-sector model predicts modest losses of around 0.2-0.3% of German Gross National Expenditure (GNE)

or around €80-120 per year per German citizen.[1] To explain what drives these low losses we provide a simple formula that points to two key sufficient statistics: first the share of energy imports in German GNI (which equals a modest 2.5%) as well as the predicted change in this share (which is determined by the elasticity of substitution). Unless the change in this share is unrealistically large (which would happen for an extremely low elasticity), the GNI loss remains small.

6. Given the uncertainty surrounding elasticities of substitution as well as the structure of production, we use our simple and transparent model to consider some potential worst-case scenarios for extremely low elasticities. We argue that economic losses from a -10% energy shock could be up to 1.5% of German GNE or €600 per year per German citizen, i.e. an order of magnitude higher than the 0.2-0.3% or €80-120 implied by the Baqaee-Farhi model.

7. When the elasticity of substitution is not just low but exactly zero (Leontief production) the economic losses can be even larger. But this case is (a) inconsistent with empirical evidence and (b) makes a number of nonsensical predictions.

8. Rather than aggregating gas, oil and coal into an aggregate " brown energy" input, we treat gas as a separate input that cannot be substituted with oil and coal. As explained in the main text, the resulting shock to gas supply is up to -30%. With an elasticity of substitution between gas and other inputs considerably below estimates in the literature of 0.1, this scenario results in GNE losses of 2.3% or €912 per year per German citizen.

9. We discuss a number of mechanisms that are outside of our model and that could potentially further amplify economic losses (depending on the policy response). To provide a "safety margin" for such missing mechanisms, we round up the 2.3% GNE losses to 3% which is the headline worst-case number featured in the paper's abstract.

10. A supplement available at `https://benjaminmoll.com/RussianGas_Substitution/` discusses in more detail the economic idea of substitution. We provide some historical real-world examples that demonstrate how firms do find ways to substitute in adversity (perhaps unexpectedly even for themselves). And we make some additional general observations on substitution in the macroeconomy, in particular that a commonly held micro "engineering view" of substitution is too narrow and misses important mechanisms through which the macroeconomy would adapt to an import stop.

11. A supplement available at `https://benjaminmoll.com/RussianGas_Literature/` reviews other studies providing quantitative estimates of an import stop.

Replication materials for all results in section 2 as well as the empirical results in section 3 can be found here `https://benjaminmoll.com/RussianGas_Replication/`.

[1] German GNE is €3,175 billion (see World Bank, 2022 `https://data.worldbank.org/indicator/NE.DAB.TOTL.CN?locations=DE`) and Germany has a population of 83 million implying a per-capita GNE of €40,000. It then follows that 0.2-0.3% of GNE are €80-120.

A.1 Fact Sheet: Energy Dependence of the German Macroeconomy

This appendix summarizes some key statistics that provide important guide posts for assessing the macroeconomic effects of an import stop.[2]

Facts on the German economy's energy dependence:

1. German consumption of gas, oil and coal is about 4% of Gross National Expenditure (GNE). For comparison German GNE was €3,175 billion in 2020 and therefore somewhat larger than German GDP of €3,097 billion (i.e. GNE was 2.5% larger than GDP) .[3]

2. Total German *imports* of gas, oil and coal are about 2.5% of GNE.[4]

3. German consumption of gas only is about 1.2% of GNE. Since all gas is imported, this is also the size of total German *imports* of gas relative to GNE.[5]

4. Table 1 summarizes the gas usage of broad economic sectors: households, industry, services, and so on. It compares this to the economic importance of these sectors in terms of employment and gross value added. For example, industry uses 36.9% of total gas while accounting for 22.6% of total employment and 25.9% of gross value added. In contrast, services, trade & commerce use only 12.8% of all gas but account for a much larger fraction of employment (72.8%) and gross value added (69.7%).

5. Table 2 lists key statistics for three industries that would likely be hardest hit by an import stop: Chemicals, Food+, and Metal. These three industries make up for 59% of gas usage within the industrial sector. The combined number of employees in these three industries is about 1.5 million (352 + 941 + 271 = 1,564). For comparison the table also lists the same statistics for the three industries that were hardest hit during the 2020 Covid-19 pandemic: Air Transportation, Hospitality, and Entertainment. All of gross value added, wages, and number of employees of the industries most likely affected by an import stop are roughly comparable in order of magnitude to the hardest hit sectors in 2020. For example, the combined number of employees in the Air Transportation, Hospitality, and Entertainment industries was about 2.6 million (66 + 1894 + 693 = 2,653) and thus higher than the 1.5 million in the industries likely to be most affected by an import stop. It is

[2] Some of the numbers are generated using simple back-of-the-envelope calculations because we were unable to find more direct data sources. Please contact b.moll@lse.ac.uk if you are aware of such more direct data sources.

[3] As discussed in Table 1 in the main text, Germany imports about 60% of its gas, oil and coal. Total and total German imports of gas, oil and coal are roughly €80 bn in 2021 (see https://www.destatis.de/DE/Themen/Wirtschaft/Aussenhandel/Tabellen/einfuhr-ausfuhr-gueterabteilungen.html;jsessionid=7345586EA38C7821B58F6C63E9DAC7A2.live731) implying that total German consumption of gas, oil and coal was €80 bn / 60% = €133 bn. German 2020 GNE is €3,175 billion (see World Bank, 2022 https://data.worldbank.org/indicator/NE.DAB.TOTL.CN?locations=DE) so that German consumption of gas, oil and coal is roughly 4% of GNE. German 2020 GDP is €3,097 billion (see World Bank, 2022 https://data.worldbank.org/indicator/NY.GDP.MKTP.KN?locations=DE).

[4] German GNE is €3,175 billion and total German imports of gas, oil and coal are roughly 80 bn in 2021.

[5] German GNE is €3,175 billion and total German imports of gas and oil are roughly 75 bn in 2021 (see https://www.destatis.de/DE/Themen/Wirtschaft/Aussenhandel/Tabellen/einfuhr-ausfuhr-gueterabteilungen.html;jsessionid=7345586EA38C7821B58F6C63E9DAC7A2.live731). According to Table 1 in the main text, gas imports are roughly the same order of magnitude in volume as oil imports. Hence we calculate the share of gas imports in GNE as $0.5 \times 73/3,175 \approx 1.2\%$

	Households	Industry	Services, T&C	Electricity Gen.	Other
Gas usage (% of total)	30.8	36.9	12.8	12.6	6.9
Employment (% of total)		22.6	72.8	0.6	2.9
Gross Value Added (%)		25.9	69.7	2.2	2.3

Table 1: Gas usage and economic importance of broad sectors of German economy

Notes: The source for gas usage is BDEW (2021). In the first row on gas usage, "Other" includes heating suppliers and transportation. The source for employment and value added is the National Accounts from Eurostat (2020): https://ec.europa.eu/eurostat/databrowser/view/NAMA_10_A64_E__custom_2410757/default/table?lang=en and https://ec.europa.eu/eurostat/databrowser/view/NAMA_10_A64__custom_2410837/default/table?lang=en, repectively. The categories "Industry", "Services, Trade and Commerce", "Electricity Generation", and "Other" are aggregated from the NACE classification of economic activities (see https://ec.europa.eu/eurostat/ramon/nomenclatures/index.cfm?TargetUrl=LST_NOM_DTL&StrNom=NACE_REV2&StrLanguageCode=EN) as follows. Industry is defined as manufacturing and construction. Services, trade & commerce includes wholesale and retail trade; repair of motor vehicles and motorcycles, transportation and storage, accommodation and food service activities, information and communication, financial and insurance activities, real estate activities, professional, scientific and technical activities, administrative and support service activities, public administration and defence; compulsory social security, education, human health and social work activities, arts, entertainment and recreation and other service activities. Other is agriculture, forestry & fishing, mining & quarrying, water supply; sewerage, waste management & remediation activities and activities of households as employers; undifferentiated goods - and services - producing activities of households for own use.

