

Putting the 'Finance' into 'Public Finance': A Theory of Capital Gains Taxation

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Tax treatment of capital gains due to changing asset prices

Tax system of typical country: tax capital gains **on realization** (i.e. sale)

But recent policy proposals:

- tax **capital gains on accrual**
(Biden-Harris administration,...)
- tax **wealth** (Piketty, Zucman, ...)



Old idea: **Haig-Simons** comprehensive income tax

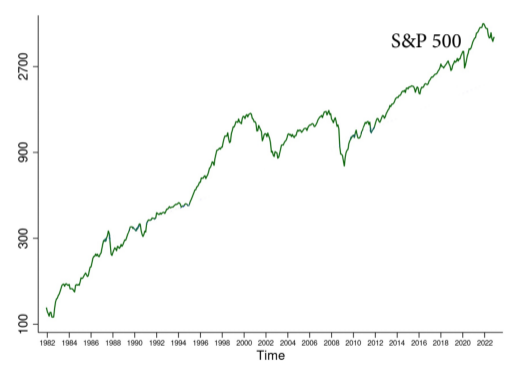
$$\text{income} = \text{consumption} + \Delta\text{wealth}$$

Auerbach (1989)

“Many of the distortions associated with the present system of capital gains taxation result from its deviation from the Haig-Simons approach.”

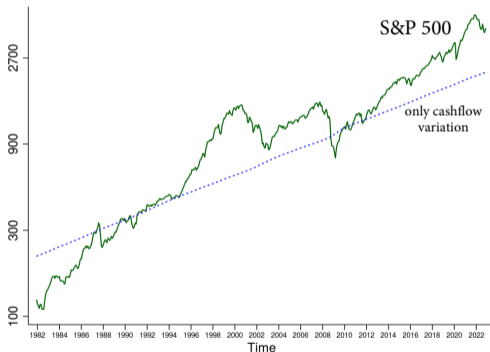
“These deviations may have historical explanations but their persistence is hard to rationalize from an economic perspective.”

Background: rising and fluctuating asset prices



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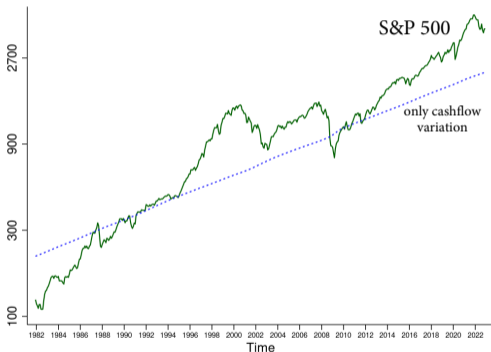
- Conventional view: asset prices move too much to be accounted for by changing cash flows alone \Rightarrow discount rate variation (Shiller, Campbell-Shiller, ...)



Source: Bordalo et al. following Shiller 1981

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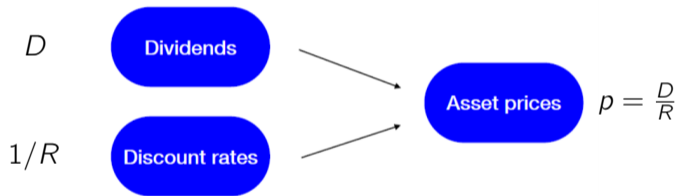
- Growing positive literature: asset prices \Rightarrow wealth inequality
Kuhn et al. (2020), Greenwald et al. (2021), Fagereng et al. (2021, 2023), Martínez-Toledano (2023)...

But what about taxation of capital gains?

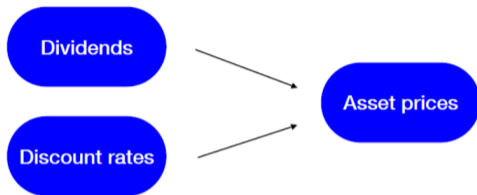
**No guidance from standard optimal capital tax theory:
No asset prices!**

To wit: no 'finance' in 'public finance'

