Putting the ‘Finance’ into ‘Public Finance’: A Theory of Capital Gains Taxation*

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Abstract

Standard optimal capital tax theory abstracts from modeling asset prices, making it unsuitable for thinking about capital gains and wealth taxation. We study optimal redistributive taxation in an environment with asset price movements, adopting the modern finance view that asset prices fluctuate not only because of changing cash flows, but also due to other factors (“discount rates”). We show that the optimal tax base (i) generally differs from the case with constant asset prices, and (ii) includes realized trades whenever asset-price changes are not exclusively driven by cash flow changes. A combination of realization-based capital gains and cash flow taxes implements the optimal allocation regardless of the source of asset-price fluctuations. These results stand in contrast to the classic Haig-Simons comprehensive income tax concept as well as recent proposals for wealth or accrual-based capital gains taxes.

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"Many of the distortions associated with the present system of capital gains taxation result from its deviation from the Haig-Simons approach. These deviations may have historical explanations but their persistence is hard to rationalize from an economic perspective." (Auerbach, 1989)

The treatment of capital gains due to changing asset prices lies at the heart of many debates regarding the taxation of capital income and wealth. While capital gains are typically taxed on realization (i.e. asset sale) in practice, a long tradition in public finance going back to von Schanz (1896), Haig (1921), and Simons (1938) advocates for taxing capital gains on accrual. This idea has recently made its way into policy proposals, including by the Biden administration.[1] In the United States, such tax policies would invariably end up in the Supreme Court which has never conclusively ruled on whether unrealized gains constitute income.[2] Because wealth changes due to asset-price movements typically dwarf ordinary saving and income flows for top wealth holders, debates about wealth taxation also often end up being about the desirability (and practicality) of taxing unrealized capital gains.

The existing public finance literature on optimal capital taxation abstracts from explicitly modelling changing asset prices, and therefore provides no guidance in these debates.[3] Our paper aims to fill this gap by "putting the 'finance' into 'public finance'." That is, we study optimal redistributive taxation in the presence of asset price fluctuations. Importantly, we do so adopting the view of the modern finance literature that asset prices change not only in response to changing cash flows but also due to changes in discount rates (Campbell and Shiller, 1988). In this dichotomy, "discount rates" simply means any sources of asset price changes other than current and expected future cash flows. Empirically, asset prices move too much to be accounted for by changing cash flows alone, both at high frequencies and over longer time horizons.[4]

Our main contribution is to show that optimal redistributive taxes (i) generally differ from the case with constant asset prices, and (ii) must target realized trades whenever asset-price changes are not exclusively driven by cash flow changes. One simple and robust tax implementation that applies without requiring knowledge of the source of asset-price changes is a combination of realization-based capital gains and dividend taxes. The intuition is that, holding constant cash flows, asset-price increases redistribute toward asset sellers who realize

[1] U.S. Office of Management and Budget (2022), U.S. Department of the Treasury (2022), Saez et al. (2021), Zucman (2024), and The Economist (2024). Leiserson and Yagan (2021) calculate that the 400 wealthiest U.S. families paid an average tax rate of only 8.2% in the years 2010 to 2018 by including unrealized capital gains in the tax base.

[2] This is despite the Supreme Court having repeatedly heard such cases since *Eisner v. Macomber* in 1920. The key question is whether unrealized gains constitute income under the 16th Amendment of the U.S. constitution. See Fox and Liscow (2024) for a useful summary of the legal arguments and the U.S. Supreme Court's position.

[3] See, for example, Atkinson and Stiglitz (1976), Chamley (1986), and Judd (1985). Like us, Piketty et al. (2023) lament the absence of asset-price effects from the literature. Interestingly, Lucas (1990) begins his review of the literature with a discussion of capital gains taxation: "When I left graduate school, in 1963, I believed that the single most desirable change in the U.S. tax structure would be the taxation of capital gains as ordinary income. I now believe that neither capital gains nor any of the income from capital should be taxed at all. My earlier view was based on what I viewed as the best available economic analysis, but of course I think my current view is based on better analysis." However, also Lucas does not explicitly consider changing asset prices.

[4] See for example Shiller (1981), Campbell and Shiller (1988), Cochrane (2011), Greenwald et al. (2019) and van Binsbergen (2020), the secular increase in many measures of price-dividend ratios, and the decline in real interest rates. While this is the conventional view, others have argued that fluctuations in cash flows are first order. Our reading of this debate is that it is imperative to understand the tax implications of both sources.

capital gains, away from asset purchasers who pay a higher price for a given dividend stream, while not directly affecting those who do not trade. Optimal redistributive taxation must take this dynamic into account, as well as accounting for any changes in relative income due to cash flows changes.

Taxes that are optimal in environments with constant asset prices may cease to be optimal, or change in counterintuitive ways, when asset prices fluctuate. While a wealth tax may be optimal with constant asset prices, its progressivity needs to change whenever asset prices move and optimal taxation may even prescribe tax cuts for the wealthiest when asset prices rise. Taxing unrealized capital gains is optimal only in restrictive knife-edge cases, so that our results also stand in contrast to the classic Haig-Simons comprehensive income tax concept.[5]

Our study of redistributive taxation with changing asset prices starts from a simple baseline environment: a small open economy in which a large number of investors trade two assets (risky capital and a risk-free bond) with exogenously given asset prices and asset returns that are homogeneous across investors. Investors begin with heterogeneous endowments of the two assets and face different income profiles. The small open economy assumption allows us to study the implications of fluctuations in asset prices and cash flows on the income distribution in a transparent manner. Later, we study various extensions, including general equilibrium and heterogeneous returns, and show that the results from this simple setting generalize.

We are interested in how the optimal tax system redistributes in response to changing asset prices. As a first step, we assume the government has access to type-specific lump-sum taxes and characterize the set of first-best tax schedules that trace out the Pareto frontier. This benchmark is useful as it generates a clear distinction in how a investor's tax burden should react to changes in discount rates versus cash flows. We then show that the principles observed in the first-best problem are present in a second-best allocation in which the government is restricted to distortive taxes à la Mirrlees (1971), so that the classic tradeoff between redistribution and efficiency arises. While the first-best is clearly not realistic and implies extreme predictions about optimal tax *rates*, it turns out to be instructive about the optimal tax *base*, i.e., what taxes should condition on depending on the sources of asset price changes. These results then generalize in a natural way to more interesting second-best tax systems.

To explain our findings, it is useful to consider the standard definition for an asset's return

$$R_{t+1} = \frac{D_{t+1} + p_{t+1}}{p_t}, \tag{1}$$

where p_t denotes the asset's price and D_t its cash flow, i.e. the return equals dividend yield plus capital gain. Suppose the economy is initially in a steady state with a constant asset price $\overline{p}$, dividend $\overline{D}$, and associated asset return $\overline{R}$. This is an example of the case typically studied in the literature and the properties of optimal capital tax systems in such steady states are well understood (see the literature discussion below). We instead allow $\{p_t, D_t, R_t\}$ to fluctuate in flexible ways. Suppose that at time $t = 0$ a shock hits the economy and results in asset

[5] After the quote at the beginning of this introduction, Auerbach (1989) adds: "It is therefore disappointing and puzzling that the debate about capital gains taxes continues to focus almost exclusively on tax rates rather than on tax structure." We wholeheartedly agree.

prices, returns and cash flows deviating from the initial steady state. For example, asset prices p_t may increase because expected future cash flows D_t increase or for other reasons that are independent of changes in cash flows D_t, i.e. discount rate changes. The question we are after is: how should the tax system redistribute in response to these changes?

A useful stepping stone for answering this question is the idea of "Slutsky compensation," defined as the change in the investor's budget that keeps the initial consumption bundle affordable at the new prices and dividends. We show that this compensation generally requires conditioning on realized trades: when asset prices rise, sellers benefit and hence need to be taxed whereas buyers lose and hence need to be compensated. Dividend income changes are similarly compensated or taxed. Building on the Slutsky-compensation logic, optimal first-best taxation is straightforward: just like Slutsky compensation, it taxes sellers, compensates buyers, and taxes dividend income changes. Importantly, it generally targets realized trades rather than asset holdings.

There are two useful polar special cases. In the first special case, the time path of asset prices $\{p_t\}$ changes while cash flows remain at the initial steady state $\overline{D}$. This case corresponds to asset price changes driven entirely by discount rates. In the second special case, asset prices and cash flows $\{p_t, D_t\}$ instead change proportionately and in such a way that the asset return remains at the initial steady state $\overline{R}$, corresponding to asset prices driven entirely by cash flows.

We show that, in the first special case with changing discount rates, the change in the tax burden depends *only* on investors' realized trades (purchases and sales) and the price changes relative to steady state—it is independent of investors' asset holdings. Intuitively, rising asset prices benefit sellers, who are therefore taxed, and hurt buyers, who are therefore subsidized. In contrast, in the second special case with changing cash flows, optimal lump-sum taxes target the investor's individual wealth gain due to the asset price change, so that it is asset holdings rather than transactions that matter. However, this is a knife-edge result: whenever asset-price changes are not exclusively driven by cash flow changes, optimal lump-sum taxes target realized trades as well. A simple implementation that works in both special (and all intermediate) cases is a realization-based capital gains tax combined with a dividend tax.

Wealth taxes are sometimes likened to taxes on "presumptive income" (Zucman, 2024, or the Dutch "box 3" wealth tax): for example, a 2% wealth tax is equivalent to a 40% tax on *presumed* capital income from a *constant* asset return of 5%. When asset values increase and the increase is entirely due to higher cashflows (the second special case), the asset return remains constant and therefore the increase in presumptive income exactly matches the increase in actual income. But in all other cases, the return falls and therefore actual income rises by less than presumptive income calculated as a constant return to the increased market value of wealth. Thus "presumptive income" is overestimated and hence wealth taxes redistribute suboptimally whenever asset valuations are not exclusively driven by cash flows.[6]

While our formula for optimal redistributive taxes is reminiscent of realization-based capital gains taxes in practice, it also differs in important ways. For example, optimal taxes (i) not only tax sellers but also compensate buyers who experience "purchasing losses" when prices

[6] Appendix F further illustrates this analogy by means of a simple numerical example.

rise; (ii) they compensate realized capital *losses* and tax "purchasing gains" when prices fall; (iii) they tax *net* rather than *gross* transactions (selling and re-investing at the same price incurs no tax liability); (iv) they adjust for inflation; and (v) the capital gain or loss is typically calculated relative to a basis that differs from the historical basis at which the investor purchased the asset. Finally, in the first-best case with lump-sum taxes, our formula corresponds to a tax rate of 100%, i.e. the government taxes away realized capital gains in their entirety and uses the proceeds to compensate the losers from rising prices.

The first-best tax scheme is designed for redistribution, not to replace missing insurance markets. In particular, the government is not providing investors with otherwise missing insurance against future asset price or cash flow fluctuations. A tax scheme focusing on net trades, however, does have advantages in terms of simplicity and robustness over ones that involve transfers only in the initial period, a point we discuss explicitly in the text.

Turning to second-best tax systems à la Mirrlees (1971), our results regarding the optimal tax *base* carry over from the first-best analysis in a natural way. In this environment, only investor *choices* can be taxed, for example asset sales, consumption, or savings. Our interest remains how the second-best optimal policy redistributes when asset prices change. We show that the tax schedule monotonically increases as a function of trading gains, albeit with a slope less than in the first-best. This is intuitive—taxing asset sellers in response to a price increase achieves a preferred distribution of income, like in the first-best, but now also distorts saving behavior. We also show that the optimal tax schedule converges to the first-best optimum as the investors' inter-temporal elasticity of substitution goes to zero. Hence, our insights from the first-best tax system are not knife-edge, but extend qualitatively to environments with more limited and realistic tax instruments.

If end-of-period wealth is taxed rather than sales, optimal taxes may become *less* progressive when asset prices rise. Intuitively, if those holding the asset at the end of the period are net *purchasers*, they should be subsidized rather than taxed (relative to the baseline tax schedule). While this example is extreme, it illustrates why the fluctuating market value of investors' asset holdings is a problematic target for redistributive taxes.

Our analysis of second-best tax systems also considers the "lock-in" effect emphasized in the capital gains taxation literature: realization-based taxes may incentivize deferring the liquidation of appreciated assets and distort optimal portfolio allocation. Using a two-asset version of our model, we show that an optimally designed second-best tax system avoids such distortions even when it targets realized capital gains. It does so by targeting total net trades rather than gains from selling individual assets: when an investor sells one asset and uses the proceeds to purchase another one, there is no tax burden, thus eliminating the lock-in effect.

Finally, we consider several extensions: general equilibrium, return heterogeneity, and bequests. While some of these features modify our optimal tax formula in natural ways, the key findings emphasized so far remain unchanged. Specifically, in contrast to the classic Haig-Simons comprehensive income tax concept, optimal redistributive taxes generally target realized trades. In fact, we show that when investors receive heterogeneous cash flows, taxes on unrealized capital gains or wealth are no longer optimal *even when asset prices are driven entirely* *by cash flows*, reinforcing our results from the baseline setting. Our model with borrowing also speaks to an issue that has received attention in the popular debate: wealthy individuals borrowing against appreciating assets rather than selling them, often aiming to take advantage of the "stepped-up basis" for bequeathed assets as part of a "buy, borrow, die" strategy. Our results suggest that basis step-up should be abolished, eliminating the viability of such plans.

Literature. Our paper contributes to the literature studying the optimal taxation of capital income and wealth.[7] To differentiate our paper, it is again useful to consider the expression for an asset's return (1). The existing literature features either a constant asset price (and hence no capital gains or losses) or works with variants of the neoclassical growth model. In this model, asset-return movements are typically small, reflecting the disappointing asset-pricing properties of the standard real business cycle model. Our analysis instead allows for flexible changes in asset returns that are independent of changes in cash flows, i.e. discount rate changes. Within the environments it has considered, the literature has shown that taxing asset *holdings* may be optimal, for example by means of a wealth tax. Our paper instead shows that such taxes are problematic whenever asset prices fluctuate and are not exclusively driven by cash flow changes. In all such cases, taxes must target realized trades, and generally involve a combination of realization-based capital gains and dividend taxes.

