

# Putting the 'Finance' into 'Public Finance'

## A Theory of Capital Gains Taxation

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Princeton

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LSE

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Zurich

# Capital gains taxes in practice

- Capital gains typically taxed upon realization

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- Capital gains typically taxed **upon realization**
- But recent policy proposals
  - tax capital gains **on accrual**  
(Biden administration...)
  - tax **wealth**  
(Piketty, Zucman...)
- Old idea: **Haig-Simons** comprehensive income tax:

$$\text{income} = \text{consumption} + \Delta \text{wealth}$$

# Classics

**Auerbach (1989):** “Many of the distortions associated with the present system of capital gains taxation result from its deviation from the Haig-Simons approach. These deviations may have historical explanations but their persistence is hard to rationalize from an economic perspective.”



SEPTEMBER 23, 2021

# What Is the Average Federal Individual Income Tax Rate on the Wealthiest Americans?

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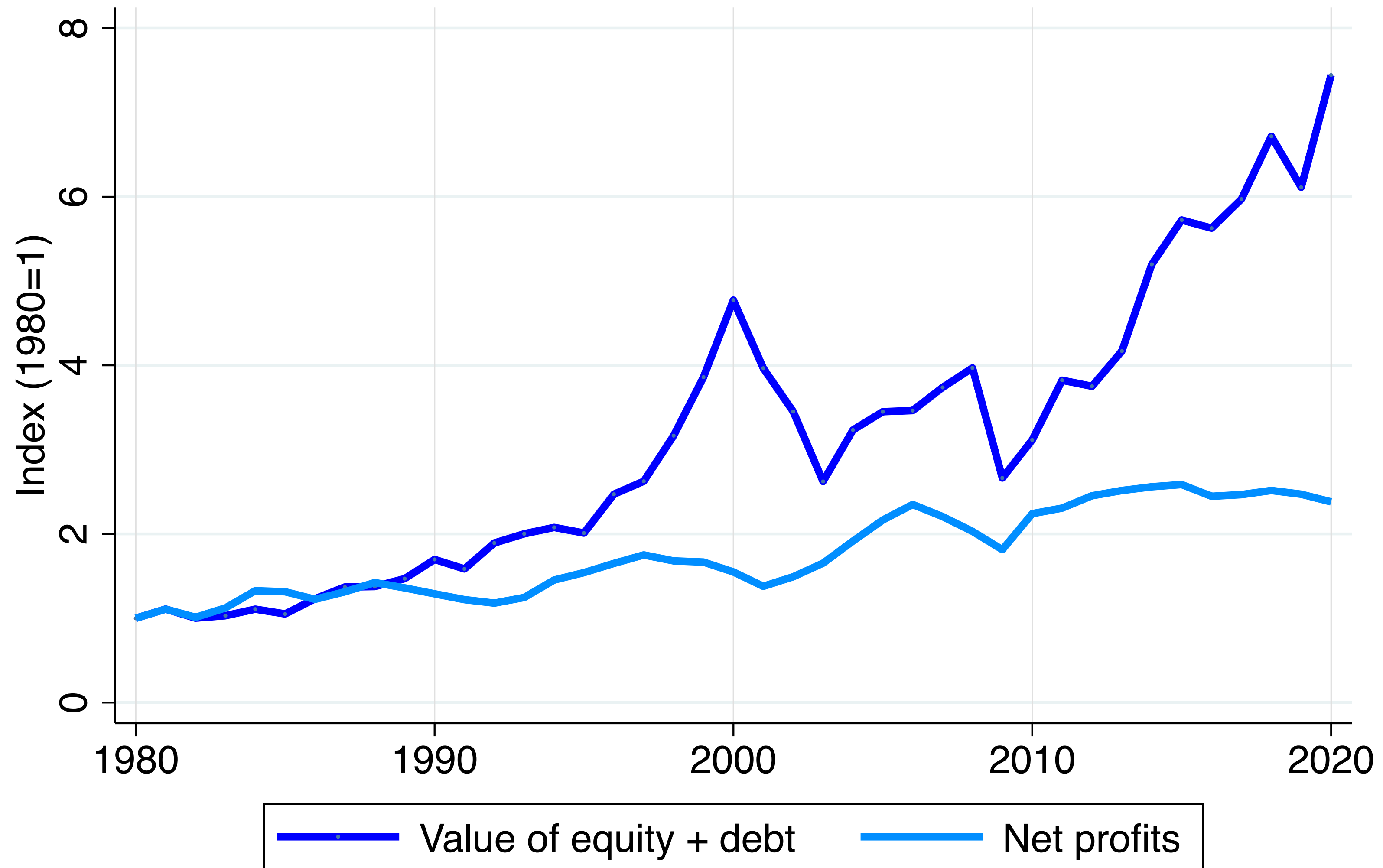
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By Greg Leiserson, Senior Economist (CEA); and Danny Yagan, Chief Economist (OMB)

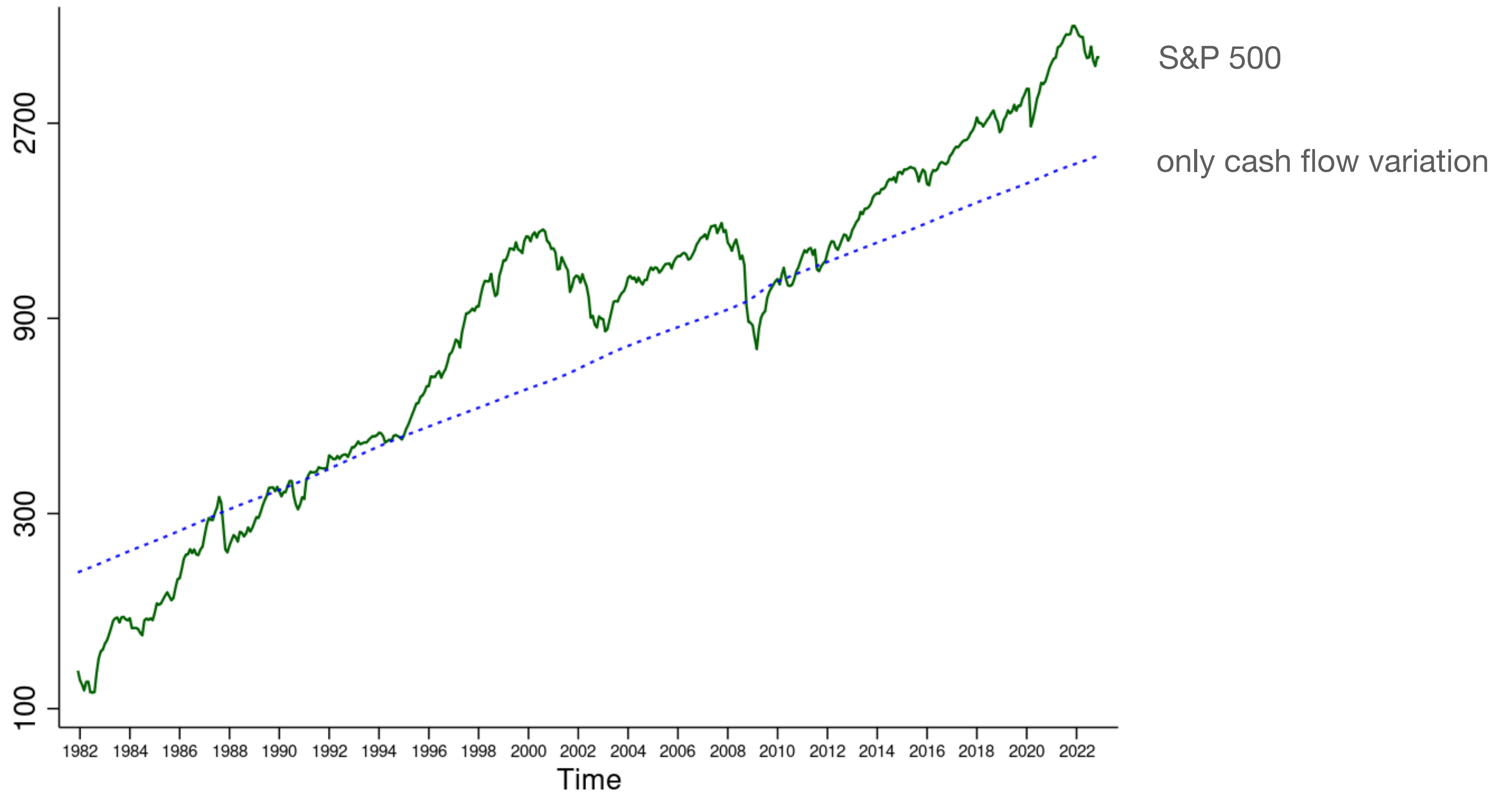
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Abstract: We estimate the average Federal individual income tax rate paid by America's 400 wealthiest families, using a relatively comprehensive measure of their income that includes income from unsold stock. We do so using publicly available statistics from the IRS Statistics of Income Division, the Survey of Consumer Finances, and Forbes magazine. In our primary analysis, we estimate an average Federal individual income tax rate of **8.2 percent** for the period 2010-2018. We also present sensitivity analyses that yield estimates in the 6-12 percent range. The President's proposals mitigate two key

# Capital gains from rising asset prices

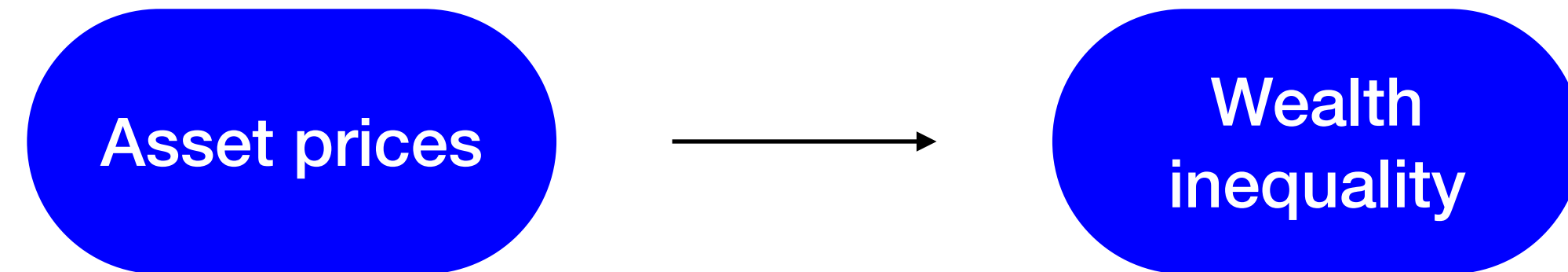


# Conventional view in asset pricing



Bordalo-Gennaioli-La Porta-OBrien-Shleifer (2023), following Shiller (1981), Campbell-Shiller (1988), ...

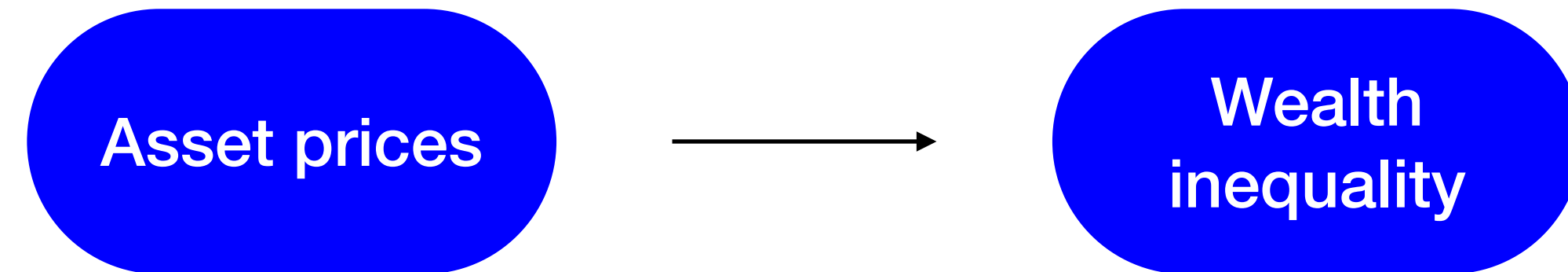
# How to tax capital gains from rising asset prices?



Kuhn et al. (2020), Greenwald et al. (2021), Fagereng et al. (2021, 2023), Martínez-Toledano (2023)...