	2022 Crisis (Import Stop)			2020 Crisis (Covid-19)		
	Chemicals	Food+	Metal	Air Trans.	Hospitality	Entert.
Gross Value Added (in € bln)	46	47	21	7	51	43
Gross Output (in € bln)	137	195	104	25	104	69
Wage Bill (in € bln)	27	35	16	5	35	21
Employees (in 1,000)	352	941	271	66	1894	693
Employees (% of total)	0.78	2.08	0.60	0.15	4.18	1.53
Share males (in %)	74	52	88	46	47	49
Capital (in € bln)	179	123	152	30	119	362
Share gas in production (%)	37	12	10			

Table 2: Key statistics for hardest hit industries

Notes: The source for the table is the Volkswirtschaftliche Gesamtrechnungen (2019)

also important that the most affected industries were essentially completely shut down during the Covid-19 pandemic but would likely be able to continue operating to some extent after an import stop.

A.2 Using simple economic theory to identify key parameters determining the macroeconomic effects

We now use simple economic theory to isolate two of the key determinants of the macroeconomic effects of cutting energy imports from Russia. These are (i) the importance of Russian imports of gas, oil and coal ("brown" energy) in production and (ii) the elasticity of substitution between these energy sources and other inputs (e.g. "green" energy).

We start by considering an extremely simple and purposely stylized setup. In this setup we assume that Germany consumes a good Y which is produced using "brown" energy (gas, oil, and coal, i.e. the energy sources imports from Russia) denoted by E as well as other inputs

X (like labor and capital) according to an aggregate production function

$$Y = F(E, X)$$

The goal is to assess the effect of a drop in energy supply E on Y and to identify what features of the production function F are important for determining the size of this effect.[6] To this end, it is useful to specialize the production function further to a constant-elasticity of substitution (CES) production function

$$Y = \left(\alpha^{\frac{1}{\sigma}} E^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} X^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\alpha > 0$ parameterizes the importance of brown energy in production and $\sigma \in [0, \infty)$ is the elasticity of substitution between brown energy and other inputs. The setup is, of course, extremely simplistic in that it only features two factors of production and no input-output linkages. However, Lemma 1 in Appendix A.5 shows that such an analysis can be a good approximation even in a much richer environment like the Baqaee-Farhi model.

The following special cases show that, depending on the value of σ, the macroeconomic effects of a decrease in energy supply E could be extremely different. The examples are complemented by Figure 1 which plots production Y as a function of energy E for different values of the elasticity σ for a simple calibration of the parameter α described in Appendix A.9.[7]

1. $\sigma = 1$, i.e. Cobb-Douglas production $Y = E^{\alpha} X^{1-\alpha}$ so that

$$\Delta \log Y = \alpha \times \Delta \log E \tag{2}$$

Hence production Y declines with energy E but with an elasticity of only α. In our calibration (see Appendix A.9) we choose $\alpha = 0.04$. Therefore, for example, a drop in energy supply of $\Delta \log E = -10\%$ (also a reasonable value, again see Appendix A.9) reduces production by $\Delta \log Y = 0.04 \times 0.1 = 0.004 = 0.4\%$. The solid purple line in Figure 1 provides a graphical illustration and shows that production is quite insensitive to energy E as expected.

2. $\sigma = 0$, i.e. Leontief production $Y = \min\{E/\alpha, X/(1-\alpha)\}$. Starting from an initial optimum, a reduction in E implies that $Y = E/\alpha$ and hence

$$\Delta \log Y = \Delta \log E \tag{3}$$

Therefore, if the elasticity of substitution is exactly zero, production Y drops one-for-one with energy supply E. This is illustrated by the dashed blue line in Figure 1 which plots production Y as a function of energy E for the Leontief case. For example, a drop

[6] In our application Y is really domestic absorption and not output (GDP). This is because energy E is an imported good and so GDP has to net imports. We ignore this distinction in the current appendix but are more careful when discussing our quantitative open-economy model in Section A.5.

[7] The code for producing the figure as well as Figures 2 and 3 below is available at `https://benjaminmoll.com/elasticity/`.

in energy supply of $\Delta \log E = -10\%$ implies a drop in production of $\Delta \log Y = -10\%$. Intuitively, the Leontief assumption means that energy is an extreme bottleneck in production: when energy supply falls by 10%, the same fraction 10% of the other factors of production X lose all their value (their marginal product drops to zero) and hence production Y falls by 10%.

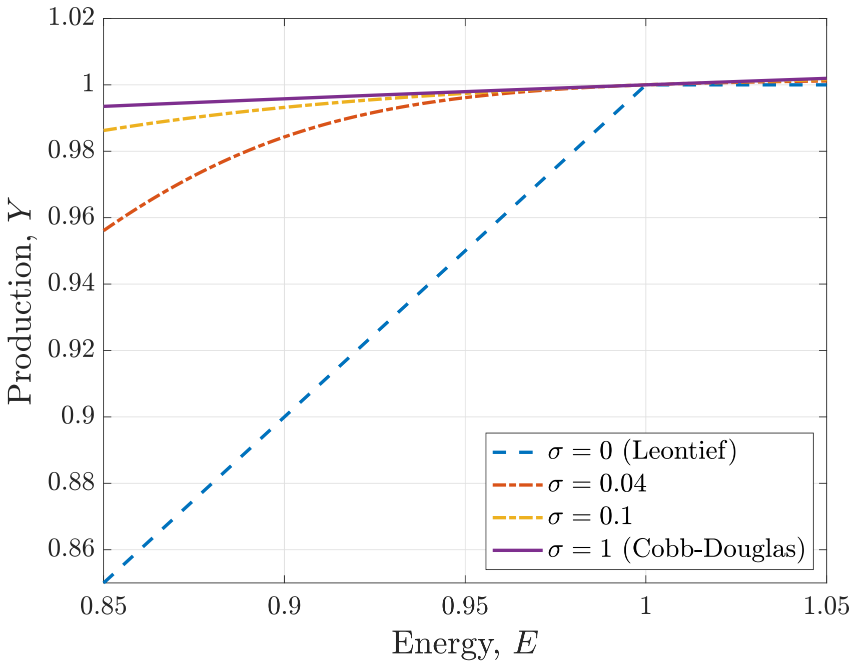


Figure 1: Output losses following a fall in energy supply for different elasticities of substitution

Outside of the simple Cobb-Douglas and Leontief cases laid out above, the dependence of production Y on energy E is more complicated. However, one can derive a simple second-order approximation to (1)

$$\Delta \log Y \approx \alpha \times \Delta \log E + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\tilde{\alpha}(1-\tilde{\alpha}) \times (\Delta \log E)^2 \tag{4}$$

where $\tilde{\alpha} = \frac{\alpha^{\frac{1}{\sigma}}}{\alpha^{\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}}}$. This approximation illustrates in a transparent fashion the importance of the elasticity of substitution σ. When $\sigma = 1$ we recover the Cobb-Douglas special case in (2). However, the formula also shows that with $\sigma < 1$ the losses can be considerably larger (the second term is negative and more so the lower is σ).