In line with our argument that it is essential to "put the 'finance' into 'public finance,'" a growing positive literature has documented an important role for asset-price and interest-rate changes in driving wealth inequality (e.g. Bonnet et al., 2014; Rognlie, 2015; Kuhn et al., 2020; Gomez, 2016; Wolff, 2022; Gomez and Gouin-Bonenfant, 2020; Cioffi, 2021; Catherine et al., 2020, 2024; Greenwald et al., 2021; Moll, 2020; Martínez-Toledano, 2022; Fagereng et al., 2023; Coven et al., 2024). The logic of our results is related to Moll (2020) and Fagereng et al. (2023) who study the welfare-relevant redistributive effects of changing asset prices. Our paper contributes to this literature by instead studying the normative implications of changing asset prices, specifically their implications for optimal capital taxation.

There is also an empirical literature studying behavioral responses, specifically of asset sales, to capital gains taxation aiming to estimate the relevant elasticities.[8] Our paper tackles optimal distortive taxation à la Mirrlees (1971) only in a stylized two-period model, which is not suitable for making quantitative predictions about optimal tax rates, but such elasticities will be key inputs in more quantitative work.

While the modern capital taxation literature provides no guidance on how to tax capital gains, an older literature anticipates some of the ideas in our paper using verbal or graphical arguments. This includes Paish (1940), Kaldor (1955) and Whalley (1979), which were partly reactions to Haig (1921) and Simons (1938) who developed the eponymous income concept.

[7] Apart from the classic contributions mentioned above, see the references in Section 1.5 and the surveys by Golosov et al. (2007), Banks and Diamond (2010), Bastani and Waldenstrom (2020), Stantcheva (2020), and Scheuer and Slemrod (2021).

[8] See for example Poterba (2002), Feldstein et al. (1980), Agersnap and Zidar (2021), and Msall and Næss (2025). There are also theoretical and quantitative studies of behavioral responses including the lock-in effect (e.g. Constantinides, 1983; Chari et al., 2005; Smith and Miller, 2023).

Roadmap. Section 1 spells out our baseline environment. Section 2 focuses on a special case with two time periods and no risk to convey our key results most transparently. Section 3 studies the first-best allocation assuming that the government has access to type-specific lump-sum taxes. In contrast, Section 4 considers the second-best problem with distortive taxation and discusses the lock-in effect. Section 5 shows how our findings carry over to the stochastic multi-period model of Section 1. Section 6 considers extensions and Section 7 concludes.

1 Baseline model

We begin by spelling out our baseline environment, which is kept purposely simple: A large number of investors trade two assets (risky capital and a risk-free bond) in a small open economy with exogenous asset prices and returns that are homogeneous across investors. In Section 6, we will consider various extensions, including general equilibrium, return heterogeneity, and intergenerational considerations. For now, we omit taxes from the analysis, which we will introduce in Section 3.

1.1 Investors

Time is discrete and indexed by $t = 0, 1, ..., T$, where T may be finite or infinite. Let s_t denote the state of nature in period t, which takes discrete values in a set $\mathcal{S}$, and let $s^t \equiv \{s_0, s_1, \ldots, s_t\}$ denote the history of states up to and including period t with associated probabilities $\pi(s^t)$. When convenient, we shall suppress the history notation and simply use the time index.

There is a continuum of heterogeneous investors indexed by their type $\theta \in [\underline{\theta}, \bar{\theta}]$, which is distributed in the population according to the cumulative distribution function $F(\theta)$. Investors have preferences over consumption sequences, $\{c_t(s^t, \theta)\}_{t,s^t}$, captured by the utility function $U(\{c_t(s^t, \theta)\})$, which is assumed to be homothetic, strictly increasing, strictly concave and differentiable in all its arguments. We embed the probability of history s^t inside the function U, which allows us to nest both expected utility and other popular specifications, such as Epstein and Zin (1989). Investors receive type-specific exogenous income flows $\{y_t(\theta)\}_{t=0}^{T}$. Since we focus on wealth and capital gains, we assume for simplicity that the income paths are deterministic for each type.

Households can transfer income across periods by saving in two assets: a potentially risky asset k that pays a dividend stream $\{D_t(s^t)\}_{t,s^t}$ and a risk-free, zero-coupon bond b. For now, we also take prices as given, with $p_t(s^t)$ denoting the price of capital and $q_t(s^t)$ the price of the bond in period t. Investors are endowed with initial assets $\{k_0(s_{-1}, \theta), b_0(s_{-1}, \theta)\}$ at time zero and, at each history s^t, choose a portfolio $\{k_{t+1}(s^t, \theta), b_{t+1}(s^t, \theta)\}$ to carry into the next period. There is no short-selling constraint, and hence asset positions may be positive or negative. In particular, investors can have negative bond holdings (i.e., borrow) while at the same time owning the capital asset. In Section 5.2 we will use this setup to discuss optimal taxation when investors borrow against appreciating assets.

The problem of an investor of type θ is to maximize her utility

$$\mathcal{U}(\theta) = \max_{\{c_t(s^t,\theta),k_{t+1}(s^t,\theta),b_{t+1}(s^t,\theta)\}_{t,s^t}} U\left(\{c_t(s^t,\theta)\}_{t,s^t}\right) \quad \text{subject to}$$

$$\begin{aligned} c_t(s^t,\theta) + p_t(s^t)(k_{t+1}(s^t,\theta) - k_t(s^{t-1},\theta)) + q_t(s^t)b_{t+1}(s^t,\theta) \\ = y_t(\theta) + D_t(s^t)k_t(s^{t-1},\theta) + b_t(s^{t-1},\theta) \quad \forall t, s^t. \end{aligned} \tag{2}$$

We impose $p_T(s^T) = q_T(s^T) = 0$ if T is finite or a No-Ponzi condition if $T = \infty$.

1.2 Aggregate economy

The economy's aggregate resource constraint is found by simply aggregating investors' budget constraints (2) across individuals. To this end, we use the convention to denote aggregate variables by capital letters, for example

$$C_t(s^t) = \int c_t(s^t,\theta)dF(\theta), \quad K_t(s^{t-1}) = \int k_t(s^{t-1},\theta)dF(\theta), \quad B_t(s^{t-1}) = \int b_t(s^{t-1},\theta)dF(\theta),$$

and so on. With this notation, the aggregate resource constraint is

$$p_t(s^t)K_{t+1}(s^t) + q_t(s^t)B_{t+1}(s^t) + C_t(s^t) = (p_t(s^t) + D_t(s^t))K_t(s^{t-1}) + B_t(s^{t-1}) + Y_t \tag{3}$$

for all t, s^t. As already noted, our benchmark analysis focuses on a small open economy with an exogenously given time path for asset prices and dividends $\{q_t(s^t), p_t(s^t), D_t(s^t)\}_{t,s^t}$. Hence, the economy's aggregate bond and capital holdings at time $t+1$ may differ from those at t as the economy as a whole may be a net buyer or net seller of B or K. In Section 6.1, we alternatively consider a closed-economy general equilibrium version of the model in which the assets are in fixed supply, so that sales or purchases are zero in the aggregate: for every seller, there is a buyer.

1.3 Sources of asset-price changes

Our interest is in the taxation of gains or losses due to changes in asset prices. The asset-pricing literature emphasizes different sources of asset price changes, in particular distinguishing between asset discount rates and cash flows. In this dichotomy, "discount rates" simply means any sources of asset price changes other than current and expected cash flows. Using a decomposition of observed asset price changes due to Campbell and Shiller (1988), much of this literature has found that discount rate shocks account for most of asset price fluctuations.[9] Other studies have argued that fluctuations in cash flows are first order.[10] Our reading of this debate is that it is imperative to understand the tax implications of both sources.

Our partial equilibrium model takes dividends $\{D_t(s^t)\}$ and asset prices $\{p_t(s^t), q_t(s^t)\}$ as given. Instead, the perspective of the asset pricing literature is to treat required asset returns or

[9] See Campbell (2018, Section 5.3.1) for an expository derivation of the Campbell-Shiller decomposition.
[10] See for example, Larrain and Yogo (2008) and Atkeson et al. (2024), but also see Nagel (2024).

stochastic discount rates as a primitive and prices as an outcome. One way of thinking about this is that, in equilibrium models, it is typically the discount factor that is pinned down which, in turn, determines asset prices. To this end, retain the small-open-economy assumption and denote by $m_{t+1}(s^{t+1})$ the stochastic discount factor of the representative counterparty in global financial markets between history s^t and s^{t+1}. To simplify notation, we drop the s^t-arguments and write m_{t+1}. Using this, the asset prices in period t satisfy

$$q_t = \mathbb{E}_t\left[m_{t+1}\right], \tag{4}$$

$$p_t = \mathbb{E}_t\left[m_{t+1}\left(D_{t+1} + p_{t+1}\right)\right]. \tag{5}$$

In words, since a bond purchased in period t pays off one unit of consumption in all states of the world in period $t+1$, its price is given by the mean stochastic discount factor. Similarly, the price of the risky asset at time t equals the expected discounted sum of dividend and price at $t+1$, i.e., it consists of dividend yield and capital gain. Dividing both sides by p_t and using the definition of the asset return R_{t+1} in equation (1) yields $1 = \mathbb{E}_t[m_{t+1}R_{t+1}]$, so the stochastic discount factor m_{t+1} and R_{t+1} are inversely related. Defining

$$m_{t\to t+k} \equiv m_{t+1} \cdot m_{t+2} \cdots m_{t+k}$$

as the stochastic discount factor between history s^t and s^{t+k} and iterating on equation (5) yields

$$p_t = \mathbb{E}_t\left[\sum_{k=1}^{T-t} m_{t\to t+k} D_{t+k}\right], \tag{6}$$

i.e., the asset price equals the expected present-discounted value of future dividends. The asset price may therefore change for two reasons: changing dividends $\{D_{t+k}\}$ or changing discount factors $\{m_{t\to t+k}\}$. Accordingly, we can consider the following two extremal cases:

1. Changes in asset prices $\{p_t, q_t\}$ driven entirely by changes in the stochastic discount factor $\{m_t\}$ while holding dividends $\{D_t\}$ fixed. An important special case occurs when discount rates change such that the bond price $\{q_t\}$ and hence the risk-free interest rate $\{1/q_t\}$ remain unchanged, which corresponds to a pure risk-premium change.

2. Changes in the asset prices $\{p_t\}$ driven entirely by changes in dividends $\{D_t\}$ while holding the stochastic discount factor $\{m_t\}$ fixed (the bond prices $\{q_t\}$ remain constant in this case). An important special case is the Gordon growth model (Gordon and Shapiro, 1956) or stochastic versions of it.

Both of these cases are the opposite extremes of the general, intermediate case, with arbitrary changes in $\{p_t, q_t\}$ and $\{D_t\}$, which corresponds to asset price changes driven by a mixture of dividend and discount rate changes.

Thus, our approach is flexible enough to capture a wide range of state-of-the-art asset pricing models. For example, since by equations (4) and (5) the asset prices $\{p_t, q_t\}$ depend on the probabilities $\pi(s^t)$ with which different histories occur (through the expectations operator), we can also allow for subjective beliefs as potential drivers of asset prices (Adam et al., 2017; Bordalo et al., 2023). More optimistic beliefs correspond to putting higher subjective probabilities $\tilde{\pi}(s^t)$ on histories in which cash flows $D_t(s^t)$ are high. This generates an increase in the asset price p_t while leaving actual cash flows unaffected. Changes in subjective beliefs are therefore equivalent to discount rate shocks (Special Case 1).

1.4 The deterministic case

While our results hold for the general model we have just introduced, some insights become particularly easy to understand in the special case without uncertainty ($|\mathcal{S}| = 1$). Then (4) and (5) imply $q_t = m_{t+1} = 1/R_{t+1}$, so the bond and the capital asset are equivalent and the model collapses to the single-asset case. Furthermore, rather than considering different realizations of random variables, the deterministic case lends itself to simple comparative statics exercises which can be interpreted as realizations of MIT shocks.

Comparative statics as MIT shocks. In the deterministic case, equation (6) simplifies to

$$p_t = \sum_{k=1}^{T-t} \frac{D_{t+k}}{R_{t\to t+k}}, \tag{7}$$

where $R_{t\to t+k}$ is the cumulative return between time t and $t+k$:

$$R_{t\to t+k} \equiv R_{t+1} \cdot R_{t+2} \cdots R_{t+k}. \tag{8}$$

Below we will often conduct comparative statics in which the time path of some variable changes from a baseline to an alternative, which then induces a change in asset prices. For example, in Special Case 2 above, dividends may change from $\{\overline{D}_t\}$ to $\{D_t\} = \{\overline{D}_t + \Delta D_t\}$ holding constant $\{R_t\} = \{\overline{R}_t\}$, resulting in a change in the time path of asset prices

$$\Delta p_t = \sum_{k=1}^{T-t} \overline{R}_{t\to t+k}^{-1} \Delta D_{t+k}. \tag{9}$$

Alternatively, in Special Case 1 above, asset prices may change without any corresponding change in dividends. One useful interpretation of such comparative statics is as MIT shocks, i.e. realizations of zero-probability events in a stochastic setting: at time t, some new information arrives which changes asset prices going forward. As we show when we analyze the fully stochastic model in Section 5, our expressions take the same form regardless of whether we conduct comparative statics in a deterministic setting or compare across histories in a stochastic setting. For this reason, our analysis in Sections 2 to 4 focuses on the deterministic case.

Haig-Simons income. As already noted, in the deterministic case, the model collapses to the single-asset case. Dropping the bond, we can write the budget constraint (2) as

$$c_t(\theta) + p_t(k_{t+1}(\theta) - k_t(\theta)) = y_t(\theta) + D_t k_t(\theta) \quad \forall t \geq 0, \tag{10}$$

which states that "consumption plus saving equals income." An equivalent way of writing this accounting identity adds unrealized capital gains $(p_t - p_{t-1})k_t(\theta)$ on both sides, thus changing the definitions of saving and income (consumption is unchanged):

$$c_t(\theta) + \underbrace{p_t k_{t+1}(\theta) - p_{t-1}k_t(\theta)}_{\text{change in wealth}} = \underbrace{y_t(\theta) + D_t k_t(\theta) + (p_t - p_{t-1})k_t(\theta)}_{\text{Haig-Simons income}} \quad \forall t \geq 0.$$

Formulation (10) features disposable income, whereas this formulation features "Haig-Simons income" which includes unrealized capital gains (Haig, 1921; Simons, 1938). Defining the market value of wealth $a_t(\theta) \equiv p_{t-1}k_t(\theta)$ and the net return including capital gains $r_t \equiv R_t - 1$, Haig-Simons income also equals $y_t(\theta) + r_t a_t(\theta)$, i.e. income including total capital income.[11] Similarly, adding $a_t(\theta)$ on both sides of the budget constraint yields the standard

$$c_t(\theta) + a_{t+1}(\theta) = y_t(\theta) + R_t a_t(\theta) \quad \forall t \geq 0, \tag{11}$$

with $a_0(\theta) = p_{-1}k_0(\theta)$ given.