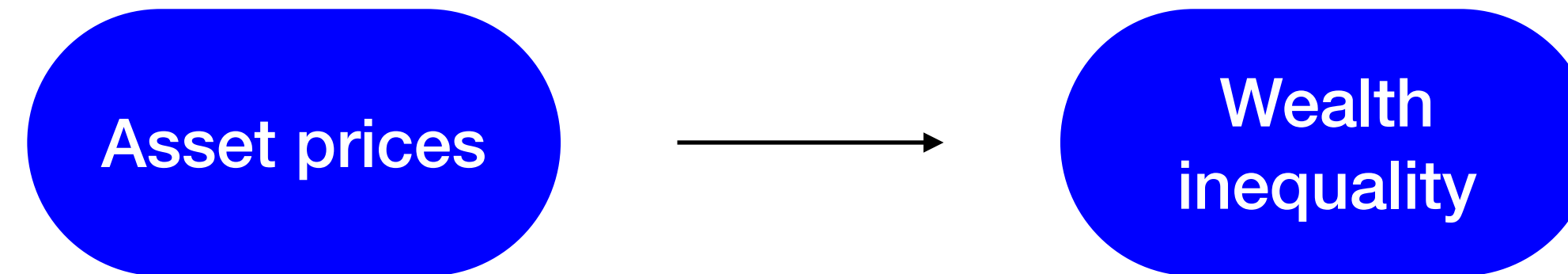
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When asset prices rise, how should optimal tax system adjust?

**No guidance from existing theories of capital taxation:**

**No asset prices!**

# What we do

Redistributive taxation with changing asset prices

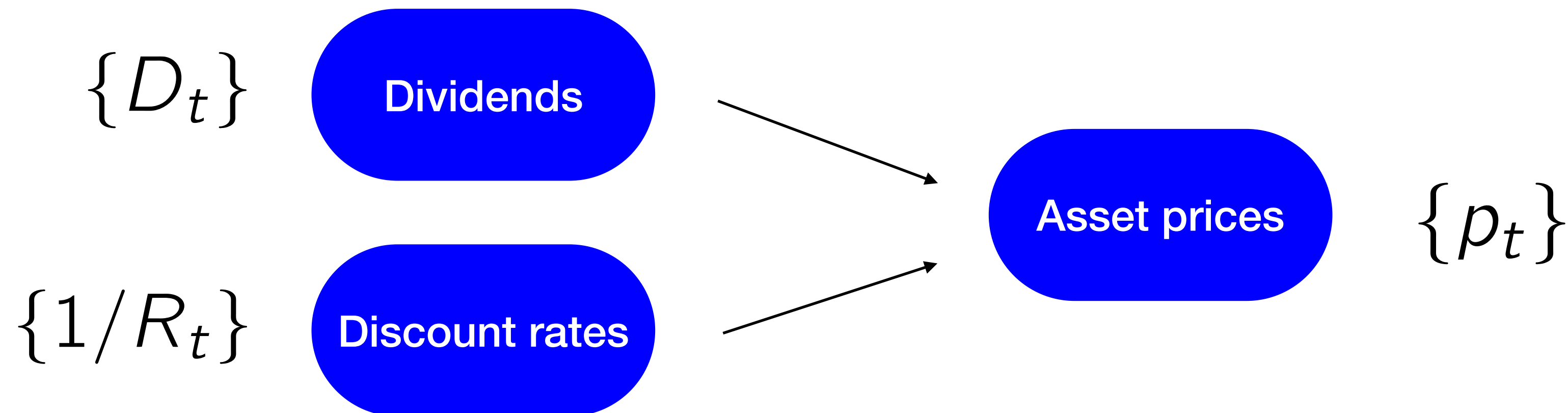
$$R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t} \quad \text{dividend yield + capital gain}$$

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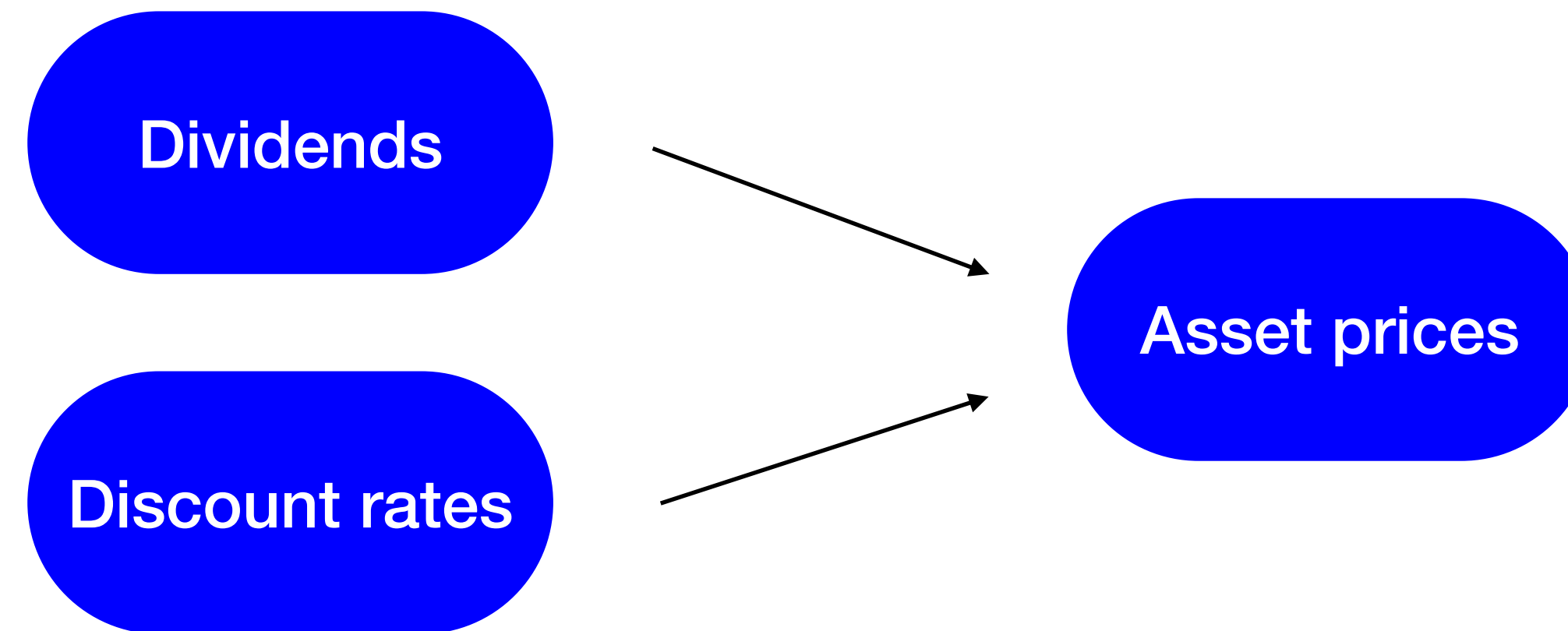
$$R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t} \quad \text{dividend yield + capital gain}$$

Asset pricing



# What we find

$$\Delta T = \tau \times \text{wealth} \times \Delta p$$

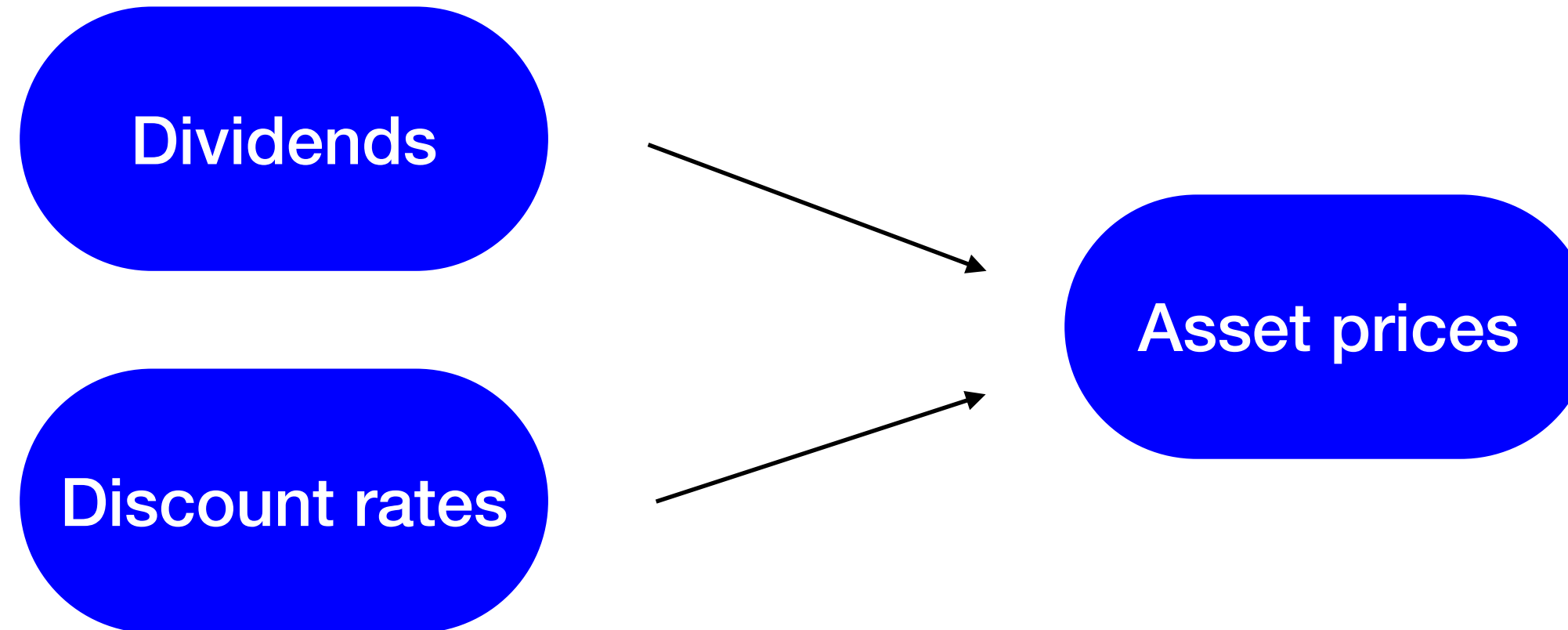


# What we find

Haig-Simons



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Dividends

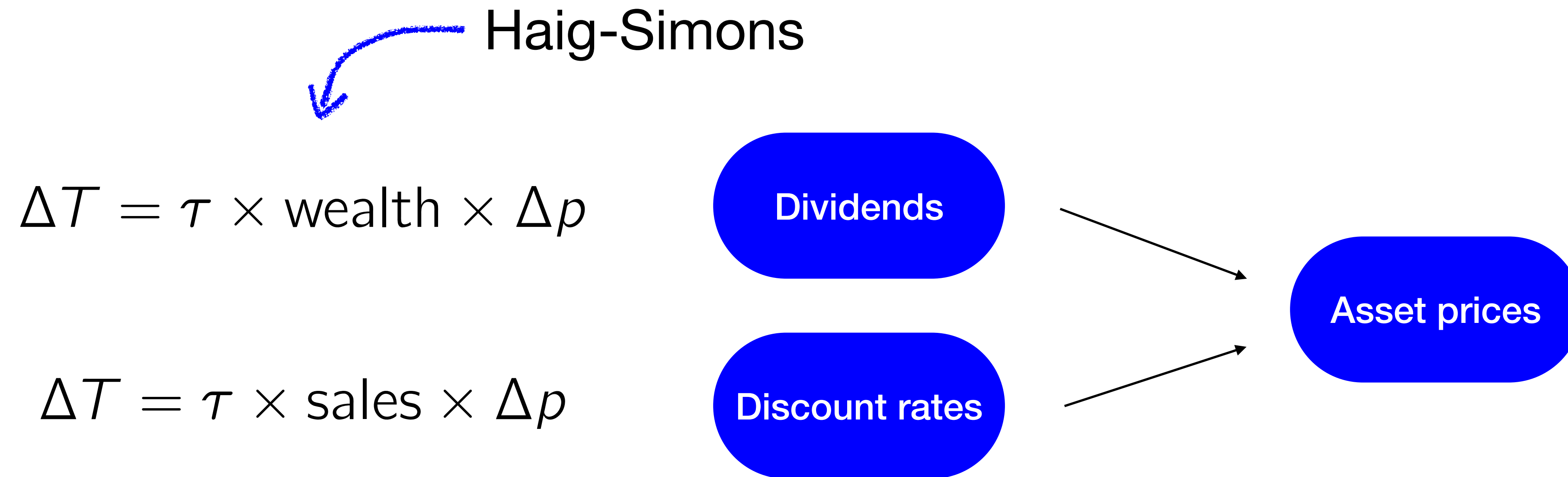
$$\Delta T = \tau \times \text{sales} \times \Delta p$$