What we do: optimal redistributive taxation with changing asset prices

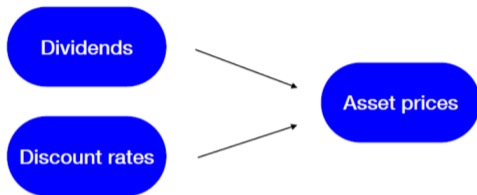


What we find



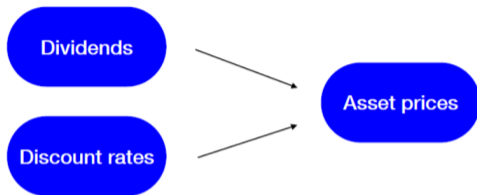
What we find

$$\Delta T = \tau \times \text{wealth} \times \Delta p$$



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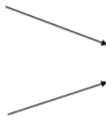
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Dividends

$$\Delta T = \tau \times \text{sales} \times \Delta p$$

Discount rates

Asset prices



What we find

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Dividends

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Discount rates

Asset prices

A diagram illustrating the relationship between different variables and asset prices. On the left, there are two blue rounded rectangular boxes. The top one is labeled 'Dividends' and the bottom one is labeled 'Discount rates'. Two arrows originate from these boxes: one from 'Dividends' pointing towards the right, and one from 'Discount rates' pointing towards the right. Both arrows converge towards a single blue rounded rectangular box on the right labeled 'Asset prices'.

Intuition: higher asset prices benefit **sellers not holders**

- unrealized capital gains are “paper gains” until you sell

What we find



Intuition: higher asset prices benefit **sellers not holders**

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In general, combination of realization-based capital gains & dividend tax

- though important differences from existing realization-based tax systems
- e.g. tax **net** trades to not distort portfolio choice (no lock-in effect)

Plan

1. Baseline model (partial equilibrium)
2. Two time periods, one asset, no risk
3. First-best
4. Second-best (Mirrlees)
 - Portfolio choice and lock-in effect
5. Back to multi-period model
6. Extensions
 - General equilibrium
 - Heterogeneous returns
 - Risk and borrowing
 - Borrowing versus selling
 - Bequests and sub-optimality of step-up in basis at death
 - Wealth in utility

Baseline model

Investors

Indexed by $\theta \sim F(\theta)$, differ in initial wealth $k_0(\theta)$, income profiles $\{y_t(\theta)\}_{t=0}^T$

$$V = \max_{\{c_t, k_{t+1}, b_{t+1}\}_{t=0}^T} \mathbb{E}_0 U(c_0, \dots, c_T) \quad \text{s.t.}$$

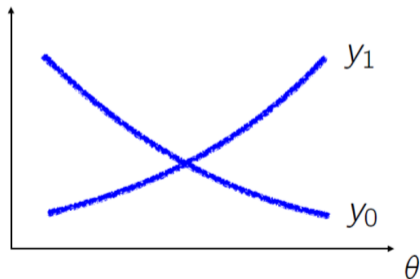
$$c_t + p_t(k_{t+1} - k_t) + q_t b_{t+1} = y_t + D_t k_t + b_t - T_t \Rightarrow R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t}$$

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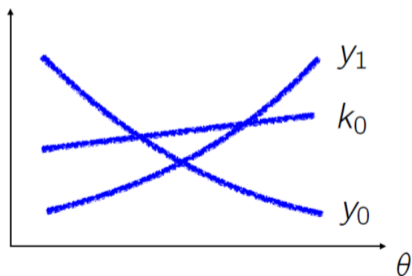


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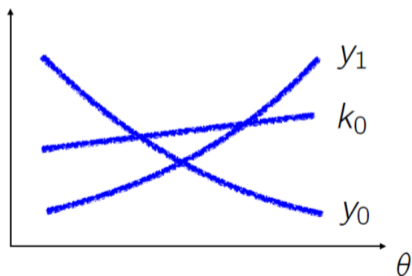


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Small open economy: $\{D_t, p_t, q_t\}$ processes exogenously given

Where do $\{p_t, q_t\}$ come from? ▶ Rep. counterparty in global financial markets with SDF $m_{t \rightarrow t+s}$

Comments

Owner-occupied housing

- $D_t =$ imputed rents

Endogenous payout policy and share repurchases

- $D_t =$ profits net of investment, $p_t =$ total value of firm

Haig-Simons income includes unrealized capital gains

- budget constraint (without b_t for simplicity)

$$c_t + p_t(k_{t+1} - k_t) = y_t + D_t k_t$$

- add unrealized capital gains $(p_t - p_{t-1})k_t$ on both sides

$$c_t + \underbrace{p_t k_{t+1} - p_{t-1} k_t}_{\text{change in wealth}} = \underbrace{y_t + D_t k_{t-1} + (p_t - p_{t-1}) k_t}_{\text{Haig-Simons income}}$$