1.5 Comparison to setups studied in the capital taxation literature

Before proceeding, we briefly connect our setup to other models in the existing literature on optimal capital taxation. These make different assumptions on the determination of asset prices and dividends $\{p_t, D_t\}$, and hence returns $\{R_t\}$.

Partial equilibrium models. This is the special case with $R_t = \overline{R}$ for all t. The most obvious way of generating this is to assume that $p_t = \overline{p}$ and $D_t = \overline{D}$ for all t. Alternatively, prices and dividends could grow at the same constant rate. This captures models of capital taxation with a linear savings technology, such as the finite-horizon models based on Atkinson and Stiglitz (1976) (e.g. Saez, 2002; Scheuer and Wolitzky, 2016; Hellwig and Werquin, 2024; Ferey et al., 2024), some of the new dynamic public finance literature (surveyed in Golosov et al. (2007)), or infinite-horizon partial equilibrium models such as Saez and Stantcheva (2018).

Neoclassical growth model. Starting with Chamley (1986), many papers have studied optimal capital taxation in variants of the growth model. Denote by $\sum_{t=0}^{T} \beta^t U(C_t)$ the preferences of the representative consumer and by $f(K_t, A_t L_t)$ the constant-returns technology for producing output, where C_t is consumption, K_t is capital, A_t is productivity, and L_t is labor with inelastic supply $L_t = 1$. How to map the growth model into our setup depends on the particular decentralization. In any case, the asset return is $R_{t+1} = f_K(K_{t+1}, A_{t+1}) + 1 - \delta$ and this asset return equals the relevant discount rate (this is the standard Euler equation):

$$R_{t+1} = \frac{1}{\beta}\frac{U'(C_t)}{U'(C_{t+1})}. \tag{12}$$

[11] The mismatching time subscripts in $a_t(\theta) \equiv p_{t-1}k_t(\theta)$ are solely due to our notational convention which uses $k_t(\theta)$ to denote asset holdings *at the beginning* of period t. Alternatively, using $k_t(\theta)$ to denote asset holdings *at the end* of period t, (2) becomes $c_t(\theta) + p_t(k_t(\theta) - k_{t-1}(\theta)) = y_t(\theta) + D_t k_{t-1}(\theta)$ so that wealth is $a_t(\theta) \equiv p_t k_t(\theta)$.

Furthermore, the unit price of capital (relative to consumption) equals one because the consumption good can be converted into investment one-for-one.[12]

In contrast, dividends and asset prices $\{D_t, p_t\}$ differ across decentralizations. For example, our asset may correspond to shares in the representative firm which are in unit fixed supply, the typical assumption in the literature studying asset pricing in production economies (e.g. Jermann, 1998).[13] The cash flows D_t are then firm profits net of investment and the asset price equals the firm's capital stock $p_t = K_{t+1}$ (see Appendix A.1) so variations in the capital stock generate capital gains and losses.

An interesting case is that of a balanced growth path (BGP) with productivity growth $A_{t+1}/A_t = G > 1$ and isoelastic preferences $U'(C) = C^{-1/\sigma}$. On this BGP, the asset return is constant and pinned down from $\overline{R} = (1/\beta)G^{1/\sigma}$ but it consists of both a dividend yield and a capital gains component:

$$\frac{D_{t+1}}{p_t} = \overline{R} - G, \quad \frac{p_{t+1}}{p_t} = G.$$

Capital income is the sum of dividend income plus (unrealized) capital gains and Chamley's result is that the long-run tax rate on this combined capital income should be zero.

Our interest is in optimal capital gains taxation in response to asset-price fluctuations away from such a balanced growth path. With the right decentralization, a stochastic version of the model above (as in, for example, Zhu, 1992; Chari and Kehoe, 1999) would feature such asset-price fluctuations. However, with standard shock processes, movements in the stochastic discount factor and hence asset return R_{t+1} would be quantitatively small, analogous to the disappointing asset-pricing properties of the real business cycle model.[14] We instead allow for flexible stochastic processes for the drivers of asset prices, including potentially large fluctuations in stochastic discount factors and asset returns.

Growth models with heterogeneous households. Going back to Judd (1985), many contributions have studied capital taxation in growth models with heterogeneous households or entrepreneurs (see Werning (2007), Shourideh (2012), Farhi et al. (2012), Straub and Werning (2020), Benhabib and Szöke (2021) and Guvenen et al. (2023, 2024) for recent examples). Despite the (often) rich heterogeneity, the backbone of all of these papers is the neoclassical growth model, so the discussion in the preceding paragraph still applies.

Our setup. In sum, the setups studied in the existing literature feature either constant asset prices or small movements of asset returns and a constant unit price of capital. We instead study optimal taxation with exogenous stochastic processes for discount factors and dividends $\{m_t, D_t\}$ and associated prices and returns $\{p_t, R_t\}$. This allows us to take on board the modern finance view that asset prices change not only because of changing cash flows but also due

[12] We discuss this property in more detail in Appendix A.1 where we also discuss how to break it.

[13] Chari et al. (2018), an earlier version of Chari et al. (2020), analyzes optimal capital taxation in such a setup.

[14] The difficulty with explaining asset returns in RBC models is connected to the assumption that the consumption good can be converted into investment one-for-one. For example, Jermann (1998) writes that "in the standard one-sector model agents can easily alter their production plans to reduce fluctuations in consumption. This suggests that the frictionless and instantaneous adjustment of the capital stock is a major weakness in this framework."

discount rates. While our baseline analysis is therefore silent on the ultimate fundamental drivers of asset prices (preferences and technology), Sections 5 and 6 show that our findings remain valid in richer environments endogenizing these fluctuations.

1.6 Efficient allocations

We conclude this section by evoking an auxiliary property of first-best Pareto efficient allocations in our general model that will be useful below. For now we do not consider the question of implementing these allocations with taxes, which will be the focus of the next sections.

Let $\omega(\theta)$ be the Pareto weight on an investor of type θ. Any Pareto efficient allocation $\{c_t^*(s^t,\theta)\}$ must satisfy the following sub-problem:

$$\max_{\{c(s^t,\theta)\}} \int \omega(\theta) U(\{c(s^t,\theta)\}) dF(\theta) \quad \text{s.t.} \quad \int c(s^t,\theta) dF(\theta) \leq C_t(s^t) \quad \forall t, s^t \tag{13}$$

for some aggregate consumption $\{C_t(s^t)\}$. The following lemma shows that, given our assumption of homothetic preferences, any optimum will obey a linear sharing rule.

Lemma 1. *Any solution $\{c_t^*(s^t,\theta)\}$ to problem (13) satisfies*

$$c_t^*(s^t,\theta) = \Omega(\theta) C_t(s^t) \quad \forall \theta, t, s^t$$

for some θ-dependent constant $\Omega(\theta)$ which is increasing in $\omega(\theta)$ and satisfies $\int \Omega(\theta) dF(\theta) = 1$.

This property will allow for particularly transparent expressions in our benchmark results below on how first-best optimal taxes respond to changes in asset prices. In Section 4, we will consider second-best optimal taxes where this property no longer necessarily holds.

2 Two time periods

To build intuition, we now turn to the case with two time periods and no risk. Our analysis of optimal taxation in the next two sections uses this model before we show in Section 5 that our findings carry over to the general multi-period case with uncertainty.

2.1 The investor's problem

With two time periods $t = 0, 1$ and no risk, we can drop the bond (as discussed in Section 1) and the investor's problem is to maximize utility $U(c_0(\theta), c_1(\theta))$ subject to:

$$c_0(\theta) + p(k_1(\theta) - k_0(\theta)) = y_0(\theta), \tag{14}$$

$$c_1(\theta) = y_1(\theta) + Dk_1(\theta). \tag{15}$$

Given that the world ends after time period 1, the asset price $p_1 = 0$. For simplicity, we also assume that the asset pays no dividend in the first period $D_0 = 0$. Given this, we then drop the time-subscripts on p_0 and D_1 to ease notation. The asset's returns in the two time periods are given by

$$R_0 \equiv \frac{p}{p_{-1}} \quad \text{and} \quad R_1 \equiv \frac{D}{p}, \tag{16}$$

which is the standard expression (1) with $D_0 = p_1 = 0$ and where p_{-1} is a baseline price.

A useful reformulation of the investors' problem is in terms of asset sales $x(\theta) \equiv k_0(\theta) - k_1(\theta)$, where $x > 0$ represents sales and $x < 0$ purchases of k. Using this, the investors solve:

$$\begin{aligned} \mathcal{U}(\theta) \equiv & \max_{\{c_0(\theta), c_1(\theta), x(\theta)\}} U(c_0(\theta), c_1(\theta)) \quad \text{s.t.} \\ & c_0(\theta) = y_0(\theta) + px(\theta) \\ & c_1(\theta) = y_1(\theta) + D(k_0(\theta) - x(\theta)) \end{aligned} \tag{17}$$

The $t = 0$ budget constraint states that consumption $c_0(\theta)$ equals exogenous income $y_0(\theta)$ plus revenue from asset sales $px(\theta)$. The $t = 1$ budget constraint states that consumption $c_1(\theta)$ equals exogenous income $y_1(\theta)$ plus capital income $D(k_0(\theta) - x(\theta))$ consisting of the dividend payments D on the assets brought forward to period 1, $k_0(\theta) - x(\theta)$.

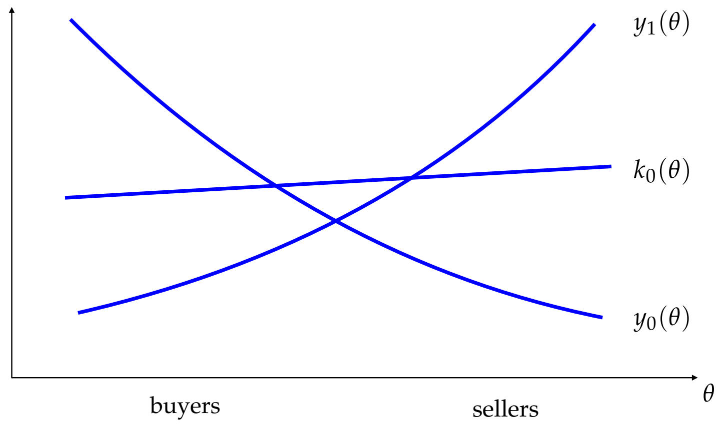


Figure 1: Example of heterogeneity in initial assets and incomes over time

Fundamentally, investors differ in initial asset holdings $k_0(\theta)$ and incomes $\{y_0(\theta), y_1(\theta)\}$. This heterogeneity generates gains from trade, with natural buyers and sellers of the asset. Figure 1 depicts an example in which high-θ types have lower initial income $y_0(\theta)$ but higher future income $y_1(\theta)$ and relatively similar initial asset holdings $k_0(\theta)$. In this example, low-θ types will be buyers of the asset with $x(\theta) < 0$ (savers with $c_0(\theta) < y_0(\theta)$) whereas high-$\theta$ types will be sellers with $x(\theta) > 0$ (effective borrowers with $c_0(\theta) > y_0(\theta)$).

It is sometimes useful to combine the two period budget constraints in (17) into a present-value budget constraint:

$$c_0(\theta) + \frac{p}{D} c_1(\theta) = y_0(\theta) + \frac{p}{D} y_1(\theta) + pk_0(\theta). \tag{18}$$

This constraint states that the present-discounted value of consumption (discounted at the asset return $R_1 = D/p$ defined in (16)) equals the present-discounted value of income plus initial wealth. This constraint can also be aggregated across all investors to yield

$$C_0 + \frac{p}{D}C_1 \leq Y_0 + \frac{p}{D}Y_1 + pK_0, \tag{19}$$

which is the economy's aggregate resource constraint in this partial equilibrium model.

Sources of asset-price changes. In this model, the discussion of cash flows and discount rates as drivers of asset-price changes in Section 1.3 becomes particularly simple. Treating the required return R_1 and dividends D as the primitives in (16), the two-period version of (7) is simply $p = D/R_1$. Special Case 1 is thus the case in which the asset price p changes holding dividends D fixed. On the opposite extreme, Special Case 2 is the case in which both p and D change proportionately such that the asset return $R_1 = D/p$ stays constant.

2.2 An Envelope Condition

The goal of our paper is to study how the optimal tax system deals with changing asset prices. As a warm up, it is useful to first consider a simpler question: what are the redistributive effects of rising asset prices and cash flows, i.e., who wins and who loses as a result of these changes? Consider small deviations of the asset price $\mathrm{d}p$ and dividends $\mathrm{d}D$. Following Moll (2020) and Fagereng et al. (2023), we use the envelope theorem to differentiate the value function $\mathcal{U}(\theta)$ of investors defined in (17) to obtain

$$\mathrm{d}\mathcal{U}(\theta) = U_{c_0}(\theta)\left(x(\theta)\,\mathrm{d}p + \frac{p}{D}k_1(\theta)\,\mathrm{d}D\right). \tag{20}$$

Consider first Special Case 1: the asset price rises ($\mathrm{d}p > 0$) but cash flows are fixed ($\mathrm{d}D = 0$). The marginal welfare effect is given by marginal utility times the extent to which this rise relaxes the budget constraint at $t = 0$, namely asset sales $x(\theta)$ times the price change $\mathrm{d}p$. Intuitively, a rising asset price benefits sellers of the asset (i.e., $x(\theta) > 0$) and hurts buyers (i.e., $x(\theta) < 0$). To first order, it does not affect individuals who do not plan to trade (i.e., $x(\theta) = 0$): for them, the increasing asset price is merely a "paper gain" with no corresponding effect on welfare. Hence, only asset *transactions* matter whereas asset *holdings* do not.

When dividends rise ($\mathrm{d}D > 0$)—as in Special Case 2—the second term in (20) becomes non-zero since this directly benefits asset holders ($k_1(\theta) > 0$). However, it remains true that the welfare effect of the asset-price change $\mathrm{d}p$ itself depends only on asset transactions $x(\theta)$.

3 First best

We are interested in how the optimal tax system redistributes in response to changing asset prices. As a first step, we will assume that the government has access to type-specific lump-sum taxes. While this implies extreme predictions about tax *rates*, it turns out to be instructive about the optimal tax *base*, i.e., what quantities taxes should target, which is our main object of interest. We will consider more realistic, second-best tax systems in Section 4.