Discount rates

Asset prices



# What we find



Beyond simplest case: ~~Haig-Simons~~ even with dividend-driven  $\Delta p$

In general, combination of realization-based capital gains & dividend tax



# Plan

1. Benchmark model (no risk, partial equilibrium)
2. Two periods
3. First-best
4. Second-best (Mirrlees)
5. Extensions
  - Back to multi-period model
  - General equilibrium
  - Heterogeneous returns
  - Risk and borrowing

Environment

# Investors

Indexed by  $\theta \sim F(\theta)$ , differ in initial wealth and income

$$V = \max_{\{c_t, k_{t+1}\}_{t=0}^T} U(c_0, \dots, c_T) \quad \text{s.t.} \quad c_t + p_t(k_{t+1} - k_t) = y_t + D_t k_t - T_t$$

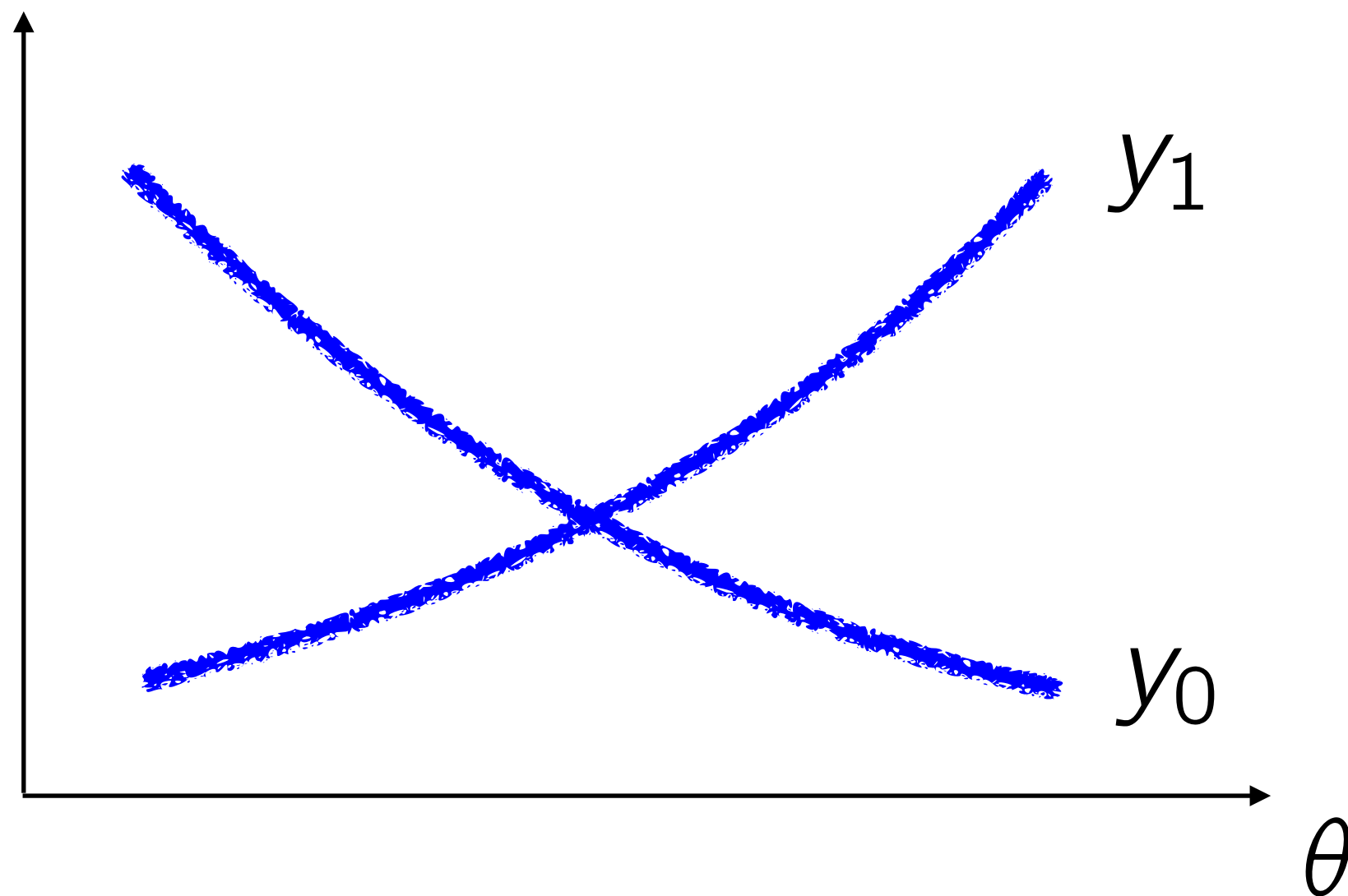
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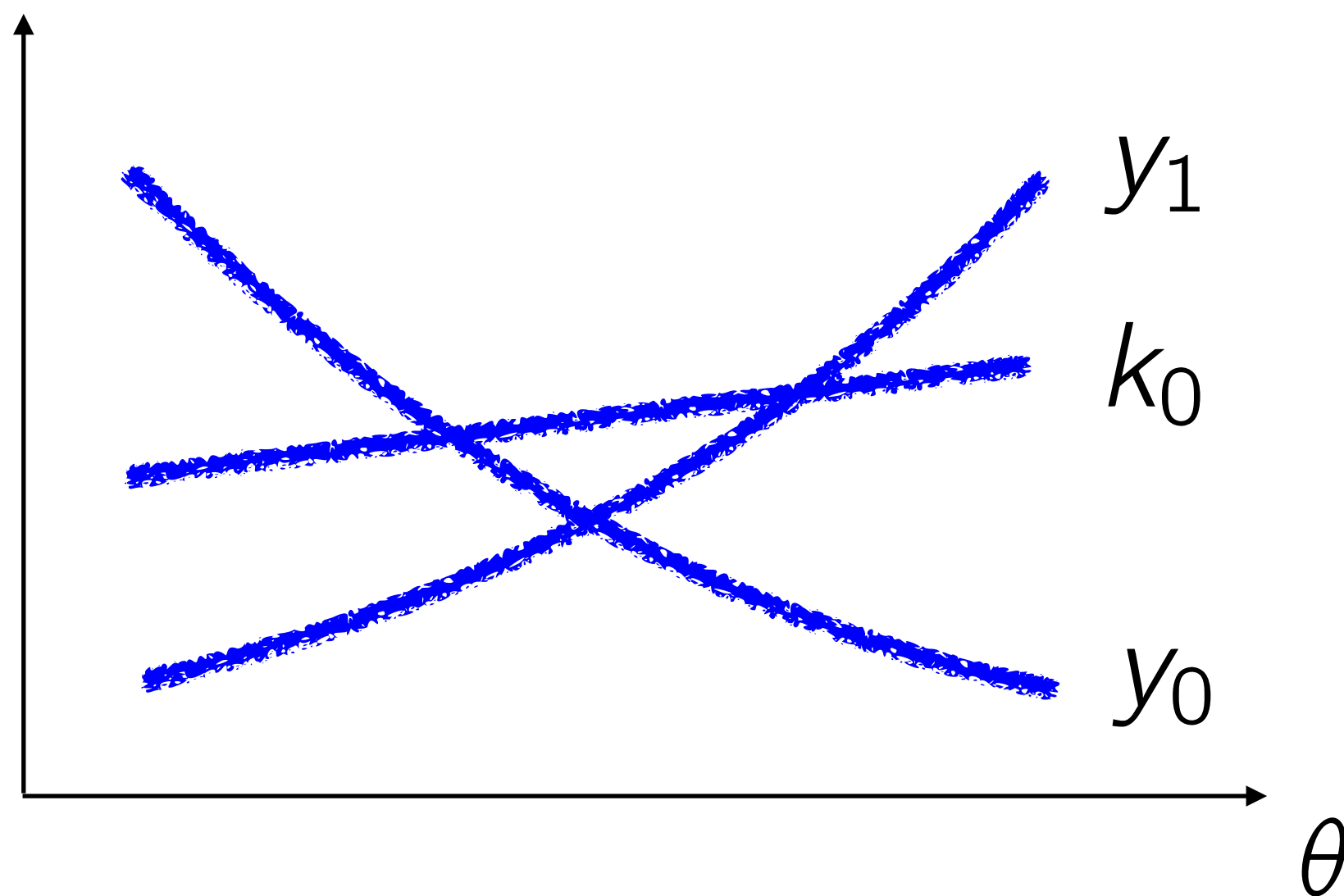


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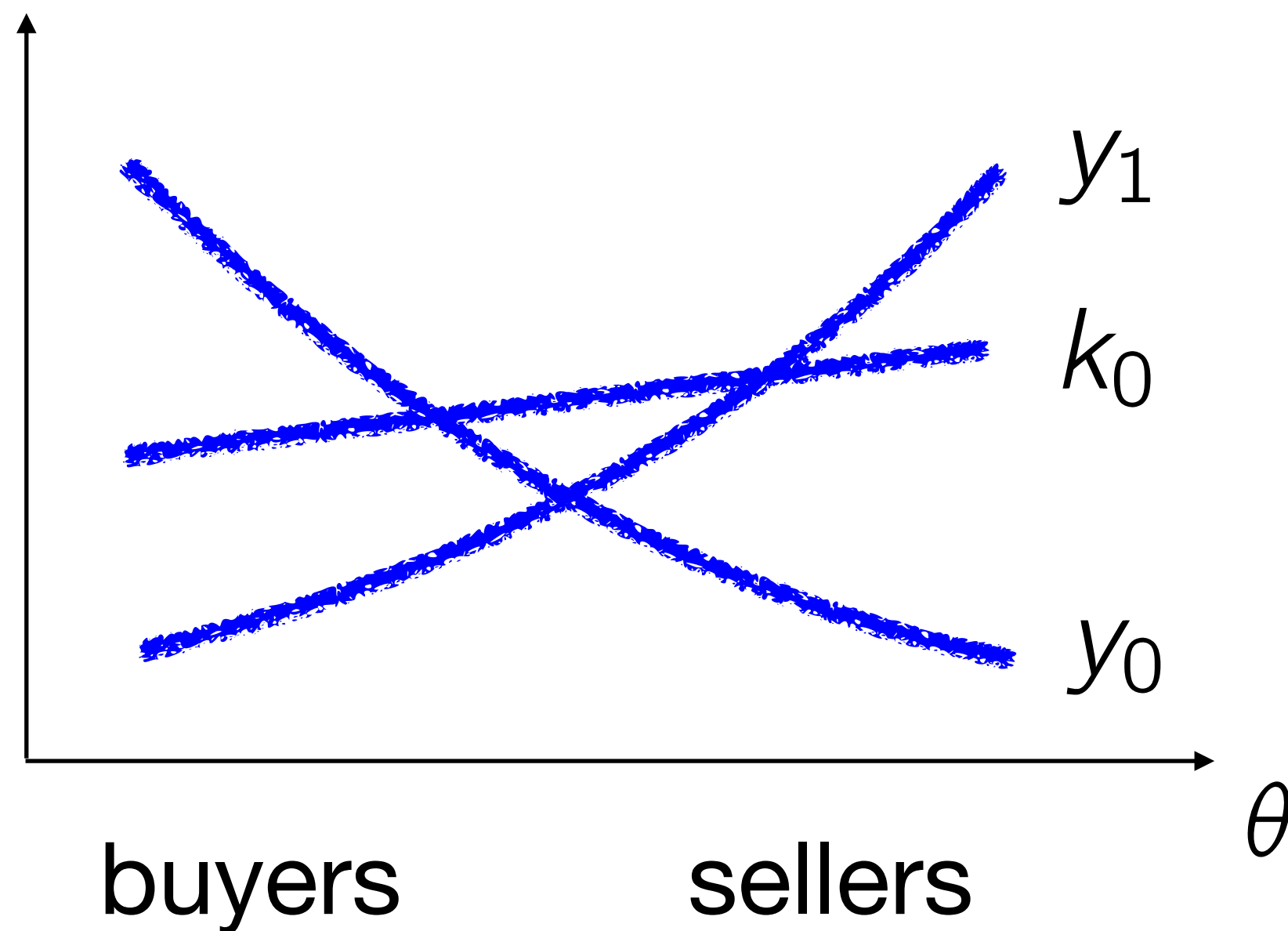