Comparison to setups in capital taxation literature

1. Partial equilibrium models with constant $R_t = \bar{R}$ (Atkinson-Stiglitz,...)
2. Neoclassical growth model (Chamley, ...): depends on decentralization
 - production-based asset pricing: shares in representative firm
 - BGP with $A_{t+1}/A_t = G$:

$$\bar{R} = (1/\beta)G^{1/\sigma} \quad \text{with} \quad \frac{D_{t+1}}{p_t} = \bar{R} - G \quad \text{and} \quad \frac{p_{t+1}}{p_t} = G$$

- **small fluctuations** in discount rate $R_{t+1} = \frac{1}{\beta} \frac{U'(C_t)}{U'(C_{t+1})}$
3. Growth models with het. households (Werning, Judd, Straub-Werning,...)
 - same as 2
 4. Our setup
 - optimal taxation with exogenous $\{p_t, D_t\}$ and hence returns $\{R_t\}$
 - allows us to take on board discount rate variation in flexible way

Two time periods, one asset, no risk

- Investors indexed by $\theta \sim F(\theta)$

$$V = \max_{c_0, c_1, k_1} U(c_0, c_1) \quad \text{s.t.}$$
$$c_0 + p(k_1 - k_0) = y_0 - T_0$$
$$c_1 = y_1 + Dk_1$$

- Asset return $R = D/p \Rightarrow p = D/R$ (Campbell-Shiller)
- Resource constraints

$$\int c_0(\theta) dF(\theta) + \frac{p}{D} \int c_1(\theta) dF(\theta) \leq Y$$
$$Y \equiv \int y_0(\theta) dF(\theta) + \frac{p}{D} \int y_1(\theta) dF(\theta) + p \int k_0(\theta) dF(\theta)$$

Two time periods, one asset, no risk

- In terms of asset sales $x \equiv k_0 - k_1$

$$V = \max_{c_0, c_1, x} U(c_0, c_1) \quad \text{s.t.}$$

$$c_0 = y_0 + px - T_0$$

$$c_1 = y_1 + D(k_0 - x)$$

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First best

Pareto problem

Individual lump-sum taxes $T_0(\theta)$

$$\max_{c_0(\theta), c_1(\theta)} \int \omega(\theta) U(c_0(\theta), c_1(\theta)) dF(\theta) \quad \text{s.t.}$$

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Prices and dividends (p, D) fluctuate around baseline (\bar{p}, \bar{D})

Design **tax rule** $T_0(\theta; p, D)$ that optimally redistributes given fluctuations

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$$U(c_0, c_1) = \frac{c_0^{1-1/\sigma}}{1-1/\sigma} + \beta \frac{c_1^{1-1/\sigma}}{1-1/\sigma} \quad \text{but want to take } \sigma \rightarrow 0 \text{ later}$$

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$$U(c_0, c_1) = G(C(c_0, c_1)), \quad C(c_0, c_1) = \left(c_0^{\frac{\sigma-1}{\sigma}} + \beta c_1^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad G(C) = \frac{C^{1-\gamma}}{1-\gamma}$$

Changing asset prices

Proposition: Suppose the asset price increases by Δp while dividends D remain unchanged. The change in the optimal tax burden $T_0(\theta)$ is

$$\Delta T_0(\theta) = x(\theta)\Delta p - \Omega(\theta)X\Delta p$$

100% tax on realized capital gains

aggregate asset sales

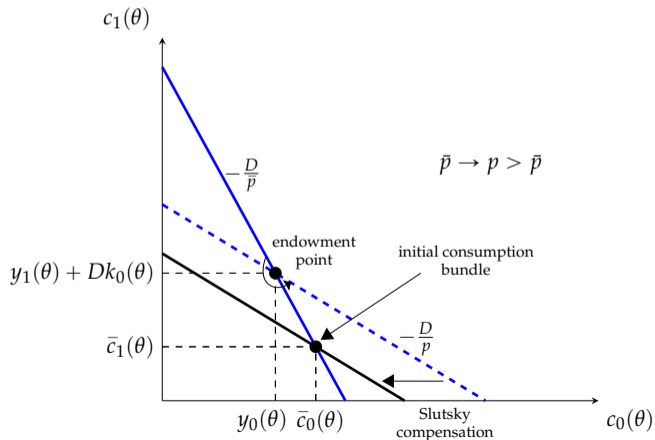
$$\frac{\omega(\theta)^{1/\gamma}}{\int \omega(\theta')^{1/\gamma} dF(\theta')}$$

- Second term: $X = 0$ for particular parameters or in general equilibrium
- Tax on **net** transactions
- **Subsidy** if $x < 0$
- Holds even for large Δp



Slutsky compensation

Change in investor's total budget that keeps initial consumption bundle affordable at new prices



Changing asset prices and dividends

Proposition: Suppose the asset price increases by Δp and dividends by ΔD . The change in the optimal tax burden $T_0(\theta)$ is

$$\Delta T_0(\theta) = x(\theta)\Delta p + \frac{p}{D}k_1(\theta)\Delta D - \Omega(\theta) \left[X\Delta p + \frac{p}{D}K_1\Delta D \right]$$

tax on realized
capital gains



tax on dividend
income



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
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Special case $\Delta D/\Delta p = D/p$? Asset price change driven **only** by dividends.