3.1 First-best consumption allocation

For a given asset price p and dividend D, any Pareto efficient allocation $\{c_0^*(\theta), c_1^*(\theta)\}$ solves

$$\max_{\{c_0(\theta), c_1(\theta)\}} \int \omega(\theta) U(c_0(\theta), c_1(\theta)) dF(\theta) \quad \text{s.t.} \quad (19). \tag{21}$$

By Lemma 1, we have $c_t^*(\theta) = \Omega(\theta) C_t^*$, so the planner assigns to each investor θ a time-invariant share of (optimally chosen) aggregate consumption C_t^*. The optimal allocation can be implemented in a decentralized equilibrium when the government is able to redistribute with type-specific lump-sum taxes $T_0(\theta)$ in period 0 and $T_1(\theta)$ in period 1. The investors' budget constraints (14) and (15) become

$$c_0(\theta) = y_0(\theta) + px(\theta) - T_0(\theta) \tag{22}$$

$$c_1(\theta) = y_1(\theta) + D(k_0(\theta) - x(\theta)) - T_1(\theta). \tag{23}$$

We impose the government budget constraints

$$\int T_0(\theta) dF(\theta) = \int T_1(\theta) dF(\theta) = 0,$$

which implies, without loss, that the government does not own assets itself.

To back out the optimal taxes from the optimal consumption allocation $\{c_0^*(\theta), c_1^*(\theta)\}$, we can use the budget constraints (22) and (23). Given the ability of an investor to move resources inter-temporally, $T_0(\theta)$ and $T_1(\theta)$ are not separately pinned down and we require a normalization. One example is to set $T_1(\theta) = 0$. Then the second-period budget constraint (23) determines $x^*(\theta)$ and we obtain $T_0(\theta)$ from the first-period budget constraint (22). However, we will also consider alternative normalizations below when this is convenient.

3.2 Taxing changing asset prices

We begin with the general case of prices and dividends (p, D) that vary relative to some baseline values $(\overline{p}, \overline{D})$, allowing for asset prices driven by a mixture of discount rate and dividend changes. The goal is to design a tax rule $T_0(\theta) = T_0(\theta; p, D)$ that optimally redistributes across investors in response to these (p, D) variations. We denote by $\overline{T}_0(\theta) = T_0(\theta; \overline{p}, \overline{D})$ the taxes that implement the Pareto efficient allocation at the baseline prices and dividends, and by $\Delta p = p - \overline{p}$ and $\Delta D = D - \overline{D}$ the changes in prices and dividends relative to the baseline.

Proposition 1. *Let the asset price change from $\overline{p}$ to $p = \overline{p} + \Delta p$ and dividends from $\overline{D}$ to $D = \overline{D} + \Delta D$. Then the optimal tax $T_0(\theta)$ (when $T_1(\theta) = 0$) is given by*

$$\begin{aligned} T_0(\theta) &= \overline{T}_0(\theta) + \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D - \Omega(\theta)\left[\overline{X}\Delta p + \frac{p}{D}\overline{K}_1 \Delta D\right] \\ &= \overline{T}_0(\theta) + x(\theta)\Delta p + \frac{\overline{p}}{\overline{D}} k_1(\theta)\Delta D - \Omega(\theta)\left[X\Delta p + \frac{\overline{p}}{\overline{D}} K_1 \Delta D\right] \end{aligned}$$

where $x(\theta)$ and $k_1(\theta)$ are investor θ's asset sales and second-period asset holdings at the new price p and dividends D, X and K_1 are the corresponding aggregate asset sales and holdings, and, similarly, $\overline{x}(\theta)$, $\overline{k}_1(\theta)$, $\overline{X}$, and $\overline{K}_1$ are asset sales and holdings at the baseline price $\overline{p}$ and dividend $\overline{D}$.

Slutsky Compensation. To build intuition for this result, it is helpful to relate it to the concept of "Slutsky compensation," which is sometimes used to define income and substitution effects of price changes. Slutsky compensation is defined as the change in the investor's total budget (i.e., the change in initial endowment y_0) that keeps the initial consumption bundle affordable at the new prices (e.g. Mas-Colell et al., 1995, pp. 29-30).[15] Using this idea, we have the following lemma:[16]

Lemma 2. *When the asset price changes from $\overline{p}$ to $p = \overline{p} + \Delta p$ and dividends change from $\overline{D}$ to $\overline{D} + \Delta D$, the corresponding Slutsky compensation $\Delta y_0(\theta)$ is given by*

$$\Delta y_0(\theta) = -\overline{x}(\theta)\Delta p - \frac{p}{D}\overline{k}_1(\theta)\Delta D.$$

This reveals that the first part of the optimal tax change characterized in the first equation in Proposition 1 coincides with the Slutsky compensation for the underlying price and dividend change. It is useful to organize the interpretation of the result and why it is related to Slutsky compensation along the special cases from Section 1.3.

3.2.1 Special Case 1: Only Discount Rate Changes

This is the first experiment discussed in Section 1.3, namely, an asset price change Δp exclusively driven by a change in the discount rate and hence holding dividends constant ($\Delta D = 0$). Then we immediately obtain the following corollary of Proposition 1:

Corollary 1. *Let the asset price change from $\overline{p}$ to $p = \overline{p} + \Delta p$ holding dividends fixed $D = \overline{D}$. Then the optimal tax $T_0(\theta)$ is given by*

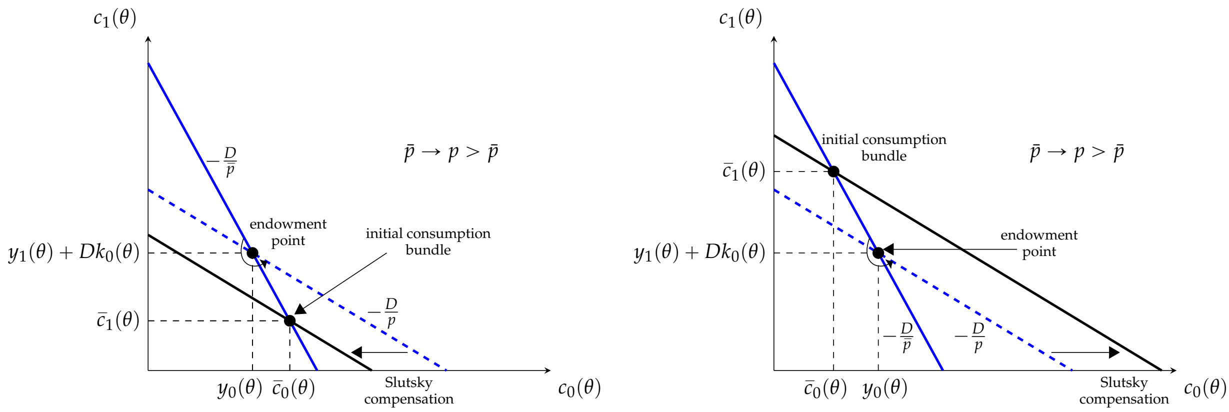
$$T_0(\theta) = \overline{T}_0(\theta) + \overline{x}(\theta)\Delta p - \Omega(\theta)\overline{X}\Delta p = \overline{T}_0(\theta) + x(\theta)\Delta p - \Omega(\theta)X\Delta p.$$

The first equation shows that the response of the optimal tax system to an asset price change is closely related to the Slutsky compensation from Lemma 2. Indeed, the two would coincide if there were no aggregate asset trade with the rest of the world, $\overline{X} = 0$. The change in $T_0(\theta)$ makes investor θ's original consumption allocation just affordable again, and then redistributes the aggregate capital gains in the optimal way, determined by the welfare weights $\Omega(\theta)$.

[15]The standard use of Slutsky compensation is to compute "Slutsky-compensated demand." In particular, the difference between Slutsky-compensated demand at the new prices and demand at the old prices is one definition of the substitution effect. An alternative definition of income and substitution effects is based on "Hicksian compensation," which is the change in total budget that restores the original level of utility. We thank Dejanir Silva for pointing out the connection of the welfare gains formula (20) to the Slutsky compensation idea. See Caramp and Silva (2023) for a related result in the context of monetary policy transmission via asset prices.

[16]The term "Slutsky compensation" is normally reserved for pure price changes. Here and elsewhere we use it to also refer to compensation of dividend changes ΔD.

According to Lemma 2, to make investors' initial consumption bundle just affordable, buyers (i.e., $\overline{x}(\theta) < 0$) are compensated for the price increase (subsidized) whereas sellers (i.e., $\overline{x}(\theta) > 0$) are taxed. Figure 2 provides a graphical representation of Slutsky compensation based on the Fisher diagram, the standard graphical apparatus for intertemporal consumption choice problems. However, we include only the budget sets, and omit the indifference curves. Panel (a) plots the case of a seller while panel (b) plots that of a buyer. In both panels, the steeper solid line is the budget constraint at the initial asset price $\overline{p}$ and the dashed line is that at the new, higher price p. A change in the asset price rotates the budget constraint through the endowment point, with an increase in price generating a shallower budget line (the slope is $-D/p$). A reference line is drawn through the initial consumption allocation $(\overline{c}_0, \overline{c}_1)$ with the slope of the new budget line. The horizontal shift between the dashed line and this parallel reference line is the amount of resources needed to be added or subtracted in period 0 to afford the initial consumption allocation at the new prices. This is the Slutsky compensation. For the seller (left panel), the rise in price moves the initial consumption point into the interior of the budget set, implying a negative Slutsky compensation. The converse is true for the buyer.



(a) Effect on seller

(b) Effect on buyer

Figure 2: Slutsky compensation after a pure asset-price increase (Special Case 1)

Notes: The figure depicts Slutsky compensation in response to an asset price increase. In both panels, the solid red line is the initial budget line, with the endowment and consumption points marked. The shallower dashed line through the endowment point is the new budget line after the price change. The solid black line parallel to the dashed line is the budget line after the Slutsky compensation, which by definition contains the initial consumption allocation at the new prices. Panel (a) depicts an initial seller of the asset while Panel (b) depicts a buyer.

The intuition for why the Slutsky compensation is relevant for the optimal tax change in Corollary 1 is that a pure discount rate change does not change aggregate resources other than through trade with the rest of the world; hence, in a closed economy, the initial consumption level of every individual remains the relevant target for optimal policy, which is precisely what Slutsky compensation is designed to deliver. The additional term in Corollary 1 then captures the optimal distribution of the aggregate gains, which is additively separable from the individual compensation under homothetic preferences.

The second equation in Corollary 1 shows that the optimal tax $T_0(\theta)$ can also be written in terms of asset sales at the new asset price p. For example, if $x(\theta) > 0$ and $\Delta p > 0$, then $T_0(\theta)$ effectively taxes the *realized* capital gains of investor θ. Because of the lump-sum nature of the tax system, these gains are in fact taxed away completely, at a rate of 100%. Note that $x(\theta)$ are the new asset sales not only at the new price but also at the new taxes. In certain cases, the old and new asset sales coincide, $x(\theta) = \overline{x}(\theta)$.[17]

When $X = 0$, as happens under some parameter configurations[18] or in the closed economy we consider in Section 6.1, the tax $T_0(\theta) = \overline{T}_0(\theta) + x(\theta)\Delta p$ corresponds to a realization-based capital gains tax (relative to the reference price $\overline{p}$), akin to the kind of capital gains taxes implemented in many countries. However, our tax formula is not limited to when the investor sells the asset ($x(\theta) > 0$) and realizes a gain ($\Delta p > 0$). It also prescribes to compensate realized capital losses ($x(\theta) > 0$ and $\Delta p < 0$) as well as purchasing gains and losses (when $x(\theta) < 0$). For instance, when the investor purchases the asset and its price falls, she benefits from a "purchasing gain" $x(\theta)\Delta p > 0$, which is also taxed away. Generally, optimal taxes target "trading gains and losses."

Importantly, when the investor does not trade ($x(\theta) = 0$), no tax change is triggered by the asset price change (except for a redistribution of the potential aggregate capital gains or losses $X\Delta p$). This reveals another difference from typical real-world capital gains taxes: the optimal tax conditions on *net* transactions only. For instance, if an individual sells a house and then buys another house of the same quality and price, and house prices go up, she realizes a capital gain on the sold house and a purchasing loss on the purchased house, which cancel out. By contrast, since typical tax systems, in practice, do not contain the second component (i.e., the subsidy on the purchasing loss), they would only tax the (gross) realized capital gains from the first transaction.[19]

3.2.2 General Case

We now return to the general case in Proposition 1. In addition to the terms discussed so far, new terms capturing the dividend change ΔD emerge. According to both formulas in Proposition 1, the additional dividend income, discounted back to period 0, must also be taxed away, and the aggregate dividend income change is redistributed optimally according to the welfare weights. In other words, the tax/subsidy on trading gains and losses is complemented by a dividend income tax.

This is again closely related to the Slutsky compensation in Lemma 2. Intuitively, investors with asset holdings $\overline{k}_1(\theta) > 0$ benefit from a higher dividend $\Delta D > 0$ and therefore need to be taxed in order to make their initial consumption bundle just affordable. Graphically, the combination of rising asset prices and rising dividends means that the budget line not only rotates around the endowment point but also shifts outwards—see Figure 3a.

While the intuition is therefore similar to the welfare gains formula (20), an important difference is that the Slutsky compensation argument follows exclusively from investors' budget constraints at the two prices. As a result, assumptions on preferences or the optimality of the

[17] This happens in the closed economy of Section 6.1 in which aggregate asset sales are zero. In this case, optimal policy simply takes everyone back to their baseline consumption allocation, which implies $x(\theta) = \overline{x}(\theta)$.

[18] For example, with preferences $u(c_0) + \beta u(c_1), u' > 0, u'' < 0$, $X = 0$ whenever $\beta\overline{D}/p = 1$ and $Y_0 = Y_1 + \overline{D}K_0$.

[19] Our theory features only real variables, so it prescribes taxing inflation-indexed capital gains (as practiced in some countries like Israel).