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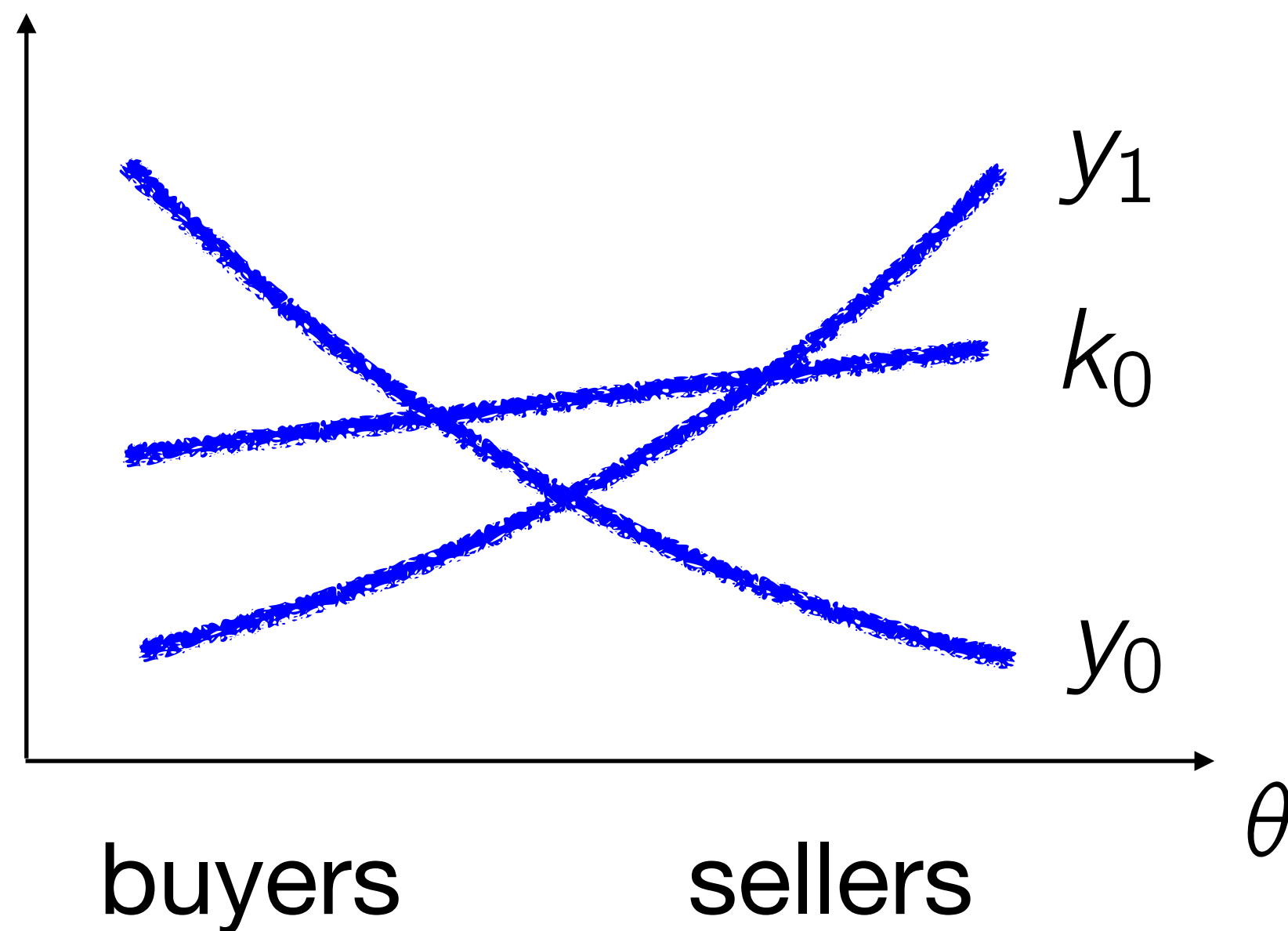


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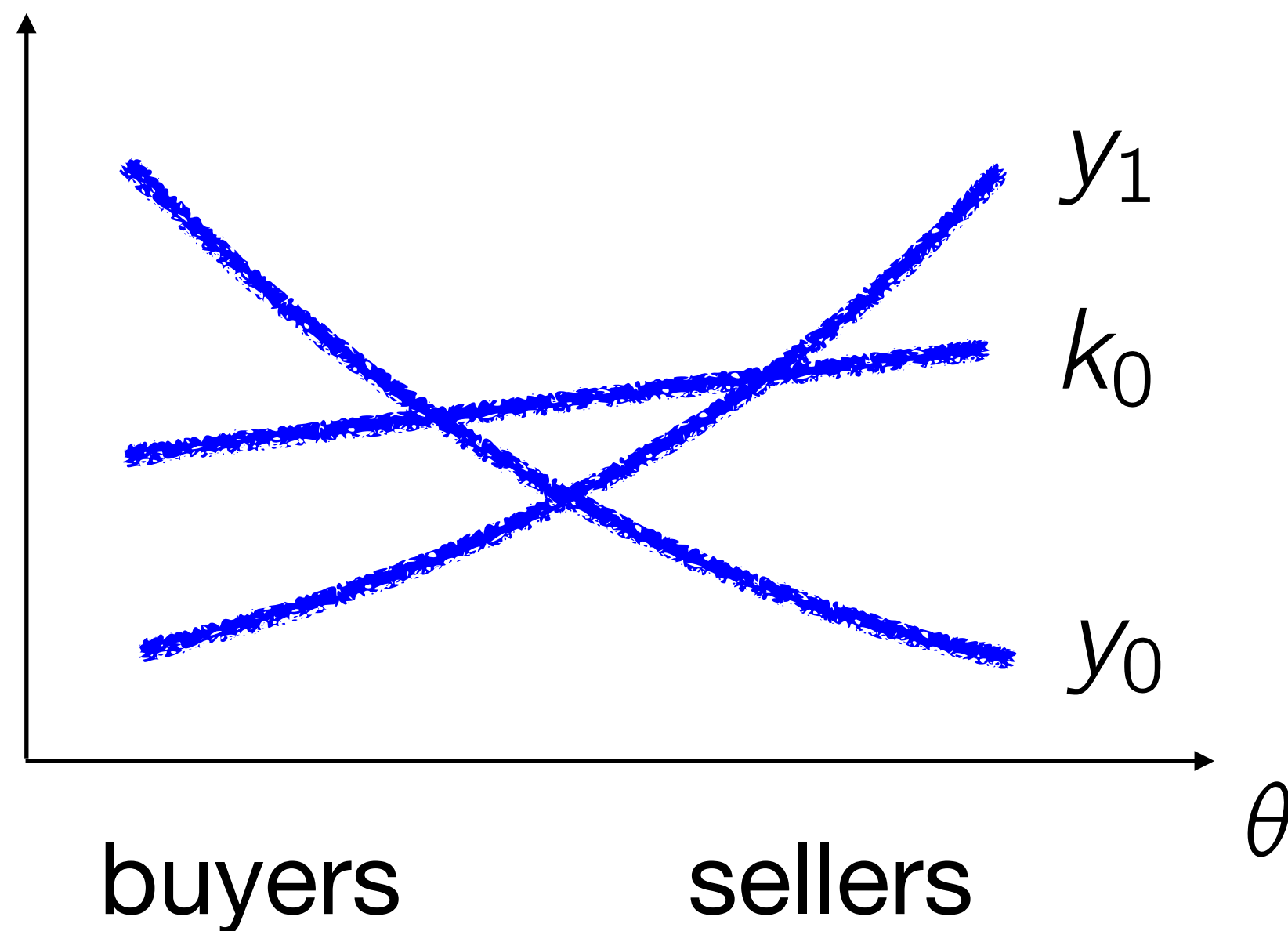


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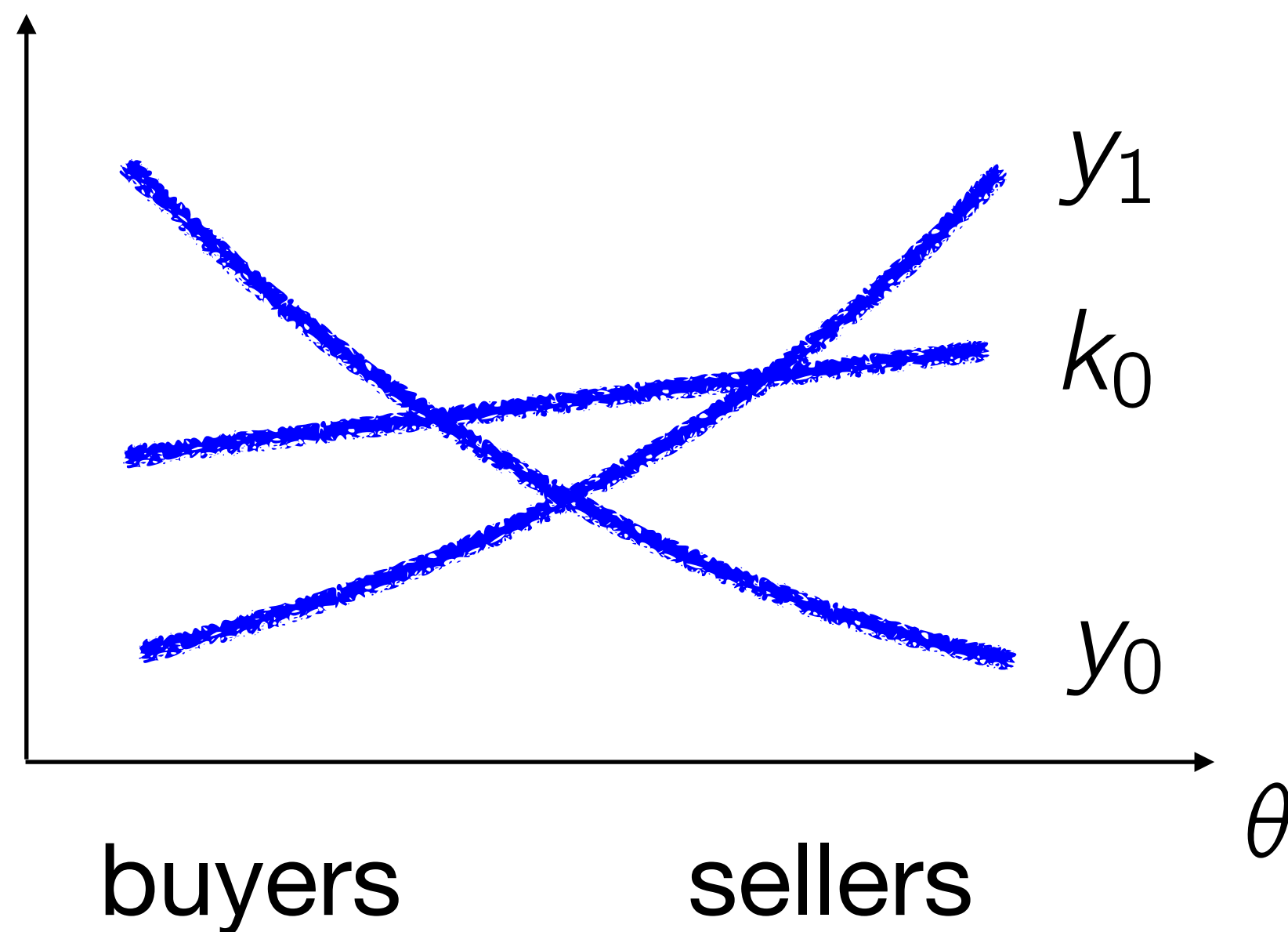