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$$= \frac{p}{D}(k_0(\theta) - x(\theta))\frac{D}{p}\Delta p$$


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100% tax on wealth increase 

 aggregate wealth

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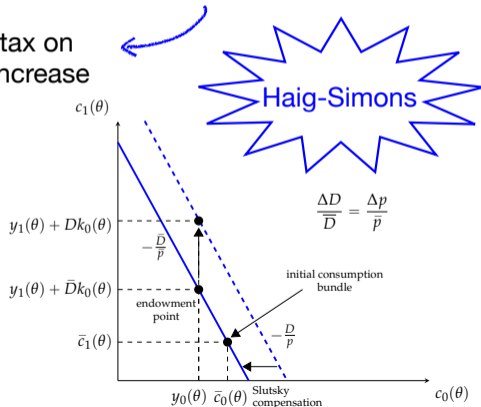


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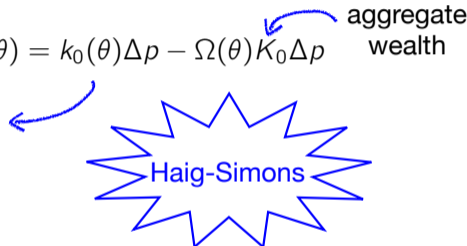
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aggregate wealth

100% tax on wealth increase



- Tax on wealth/unrealized gains is **knife-edge!**
- Later: multi-period or heterogeneous returns
⇒ **don't work** in general **even with dividend-driven p -changes**
- In general, tax must depend on realizations
- Consumption tax Tax on total returns