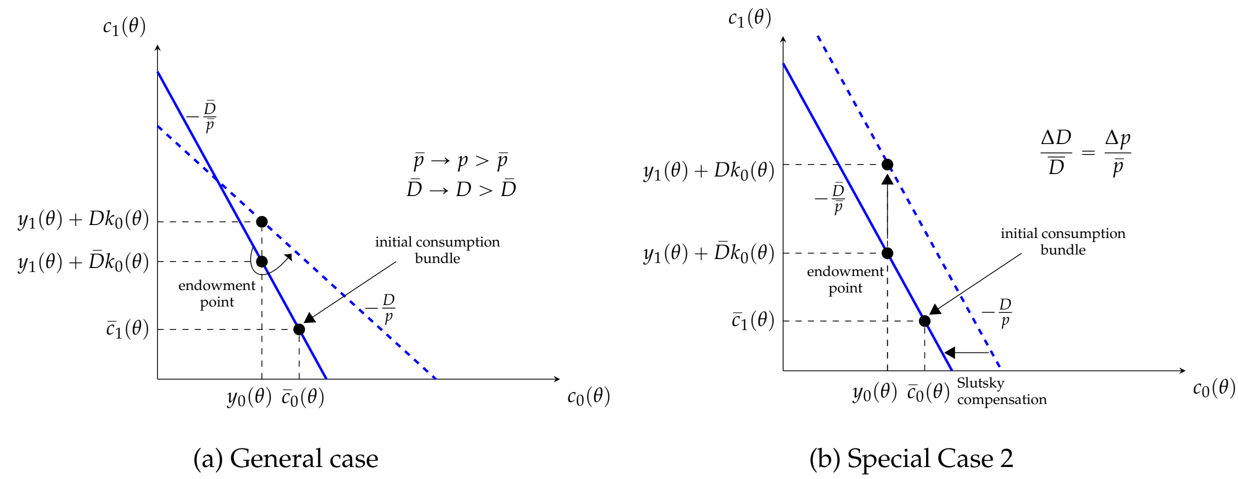


Figure 3: Slutsky compensation of combined asset-price and cash-flow changes

Notes: This figure is similar in composition to Figure 2, but allows for both price and dividend changes. Relative to the previous figure, in this case the endowment point shifts by the change in dividend income. The figure omits the new budget line absent the Slutsky compensation.

initial allocation (used in applying the envelope theorem in equation (20)) play no role. Since budget constraints are linear in prices, Lemma 2 holds for arbitrary non-infinitesimal asset price and dividend changes. This property translates to the optimal tax result in Proposition 1.

Similar to the special case in Corollary 1, Proposition 1 shows that $T_0(\theta)$ can be written both in terms of asset sales $\overline{x}(\theta)$ and asset holdings $\overline{k}_1(\theta)$ under the *old* prices (in which case dividend income must be discounted using the *new* discount rate p/D) or in terms of asset sales $x(\theta)$ and asset holdings $k_1(\theta)$ under the *new* prices (in which case the *old* discount rate $\overline{p}/\overline{D}$ must be used). In fact, when we allow the lump-sum taxes in both periods to adjust (rather than using the normalization $T_1(\theta) = 0$), we can write the optimal tax as

$$
\begin{aligned}
T_0(\theta) &= \overline{T}_0(\theta) + \overline{x}(\theta)\Delta p - \Omega(\theta)\overline{X}\Delta p = \overline{T}_0(\theta) + x(\theta)\Delta p - \Omega(\theta)X\Delta p \\
T_1(\theta) &= \overline{T}_1(\theta) + \overline{k}_1(\theta)\Delta D - \Omega(\theta)\overline{K}_1\Delta D = \overline{T}_1(\theta) + k_1(\theta)\Delta D - \Omega(\theta)K_1\Delta D,
\end{aligned}
$$

where $\{\overline{T}_t(\theta)\}$, $t = 0, 1$, are some optimal baseline taxes. Hence $T_0(\theta)$ deals with the pure asset price change in the form of a realization-based tax on capital gains just like in Corollary 1, whereas $T_1(\theta)$ acts as a tax on the changed dividend income in $t = 1$. In particular, no discounting is needed under this alternative normalization.

3.2.3 Special Case 2: Only Cash Flow Changes

Finally, we consider the second extreme case from Section 1.3 in which asset prices change exclusive because of future dividends. For simplicity, we return to the normalization $T_1(\theta) = 0$.

Corollary 2. *Let the asset price change from $\overline{p}$ to $p = \overline{p} + \Delta p$ and let dividends change from $\overline{D}$ to $D = \overline{D} + \Delta D$ such that $\Delta D/\Delta p = \overline{D}/\overline{p}$. Then the optimal tax $T_0(\theta)$ is given by*

$$T_0(\theta) = \overline{T}_0(\theta) + k_0(\theta)\Delta p - \Omega(\theta)K_0\Delta p$$

Since dividends and the asset price grow by the same percentage, the asset return $R_1 = D/p$ remains unchanged, i.e., the new return $R_1 = (\overline{D} + \Delta D)/(\overline{p} + \Delta p)$ equals the old return $\overline{R}_1 = \overline{D}/\overline{p}$. According to the corollary, the optimal tax $T_0(\theta)$ then taxes the investor's individual wealth gains $k_0(\theta)\Delta p$ due to the asset price change. Hence, in this case, the first-best optimal tax system conditions on the investor's *unrealized* capital gains. This tax base is therefore consistent with an accrual-based capital gains tax, as under the Haig-Simons comprehensive income tax (Haig, 1921, and Simons, 1938), or a wealth tax.

Of course, since Proposition 1 continues to apply, we could still express the optimum as a combination of a tax on trading gains and a dividend income tax. Why do the two collapse to the accrual-based tax in Corollary 2, which only depends on initial wealth in $t = 0$? The reason can be understood as follows: If the investor sells all her assets, then $x(\theta) = k_0(\theta)$ and $k_1(\theta) = 0$. Hence, there is no dividend income in $t = 1$, and realized capital gains in $t = 0$ are given by $k_0(\theta)\Delta p$, just as in the corollary. Now suppose instead that the investor decides not to sell all her assets. This results in some dividend income in $t = 1$ (since now $k_1(\theta) = k_0(\theta) - x(\theta) > 0$), but at the same time in reduced realized capital gains in $t = 0$. When the price and dividend changes happen to be proportional, the two effects exactly offset each other and the overall income change is still given by $k_0(\theta)\Delta p$, no matter how much the individuals sells. Formally, we can use the first equation in Proposition 1 to obtain

$$\overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D = \overline{x}(\theta)\Delta p + \frac{p}{D}(k_0(\theta) - \overline{x}(\theta))\Delta p\frac{D}{p} = k_0(\theta)\Delta p.$$

Since this holds for all investors, the aggregate quantities collapse in the same way.

Figure 3b relates this case graphically to the corresponding Slutsky compensation. In contrast to Figures 2 and 3a, the budget line does not change slope (which remains unchanged at $-\overline{D}/\overline{p}$) and instead shifts outward. Specifically, the increase in dividends means that the endowment point $(y_0(\theta), y_1(\theta) + Dk_0(\theta))$ shifts upward. In the lifetime budget constraint (18), the return D/p remains unchanged and therefore the only effect of the joint asset price and dividend change is the revaluation of initial wealth $pk_0(\theta)$.

While this special case therefore provides a justification for using the Haig-Simons income concept as the tax base, this logic demonstrates that it is knife-edge. Whenever capital gains are not entirely due to dividend changes, i.e. as soon as discount rate changes are part of the story as well—in the Fisher diagram of Figures 2 and 3, as soon as the budget line rotates even a little bit—, the additional dividend income and capital gains no longer cancel out. Moreover, we will show in Section 6.2 that the cancellation result will break down, even when asset prices are exclusively driven by dividend changes, in a richer model with heterogeneous returns.

3.3 Discussion

Baseline asset price. While the "trading gains and losses" $x(\theta)\Delta p$ bear similarities to realized capital gains (in case of an asset sale), an important difference is that the price change Δp is relative to some baseline price $\overline{p}$, which does not necessarily coincide with the historical basis at which the investor purchased the asset. Instead, one needs to decide which price (and dividend) change the tax system should compensate. This becomes particularly clear in the case of a purchasing gain or loss (with $x < 0$): in this case, Proposition 1 prescribes a tax or subsidy, but since the investor has not owned the asset prior to purchasing it, there is no historical basis to go back to when computing the price change.

In the general model with uncertainty from Section 1, a natural baseline price and dividend would be given by the corresponding means. Hence, the old allocation can be interpreted as the optimum under these expected prices and dividends whereas the new prices and dividends p and D would be the ex-post realized ones, so the tax system is tasked to compensate the winners and losers relative to the ex-ante expectations. Another natural baseline is a Gordon growth model or BGP in which the asset return is constant and dividends grow at a constant rate (see Section 1).

Baseline taxes. Proposition 1 also assumes that taxes are set optimally for given Pareto weights at the baseline prices and dividends. If baseline taxes were not set optimally (or based on different Pareto weights), we could always decompose the overall change in taxes into two steps: First, holding baseline prices and dividends fixed, a reform of the baseline taxes towards the optimum according to the new Pareto weights. Second, holding Pareto weights fixed, a move towards the optimum under the new prices and dividends. Our analysis isolates the second step. The first step has nothing to do with asset prices and is completely standard, namely, a tax reform moving the allocation from the interior of the Pareto frontier (or along the frontier) towards a particular point on that frontier in a given economy.

Endogenous payout policy and share repurchases. Businesses have control over their dividend payments and may have alternative means of distributing their profits to shareholders, specifically via share repurchases. Appendix B.3 provides an alternative, capital-structure neutral formulation of our setup in which such distinctions are immaterial. The key idea of this formulation is to consolidate the firm and investor budget constraints, in particular to consider profits net of investment as the relevant measure of cash flows D_t regardless of whether they are distributed to investors via dividend payments or share repurchases and to consider the firm's total value as the relevant measure of the share price p_t.

Owner-occupied housing. Owner-occupied housing generates a flow of housing services and implementing our tax formula therefore requires valuing this "dividend." The solution is to measure the dividend D as imputed rents, i.e., to value owner-occupied housing services as the rental income the homeowner could have received if the house had been let out. Thus, if part of the house-price increase in New York City was due to the city's amenities improving, rents would rise so that $\Delta D_t > 0$ in addition to $\Delta p_t > 0$ and our formula would prescribe taxing the additional imputed rents. This approach is already used by some countries such as Denmark and Switzerland.

3.4 Alternative implementations: taxes on total capital income or expenditure

In Proposition 1, we have expressed the first-best tax response to asset price and dividend changes in terms of investors' realized capital gains and additional dividend income. We now show that the optimal tax change can be equivalently understood in two alternative ways: one based on total capital income and another one based on consumption.

3.4.1 A tax on total capital income

In Section 1, we demonstrated that equation (11) is an equivalent way of writing the budget constraint (2) using an investor's market value of wealth $a_t(\theta) \equiv p_{t-1}k_t(\theta)$. Thus, as the following proposition shows, an alternative way of writing the first-best tax response in Proposition 1 is in terms of these wealth holdings and the changes in the total returns R_0 and R_1.

Proposition 2. *Let the asset price change from $\overline{p}$ to $p = \overline{p} + \Delta p$ and dividends from $\overline{D}$ to $D = \overline{D} + \Delta D$ resulting in return changes $\Delta R_0 = R_0 - \overline{R}_0$ and $\Delta R_1 = R_1 - \overline{R}_1$. Then the optimal tax $T_0(\theta)$ (when $T_1(\theta) = 0$) is given by*

$$\begin{aligned} T_0(\theta) &= \overline{T}_0(\theta) + a_0(\theta)\Delta R_0 + \frac{1}{R_1}\overline{a}_1(\theta)\Delta R_1 - \Omega(\theta)\left[A_0\Delta R_0 + \frac{1}{R_1}\overline{A}_1\Delta R_1\right] \\ &= \overline{T}_0(\theta) + a_0(\theta)\Delta R_0 + \frac{1}{\overline{R}_1}a_1(\theta)\Delta R_1 - \Omega(\theta)\left[A_0\Delta R_0 + \frac{1}{\overline{R}_1}A_1\Delta R_1\right] \end{aligned}$$

where $a_1(\theta)$ is investor θ's period-1 wealth at the new returns, $\overline{a}_1(\theta)$ that at the baseline returns, $a_0(\theta) = \overline{a}_0(\theta)$ since p_{-1} is fixed, and A_0, $A_1, \overline{A}_1$ are the corresponding aggregate wealth holdings.

At first glance, the tax change in Proposition 2 appears related to a Haig-Simons notion of income: in each period, the additional total capital income $a_t(\theta)\Delta R_t$, including unrealized gains, is taxed. However, there is an important difference. This is easiest to see by considering an increase in the asset price p holding dividends fixed (Special Case 1). In this case, we have

$$\Delta R_0 = \frac{\Delta p}{p_{-1}} > 0 \quad \text{and} \quad \Delta R_1 = \frac{D}{p} - \frac{D}{\overline{p}} < 0$$

since $p = \overline{p} + \Delta p > \overline{p}$. Hence, the investor faces a tax in period 0 (due to the higher return from the increased asset price) but a subsidy in period 1. The reason for the latter is that, whereas the asset price has increased, dividends have not, so the dividend-price ratio and thus the return in period 1 has been reduced, which needs to be compensated.

While the former tax increase (due to the unrealized capital gains in the initial period) indeed corresponds to a Haig-Simons income tax, the latter subsidy (due to the lower dividend-price ratio subsequently) does not. But Proposition 2 shows that they belong together. Therefore, due to these opposing effects, the total change in taxes is generally ambiguous. In fact, we know from Proposition 1 that it depends solely on whether the investor is a buyer or seller. For instance, when $x(\theta) = 0$ so the individual is not trading, the additional tax in period 0 and the subsidy in period 1 precisely cancel out.[20]

[20] In the multi-period model, a one-off, permanent increase in the asset price would trigger a one-off tax followed

3.4.2 An expenditure tax

There is a long-standing debate in public finance about the potential advantages of taxes on consumption or expenditures, notably in the context of capital gains. For instance, when discussing the Haig-Simons income concept, Kaldor (1955, p. 44) writes:

"We may now turn to the other type of capital appreciation which reflects a fall in interest rates rather than the expectation of higher earning power. [...] The rise in capital values in this case [comes] without a corresponding increase in the flow of real income accruing from that wealth. [...] For in so far as a capital gain is realized [...] the benefit derived from the gain is equivalent to that of any other casual profit. If however it is not so realized, there is clearly only a smaller benefit. [Therefore] treating the two kinds of capital gains in the same way is not an equitable method of measuring taxable capacities."