One can also simply plot the production function for different values of σ. To this end, consider the red and yellow dash-dotted lines in Figure 1 which plot the cases $\sigma = 0.04$ and $\sigma = 0.1$.[8] Unsurprisingly, the two cases lie in between the cases $\sigma = 0$ and $\sigma = 1$. Somewhat more interestingly, even though both of these two elasticities $\sigma = 0.04$ and $\sigma = 0.1$ are numerically close to zero, the figure reveals that the implications for the dependence of production on

[8]The figure is generated using the Matlab code referenced in footnote 7 (also see the replication materials `https://benjaminmoll.com/RussianGas_Replication/`). In particular we do *not* use the second-order approximation (4) to compute any of our numerical results for the simplified model. The reason is that the second-order approximation is potentially inaccurate for values of the elasticity of substitution σ very close to zero.

energy are potentially quite different from the Leontief case with $\sigma = 0$: even the case $\sigma = 0.04$ lies considerably closer to the Cobb-Douglas case $\sigma = 1$ than the Leontief case $\sigma = 0$. We will return to this point in Appendix A.6 below.

Besides showcasing the importance of the elasticity of substitution, these examples show that (outside of the extreme cases of zero or infinite substitutability) the parameter α also plays a key role for determining the size of economic losses (see the Cobb-Douglas special case (2)). In richer multi-sector models like that of Appendix A.5 there is also another important determinant of macroeconomic losses, namely whether factors of production are stuck in their sectors or can reallocate across sectors. In such models, a low elasticity can be compensated for if resources can be reallocated to maintain production in the critical sector. However, in the short-run, factors are likely relatively immobile and we therefore focus on that case.

For future reference, we also provide another version of the approximation (4). In particular, one can show that the expenditure share of energy $\frac{p_E E}{PY}$ (see Appendix A.9 for the definition) satisfies $\Delta\left(\frac{p_E E}{PY}\right) \approx \left(1 - \frac{1}{\sigma}\right)\tilde{\alpha}(1-\tilde{\alpha})\Delta \log E$.[9] Therefore, we can write (4) as

$$\Delta \log Y \approx \frac{p_E E}{PY} \times \Delta \log E + \frac{1}{2} \times \Delta\left(\frac{p_E E}{PY}\right) \times \Delta \log E. \tag{5}$$

This formula says that the change in the energy expenditure share is informative about the elasticity of substitution σ and hence in turn the output losses from a negative energy shock. An advantage of this formula over (4) is that it is likely easier to decide on what is a reasonable change in the expenditure share than what is a reasonable elasticity of substitution. This is a point we will return to in appendix A.6 below.

These examples show that, even in an extremely simple model like the one above, depending on the value of the elasticity of substitution σ, economic losses of an embargo on Russian energy imports can be very small or large. One main implication of this result is that any macroeconomic analysis of the size of these effects is necessarily subject to a large degree of uncertainty. The reason is that the relevant elasticities of substitution are very hard to discipline empirically, especially for large changes in the economy's input mix of the type we are concerned with.

A.3 Time-dependence of the elasticity of substitution.

A classic result in economic theory is that elasticities tend to be larger in the long run than the short run. This result also applies to elasticities of substitution. Intuitively, in the very short run, production processes can be quite inflexible, i.e. the elasticity of substitution is low; however, over time, production processes can at least partially adapt to the different environment without Russian energy imports, i.e. the elasticity of substitution increases over time. This idea immediately implies that the size of economic losses depends crucially on the time frame over which adjustments take place, with economic losses likely being smaller in the medium- and long-run.

[9] For example in the Cobb-Douglas case $\sigma = 1$, $\frac{p_E E}{PY} = \alpha$ and so $\Delta\left(\frac{p_E E}{PY}\right) = 0$.

As already noted, another determinant of economic losses is how easy it is to reallocate resources across sectors. This likely also differs between the short- and long run. Thus, even if structural (micro) elasticities of substitution do not depend on time horizon, more macro elasticities can depend on the time horizon (because the long-run macro elasticities also capture reallocation across sectors).

A.4 Empirical evidence on elasticities of substitution

In this section, we provide a summary of existing estimates on price elasticities for energy demand. Below, we also explain how to relate them to the elasticity of substitution between inputs that is the parameter of interest for our analysis.

Labandeira et al. (2017) provide a comprehensive overview of the existing estimates in their meta-analysis of existing elasticity estimates for energy demand with a sample of estimates starting in the 1970s. Their analysis distinguishes carefully between short-run and long-run elasticity estimates where they consider all demand changes within one year as short-run and otherwise as long run. In total, their sample contains 966 short-run elasticity estimates and 1010 long-run elasticity estimates and they report an average short-run elasticity of -0.236 and a long-run elasticity of -0.596. After dropping outliers the respective mean (median) elasticities are -0.186 (-0.140) and -0.524 (-0.429). Hence, the long-run elasticity is about three times larger than the short-run elasticity. Their meta-analysis controls then for characteristics of the respective study from which the elasticity estimate is taken. For the 230 studies that consider only natural gas and controlling for the characteristics of the studies, Labandeira et al. (2017) find an average short-run elasticity for natural gas of -0.18 and a long-run elasticity of -0.684. For heating oil, the average short- and long-run estimates across the 44 studies are -0.017 and -0.185, respectively. For the 376 studies that consider energy in general, the estimates are similar with a short-run elasticity of -0.221 and a long-run elasticity of -0.584. They also report differences between industrial consumers and residential consumers but the differences between consumer groups are within 10% of the average estimates.[10]

The paper by Auffhammer and Rubin (2018) provides cleanly identified residential household demand elasticities for natural gas. They find find price elasticities between -0.17 and -0.2 in line with the estimates for short-run demand elasticities in Labandeira et al. (2017). Notably price elasticities have a strong seasonal component. During the summer, Auffhammer and Rubin (2018) find households to be inelastic to price changes whereas elasticities are high during the winter. These seasonal differences can be important for policy if policy wants to induce households to invest in substitution technologies during the summer. Although it could be that high demand elasticities during the winter could result from households expectations of high elasticties during the winter months.

The analysis in Steinbuks (2012) focuses on energy demand elasticities in manufacturing. The study is particularly interesting as it considers in great detail also different production

[10]They also survey the older literature on energy demand elasticities. Short-run demand elasticities in the older literature for natural gas and oil vary over similar ranges as the results reported in (see Table 1 in Labandeira et al. (2017)).

processes in the manufacturing production process such as heating, cooling, or electricity generation. When looking at all processes, the estimated short-run own-price demand elasticity for natural gas is -0.16 and -0.24 in the long-run. For heating processes, the estimated elasticities are more than three times larger in absolute value. The estimates for all processes align with the average short-run estimates in Labandeira et al. (2017).