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Add  $(p_t - p_{t-1})k_t$  on both sides:

$$c_t + \underbrace{p_t k_{t+1} - p_{t-1} k_t}_{\text{change in wealth}}$$

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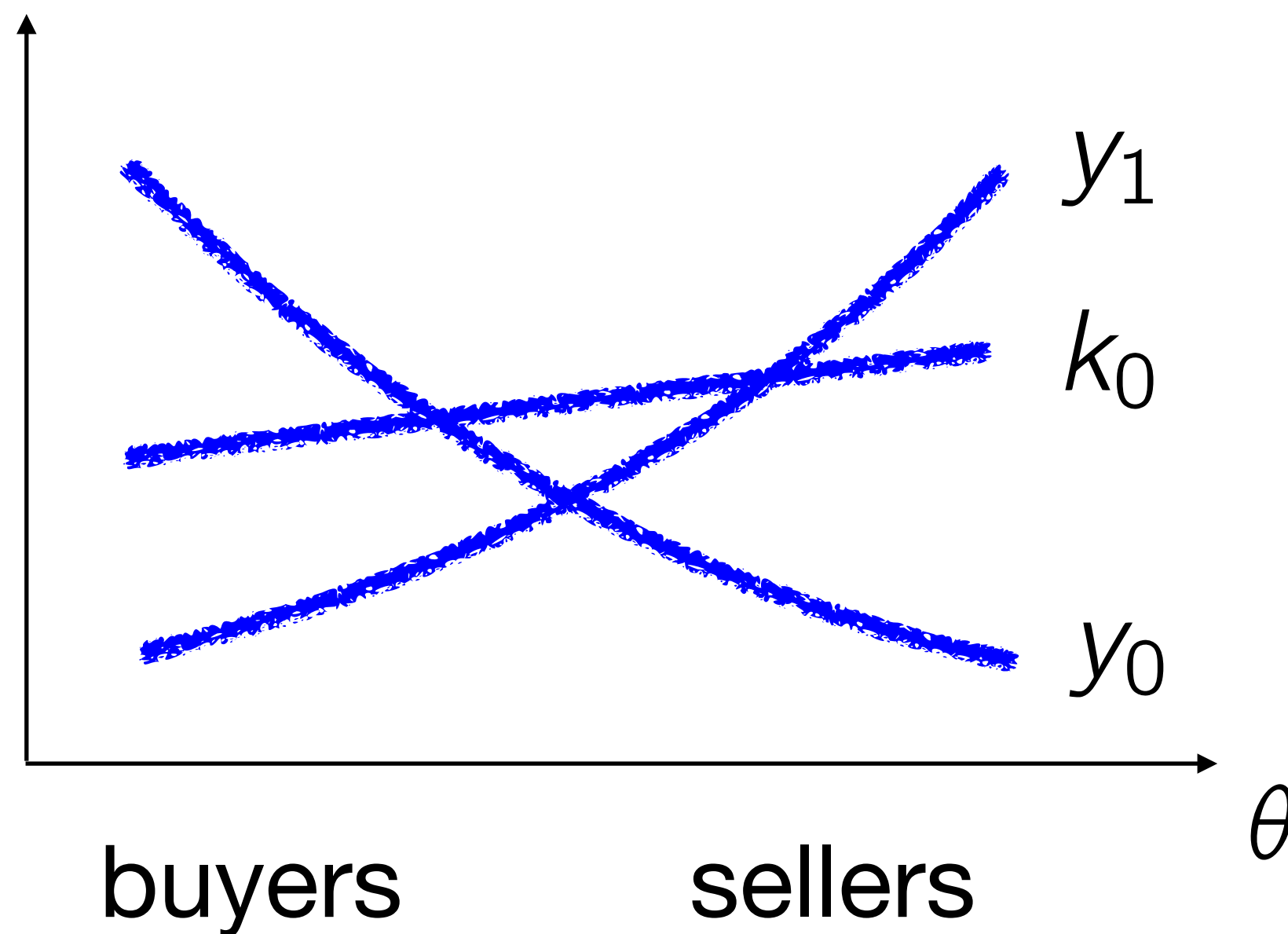
# Investors

Literature

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Resource constraints

$$\int c_0(\theta) dF(\theta) + \frac{p}{D} \int c_1(\theta) dF(\theta) \leq Y$$

$$Y \equiv \int y_0(\theta) dF(\theta) + \frac{p}{D} \int y_1(\theta) dF(\theta) + p \int k_0(\theta) dF(\theta)$$

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First-best

# Pareto problem

Individual lump-sum taxes  $T_0(\theta)$

$$\max_{c_0(\theta), c_1(\theta)} \int \omega(\theta) U(c_0(\theta), c_1(\theta)) dF(\theta) \quad \text{s.t.}$$

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$$U(c_0, c_1) = G(C(c_0, c_1)), \quad C(c_0, c_1) = \left( c_0^{\frac{\sigma-1}{\sigma}} + \beta c_1^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad G(C) = \frac{C^{1-\gamma}}{1-\gamma}$$

# Changing asset prices

**Proposition:** Suppose the asset price increases by  $\Delta p$  while dividends  $D$  remain unchanged. The change in the optimal tax  $T_0(\theta)$  is

$$\Delta T_0(\theta) = x(\theta)\Delta p - \Omega(\theta)X\Delta p$$

100% tax on realized capital gains

aggregate asset sales

$$\frac{\omega(\theta)^{1/\gamma}}{\int \omega(\theta')^{1/\gamma} dF(\theta')}$$

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- Sales  $x$  at **new** price
- Tax on **net** transactions
- **Subsidy** if  $x < 0$

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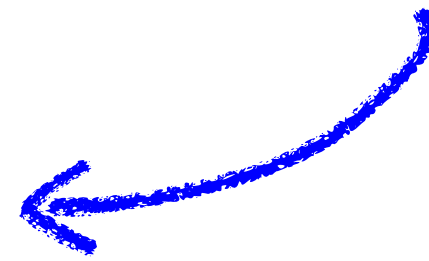
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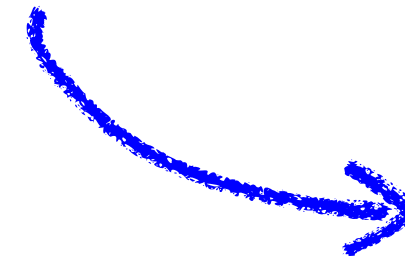
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tax on realized  
capital gains



tax on dividend  
income



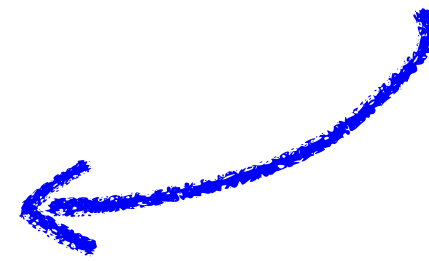
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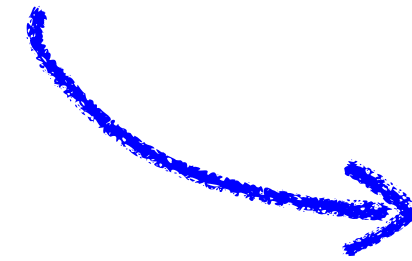
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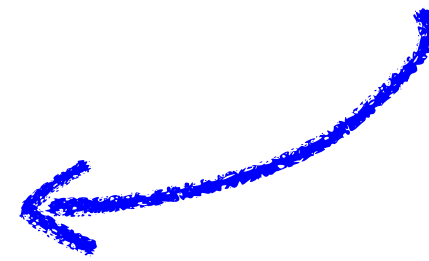
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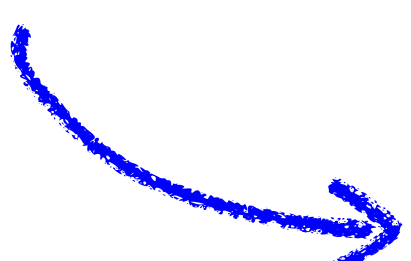
Special case  $\Delta D/\Delta p = D/p$ ? Asset price change driven *only* by dividends



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$$= \frac{p}{D}(k_0(\theta) - x(\theta))\frac{D}{p}\Delta p$$

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# Special case: fixed discount rates

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aggregate  
wealth

100% tax on  
wealth increase

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- In general, tax must depend on realizations