Second best

Distortive nonlinear taxes

1. Capital sales tax $T_x(px)$

2. Wealth tax $T_k(pk_1)$

$$c_0 = y_0 + px - T_x(px)$$

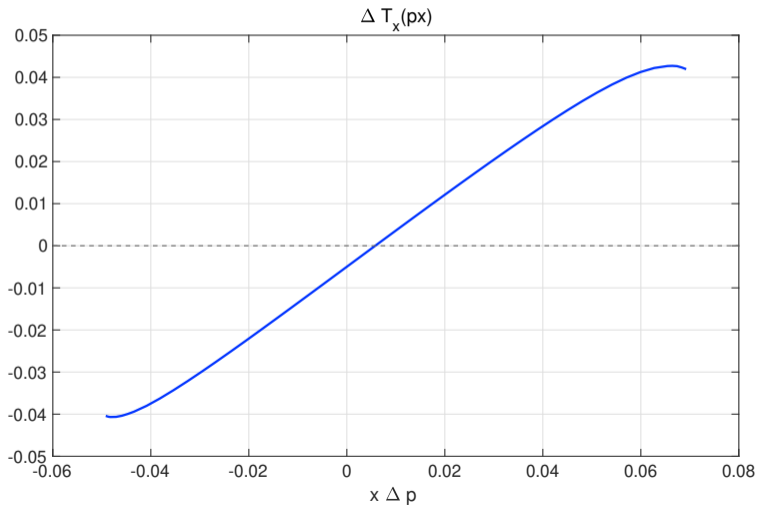
$$c_1 = Dk_1 + y_1 - T_k(pk_1)$$

$$k_1 = k_0 - x$$

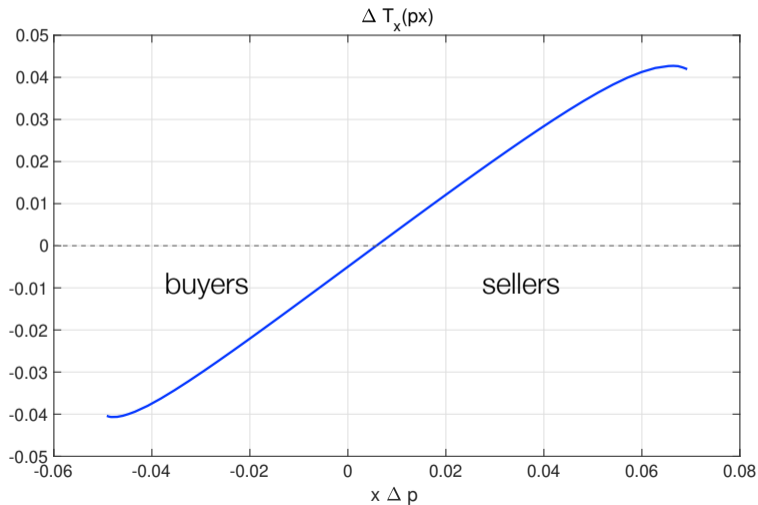
Other instruments similar, e.g. dividend/capital income tax $T_D(Dk_1)$

Portfolio choice? Momentarily...

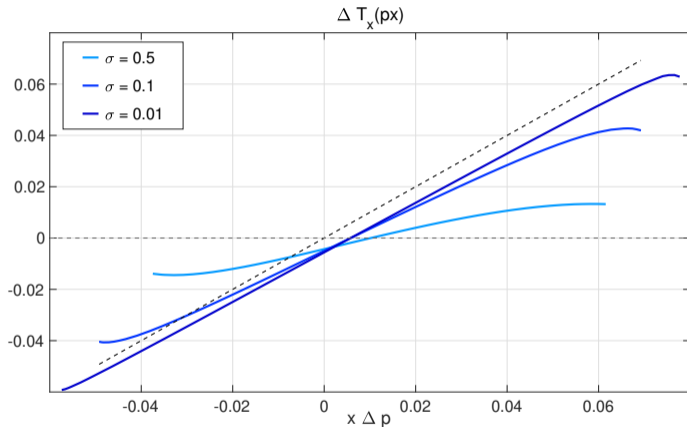
How the optimal tax responds to a rising asset price



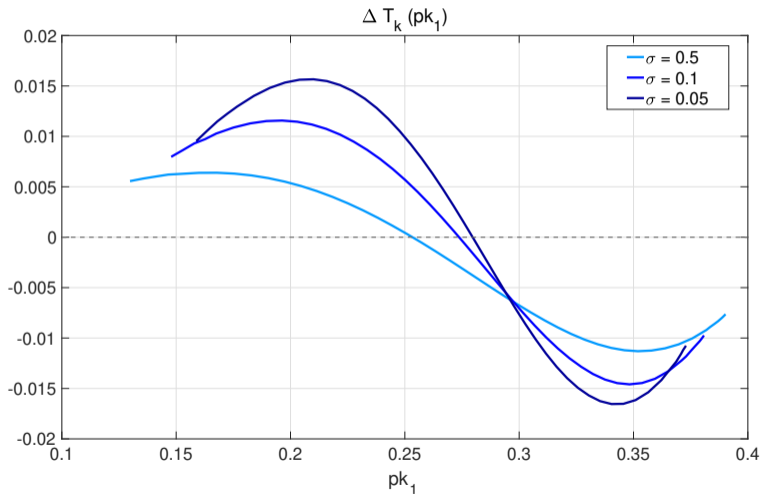
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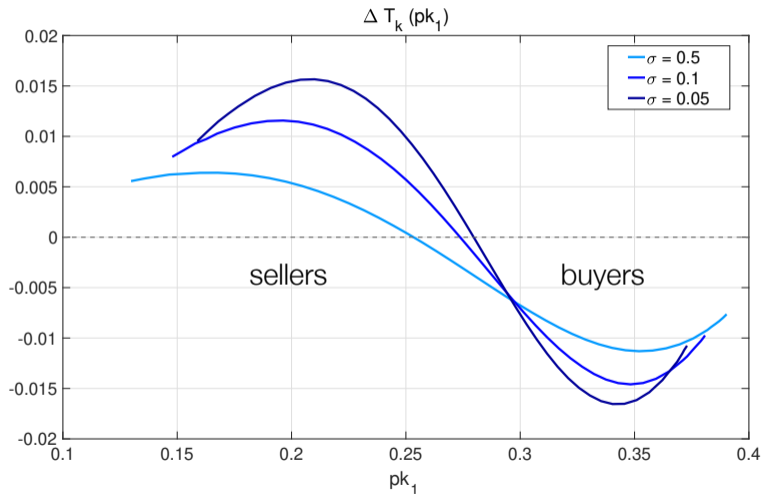


Role of the IES





Proposition: Suppose $V'_{FB}(\theta) \in [y'_0(\theta), D'_k(\theta) + y'_1(\theta)] \forall \theta$. Then the solution to the second-best problem converges to the first-best allocation as $\sigma \rightarrow 0$.