Given this problem with using Haig-Simons income as the tax base, Kaldor instead advocates for an expenditure-based tax. This raises the question whether the optimal tax response to changing asset prices and dividends in Proposition 1 could also be understood as an expenditure-based tax. Proposition 9 in Appendix B formalizes this conjecture. It shows that the new optimum after a change in asset price and dividends can be implemented with a combination of lump-sum taxes and transfers targeting consumption changes. Notably, if the parameter changes Δp and ΔD are "zero-sum," so that optimal aggregate consumption C_t does not change, then optimal redistributive taxation simply taxes away any increase in consumption from the asset-price and dividend changes (or compensates the corresponding reduction in consumption), i.e. a "pure" expenditure tax. In line with Kaldor's logic, just like Proposition 1, this works for any combination of asset price and dividend changes, i.e. regardless of the source of capital gains.

4 Second best

We now turn to the case where the government's tax instruments are more limited. Specifically, they are restricted to condition on investors' choices, such as their asset sales, wealth, or consumption. This distorts investors' behavior, inducing the classic tradeoff between redistribution and efficiency. Our main conclusion is that the previous results on the tax base generalize in a natural way. We first consider a one-asset setup as above, isolating savings distortions. Section 4.4 then considers multiple assets and the question whether taxes create a "lock-in" effect that distorts portfolio choice.

by a subsidy forever, in a way that their present value sum is zero for an investor who is not trading (see Proposition 10 in Appendix D.6). Thus, the alternative implementation in Proposition 2 can lead to very volatile taxes over time compared to Proposition 1.

4.1 Mirrlees Problem

An asset sales tax. We begin with a (non-linear) tax $T_x(px)$ on asset sales, paid in period 0, similar to a realization-based capital gains tax. The investors' budget constraints become

$$c_0(\theta) = y_0(\theta) + px(\theta) - T_x(px(\theta)) \quad \text{and} \quad c_1(\theta) = y_1(\theta) + D(k_0(\theta) - x(\theta)).$$

This corresponds to a situation where $x(\theta)$ (and hence $z_x(\theta) \equiv px(\theta) - T_x(px(\theta))$) is observable but $k_0(\theta)$, $y_0(\theta)$ and $y_1(\theta)$ are not. The incentive constraints are therefore

$$\mathcal{U}(\theta) \equiv U(z_x(\theta) + y_0(\theta), D(k_0(\theta) - x(\theta)) + y_1(\theta)) \geq U(z_x(\hat{\theta}) + y_0(\theta), D(k_0(\theta) - x(\hat{\theta})) + y_1(\theta))$$

for all $\theta, \hat{\theta}$. Abstracting from bunching, we work with the local version of the incentive constraints. By the envelope theorem,

$$\mathcal{U}'(\theta) = U_{c_0}(c_0(\theta), c_1(\theta))y_0'(\theta) + U_{c_1}(c_0(\theta), c_1(\theta))(Dk_0'(\theta) + y_1'(\theta)) \ \ \forall \theta. \tag{24}$$

Hence, the second-best problem corresponding to the optimal asset sales tax is

$$\max_{\{c_0(\theta), c_1(\theta), \mathcal{U}(\theta)\}} \int \omega(\theta)\mathcal{U}(\theta) dF(\theta) \tag{25}$$

s.t. $\mathcal{U}(\theta) = U(c_0(\theta), c_1(\theta))$, the resource constraint (19) and the incentive constraints (24).

A wealth tax. Alternatively, consider a tax $T_k(pk_1(\theta))$ on investors' wealth in period 1.[21] This corresponds to a setting where $k_1(\theta)$ (and hence $z_k(\theta) \equiv Dk_1(\theta) - T_k(pk_1(\theta))$) is observable, resulting in the global incentive constraints

$$\mathcal{U}(\theta) \equiv U(p(k_0(\theta) - k_1(\theta)) + y_0(\theta), z_k(\theta) + y_1(\theta)) \geq U(p(k_0(\theta) - k_1(\hat{\theta})) + y_0(\theta), z_k(\hat{\theta}) + y_1(\theta))$$

for all $\theta, \hat{\theta}$. The local incentive constraints can therefore be written in the same general form as in the case of the asset sales tax, namely

$$\mathcal{U}'(\theta) = U_{c_0}(c_0(\theta), c_1(\theta))A(\theta) + U_{c_1}(c_0(\theta), c_1(\theta))B(\theta) \ \ \forall \theta, \tag{26}$$

with the only difference that, now, $A(\theta) = pk_0'(\theta) + y_0'(\theta)$ and $B(\theta) = y_1'(\theta)$. Hence, the second-best problem for the wealth tax is the same as above, except for the incentive constraints (26) instead of (24).[22]

Other taxes. We can allow for other second-best tax instruments, such as consumption taxes, in an analogous way. Notably, we show in Appendix C that the general incentive constraints

[21] This is equivalent to a tax on dividend income $Dk_1(\theta)$ since dividends D are the same for all investors in our baseline model. By contrast, a tax on period-0 wealth $pk_0(\theta)$ would be lump-sum and return us to the first-best case when $k_0(\theta)$ is invertible.

[22] In fact, as can be seen immediately from the incentive constraints, the two second-best problems are identical when investors only differ in their incomes $y_0(\theta)$ and $y_1(\theta)$ but not in their initial wealth $k_0(\theta)$. In other words, in this case, a wealth tax and an asset sales tax are two decentralizations of the same optimal allocation.

(26) still apply, with the only difference that the terms $A(\theta)$ and $B(\theta)$ are modified. In the case of a nonlinear tax on c_0, we have $A(\theta) = 0$ and $B(\theta) = Dk_0'(\theta) + \frac{D}{p}y_0'(\theta) + y_1'(\theta)$, whereas a tax on c_1 implies $A(\theta) = pk_0'(\theta) + y_0'(\theta) + \frac{p}{D}y_1'(\theta)$ and $B(\theta) = 0$. More generally, this structure extends to any such second-best problem, including when combinations of taxes are available.[23]

4.2 Taxing changing asset prices

We consider an example economy with investors θ uniformly distributed on the unit interval and $y_0(\theta) = 1 - \theta$, $y_1(\theta) = \theta$ and $k_0(\theta) = 0.1$ for all $\theta \in [0, 1]$. Thus, similar to Figure 1, higher-θ investors feature a more backloaded income profile (while there is no heterogeneity in the initial asset endowment), making them natural sellers (borrowers), whereas lower-θ investors are buyers (savers). We use preferences $U(c_0, c_1) = G(\mathcal{C}(c_0, c_1))$ where

$$\mathcal{C}(c_0, c_1) = \left(c_0^{\frac{\sigma-1}{\sigma}} + \beta c_1^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad G(\mathcal{C}) = \frac{\mathcal{C}^{1-\gamma}}{1-\gamma} \quad \text{with} \quad \sigma, \gamma > 0. \tag{27}$$

Here, $\mathcal{C}$ is a composite commodity in which β is the discount factor used to discount consumption in the second time period and σ is the intertemporal elasticity of substitution. The parameter γ governs curvature over this composite commodity.[24] To start with, we set $\sigma = 0.5$, $\gamma = 1$ (so $G(\mathcal{C}) = \log(\mathcal{C})$) and $\beta = 0.5$.

As a baseline, we consider an asset price $\overline{p} = 1$ and dividends $\overline{D} = 2$ (so $\overline{D}/\overline{p} = 1/\beta$). We compute the utilitarian optimum (with $\omega(\theta) = 1$ for all θ) for this baseline and then compare it to a situation where the asset price rises by 30% to $p = 1.3$, holding dividends fixed at $\overline{D}$. Hence, this illustrates Special Case 1, an asset price change driven by a discount rate change.

Asset sales tax. The left panel in Figure 4 shows the optimal asset sales tax schedules $T_x(px)$ in both of these economies, which are decreasing. The reason is that, in this specification, higher-θ individuals have the lower present-value of income, so the direction of redistribution runs from low- to high-θ types. As discussed above, asset sales x are naturally increasing in θ, so the optimum puts a tax on the (richer) buyers and a subsidy on the (poorer) sellers.

Our main interest is in how the optimal tax *changes* in response to the asset price increase. This is depicted in the right panel of Figure 4, where we plot the change in the tax $\Delta T_x(px)$ as a function of the trading gains and losses $x\Delta p$. It reveals a positive relationship, just like in Corollary 1, albeit with a slope of less than one. This is intuitive: the solution now balances the optimal redistribution, which works in the same way as in the first-best case (namely, increasing the tax burden on the sellers, who gain from the asset price increase, and lowering

[23]In Appendix C, we characterize the Mirrlees (1971) Pareto optima for all these tax instruments and develop a numerical algorithm to compute them for changing asset prices.

[24](27) is a monotone transformation of the more standard intertemporally separable utility function $\sum_t \beta^t u(c_t)$ with $u(c) = c^{1-1/\sigma}/(1-1/\sigma)$. The reason for working with this monotone transformation is that, below, we will be interested in the limit as the intertemporal elasticity of substitution σ goes to zero, which is ill-defined for the standard specification. For example $c^{1-1/\sigma}/(1-1/\sigma) \to 0$ as $\sigma \to 0$ for all $c > 1$ (the numerator converges to zero and the denominator to $-\infty$). In contrast, (27) satisfies $U(c_0, c_1) \to G(\min\{c_0, c_1\})$ as $\sigma \to 0$, i.e., it converges to a (monotone transformation of a) Leontief utility function, as expected.

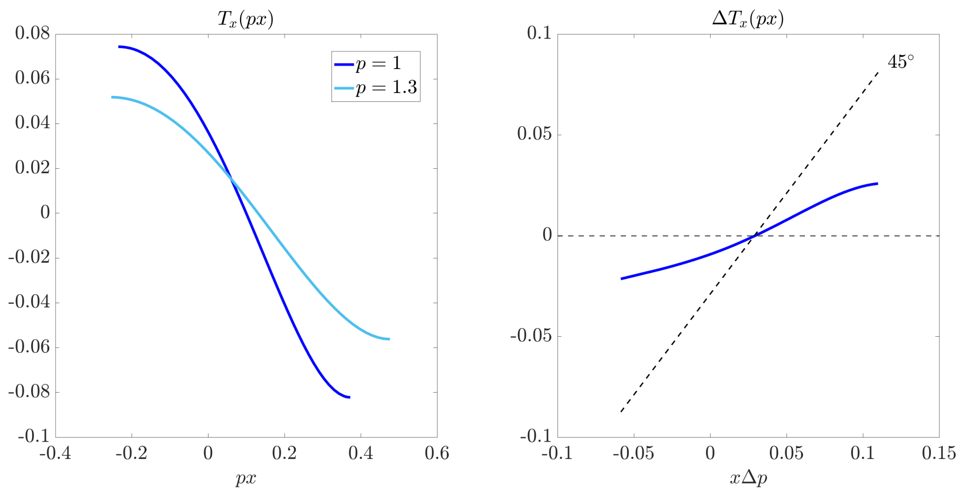


Figure 4: Optimal asset sales tax with increasing asset prices

Notes: The left panel depicts the second-best tax schedule, as a function of asset sales $px(\theta)$, for two alternative prices of the asset. The right panel depicts the difference between the schedule associated with p=1.3 and the baseline p=1.0 schedule, ΔT_x. The right panel plots $\Delta T_x(px(\theta))$ against the net trading gains $x(\theta)\Delta p$, depicting that the change in taxes increases in net gains.

it for the buyers), with the distortive effects of a positive marginal tax rate on investors' savings behavior.

Wealth tax. Figure 5 shows the respective graphs for the alternative implementation of the optimum with a wealth tax. In this example, the wealth tax is increasing in period-1 wealth pk_1: since richer, low-θ investors have a more front-loaded income stream, they buy more assets and hence own more wealth at the beginning of the second period. Thus, in terms of levels, a progressive wealth tax with a positive tax burden on the rich and a subsidy on the poor is optimal. However, the right panel shows that the optimal response to increasing asset prices is to make the wealth tax *less* progressive. This is because, again, wealthy individuals are buyers in this case, who lose from the asset price increase, so their tax burden should fall as a compensation. Conversely, low-wealth borrowers are sellers of the asset, and hence benefit from the asset price increase, so their tax burden should increase.

An example of such a configuration would be housing markets where relatively well-off households, who already own a house, want to upsize (for instance because of a growing family). Thus, despite being in the upper percentiles of the wealth distribution, these households are net buyers. As a result, when house prices rise, they are worse off, and introducing a progressive wealth tax in this situation would not achieve the desired direction of redistribution.

In sum, while a wealth tax in this example can be used to implement the constrained optimum, its comparative statics in response to an asset price increase are counter-intuitive. This is because the wealth tax (or related accrual-based tax instruments) is an *indirect* way of targeting buyers versus sellers, which is what ultimately drives the welfare effects. By contrast, the comparative statics of the asset sales tax are always the same (as in the right panel of Figure 4) since conditioning on realized capital gains *directly* targets the correct tax base.

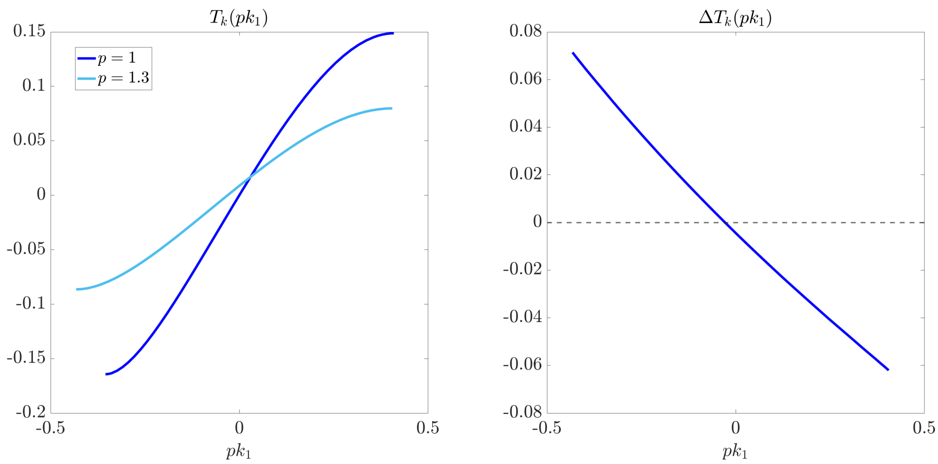


Figure 5: Optimal wealth tax with increasing asset prices

Notes: The left panel depicts the second-best tax schedule, as a function of period one wealth, pk_1, for two alternative prices of the asset. The right panel depicts the difference between the schedule associated with p=1.3 and the baseline p=1.0 schedule, ΔT_k. The right panel plots $\Delta T_k(pk_1)$ against period one wealth, depicting that the change in taxes decreases in period one wealth.