Overall, we find a range of estimates for own-price short-run elasticities of gas and energy demand that are mainly in the range from -0.15 and -0.25.

To see how the estimated own-price elasticities relate to the elasticity of substitution between inputs, denote the price of energy by p_E and that of other inputs by p_X. It is easy to show that the CES production function (1) implies the following demand curve

$$\frac{E}{X} = \frac{\alpha}{1-\alpha}\left(\frac{p_E}{p_X}\right)^{-\sigma}$$

Assuming that X and p_X are constant, the elasticity of substitution σ is therefore also the own-price elasticity of demand of the energy input. For example, Leontief production $\sigma = 0$ would imply a perfectly inelastic demand curve. Given this result, we can map evidence on this own-price elasticity directly into the elasticity of substitution σ.

A.5 Baqaee-Farhi Multi-Sector Open-Economy Model

A.5.1 Brief description of the model

We briefly describe the main features of the computational model of Baqaee and Farhi (2021). For a more detailed description see their paper and in particular Section 8 and Appendix K. The Baqaee-Farhi model is a state-of-the-art multi-sector model with rich input-output linkages and in which energy is a critical input in production. The model is *designed* to address questions in which production chains play a key role (the words "input-output linkages", "production networks" and "production chains" all mean the same thing), and to think about the propagation of shocks along said production chains, i.e. the "production cascades" that have featured prominently in the popular debate. Put slightly differently (and with apologies for being repetitive): the model is designed to examine a shock to an upstream product (e.g. an energy input) and to make predictions about how this shock propagates downstream through the production chain.

Besides production chains, the Baqaee-Farhi model also features another important ingredient: international trade. This generates an important substitution possibility: when downstream goods become expensive to produce domestically following a stop of Russian energy imports, they can potentially be imported instead. The original application of Baqaee and Farhi (2021) was to examine gains from trades in the presence of said production chains and one the paper's main finding is that "accounting for nonlinear production networks significantly raises the gains from trade." This fact is precisely why we chose to work with the Baqaee-Farhi model: it is known to generate large effects of trade barriers (for example a complete import stop), in particular relative to other models in the literature.

In summary, relative to the simple model in Section A.2, the Baqaee-Farhi model is much richer. In particular, it adds production chains and international trade. These two ingredients have opposite effects on the size of economic losses of an import stop: on the one hand, production chains amplify the effects; but on the other hand, the ability to substitute via international trade dampens the effects. As any model, the Baqaee-Farhi model has some limitations which we discuss in Appendix A.5.5.

The model features 40 countries as well as a "rest-of-the-world" composite country, and 30 sectors with interlinkages that are disciplined with empirical input-output matrices from the World Input-Output Database (Timmer et al., 2015). Each entry of the World Input-Output matrix represents a country-sector pair, e.g. we use data on the expenditure of the German "Chemicals and Chemical Products" sector on "Electricity, Gas and Water Supply" and how much of this expenditure goes to different countries, say how much goes to Germany itself and how much to Russia. The model features a nested CES structure. Besides the input-output matrices, the key parameters of the model are the elasticities σ, θ, γ and ε

- σ is the elasticity of substitution across consumption sectors (30 sectors)
- θ is the elasticity of substitution across value-added and intermediate inputs
- γ is the elasticity of substitution across primary factors
- ε is the elasticity of substitution across intermediate input sectors

In addition to the parameterizations used in Baqaee and Farhi (2021), we also experiment with lower values for these elasticities so as to be conservative.

A.5.2 Which metric for macroeconomic losses? GNE vs GDP

We follow Baqaee and Farhi (2021) and focus on Gross National Expenditure (GNE) or domestic absorption as our main metric for judging macroeconomic damage to the German domestic economy. The main reason is that in many macroeconomic and trade models including the Baqaee-Farhi model, GNE has a welfare interpretation; in contrast, GDP does not. We also note that in the Baqaee-Farhi model, nominal GNE is equivalent to nominal Gross National Income (GNI) so our numbers can also be interpreted as GNI losses.

A.5.3 Theoretical results and back-of-the-envelope calculations

The following theoretical results show which model features and predictions are most informative about the size of GNE losses. These are: (i) the share of brown energy imports (gas, oil and coal) in German GNE, and (ii) by how much this share rises following an embargo of Russian imports. The data show that this share is small at about 2.5% of GNE and the model simulations in the next section imply that, while this share rises considerably, it does not rise by an unreasonably large amount. This will imply that the GNE losses of an embargo on Russian energy are small. These results are new and are not featured in (Baqaee and Farhi, 2021).

Notation: Let W be real GNE, b_i be the share of good i in GNE, and c_i be quantity of good i in GNE. Let x_{ij} be purchases by i of good j. Let y_i be gross production of good i. Let x_i^X be exports of good i. Let D be the set of domestic producers.

Lemma 1. *To first order*

$$\Delta \log W = \sum_{j \notin D} \frac{p_j m_j}{GNE} \Delta \log m_j - \sum_{i \in D} \frac{p_i x_i^X}{GNE} \Delta \log x_i^X \quad \text{where} \quad m_j = \left(\sum_{i \in D} x_{ij} + c_j \right) \text{ for } j \notin D.$$

Hence the change in domestic real GNE is the change in imports minus the change in exports. Additionally assuming that real GNE is homothetic, we can go one step further and obtain a second-order approximation:

$$\begin{aligned} \Delta \log W = & \sum_{j \notin D} \frac{p_j m_j}{GNE} \Delta \log m_j - \sum_{i \in D} \frac{p_i x_i^X}{GNE} \Delta \log x_i^X \\ & + \frac{1}{2} \left[\sum_{j \notin D} \Delta \frac{p_j m_j}{GNE} \Delta \log m_j - \sum_{i \in D} \Delta \frac{p_i x_i^X}{GNE} \Delta \log x_i^X \right]. \end{aligned} \tag{6}$$

As we will explain in more detail below, equation (6) in Lemma 1 is the natural generalization of the approximation (5) for the simple model in appendix A.2. A surprising implication of Lemma 1 is that one can approximately ignore the economy's input-output structure: the economy's input-output matrix does not make an appearance in the equations. Instead, the economy as a whole "behaves like one large representative producer."

It is important to note that this result does *not* mean that "the economy's input-output structure does not matter for the macroeconomy" or the like (which would obviously defeat the purpose of working with a rich multi-sector model like the Baqaee-Farhi model to begin with); instead, the input-output structure will determine how large the changes in the expenditure shares $\Delta \frac{p_j m_j}{GNE}$ are that are important determinants of the economy's overall response to shocks like an import stop – see the second line of (6). Put differently, this is a sufficient statistics result: of course input-output linkages matter but their role is captured by how these expenditure shares respond to shocks.[11]

Application of Lemma 1 to cutting imports from Russia. Denote energy imports by m_E and their price by p_E. Assume that the only import which falls is energy, i.e. $\Delta \log m_j = 0$ for all $j \neq E$. Also assume that other exports are not affected: $\Delta \log x_i^X = 0$.[12] Then the first-order approximation is $\Delta \log W \approx \frac{p_E m_E}{GNE} \Delta \log m_E$ and the second-order approximation is

$$\Delta \log W \approx \frac{p_E m_E}{GNE} \Delta \log m_E + \frac{1}{2} \Delta \frac{p_E m_E}{GNE} \Delta \log m_E. \tag{7}$$

[11] It is also worth noting that this result is not special to our model; instead it is a consequence of production efficiency and therefore holds in a larger class of models with this feature.