Portfolio choice and lock-in effect

Realization-based tax may distort portfolio choice

bond  adjustment cost 

$$c_0 = px - qb - \chi(x) + y_0 - T_0$$
$$c_1 = D(k_0 - x) + b + y_1$$

With observable trades x and b , optimum can be implemented with a tax

$$T(px - qb - \chi(x))$$

- Undistorted portfolio choice \Rightarrow no lock-in effect
- Tax on **net** trades

Back to multi-period model

Investors

$$\max_{\{c_t, k_{t+1}\}} \frac{1}{1-\gamma} \left(\sum_{t=0}^T \beta^t c_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma(1-\gamma)}{\sigma-1}} \quad \text{s.t.}$$

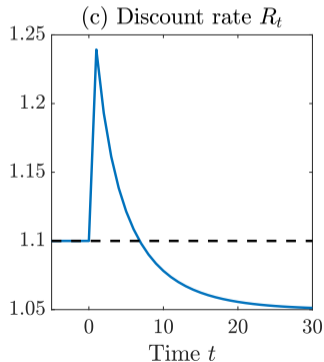
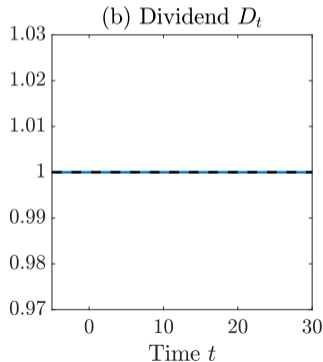
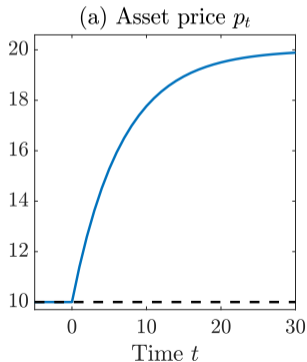
$$p_t k_{t+1} + c_t = y_t + D_t k_t + p_t k_t - T_t$$

Rates of return:

$$R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t}, \quad R_{0 \rightarrow t} = R_1 \cdot R_2 \cdots R_t$$

Experiment

- Start at some baseline (steady state or BGP) with tax system $\bar{T}_t(\theta)$
- Then prices and dividends $\{p_t, D_t\}_{t=0}^T$ change
- Example:



Taxing changing asset prices in multi-period model

Proposition: Suppose asset prices change by $\{\Delta p_t\}_{t=0}^T$ and dividends by $\{\Delta D_t\}_{t=0}^T$. The change in the optimal taxes $\{\Delta T_t(\theta)\}_{t=0}^T$ is such that

$$\sum_{t=0}^T R_{0 \rightarrow t}^{-1} \Delta T_t(\theta) = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} [x_t(\theta) \Delta p_t + k_t(\theta) \Delta D_t - \Omega(\theta)(X_t \Delta p_t + K_t \Delta D_t)]$$

Example:

$$\Delta T_t(\theta) = x_t(\theta) \Delta p_t + k_t(\theta) \Delta D_t - \Omega(\theta)(X_t \Delta p_t + K_t \Delta D_t) \quad \forall t$$

Special case: constant discount rates

$$\frac{\Delta D_{t+1} + \Delta p_{t+1}}{\Delta p_t} = \frac{D_{t+1} + p_{t+1}}{p_t} \quad \text{i.e., } R_{t+1} \text{ unchanged, only } R_0 \text{ affected}$$

$$\Rightarrow \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \Delta T_t(\theta) = [k_0(\theta) - \Omega(\theta)K_0](\Delta D_0 + \Delta p_0)$$

Tax unrealized gain at $t = 0$ but tax on all future gains = 0 \Rightarrow **Haig-Simons**

- perfect foresight so Δp_0 already incorporates all news about $\{\Delta D_t\}_{t=1}^T$

Even more special case:

- constant discount rates
- at each $t \geq 0$, MIT shock to $\{D_{t+s}\}$ so that **realized** $R_t = \frac{D_t + p_t}{p_{t-1}}$ moves

$$\sum_{s \geq t} R_{t \rightarrow t+s}^{-1} \Delta T_s(\theta) = [k_t(\theta) - \Omega(\theta)K_t](\Delta p_t + \Delta D_t)$$