4.3 Role of the intertemporal elasticity of substitution

In Figure 6, we return to the asset sales tax and show its optimal response to an asset price increase for different values of the intertemporal elasticity of substitution σ (the dark blue schedule is the same as in the right panel of Figure 4). It illustrates that the optimal second-best policy converges to the first-best solution in Corollary 1, with a 100% marginal tax rate on realized capital gains, as σ approaches zero. The intuition is simply that a vanishing substitution elasticity implies a vanishing savings distortion from the tax, which therefore becomes equivalent in the limit to a lump-sum tax instrument. This demonstrates that our first-best results from the previous Section 3 are not knife-edge, but extend qualitatively to the case of more realistic and limited tax instruments as long as the distortive effects remain small.

The next proposition formalizes this result. Denote by $\Gamma^*(\sigma) \equiv \{(c_0^*(\theta,\sigma), c_1^*(\theta,\sigma))\}$ the optimal first-best allocation solving (21) subject to (19) when preferences are given by (27). We are interested in the limit as $\sigma \to 0$, so that $\mathcal{C}(c_0, c_1) = \min\{c_0, c_1\}$, which, as we show in the appendix, implies $c_0^*(\theta, 0) = c_1^*(\theta, 0) \equiv c^*(\theta)$. We need the following regularity assumption to obtain our convergence result:

Assumption 1. *(i) There exists a function α with $0 < \alpha(\theta) < 1$ for all θ such that*

$$c^{*\prime}(\theta) = \alpha(\theta)A(\theta) + (1 - \alpha(\theta))B(\theta)$$

and (ii) letting $g(\theta) \equiv \log\left(\frac{1-\alpha(\theta)}{\beta\alpha(\theta)}\right)$, $g'(\theta)$ exists and is bounded for all θ.

Part (i) amounts to a restriction on the Pareto weights $\omega(\theta)$ for any given heterogeneity $\{k_0(\theta), y_0(\theta), y_1(\theta)\}$. We show in the appendix that it follows from the first-best allocation $\Gamma^*(0)$ being incentive compatible when $\sigma = 0$, which is needed for the second-best allocation to be able to approach it. Part (ii) is a technical regularity condition on these Pareto weights.

Denote by $\Gamma^M(\sigma) \equiv \{(c_0^M(\theta,\sigma), c_1^M(\theta,\sigma))\}$ the solution to the general Mirrlees problem (25)

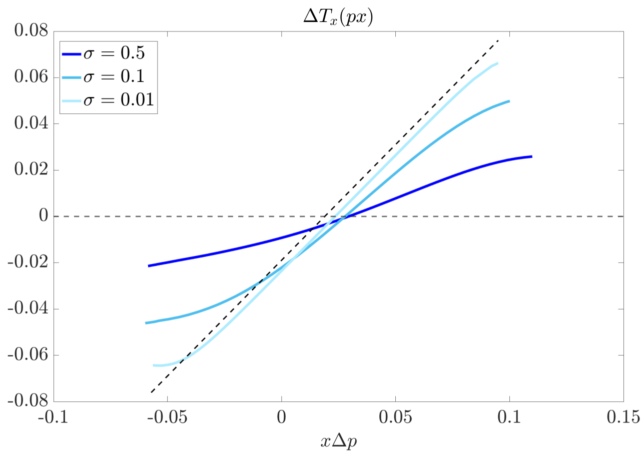


Figure 6: Capital gains tax with a decreasing intertemporal elasticity of substitution

subject to (19) and (26) for the same Pareto weights $\{\omega(\theta)\}$. This yields the following result:

Proposition 3. *Under Assumption 1 and preferences (27), $\Gamma^M(\sigma) \to \Gamma^*(\sigma)$ uniformly as $\sigma \to 0$.*

Hence, for small σ and hence small distortions, our first-best results are informative about general second-best tax instruments, since the respective allocations are close.

4.4 Portfolio choice with distortive taxes and the lock-in effect

Our analysis in this section thus far was based on a single-asset environment and emphasized the savings distortions from taxes. Therefore, it abstracted from taxes distorting portfolio choice. An important example of such portfolio distortions is the "lock-in" effect emphasized in the capital gains taxation literature (e.g. Holt and Shelton, 1962; Constantinides, 1983; Auerbach, 1991; Chari et al., 2005). Specifically, realization-based taxes incentivize deferring the liquidation of appreciated assets and thus distort optimal portfolio re-balancing in response to asset price changes.

We now show that an optimally designed second-best tax system does not introduce such distortions, even when it targets realized capital gains. The reason is that, in a multi-asset setting, it is always optimal to tax total net trades, i.e., to net all sales and purchases across the entire portfolio of assets, rather than taxing the gross gains from selling individual assets. As a result, when an investor sells one asset and uses the proceeds to purchase another one, there is generally no tax burden and therefore no lock-in effect.

As in Section 1, we introduce a second asset in the form of a bond. For simplicity, we consider a deterministic setup and therefore include a trading friction to prevent portfolio choice from being indeterminate. An investor's budget constraints are

$$c_0 = px - qb - \chi(x) + y_0 - T(x,b) \quad \text{and} \quad c_1 = D(k_0 - x) + b + y_1.$$

Trading frictions are captured by an adjustment cost $\chi(x)$ that is increasing in $|x|$ and convex. We allow for a general tax $T(x,b)$ on all trades x and b. In principle, such a tax could distort the investor's portfolio choice between capital and the bond, but we show that this is never optimal. To do so, we compare to a tax $T(z)$ on the total net trades $z \equiv px - qb - \chi(x)$, which by construction leaves the portfolio choice undistorted.[25]

Proposition 4. *Any optimum achieved by a tax $T(x, b)$ leaves portfolio choice undistorted and can be implemented with a tax on total net trades $T(z)$. Thus, there is no lock-in effect at the optimum.*

Hence, even if it is possible to condition taxes on individual (gross) trades, any optimum will not do so and instead will simply tax total net trades in or out of the portfolio. This result is akin to a production efficiency result (Diamond and Mirrlees, 1971). It implies that the second-best optimal policy does not introduce a lock-in effect. The lock-in effect results from the fact that, in practice, the capital gains from individual gross trades are taxed. Instead, by Proposition 4, pure portfolio re-balancing trades should not trigger a tax liability. Note that this result is not specific to our setting, i.e. taxing net transactions eliminates the lock-in effect also in other settings.[26]

5 Optimal taxation in the general model

We now return to the case of first-best tax instruments and show how our results on optimal taxation from the deterministic two-period model extend to the general model from Section 1. In a first step, we allow for risk and borrowing but remain in our two-period framework. We then show that these results generalize to the multi-period model with uncertainty and arbitrary time horizon.

5.1 Risk and Borrowing in the Two-Period Model

Consider our benchmark environment from Section 1 and assume, for now, $T = 1$. As in Section 1.3, denote by $m(s^1)$ the stochastic discount factor of the representative counterparty in global financial markets where $s^1 = (s_0, s_1)$. In particular, the period-0 Arrow-Debreu price of a unit of consumption delivered in state s_1 is $\pi(s_1)m(s^1)$. No arbitrage implies:

$$p(s_0) = \sum_{s_1} \pi(s_1)m(s^1)D(s^1), \qquad q(s_0) = \sum_{s_1} \pi(s_1)m(s^1). \tag{28}$$

The investors' flow budget constraints (2) specialize to

$$\begin{aligned} c_0(\theta, s_0) &= p(s_0)(k_0(\theta, s_{-1}) - k_1(\theta, s_0)) - q(s_0)b(\theta, s_0) + y_0(\theta) - T_0(\theta, s_0) \quad \forall s_0 \\ c_1(\theta, s^1) &= D(s^1)k_1(\theta, s_0) + b(\theta, s_0) + y_1(\theta) - T_1(\theta, s^1) \quad \forall s^1 \end{aligned}$$

[25] Whether the tax is imposed in period 0 or 1 makes no difference for our argument. In particular, a tax on net trades in period 1, $b - Dx$, would be equivalent. We will relate our results to the deferral advantage, i.e., the interest advantage from deferring realization, in our general dynamic model in Section 5.

[26] For example, Auerbach (1991) starts with a simple two-period illustration: "An investor, having accrued a first-period gain, g, must decide whether to realize the gain and reinvest at the rate of return, i, or hold the asset for an additional rate of return r. [A tax on realized capital gains...] makes the investor willing to hold even for a range of returns $r < i$." The investor's reinvestment decision is a case of pure portfolio re-balancing. Therefore not taxing such re-balancing eliminates the lock-in effect. Magnus (2024) makes a related proposal.

where, just like in the deterministic model, we assume $D_0 = b_0 = p_1 = 0$. $p(s_0)$ denotes the period-0 price of risky capital, $q(s_0)$ the price of the bond, $b(\theta, s_0)$ the amount of the bond purchased in period 0 and $D(s^1)$ the dividends paid to capital in period 1.

We allow taxes and transfers in both periods $t = 0, 1$ to be indexed by s^t. In order to ensure that risk is relevant, we assume

$$\int T_0(\theta, s_0) dF(\theta) = \int T_1(\theta, s^1) dF(\theta) = 0 \quad \forall s_0, s^1,$$

so the economy cannot insure itself other than through trading capital and the bond with the rest of the world. In other words, it does not have access to the full set of Arrow-Debreu insurance markets. Hence, aggregate second-period allocations are spanned by the two assets:

$$C_1(s^1) \equiv \int c_1(\theta, s^1) dF(\theta) = D(s^1) K_1(s_0) + Y_1 + B(s_0), \quad \forall s^1,$$

where $K_1(s_0) = \int k_1(\theta, s_0) dF(\theta)$, $Y_1 = \int y_1(\theta) dF(\theta)$, and $B(s_0) = \int b(\theta, s_0) dF(\theta)$.[27]

First Best. The first-best allocation is the solution to

$$\max_{\{c_0(\theta,s_0), \{c_1(\theta,s^1)\}, X(s_0)\}} \mathbb{E} \int \omega(\theta) U(\{c_0(\theta, s_0), c_1(\theta, s^1)\}) dF(\theta) \quad \text{s.t.}$$

$$\int c_0(\theta, s_0) dF(\theta) + q(s_0) \int c_1(\theta, s^1) dF(\theta) = Y_0 + q(s_0) Y_1 + p(s_0) X(s_0) + q(s_0) D(s^1)(K_0 - X(s_0))$$

for all s_0, s^1 and where $X(s_0) \equiv \int (k_0(\theta) - k_1(\theta, s_0)) dF(\theta)$. As in the benchmark environment, the fact that individuals can trade assets generates an indeterminacy in the tax system that decentralizes the first-best allocation. With two assets, there are two dimensions of indeterminacy, spanned by the payoffs to the risk-free bond and risky capital. Specifically:

Lemma 3. *There exists a first-period tax schedule $T_0(\theta, s_0)$ that implements the first-best allocation when combined with any second-period tax schedule of the form*

$$T_1(\theta, s^1) = \alpha(\theta, s_0) + \gamma(\theta, s_0) D(s^1) \quad \forall \theta, s^1,$$

where, for any given s_0, $\alpha(\theta, s_0)$ and $\gamma(\theta, s_0)$ are arbitrary functions of θ that satisfy $\int \alpha(\theta, s_0)) dF(\theta) = \int \gamma(\theta, s_0)) dF(\theta) = 0$.

That is, the second-period tax schedule can be an arbitrary linear function of the payoffs to risky capital. This follows from the fact that individuals can always adjust their private holding of the two assets to account for differences in the tax system that are spanned by the payoffs to the bond and capital.

Shocks to asset prices and dividends. We are now in position to revisit how shocks to asset prices and cash flows induce changes in the optimal tax burden. In period 0, consider a base-

[27] Without loss of generality, we assume that the government does not own bonds or capital directly, and thus the aggregates B and K_1 reflect the aggregated holdings of private individuals.

line state $\bar{s}_0$ (with corresponding pricing kernel $m(\bar{s}_0, s_1)$ and dividends $D(\bar{s}_0, s_1)$) and compare it to another, new shock s_0 with stochastic discount factor $m(s_0, s_1)$ and dividends $D(s_0, s_1)$. For example, suppose attitudes toward risk change, or the time discounting inherent in m changes. By expression (28), this induces changes in the price of capital $p(s_0)$ and risk-free bonds $q(s_0)$ even in the absence of changes to dividends. Additionally, cash flows $D(s_0, s_1)$ themselves may change. The next result extends Proposition 1 to the case with risk.

Proposition 5. *Suppose shock s_0 is realized so that the pricing kernel changes by $\Delta m(s^1) = m(s_0, s_1) - m(\bar{s}_0, s_1)$ and dividends change by $\Delta D(s^1) = D(s_0, s_1) - D(\bar{s}_0, s_1)$. Let*

$$\Delta p = \sum_{s_1} \pi(s_1) \left[m(s_0, s_1) D(s_0, s_1) - m(\bar{s}_0, s_1) D(\bar{s}_0, s_1) \right] \quad \textit{and} \quad \Delta q = \sum_{s_1} \pi(s_1) \Delta m(s^1).$$

Then the following tax change

$$\Delta T_0(\theta, s_0) = T_0(\theta, s_0) - T_0(\theta, \bar{s}_0) \quad \textit{and} \quad \Delta T_1(\theta, s^1) = T_1(\theta, s_0, s_1) - T_0(\theta, \bar{s}_0, s_1) \ \forall s_1$$

is an optimal response:

$$\begin{aligned} \Delta T_0(\theta, s_0) &= x(\theta, s_0) \Delta p - b(\theta, s_0) \Delta q - \Omega(\theta) \left[X(s_0) \Delta p - B(s_0) \Delta q \right] \\ \Delta T_1(\theta, s^1) &= k_1(\theta, s_0) \Delta D(s^1) - \Omega(\theta) K_1(s_0) \Delta D(s^1). \end{aligned}$$

As before, it is easiest to discuss this result separating changes in discount factors from cash flows. For the case of an asset-price change exclusively due to discount-rate variation (Special Case 1), the counterpart to Corollary 1 is:

Corollary 3. *Suppose shock s_0, relative to the baseline $\bar{s}_0$, changes the pricing kernel by $\Delta m(s_0, s_1) \neq 0$ for some $s_1 \in S$, while dividends are unchanged: $\Delta D(s_0, s_1) = 0$ for all $s_1 \in S$. Then an optimal change in the tax schedule is to maintain second-period taxes $\Delta T_1(\theta, s^1) = 0 \ \forall s_1, \theta$, and set*

$$\Delta T_0(\theta, s_0) = x(\theta, s_0) \Delta p - b(\theta, s_0) \Delta q - \Omega(\theta) \left[X(s_0) \Delta p - B(s_0) \Delta q \right].$$

Compared to Corollary 1, the only difference is the additional compensation for changes in the bond price Δq. The intuition is simply that a change in the interest rate on the bond redistributes between borrowers and savers, and the first-best tax response counteracts this.