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Consumption  
Tax

Second-best

# Distortive nonlinear taxes

1. Capital sales tax  $T_x(px)$
2. Wealth tax  $T_k(pk_1)$

$$c_0 = y_0 + px - T_x(px)$$

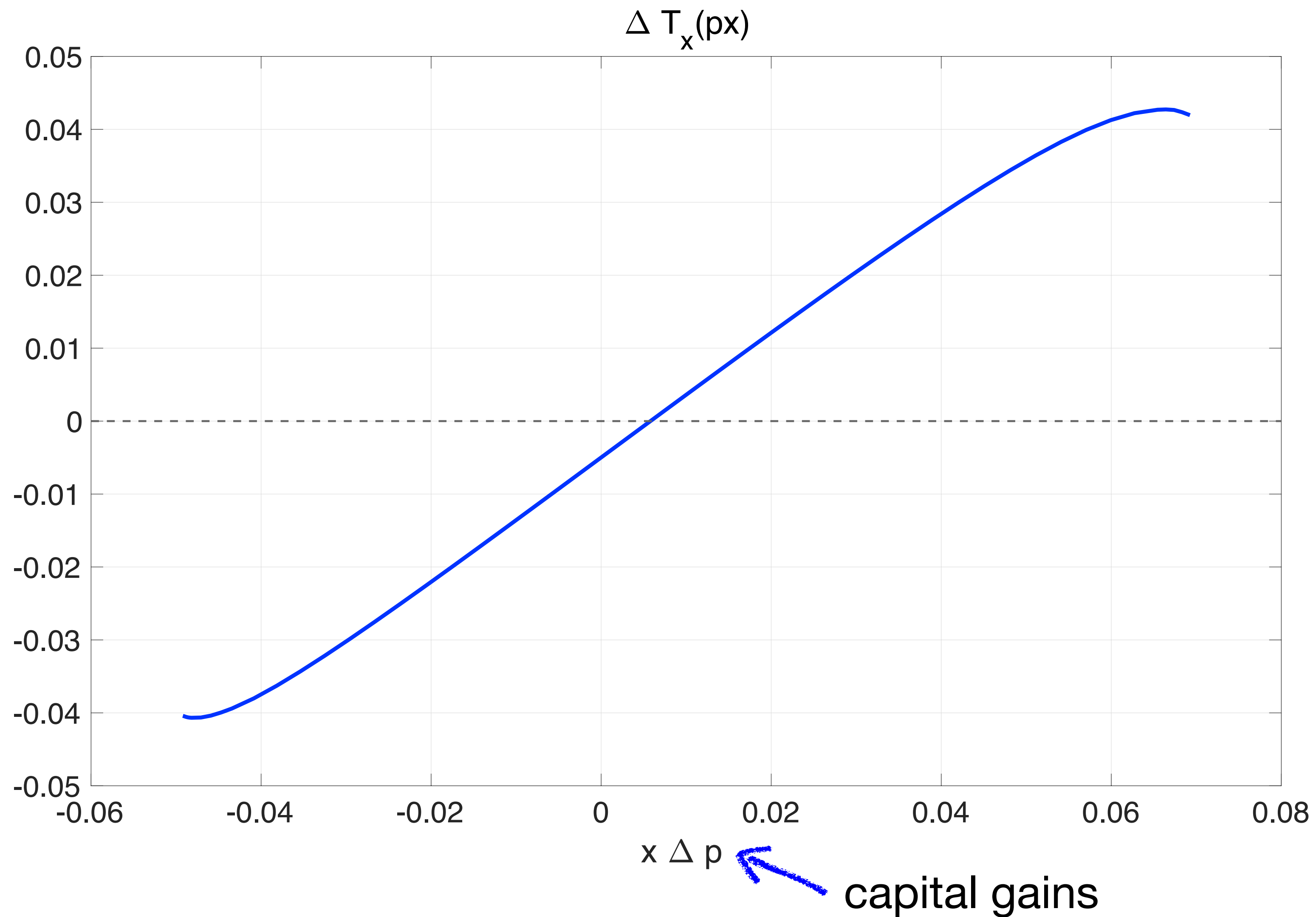
$$c_1 = Dk_1 + y_1 - T_k(pk_1)$$

$$k_1 = k_0 - x$$

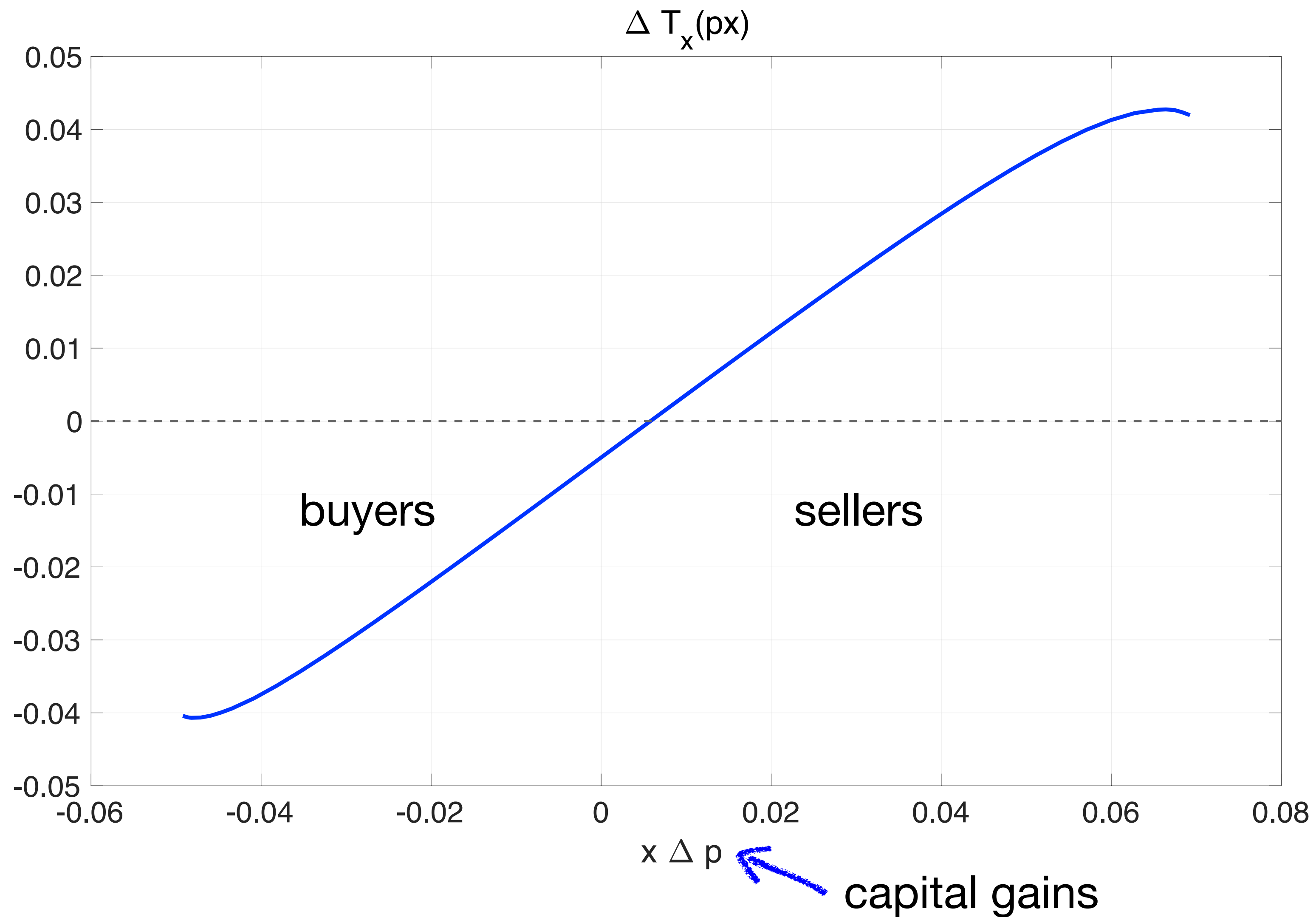
Other instruments similar, e.g. dividend/capital income tax  $T_D(Dk_1)$



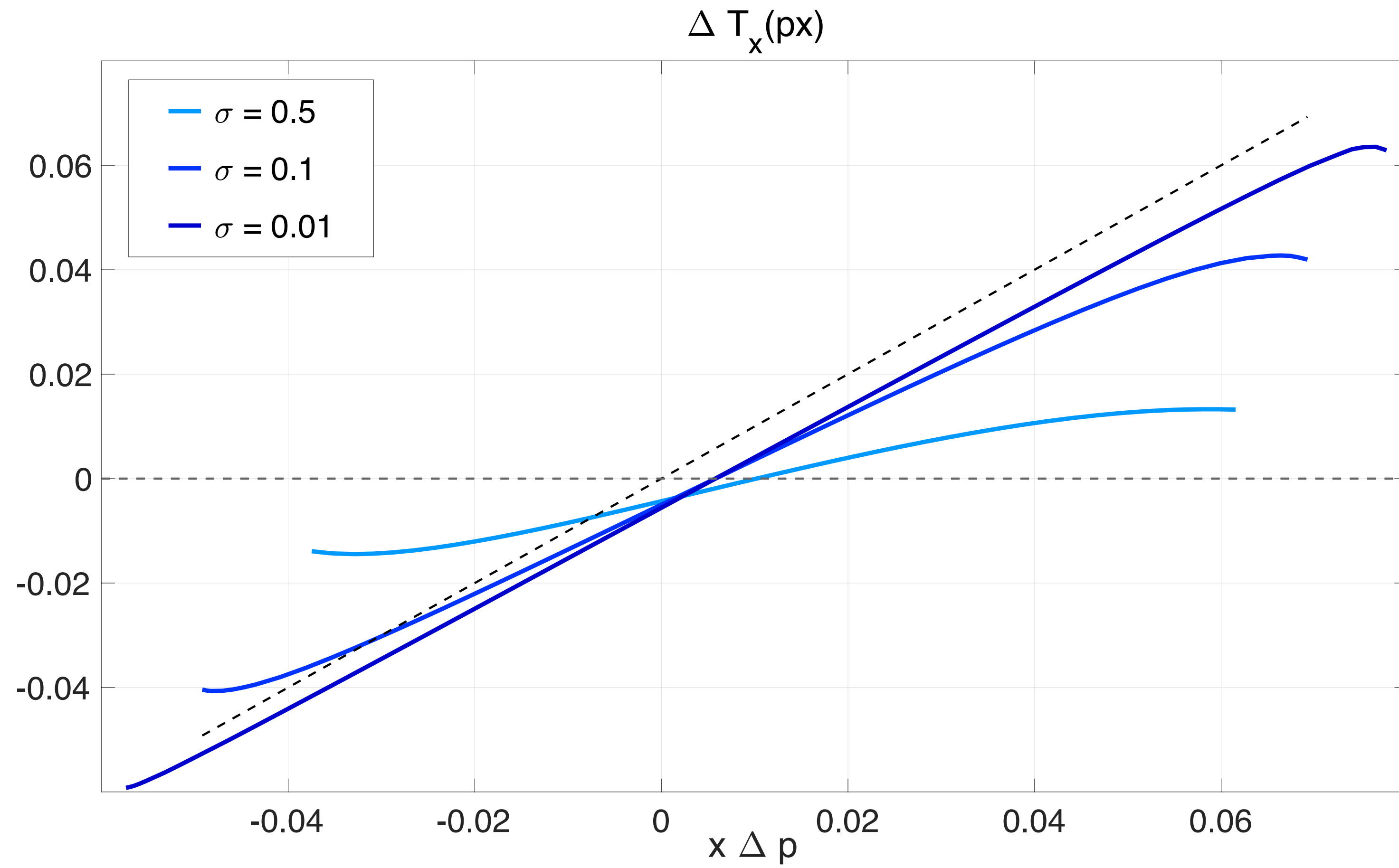
# How the optimal tax responds to a rising asset price



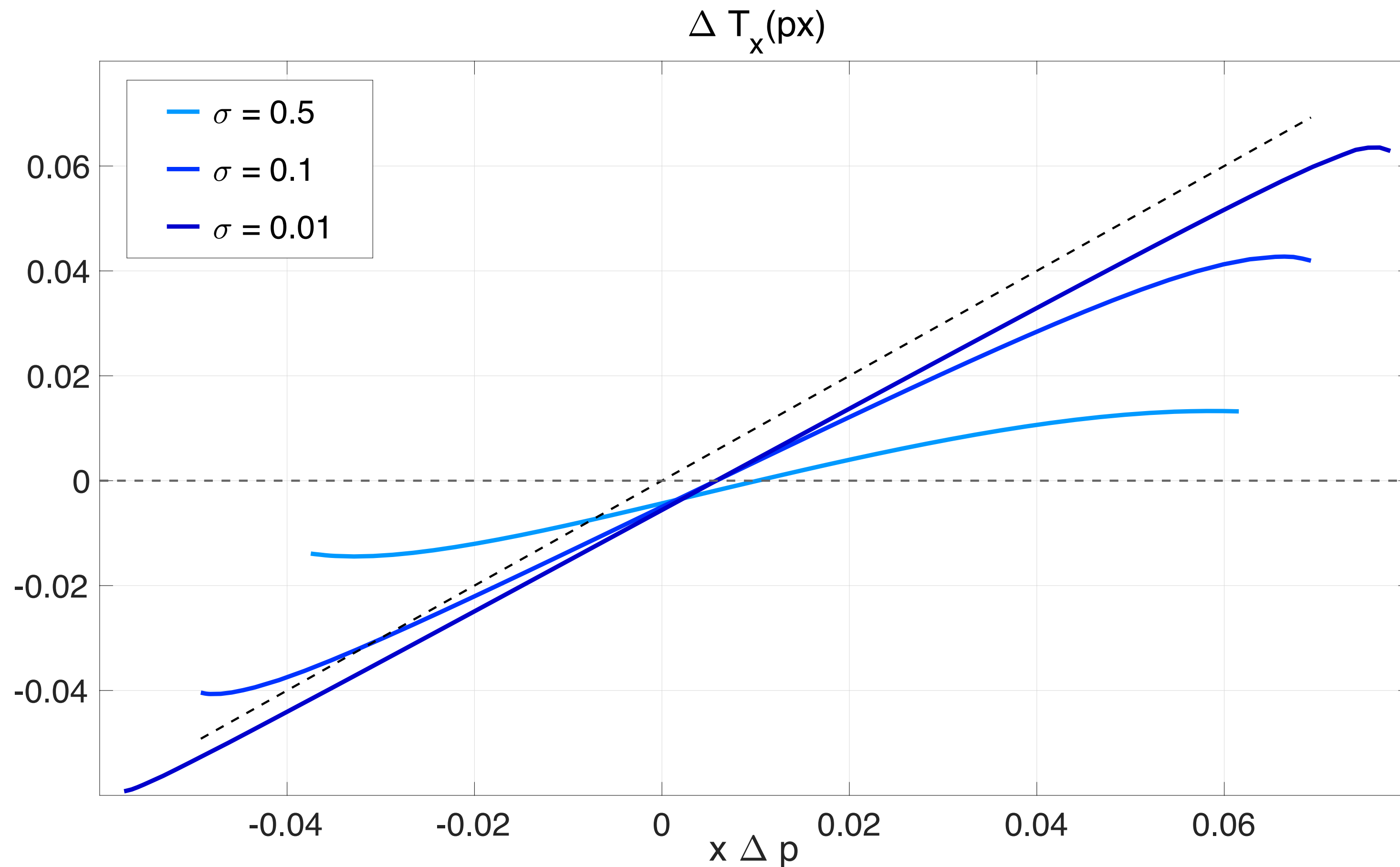
# How the optimal tax responds to a rising asset price



# Role of the IES



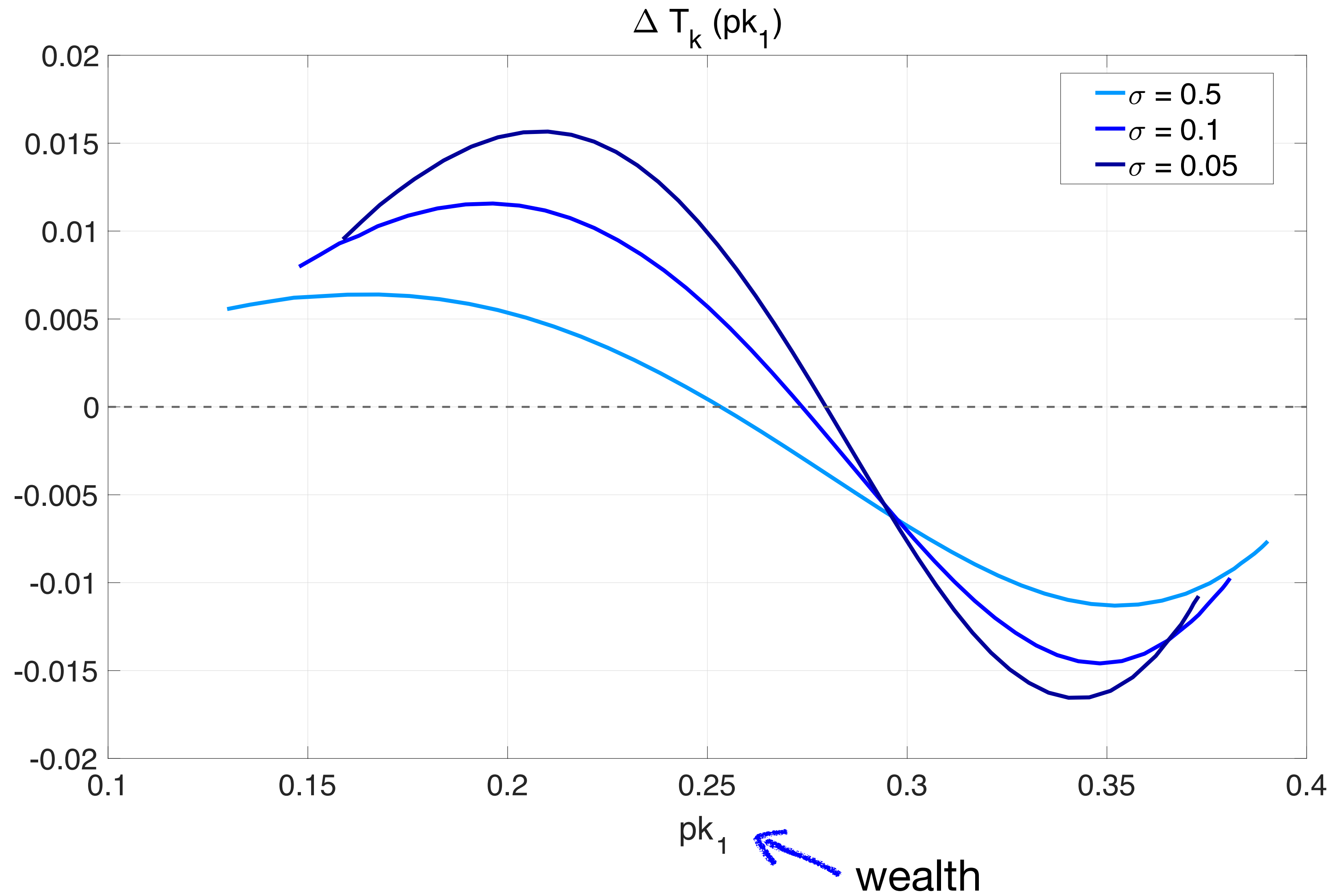
# Role of the IES



**Proposition:** Suppose  $V'_{FB}(\theta) \in [y'_0(\theta), Dk'_0(\theta) + y'_1(\theta)] \forall \theta$ . Then the solution to the second-best problem converges to the first-best allocation as  $\sigma \rightarrow 0$ .