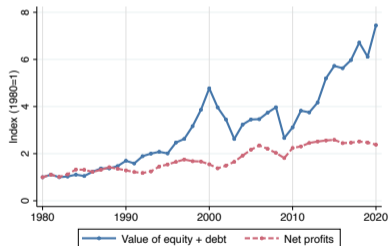
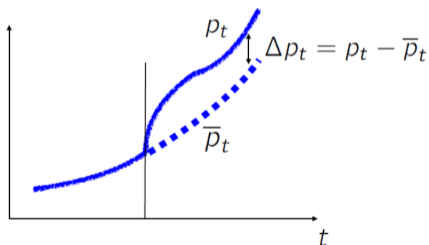
100 % tax on unrealized capital gains at each $t \geq 0$ = **Haig-Simons**

What are Δp and ΔD ? An example

$$\Delta T_t(\theta) = x_t(\theta)\Delta p_t + k_t(\theta)\Delta D_t - \Omega(\theta)(X_t\Delta p_t + K_t\Delta D_t) \quad \forall t$$

Old BGP:

$$\bar{D}_t = G^t \bar{D}_0 \quad \bar{R}_{t+1} = \bar{R} \quad \bar{p}_t = G^t \bar{p}_0$$



Extensions

- General equilibrium
- Heterogeneous returns
- Risk and borrowing
- Borrowing versus selling
- Bequests and sub-optimality of step-up in basis at death
- Wealth in utility

Conclusion

When asset valuations change, optimal taxes change by

$$\Delta T = \tau \times \text{sales} \times \Delta p$$

In general, combination of realization-based capital gains + dividend taxes

- no need to know source of capital gains
- important differences from existing realization-based tax systems

Wealth or accrual-based taxes are knife-edge at best

- beyond baseline: may not work even with only cashflow-driven Δp
- may redistribute in “wrong” direction

Linked backup slides

Counterparty has stochastic discount factor

$$m_{t \rightarrow t+s}$$

which prices the two assets:

$$p_t = \mathbb{E}_t \left[\sum_{s=1}^{T-t} m_{t \rightarrow t+s} D_{t+s} \right] \quad \text{and} \quad q_t = \mathbb{E}_t \left[\sum_{s=1}^{T-t} m_{t \rightarrow t+s} \right]$$

Proposition: Suppose the asset price increases by Δp and dividends by ΔD . The change in the optimal taxes $T_0(\theta)$ and $T_1(\theta)$ is

$$\Delta T_t(\theta) = \Delta \hat{c}_t(\theta) - \Omega(\theta) \Delta C_t$$

where $\Delta \hat{c}_t$ is the change in consumption holding taxes fixed.

No need to know source of capital gains: Δp vs. ΔD !

Kaldor's expenditure tax!

Tax on total returns ▶ back

$$c_0 + a_1 = y_0 + R_0 a_1 - T_0, \quad c_1 = y_1 + R_1 a_1$$

where $R_0 = p/p_{-1}$, $R_1 = D/p$ which are $R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t}$ with $D_0 = p_1 = 0$

- note: $p \uparrow$ holding D fixed $\Rightarrow R_0 \uparrow$ but $R_1 \downarrow$

Proposition: Suppose the asset price increases by Δp and dividends by ΔD resulting in return changes ΔR_0 and ΔR_1 . Then

$$\Delta T_0(\theta) = a_0(\theta)\Delta R_0 + \frac{1}{R_1} a_1(\theta)\Delta R_1 - \Omega(\theta) \left[A_0(\theta)\Delta R_0 + \frac{1}{R_1} A_1(\theta)\Delta R_1 \right]$$

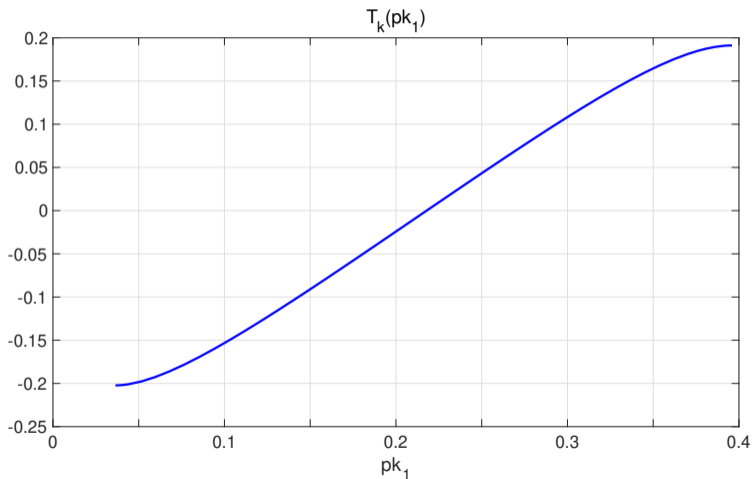
Alternatively, set $\Delta T_0 = a\Delta R_0 - \Omega(\theta)A_0\Delta R_0$ and $\Delta T_1 = a_1\Delta R_1 - \Omega(\theta)A_1\Delta R_1$