[12] Alternatively, we could assume that exports do not rise following the shock, $\Delta \log x_i^X \leq 0$, and that imports of other goods do not fall, $\Delta \log m_j \geq 0$ for $j \neq E$, in which case $\Delta \log W \geq \frac{p_E m_E}{GNE} \Delta \log m_E + \frac{1}{2} \Delta \frac{p_E m_E}{GNE} \Delta \log m_E$, i.e. equation (7) provides an upper bound on GNE losses $|\Delta \log W|$.

Note that the approximation (7) takes exactly the same form as the approximation (5) for the simple model in appendix A.2. The differences are that (i) it holds in a much richer open-economy model with a complex production network, (ii) it features the share of energy *imports* in GNE rather than total energy purchases (because the model is an open-economy model). The intuition for the second-order term is also the same: the change in the GNE share of energy imports $\Delta \frac{p_E m_E}{GNE}$ summarizes in a succinct fashion the substitutability implied by model choices about elasticities, the input-output structure, and so on.

We now conduct some simple back-of-the-envelope calculations to gauge the GNE losses of cutting imports from Russia. Total German imports of gas, oil and coal as a fraction of GNE were around 2.5% – see Fact 2 in Appendix A.1.

Consider first an extreme case in which all energy imports from Russia are cut (all of gas, oil and coal) and Germany cannot substitute any of it (in contrast in the main text we argued that it should be possible to substitute oil and coal). As explained in the main text this accounts for roughly 30% of German energy imports, i.e. $\Delta \log m_E = -30\%$. The second-order approximation also requires a prediction for the change in the energy share of GNE following the embargo $\Delta \frac{p_E m_E}{GNE}$.[13] An extreme scenario would be that this share triples from 2.5% to 7.5%, i.e. $\Delta \frac{p_E m_E}{GNE} = 5\%$. Then

$$\Delta \log W \approx 2.5\% \times -30\% + \frac{1}{2} \times 5\% \times -30\% = -0.75\% - 0.75\% = -1.5\%$$

Thus, even in the case of an extreme scenario of cutting all Russian energy imports and not being able to substitute for any of them and an extreme tripling in the share of energy imports (which reflects a very low elasticity of substitution), the GNE loss would only be 1.5%.

Next consider a case in which Germany manages to substitute for Russian oil and coal but not gas, the main scenario we argued for in Section 1 of the main text. This corresponds to a reduction in energy imports of $\Delta \log m_E = -17\%$.[14] Now assume that the GNE share of energy imports doubles from 2.5% to 5% so that $\Delta \frac{p_E m_E}{GNE} = 2.5\%$. Then

$$\Delta \log W \approx 2.5\% \times -17\% + \frac{1}{2} \times 2.5\% \times -17\% = -0.42\% - 0.21\% = -0.63\%$$

Thus, even in a scenario where substitutability is so low that the GNE share of energy imports doubles, GNE losses are relatively modest at 0.63%. This number is of the same order of magnitude as (though somewhat higher than) the computational results in Table 3 below.

Finally, an important possibility is that gas is a separate input that cannot be substituted with oil and coal. See Appendix A.7 for more on this point. Total German imports of only gas as a fraction of GNE were around 1.2% and total gas imports would likely fall by $\Delta \log m_E = -30\%$.[15] Now assume, very pessimistically, that the GNE share of gas imports triples from

[13] In contrast, the first-order approximation requires only the initial GNE share, i.e. $\Delta \log W \approx 2.5\% \times -30\% = -0.75\%$. But as we will see, second-order terms can be large.

[14] As we explained in the main text, in this scenario, German energy consumption falls by 10%. Germany imports roughly 60% of its energy so that the reduction in energy imports is 10%/60% = 17%.

[15] See Fact 3 in Appendix A.1 for the size of German gas imports. As we explained in the main text, in this scenario, German gas consumption falls by 30%. Germany imports essentially all of its gas so that the reduction in

1.2% to 3.6% so that $\Delta \frac{p_E m_E}{GNE} = 2.4\%$. This yields our preferred back-of-the-envelope calculation:

$$\Delta \log W \approx 1.2\% \times -30\% + \frac{1}{2} \times 2.4\% \times -30\% = -0.36\% - 0.36\% = 0.72\% \quad (8)$$

Thus, even in a scenario where gas is a separate input in production and substitutability is so low that the GNE share of gas imports triples, GNE losses are relatively modest at 0.72%. This number is again of the same order of magnitude as (though somewhat higher than) the computational results in Table 3 below.

A.5.4 Computational Experiment

In all our computational experiments, we make choices that are designed to deliberately make the economic losses to Germany as large as possible.

We run the following experiment: the EU raises trade barriers against all imports from Russia (including energy) that are high enough to choke off of all imports from Russia into the EU. The experiment is therefore more extreme than the one we consider in the rest of the paper for two reasons: first, all imports from Russia are choked off; second, the entire EU implements these trade barriers and not just Germany. The trade barriers take the form of iceberg costs rather than tariffs (tariffs would generate revenues). We also assume that each country has a sector-specific factor endowment that cannot move across sectors, thereby capturing that sectoral reallocation is difficult in the short run. These rigid factor markets mean for example that energy is produced with strong decreasing returns to scale. As already noted these modeling choices make the numbers as big as possible.

Table 3: German GNE losses predicted by Baqaee-Farhi multi-sector model

	Parameterization 1 (as in Baqaee-Farhi)	**Parameterization 2 (low elasticities)**	**Parameterization 3 (very low elast's I)**	**Parameterization 4 (very low elast's II)**
	A. Parameter Values			
θ	0.5	0.1	0.05	0.05
ε	0.2	0.2	0.05	0.05
σ	0.9	0.9	0.9	0.1
	B. German GNE Loss			
DEU	0.19%	0.22%	0.26%	0.30%

We now turn to the parameterization of the elasticities σ, θ, γ and ε we already discussed in appendix A.5.1. The elasticity γ is irrelevant for our experiment of our assumption that factors of production (the three types of labor and capital) are stuck in their respective sectors: γ governs how substitutable factors of production are across sectors, but since these are assumed stuck to begin with γ does not matter. We therefore keep the value $\gamma = 0.5$ of Baqaee and Farhi (2021). In contrast, the elasticities σ and particularly θ and ε are extremely important. We therefore present computational results for four different parameterizations that differ according to the values we choose for θ, ε and σ. Table 3, panel A summarizes the parameter choices.

gas imports is also 30%.

Parameterization 1 is the same as Baqaee and Farhi (2021). Parameterizations 2 to 4 purposely pick lower elasticities, again in the spirit of being as conservative as possible.

Table 3, panel B states the main computational results, namely the losses of German GNE predicted by the model. With the Baqaee-Farhi baseline parameterization the GNE loss is 0.19%; with the lower elasticities in parameterization 2 this number increases to 0.22%; with the even lower elasticities in parameterizations 3 and 4 GNE losses rise to 0.26% and 0.3% respectively. In summary, even for very low elasticities of substitution (as in parameterizations 2 and 3), the Baqaee-Farhi multi-sector model predicts modest losses of around 0.2-0.3% of German Gross National Expenditure (GNE) or around €80-120 per year per German citizen.