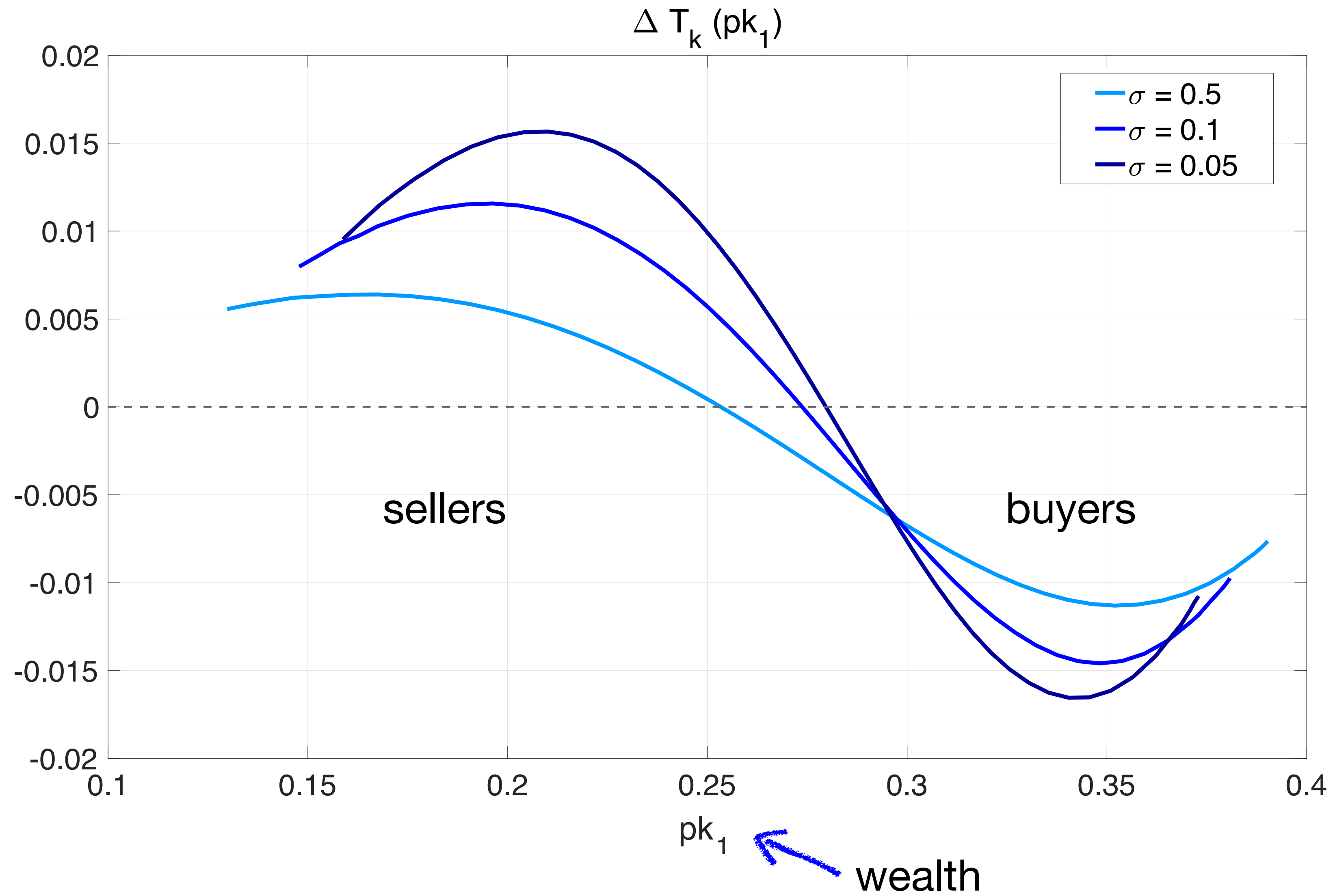
# Wealth tax

Taxes  
in levels



# Wealth tax

Taxes  
in levels



# Extensions

1. Back to multi-period model
2. General equilibrium
3. Heterogeneous returns
4. Risk and borrowing

# Conclusion

- When asset valuations change, optimal taxes change by

$$\Delta T = \tau \times \text{sales} \times \Delta p$$

- In general, combo of realization-based capital gains + dividend taxes works
- Wealth or accrual-based taxes are at best knife-edge
  - ▶ Don't work in general even with dividend-driven asset price changes
  - ▶ Often redistribute in “wrong” direction



# Lifecycle

# Investors

$$\max_{\{c_t, k_t\}} \frac{1}{1-\gamma} \left( \sum_{t=0}^T \beta^t c_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma(1-\gamma)}{\sigma-1}} \quad \text{s.t.}$$

$$p_t k_{t+1} + c_t = y_t + D_t k_t + p_t k_t - T_t$$

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Rates of return:

$$R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t}, \quad R_{0 \rightarrow t} = R_1 \cdot R_2 \cdots R_t$$

# Lifecycle

**Proposition:** Suppose asset prices change by  $\{\Delta p_t\}_{t=0}^T$  and dividends by  $\{\Delta D_t\}_{t=0}^T$ . The change in the optimal taxes  $\{\Delta T_t(\theta)\}_{t=0}^T$  is such that

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**Example:**  $\Delta T_t(\theta) = x_t(\theta) \Delta p_t + k_t(\theta) \Delta D_t - \Omega(\theta)(X_t \Delta p_t + K_t \Delta D_t) \forall t$

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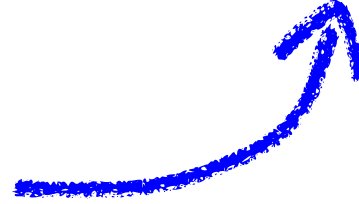
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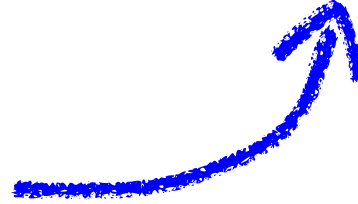
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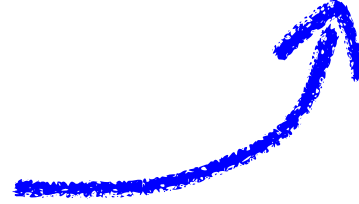
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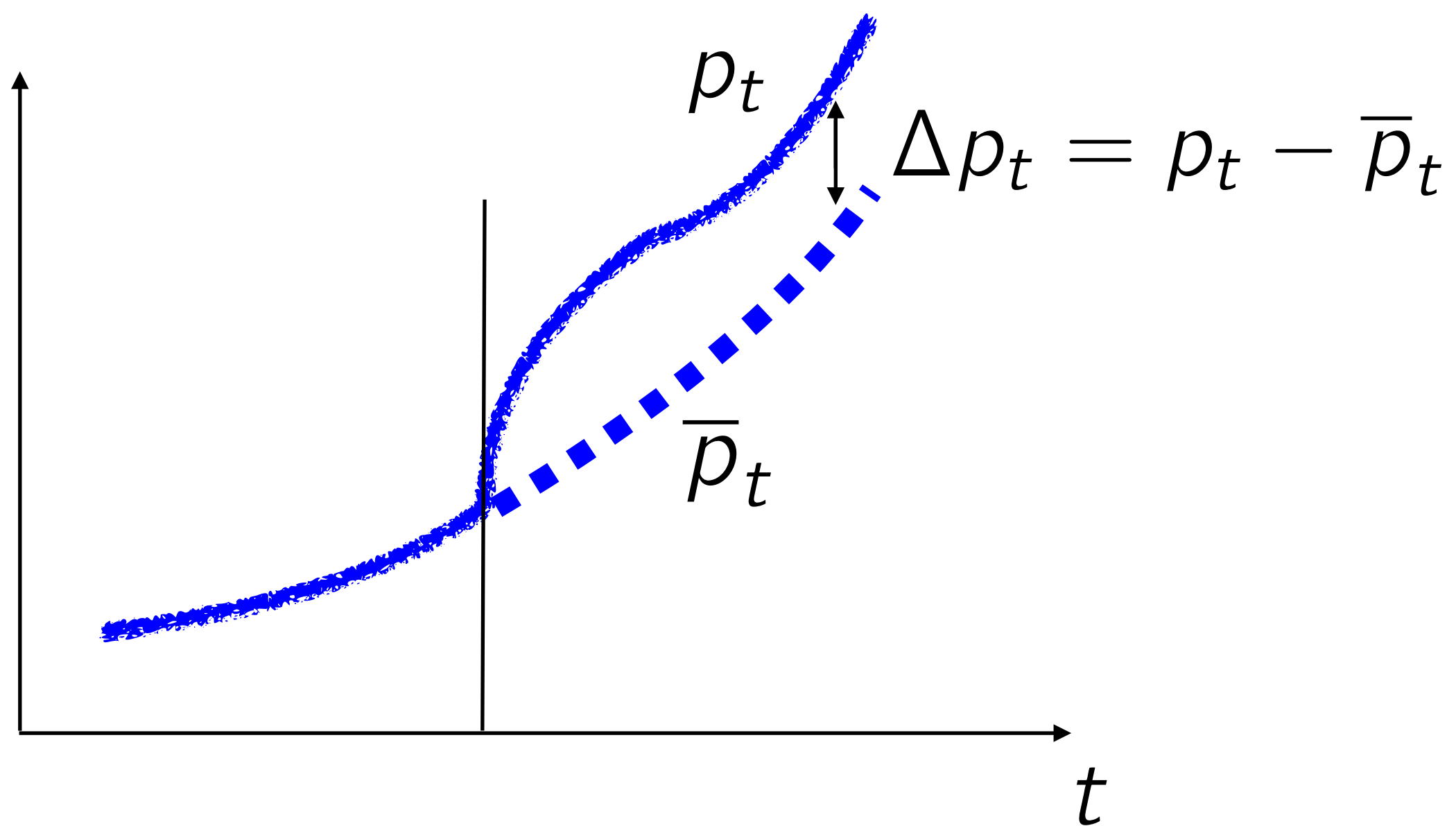


# What are $\Delta p$ and $\Delta D$ ? An example

Back

$$\Delta T_t(\theta) = x_t(\theta)\Delta p_t + k_t(\theta)\Delta D_t - \Omega(\theta)(X_t\Delta p_t + K_t\Delta D_t) \quad \forall t$$

Old BGP:  $\bar{D}_t = G^t \bar{D}_0$      $\bar{R}_{t \rightarrow t+1} = \bar{R}$      $\bar{p}_t = G^t \bar{p}_0$