Special case: constant discount rate $\Delta R_1 = 0 \Rightarrow$ Haig-Simons

But **Haig-Simons** in all other cases $\Delta R_1 \neq 0$

Tax payments potentially volatile: large tax, followed by large rebate

Optimal wealth tax schedule [▶ back](#)



Extensions

Extensions

1. General equilibrium
2. Heterogeneous returns
3. Aggregate Risk
4. Borrowing versus Selling
5. Bequests and Suboptimality of Step-Up in Basis at Death

General Equilibrium

Equilibrium asset price [▶ back](#)

- Suppose capital is in fixed supply $K_0 = K_1 = K$
- Asset price p^* adjusts to clear market:

$$p^* = \beta D \left(\frac{Y_0}{Y_1 + DK} \right)^{\frac{1}{\sigma}}$$

Proposition: Suppose the asset price increases by Δp^* while dividends D remain unchanged. The change in the optimal tax burden $T_0(\theta)$ is

$$\Delta T_0(\theta) = x(\theta) \Delta p^*$$

Heterogeneous Cashflows

Trading with adjustment costs

$$c_0 + qb = p(k_0 - k_1) - \chi(k_0 - k_1) + y_0 - T_0$$

$$c_1 = D(\theta)k_1 + b + y_1$$

- heterogeneous dividends $D(\theta), \theta \sim F(\theta)$
- convex adjustment cost

Proposition: Suppose the asset price increases by Δp while dividends $D(\theta)$ remain unchanged. The change in the optimal tax $T_0(\theta)$ is

$$\Delta T_0(\theta) \approx x(\theta)\Delta p - \Omega(\theta)x\Delta p - \frac{1}{2}\chi''(x(\theta))\Delta x(\theta)^2$$

Heterogeneous returns in GE

Suppose $\chi(x) = \kappa x^2$ and capital is in fixed supply

Then

$$p^* = q \int D(\theta) dF(\theta)$$

Asset price changes for everyone when **some** dividends change...

... even for investors whose dividends did not change!

⇒ **Haig-Simons**

Risk and borrowing

Two assets

Aggregate return risk $D(s)$, $s \in S$, probabilities $\pi(s)$

$$\begin{aligned}c_0 &= p(k_0 - k_1) + qb + y_0 - T_0 \\c_1(s) &= D(s)k_1 - b + y_1 - T_1(s)\end{aligned}$$

Asset prices:

1. capital $p = \mathbb{E}[M(s)D(s)]$
2. bond $q = \mathbb{E}[M(s)]$

where $M(s) =$ SDF of rep counterparty in global financial markets

First-best problem

Individual lump-sum taxes $T_0(\theta), T_1(\theta, s)$ with $\int T_1(\theta, s)dF(\theta) = 0$, all s

$$\max_{c_0(\theta), c_1(\theta, s), \mu(\theta)} \int \omega(\theta) U(c_0(\theta), \mu(\theta)) dF(\theta) \quad \text{s.t.}$$
$$\int c_0(\theta) dF(\theta) + q \int c_1(\theta, s) dF(\theta) = Y(s) \quad \forall s$$

$$U(c_0, \mu) = \frac{C(c_0, \mu)^{1-\gamma}}{1-\gamma}, \quad C(c_0, \mu) = \left(c_0^{\frac{\sigma-1}{\sigma}} + \beta \mu^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \mu = \left(\sum_s c_1(s)^{1-\alpha} \pi(s) \right)^{\frac{1}{1-\alpha}}$$

Special case: changing discount rates (SDF)

Proposition: Suppose the SDF $M(s)$ changes such that asset prices change by $(\Delta p, \Delta q)$. Holding fixed $\mathbb{E}[T_1(\theta, s)M(s)/q]$, the change in the optimal tax burden $T_0(\theta)$ is

$$\Delta T_0(\theta) = x(\theta)\Delta p + b(\theta)\Delta q - \Omega(\theta)[X\Delta p + B\Delta q]$$

- Borrowers/savers are winners/losers from change in q
- No borrowing constraint (would not matter with first-best tax instruments)