A.5.5 Limitations of applying the Baqaee-Farhi model to the particular question of a stop of Russian energy imports

While the Baqaee and Farhi (2021) model is a state-of-the-art multi-sector model with rich input-output linkages, we took it "off the shelf" from an existing paper. It was therefore not "custom-built" for answering the particular policy question at hand: to assess the macroeconomic effects of a stop of energy imports from Russia on the German economy. This implies the following potential limitations which need to be kept in mind when interpreting the GNE losses of 0.2-0.3% reported in Table 2, column 1 in the main text as well as Appendix Table 3:

1. **Gas is not a separate input.** The model features 30 sectors that are based on the classification in the World Input-Output Database (Timmer et al., 2015) and which are listed in Table 5 of Baqaee and Farhi (2021). As stated there, the model features an aggregated "Electricity, Gas and Water Supply" rather than a separate "Gas" sector, i.e. gas is not a separate input in production. In reality, however, gas cannot be substituted with electricity and water in many production processes (e.g. in the chemicals industry). The aggregation therefore means that the GNE losses of 0.2-0.3% generated by the Baqaee-Farhi model are likely an underestimate. Consistent with this, our back-of-the-envelope calculation (8) which covers precisely the case of gas as a separate and critical input in production generates larger GNE losses of 0.72%.

   Appendix A.7 discusses this point further through the lens of our simplified model. The table with our main results, Table 2 in the main text, reports the corresponding results in column 3, labelled "Simplified model, 30% gas shock".

2. **No Keynesian demand effects.** We discuss this limitation further in Appendix A.8. At the same time, a complementary analysis by Bayer et al. (2022) shows that, even taking into account such demand effects, the overall costs would still remain below 3%.

Regarding point 1 about gas not being a separate input in the computational model, it is worth emphasizing again that the back-of-the-envelope calculations in Section A.5.3 are not subject to this criticism. Indeed, our preferred back-of-the-envelope calculation (8) precisely covers the scenario where gas is a separate input in production. More generally, it is also worth repeating what we wrote at the beginning of Appendix A.5.4: within the possibilities of the "off the shelf" Baqaee-Farhi model, we make choices that are designed to deliberately make the economic losses to Germany as large as possible. In particular, the computational exercise is fairly dramatic: it amounts to a total collapse of EU imports from Russia and not just stopping German gas imports.

A.6 Extreme scenarios with low elasticities of substitution and why Leontief production at the macro level is nonsensical

As discussed in section A.5, our simulations and back-of-the-envelope calculations using the Baqaee-Farhi multi-sector model imply that, even for low values of elasticities of substitution, German GNE losses from an embargo of Russian energy imports would likely be modest and below 1%.

However, we have also seen in Section A.2 that *in principle* these losses can be much larger: if the elasticity of substitution σ between brown energy and other inputs were literally zero (Leontief) then production would fall one-for-one with energy supply. Here we examine some other predictions of this simple model and use them to gauge what values of elasticities should be considered reasonable.

Our main takeaways are:

1. The strict Leontief case makes nonsensical predictions with regard to the evolution of marginal products, prices and expenditure shares.

2. Models with elasticities very close to zero make similarly nonsensical predictions.

3. For a calibrated version of the simple model in Section A.2, a reasonable worst-case scenario may be the case $\sigma = 0.04$, i.e. values of σ below 0.04 are nonsensical. An elasticity of 0.04 is also very conservative compared to the empirical evidence in appendix A.4.

4. As we report in appendix A.7, in this extreme case with $\sigma = 0.04$, the simple model predicts output losses following a -10% energy supply shock of 1.5%.

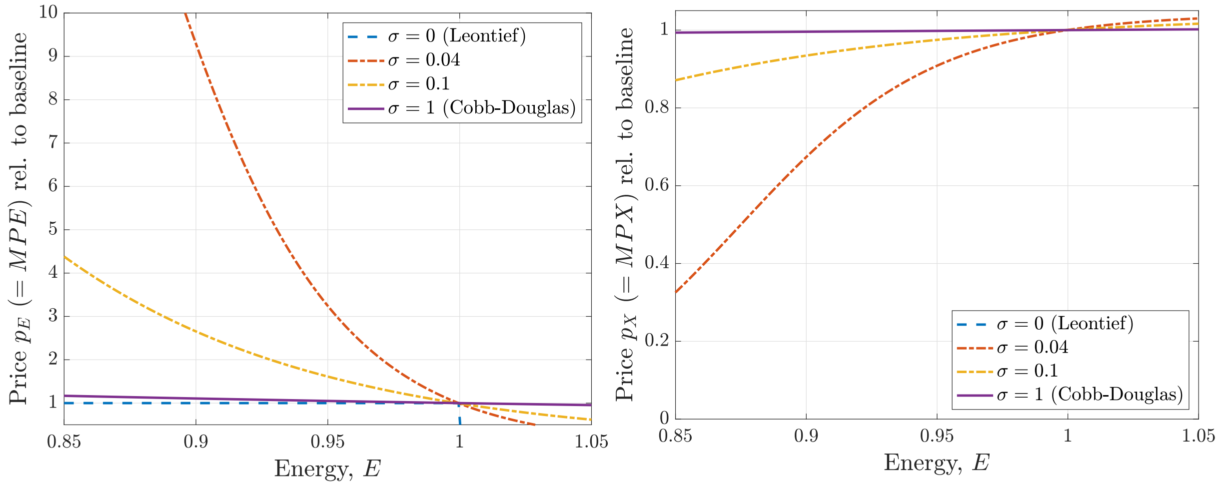
A.6.1 Leontief production $\sigma = 0$ makes nonsensical predictions

The blue dashed line in Figure 1 showed that output falls one-for-one with energy supply in the Leontief case. The blue dashed lines in Figures 2 and 3 plot additional implications of falling energy supply with Leontief production. Figure 2 shows that the marginal product of energy $\partial F(E, X)/\partial E$ jumps to $1/\alpha$ while the marginal product of other factors $\partial F(E, X)/\partial X$ falls to zero. If factors markets are competitive so that factor prices equal marginal products, this then implies that similarly the price of energy jumps to $1/\alpha$ and the prices of other factors fall to zero. Figure 3 shows that this then also implies that the expenditure share on energy jumps to 100% whereas the expenditure share on other factors falls to 0%. We consider these predictions to be economically nonsensical.

A.6.2 What values of σ are still reasonable?

This raises the question: what values of elasticities of substitution are still reasonable? To this end, Figures 2 and 3 plot the behavior of marginal products/prices and the expenditure share for two different values of σ that are close to zero. An elasticity of $\sigma = 0.1$ (yellow dashed line) implies that, following a negative energy supply shock of 10%, the marginal product of energy and hence its price rise by a factor of 2.6, the marginal product/price of other factors falls by roughly 7%, and the expenditure share of energy rises from 4% to 9%. While these numbers are large, they do not seem unreasonable.

Next, an elasticity of $\sigma = 0.04$ (red dashed line) implies that the marginal product of energy and hence its price rise by a factor of almost 10, the marginal product/price of other factors falls by more than 30%, and the expenditure share of energy rises from 4% to 26%, an increase by a factor of 6.5. We consider these huge price and expenditure share movements “borderline reasonable”. We therefore conclude that, for a calibrated version of the simple model in Section A.2, a reasonable worst-case scenario may be the case $\sigma = 0.04$: lower values of σ yield nonsensical results. This value for the elasticity of substitution is also considerably below the range of empirical estimates reported in Appendix A.4.