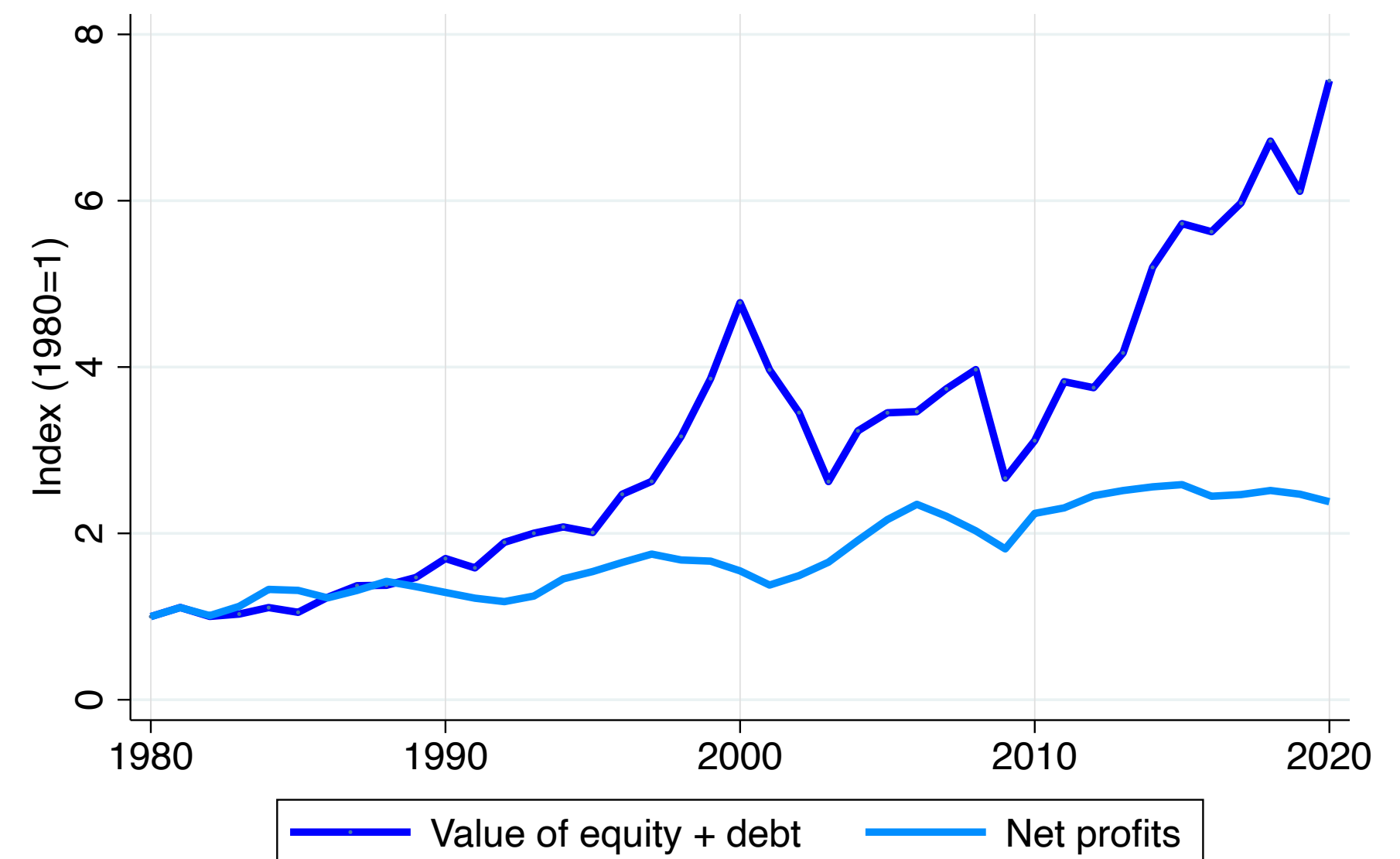
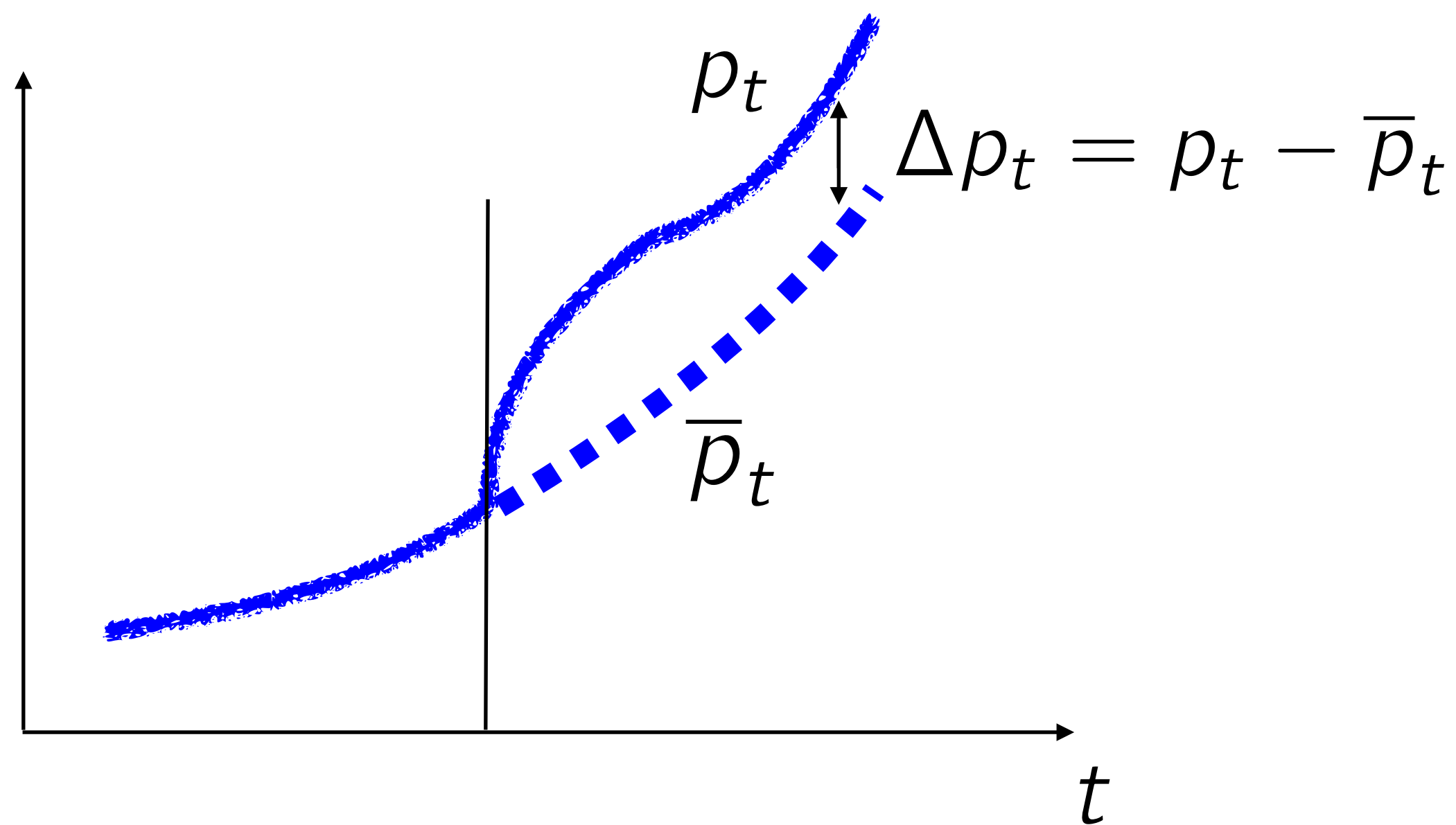


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# General equilibrium

# Equilibrium asset price

Back

Suppose capital is in fixed supply  $K_0 = K_1 = K$

Asset price  $p^*$  adjusts to clear market:

$$p^* = \beta D \left( \frac{Y_0}{Y_1 + DK} \right)^{\frac{1}{\sigma}}$$

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**Proposition:** Suppose the asset price increases by  $\Delta p^*$  while dividends  $D$  remain unchanged. The change in the optimal tax  $T_0(\theta)$  is

$$\Delta T_0(\theta) = x(\theta) \Delta p^*$$

# Heterogeneous Cashflows

# Trading with adjustment costs

$$c_0 + qb = p(k_0 - k_1) - \chi(k_0 - k_1) + y_0 - T_0$$

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**Proposition:** Suppose the asset price increases by  $\Delta p$  while dividends  $D(\theta)$  remain unchanged. The change in the optimal tax  $T_0(\theta)$  is

$$\Delta T_0(\theta) \approx x(\theta)\Delta p - \Omega(\theta)\chi\Delta p - \frac{1}{2}\chi''(x(\theta))\Delta x(\theta)^2$$

# Heterogeneous returns in GE

Back

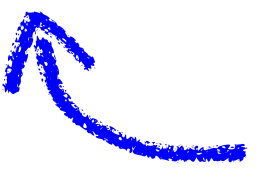
Suppose  $\chi(x) = \kappa x^2$  and capital is in fixed supply

Then  $p^* = q\bar{D}$   
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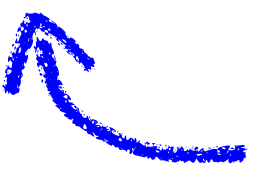
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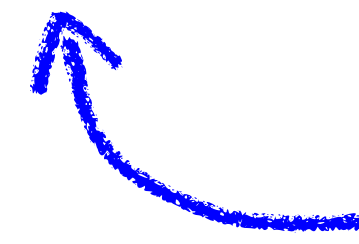
# Risk and borrowing

# Two assets

Aggregate return risk  $D(s)$ ,  $s \in S$ , probabilities  $\pi(s)$

$$c_0 = p(k_0 - k_1) + qb + y_0 - T_0$$

$$c_1(s) = D(s)k_1 - b + y_1 - T_1(s)$$

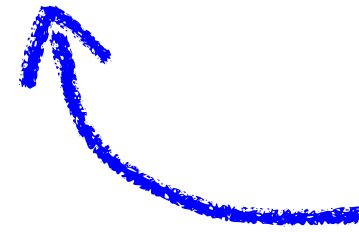


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Asset prices:

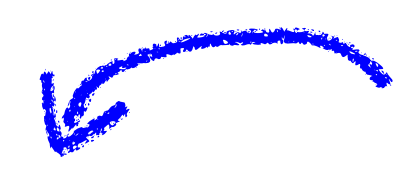
1. capital  $p = \mathbb{E}[\tilde{q}(s)D(s)]$

2. bond  $q = \mathbb{E}[\tilde{q}(s)]$

 Arrow-Debreu prices



# First-best problem

Individual lump-sum taxes  $T_0(\theta), T_1(\theta, s)$    $\int T_1(\theta, s) dF(\theta) = 0 \forall s$

$$\max_{c_0(\theta), c_1(\theta, s), \mu(\theta)} \int \omega(\theta) U(c_0(\theta), \mu(\theta)) dF(\theta) \quad \text{s.t.}$$

$$\int c_0(\theta) dF(\theta) + q \int c_1(\theta, s) dF(\theta) = Y(s) \forall s$$

$$U(c_0, \mu) = \frac{C(c_0, \mu)^{1-\gamma}}{1-\gamma} \quad C(c_0, \mu) = \left( c_0^{\frac{\sigma-1}{\sigma}} + \beta \mu^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \mu = \left( \sum_s c_1(s)^{1-\alpha} \pi(s) \right)^{\frac{1}{1-\alpha}}$$

# Changing Arrow-Debreu prices

Back

**Proposition:** Suppose Arrow-Debreu prices  $\tilde{q}(s)$  change such that asset prices change by  $(\Delta p, \Delta q)$ . Holding fixed  $\mathbb{E}[T_1(\theta, s)\tilde{q}(s)/q]$ , the change in the optimal tax  $T_0(\theta)$  is

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 aggregate  
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- Borrowers/savers are winners/losers from change in  $q$
- No borrowing constraint (would not matter with first-best tax instruments)

# Comparison to capital taxation literature

Back

1. Partial equilibrium models (Atkinson-Stiglitz...) with constant  $R_t = \bar{R}$
2. Neoclassical growth model (Chamley...): depends and decentralisation

- always: unit price of capital = 1,  $R_{t+1} = \frac{1}{\beta} \frac{U'(c_t)}{U'(c_{t+1})}$
- asset = capital:  $p_t = 1 \Rightarrow$  no capital gains
- asset = shares in representative firm, BGP with  $A_{t+1}/A_t = G$

$$\bar{R} = (1/\beta)G^{1/\sigma} \quad \text{with} \quad \frac{D_{t+1}}{p_t} = \bar{R} - G \quad \text{and} \quad \frac{p_{t+1}}{p_t} = G$$

3. Growth models with heterogeneous households (Judd, Werning, Straub-Werning...)
  - same as 2.
4. Our setup: allow flexibly for discount rate variation

# Consumption tax

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Kaldor's  
Expenditure  
Tax

# Optimal wealth tax schedule

Back