(a) Energy price

(b) Price of other inputs

Figure 2: Price of energy and other inputs following a fall in energy supply for different elasticities of substitution

A.7 Computational results from simple model in Table 2 in main text

Here we briefly explain how we obtain the computational results in the third and fourth columns in Table 2 in the main text.

Third column: 10% oil, gas, coal shock. Figure 1 plots the output loss for the worst-case scenario with $\sigma = 0.04$ we just discussed in appendix A.6.2. We use the calibration in Appendix A.9. For a 10% energy supply shock, the implied output loss is 1.5% or €600 per year per German citizen. This number is substantially higher than the less than 1% or €400 losses using

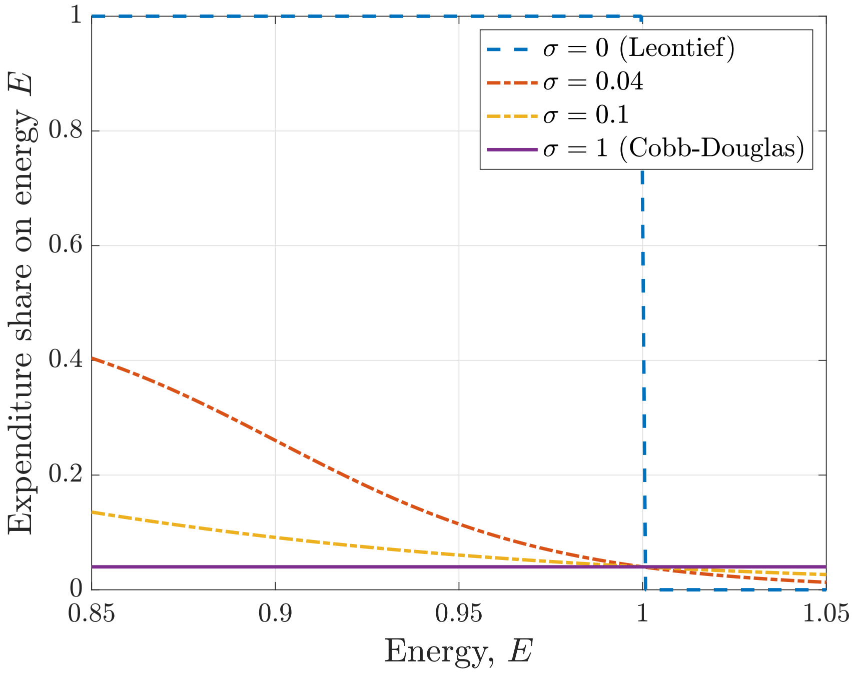


Figure 3: Expenditure share on energy following a fall in energy supply for different elasticities of substitution

the sufficient-statistics approach in column 1 of Table 2 or the 0.2-0.3% or €80-120 implied by the simulations from the Baqaee-Farhi model in column 2.

Fourth column: 30% gas shock. In the computational experiment in column 3 of Table 2, we aggregated gas, oil and coal into an aggregate " brown energy" input. This implicitly assumes that gas can be perfectly substituted with oil and coal which is implausible. We therefore conduct an additional exercise in which we treat gas as a separate input that cannot be substituted with oil and coal. As explained in the main text, the resulting shock to gas supply is up to -30%. We calibrate the model as described in Appendix A.9 and use an elasticity of substitution between gas and other inputs considerably below estimates in the literature of 0.1 (e.g. Steinbuks, 2010, estimates an elasticity of 0.16 to 0.5). As reported in column 4 of Table 2, the 30% gas shock results in GNE losses of 2.3% or €912 per year per German citizen.

A.8 Mechanisms outside the model

A.8.1 Keynesian Demand Effects

The model we use is a real model with no further business cycle amplification stemming from Keynesian demand-side effects in the presence of nominal rigidities. For example, the following mechanism is absent from the model: rising gas prices mean that households have less disposable income; they therefore spend less so that aggregate demand decreases and this sets in motion a standard Keynesian multiplier effects. That is, because of nominal rigidities the decrease in aggregate demand is met by a decrease in aggregate supply (firm production and hiring) which results in a decrease in household labor incomes; this then means that households have less disposable income and spend less; and so on.

The reason we abstract from such Keynesian aggregate demand effects is that they can, in principle, be undone by appropriate monetary and fiscal policy. However, it is important to stress that this appropriate policy response must not be taken for granted. Instead, it requires active intervention by the European Central Bank and the German fiscal authority. On the monetary side, a firm commitment to stable prices can soften the potential trade off between stabilising output and inflation. At the same time, fiscal policy needs and can, through insurance mechanisms like e.g. short term work, take care of second-round demand effects.

With regard to monetary policy, one can potentially view the energy price shocks as akin to a productivity shock. This view would then require the central bank to raise interest rates in order to stabilise inflation. Though dampening economic activity somewhat, this would also alleviate further the direct energy supply problem. Given that the shock also has the potential to increase the profit share of foreign energy importers, the shock has some elements of a shock to markups. In standard theories, these shocks are more difficult to deal with for the central bank because they raise a conflict between stabilising output and inflation.

It is arguably unrealistic to assume that macro stabilization policy can undo such Keynesian demand effects. In this case, the resulting costs need to be added on top of the costs of 0.3 to 2.2% of GDP reported in Table 1 in the main text (note: to arrive at our headline worst-case scenario of 3% in the main text we rounded up 2.2% so as to leave a "safety margin"). A complementary analysis by one of the coauthors of this paper and his collaborators (Bayer et al., 2022) shows that, even taking into account such demand effects, the overall costs would still remain below 3% of GDP.

A.8.2 Financial Amplification Effects

The model also does not include any financial amplification effects. For example, one could imagine that, in the event of an import stop, firms that are heavily gas-reliant could experience short-run liquidity problems and hits to their balance sheets. This may be the case even for firms that remain viable in the long-run because they are able to substitute for gas or other intermediate inputs affected by an import stop over time. In the event that such problems occur, policy should likely step in to minimize such financial amplification effects, e.g. by temporarily bailing out affected firms. If necessary, the government could acquire equity stakes in the affected companies (as happened in the case of Lufthansa during the Covid-19 pandemic).

A.9 Calibration of Simple CES Production Function in Appendix A.2

Calibration of α. As explained in Appendix A.7 we conduct two computational experiments using our simplest model (CES production function): a 10% energy shock in a model in which oil, gas and coal are aggregated into a common energy input and a 30% gas shock in a model in which gas is a separate input in production. Depending on the experiment, we choose the parameter α in the CES production function (1) so as to match the share of consumption of gas, oil and coal in German GNE which is given by about 4% – see Fact 1 in Appendix A.1 – or just gas which is given by about 1.2% – see Fact 3.

The calibration proceeds as follows. Importantly, our calibration strategy ensures that the model fits the share of energy imports in German GNE for any value of the elasticity substitution σ, i.e. we can vary σ while always matching this import share by construction. Cost minimization of (1) implies the following optimal factor demands

$$E = \frac{\alpha p_E^{-\sigma}}{\alpha p_E^{1-\sigma} + (1-\alpha) p_X^{1-\sigma}} PY, \qquad X = \frac{(1-\alpha) p_X^{-\sigma}}{\alpha p_E^{1-\sigma} + (1-\alpha) p_X^{1-\sigma}} PY \tag{9}$$

where p_E is the price of energy, p_X is the price of the other input and $P = \left(\alpha p_E^{1-\sigma} + (1-\alpha) p_X^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ is a price index. Therefore expenditure shares are

$$\frac{p_E E}{PY} = \frac{\alpha p_E^{1-\sigma}}{\alpha p_E^{1-\sigma} + (1-\alpha) p_X^{1-\sigma}}, \qquad \frac{p_X X}{PY} = \frac{(1-\alpha) p_X^{1-\sigma}}{\alpha p_E^{1-\sigma} + (1-\alpha) p_X^{1-\sigma}}$$

In the simulations below we normalize $p_E = p_X = 1$. This implies

$$\frac{p_E E}{PY} = \alpha, \qquad \frac{p_X X}{PY} = 1 - \alpha.$$

To match the GNE share of energy imports of 4% in the first experiment we then set $\alpha = 0.04$. In particular note that the CES specification in (1) together with this calibration strategy implies that the model fits the share of energy imports in German GNE for any value of the elasticity substitution σ. Similarly, to match the GNE share of gas of 1.2% we set $\alpha = 0.012$.

Calibration of σ. For the calibration of the elasticity σ we make use of the empirical evidence in Appendix A.4 and additionally apply the reasoning in Appendix A.6.2. In the first experiment (10% energy shock) we use $\sigma = 0.04$. In the second experiment (30% gas shock) we use $\sigma = 0.1$. Both values lie considerably below the range of empirical estimates reviewed in Appendix A.4.

A.10 Proof of Lemma 1

The proof uses the notation of Baqaee and Farhi (2021) and appendix A.5 which we briefly recap for the reader's convenience:

- W is real GNE
- b_i is the share of good i in GNE
- c_i is quantity of good i in GNE
- x_{ij} is purchases by i of good j
- y_i is gross production of good i
- x_i^X is exports of good i
- D is the set of domestic producers

With this notation, we have that the change in real GNE satisfies

$$d\log W = \sum_i b_i d\log c_i.$$

Production of good i is used either for consumption c_i, as an intermediate in domestic production $x_{ji}, j \in D$, or exported x_i^X (i.e. good i is either purchased by domestic or foreign customers)

$$y_i = c_i + \sum_{j\in D} x_{ji} + x_i^E.$$

Therefore

$$d\log c_i = \frac{p_i y_i}{p_i c_i} d\log y_i - \sum_j \frac{p_i x_{ji}}{p_i c_i} d\log x_{ji} - \frac{p_i x_i^X}{p_i c_i} d\log x_i^X,$$

where for example $(p_i y_i)/(p_i c_i)$ is nominal production of good i divided by nominal consumption of the same good. Finally production of good i satisfies

$$d\log y_i = \sum_{j\in D} \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} + \sum_{j\notin D} \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij}$$

where $(p_j x_{ij})/(p_i y_i)$ is the share of good i that is used by firm j which is either domestic $j \in D$ or foreign $j \notin D$.

Using these relationships we have:

$$\begin{aligned}
d\log W &= \sum_{i\in D} \frac{p_i c_i}{GNE} \left[\frac{p_i y_i}{p_i c_i} d\log y_i - \sum_{j\in D} \frac{p_i x_{ji}}{p_i c_i} d\log x_{ji} - \frac{p_i x_i^X}{p_i c_i} d\log x_i^X \right] + \sum_{i\notin D} \frac{p_i c_i}{GNE} d\log c_i \\
&= \sum_i \left[\frac{p_i y_i}{GNE} d\log y_i - \sum_{j\in D} \frac{p_i y_i}{GNE} \frac{p_i x_{ji}}{p_i y_i} d\log x_{ji} - \frac{p_i y_i}{GNE} \frac{p_i x_i^X}{p_i y_i} d\log x_i^X \right] + \sum_{i\notin D} \frac{p_i c_i}{GNE} d\log c_i \\
&= \left[\sum_{i\in D} \sum_{j\in D} \frac{p_i y_i}{GNE} \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} + \sum_{i\in D} \sum_{j\notin D} \frac{p_i y_i}{GNE} \frac{p_j x_{ij}}{p_i y_i} d\log x_{ij} \right] \\
&- \sum_{i\in D} \sum_{j\in D} \frac{p_i y_i}{GNE} \frac{p_i x_{ji}}{p_i y_i} d\log x_{ji} - \sum_{i\in D} \frac{p_i y_i}{GNE} \frac{p_i x_i^X}{p_i y_i} d\log x_i^X + \sum_{i\notin D} \frac{p_i c_i}{GNE} d\log c_i \\
&= \sum_{i\in D} \sum_{j\notin D} \frac{p_i y_i}{GNE} \frac{p_j x_{ji}}{p_i y_i} d\log x_{ij} - \sum_{i\in D} \frac{p_i y_i}{GNE} \frac{p_i x_i^X}{p_i y_i} d\log x_i^X + \sum_{i\notin D} \frac{p_i c_i}{GNE} d\log c_i \\
&= \sum_{j\notin D} \frac{p_j}{GNE} d\left(\sum_{i\in D} x_{ij} \right) - \sum_{i\in D} \frac{p_i x_i^X}{GNE} d\log x_i^X + \sum_{i\notin D} \frac{p_i c_i}{GNE} d\log c_i \\
&= \sum_{j\notin D} \frac{p_j}{GNE} d\left(\sum_{i\in D} x_{ij} + c_j \right) - \sum_{i\in D} \frac{p_i x_i^X}{GNE} d\log x_i^X. \\
&= \sum_{j\notin D} \frac{p_j m_j}{GNE} d\log m_j - \sum_{i\in D} \frac{p_i x_i^X}{GNE} d\log x_i^X \quad \text{where} \quad m_j = \left(\sum_{i\in D} x_{ij} + c_j \right) \text{ for } j \notin D. \square
\end{aligned}$$

References in Appendix A

Auffhammer, Maximilian and Edward Rubin, "Natural gas price elasticities and optimal cost recovery under consumer heterogeneity: Evidence from 300 million natural gas bills," Technical Report, National Bureau of Economic Research 2018.

Baqaee, David and Emmanuel Farhi, "Networks, Barriers, and Trade," Working Paper, UCLA 2021.

Bayer, Christian, Alexander Kriwoluzky, and Fabian Seyrich, "Energieembargo gegen Russland würde Wirtschaft in Deutschland kalkulierbar belasten, Fiskalpolitik wäre in der Verantwortung," Nr. 80, DIW Aktuell 2022.

Labandeira, Xavier, Jose M. Labeaga, and Xiral Lopez-Otero, "A meta-analysis on the price elasticity of energy demand," *Energy Policy*, 2017, *102*, 549–568.

Steinbuks, Jevgenijs, "Interfuel substitution and energy use in the UK manufacturing sector," *The Energy Journal*, 2012, *33* (1).

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. Vries, "An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production," *Review of International Economics*, August 2015, *23* (3), 575–605.