A noteworthy example is a shock to the pricing kernel $\Delta m(s^1)$ such that $\sum_{s_1} \pi(s_1) \Delta m(s^1) = 0$. Then, by (28), $\Delta q = 0$, so the risk-free rate is unchanged, and since we also hold dividends fixed, this corresponds to a pure risk-premium change. In this case, we collapse back to Corollary 1. In other words, even in this richer setting, the optimal tax response to an asset price change induced by a risk-premium change targets the realized trading gains and losses exactly like in our deterministic benchmark model.

For the case of asset price changes driven by only cash-flow changes (Special Case 2), we show that Corollary 2 from the deterministic model also goes through:

Corollary 4. *Suppose $\Delta D(s_0, s_1) \neq 0$ for some $s_1 \in S$, while $\Delta m(s_0, s_1) = 0$ for all $s_1 \in S$. Then an optimal change in the tax schedule is to keep $T_1(\theta, s^1)$ unchanged and set*

$$\Delta T_0(\theta, s_0) = k_0(\theta)\Delta p - \Omega(\theta) K_0 \Delta p.$$

Hence, a Haig-Simons tax on unrealized gains can be applied in period 0 in this case, but, as before, it is knife-edge and breaks down whenever the pricing kernel changes.

Role of the government. Lemma 3 speaks to the fact that there are many tax schemes that implement the same optimal allocation. In fact, setting $\alpha(\theta, s_0) = \gamma(\theta, s_0) = 0$ for all θ implies that all taxation can take place in period 0. This reflects that the tax schemes are spanned by available assets. In particular, in the initial period, the government can set

$$T_0(\theta, s_0) = p(s_0)\left(k_0(\theta) - \Omega(\theta)K_0\right) + y_0(\theta) + q(s_0)y_1(\theta) - Y_0 - q(s_0)Y_1.$$

An investor of type θ then buys $k_1(\theta, s_0) = \Omega(\theta)K_1$ units of the risky asset and $\Omega(\theta)(Y_1 + B(s_0)) - y_1(\theta)$ of the risk-free bond, meaning that everyone holds shares of the market portfolio. As a result, all investors are equally affected by future shocks to prices and dividends and there is no scope for redistributive taxation going forward.

This begs the question: why focus on tax implementations beyond the initial period? One reason is that they are simpler. Tracking cash flows and trades is easier than initial stocks of portfolio holdings. Second, in practice, it may not be feasible for all investors to purchase the full market portfolio. For example, and outside our model, an asset (e.g. a startup) may not be publicly traded in the initial period, rendering an implementation that redistributes only in period zero infeasible. However, when the founder takes the company public and sells part of their shares, a straightforward alternative implementation, which we focus on here, is to tax (or subsidize) those sales when they happen.

5.2 Borrowing versus Selling

An argument that frequently comes up in discussions about the redistributive effects of asset-price changes is that wealthy individuals do not necessarily need to sell their appreciated assets by borrowing against them. The Economist (2024) provides an instructive example:

"Say you own a successful business – so successful that your stake in it is worth \$1bn. How should you finance your spending? If you [...] sell \$20m-worth of shares [...], the entire sum represents capital gains and will be taxed at 20%, which would mean a \$4m hit. What if, instead, you called up your wealth manager and agreed to put up \$100m-worth of equity as collateral for a \$20m loan. [...] Returns from holding the equity, rather than selling it, would easily have covered the cost of servicing the borrowing. Because the proceeds of loans, which must be eventually repaid, are not considered income, doing so would have incurred no tax liability at all."

Our analysis in the preceding subsection is useful for determining how optimal taxation should treat borrowing versus selling. The most instructive case is that of positive capital gains $\Delta p > 0$ but without a corresponding change in interest rates $\Delta q = 0$, which is the case of a pure risk-premium change—see equation (28). Setting $\Delta q = 0$ in Corollary 3 yields

$$\Delta T_0(\theta, s_0) = x(\theta, s_0)\Delta p - \Omega(\theta)X(s_0)\Delta p.$$

Perhaps surprisingly, the tax formula is *independent of* whether and how much investors borrow when their assets appreciate and is, in fact, identical to Corollary 1. Contrary to a recent proposal by Fox and Liscow (2024), it is not necessary to tax borrowing.[28]

The intuition (sometimes missed in the popular debate) is that, also with the option to borrow, investors need to sell their appreciating assets *at some point* in order to repay their loans and benefit from rising asset prices. If investors never sell their assets, they will need to repay their loans out of income they could have otherwise consumed and hence they do not benefit from the capital gains.[29] On the other hand, if investors do sell to repay the loan, the realized trade should be taxed at that point.

The Economist (2024) quote above emphasized an important motive for borrowing rather than selling an asset: the asset's return often exceeds the rate at which investors can borrow. While the model here does not allow for this possibility, Section 6.2 considers a setup with heterogeneous returns with precisely this feature – see equation (33) below. Still, it turns out that the optimal tax formula is unaffected. The intuition is that, while such return differences are undoubtedly important, they are not specific to the case of wealthy individuals borrowing against appreciating assets. Instead, they are a feature of *any* levered investment strategy. For example, many homeowners with an outstanding mortgage invest some of their income in the stock market rather than pre-paying their mortgage, precisely because stock returns exceed mortgage interest rates. Investors using levered investment strategies to take advantage of such return differences is an orthogonal issue that should not be considered tax avoidance.

5.3 General model

We now turn to the fully general model from Section 1 with T periods. As before, we consider a baseline history $\bar{s}^t$ and compare to an arbitrary alternative history s^t with a corresponding change in asset prices and dividends. To fix ideas, consider Figure 7: the baseline is a steady state with constant dividend $D_t(\bar{s}^t) = \overline{D}$, discount factor $m_t(\bar{s}^t) = \overline{R}^{-t}$, and asset prices $p_t(\bar{s}^t) = \overline{p}$ and $q_t(\bar{s}^t) = \overline{q}$.[30] At time $t = 0$, a shock is realized and $D_t(s^t), m_t(s^t), p_t(s^t), q_t(s^t)$ deviate from the initial steady state. The question we consider is: how does the optimal tax system redistribute in response to this different shock realization?

For example, as in Figure 7, $p_t(s^t)$ starts increasing but without any corresponding increase in cash flows $D_t(s^t)$ (panels (a) and (b)); equivalently, the asset discount rate jumps up initially

[28] An analogous result holds in the fully general multi-period model we consider in the next subsection.

[29] An exception is "stepped-up basis" which we discuss in Section 6.3. Moreover, investors may benefit from capital gains even without selling if a collateral constraint gets relaxed by a rising asset price (Fagereng et al., 2023). Here, given our focus on the top of the wealth distribution, we abstract from such binding constraints.

[30] An alternative baseline is a BGP where dividends grow at some constant rate G, so $D_t(\bar{s}^t) = G^t\overline{D}_0$, and rates of return are constant $m_t(\bar{s}^t) = \overline{R}^{-t}$. This could be the BGP of an equilibrium model with a neoclassical production side and productivity growth of the type discussed in Section 1.5.

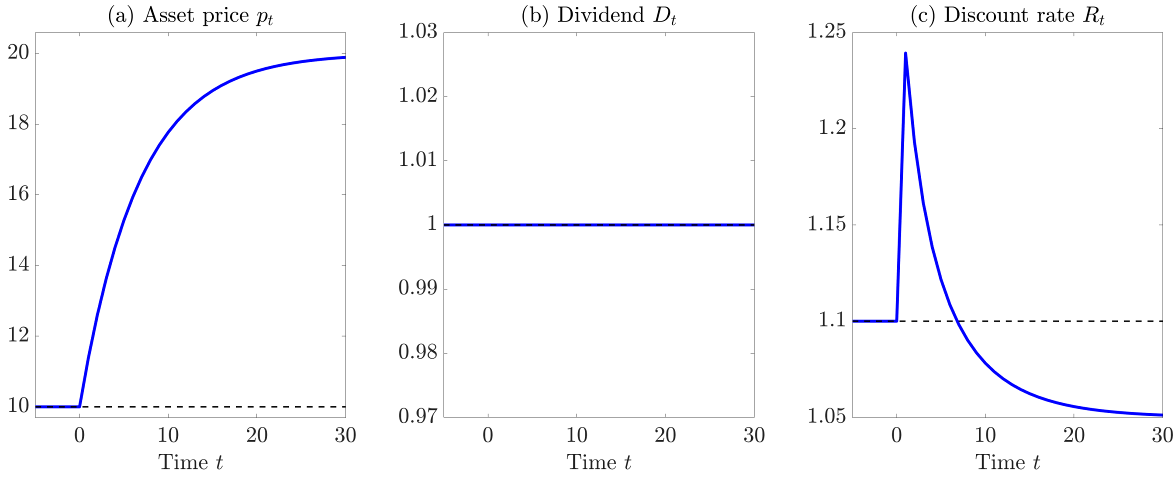


Figure 7: Example time paths for asset price, dividends, and discount rate

and then declines secularly to a lower long-run level (panel (c)), which generates capital gains. The example in Figure 7 therefore corresponds to Special Case 1 from Section 1.3 with an asset price change driven entirely by a discount rate change.

To obtain compact expressions, we suppress s^t and simply write $\Delta p_t = p_t(s^t) - p_t(\bar{s}^t)$ and analogously for ΔD_t, Δq_t and $\Delta T_t(\theta)$. Similarly, we write $k_t(\theta)$ as a shorthand for $k_t(\theta, s^{t-1})$, $b_t(\theta)$ for $b_t(\theta, s^{t-1})$, $x_t(\theta)$ for $x_t(\theta, s^t)$, and analogously for the corresponding aggregates K_t, B_t, and X_t. Let $\overline{m}_{0\to t}$ denote the stochastic discount factor evaluated at the baseline history $\bar{s}_t$. Our most general result concerns the difference in the present-discounted value of taxes in history s^t compared to the average over all possible histories $\bar{s}^t$:

$$\mathbb{E}_0\left[\sum_{t=0}^{T}\overline{m}_{0\to t}\Delta T_t(\theta)\right] = \mathbb{E}_0\left[\sum_{t=0}^{T} m_{0\to t}(\bar{s}^t)T_t(\theta, s^t)\right] - \mathbb{E}_0\left[\sum_{t=0}^{T} m_{0\to t}(\bar{s}^t)T_t(\theta, \bar{s}^t)\right],$$

where the expectation $\mathbb{E}_0$ is taken over all possible baseline histories $\bar{s}^t$.[31]

Proposition 6. *Consider a shock that changes asset prices by $\{\Delta p_t, \Delta q_t\}$ and dividends by $\{\Delta D_t\}$ relative to some baseline $\{\bar{s}^t\}$. Then the optimal change in taxes $\{\Delta T_t(\theta)\}$ is such that*

$$\begin{aligned}
&\mathbb{E}_0\left[\sum_{t=0}^{T}\overline{m}_{0\to t}\Delta T_t(\theta)\right] \\
&= \mathbb{E}_0\left[\sum_{t=0}^{T}\overline{m}_{0\to t}\left[x_t(\theta)\Delta p_t + k_t(\theta)\Delta D_t - b_{t+1}(\theta)\Delta q_t - \Omega(\theta)\left(X_t\Delta p_t + K_t\Delta D_t - B_{t+1}\Delta q_t\right)\right]\right].
\end{aligned}$$

Thus, our results from the two-period case generally apply to the expected present value of taxes. So far, we have not normalized taxes, so Proposition 6 leaves room for many different implementations. A natural example is to set

$$\Delta T_t(\theta) = x_t(\theta)\Delta p_t + k_t(\theta)\Delta D_t - b_{t+1}(\theta)\Delta q_t - \Omega(\theta)\left(X_t\Delta p_t + K_t\Delta D_t - B_{t+1}\Delta q_t\right) \tag{29}$$

[31] $\mathbb{E}_0[m_{0\to t}(\bar{s}^t)]$ is the value of a risk-free bond at time $t = 0$ that pays off one unit of consumption at time t in all possible states $\bar{s}^t$.

for all $s^t, \overline{s}^t$, meaning that we tax or subsidize the trading gains and losses as well as the change in dividend and interest income relative to the baseline period by period.

The deterministic case. The case without uncertainty is again particularly instructive because the two assets collapse to a single one and we can think of Proposition 6 as a comparative static exercise comparing taxes under two different time paths for discount rates and dividends $\{R_t, D_t\}_{t=0}^T$ and associated asset prices $\{p_t\}_{t=0}^T$ satisfying (7). Since $\overline{m}_{0\to t} = \overline{R}_{0\to t}^{-1}$, we obtain the following corollary of Proposition 6:

Corollary 5. *Suppose asset prices change by $\{\Delta p_t\}_{t=0}^T$ and dividends by $\{\Delta D_t\}_{t=0}^T$. Then optimal taxes $\{T_t(\theta)\}_{t=0}^T$ change such that*

$$\sum_{t=0}^{T} \overline{R}_{0\to t}^{-1} \Delta T_t(\theta) = \sum_{t=0}^{T} \overline{R}_{0\to t}^{-1} [x_t(\theta)\Delta p_t + k_t(\theta)\Delta D_t - \Omega(\theta)(X_t \Delta p_t + K_t \Delta D_t)].$$

Proposition 6 now applies to the comparison between any two price and dividend paths.[32] In Special Case 1, where asset prices change exclusively because of a change in discount rates, i.e. $\Delta D_t = 0$ for all t, we see that all terms involving ΔD_t in Corollary 5 drop out and optimal redistributive taxes condition only on realized trades $\{x_t(\theta), X_t\}$ and not on asset holdings $\{k_t(\theta), K_t\}$.[33]

In Special Case 2, where asset prices change exclusively because of a change in future dividends, the returns $\overline{R}_{t+1}$ remain unchanged for all $t \geq 1$ and the asset price change satisfies (9). Analogous to the example in Figure 7, the economy could initially be in a steady state with constant $\overline{D}, \overline{p}$ and $\overline{R}$ but then there are capital gains that are instead driven exclusively by a change in future dividends $\{\Delta D_t\}$. Then we obtain the following result:

Corollary 6. *Suppose the change in prices $\{\Delta p_t\}_{t=0}^T$ is exclusively driven by the change in dividends $\{\Delta D_t\}_{t=0}^T$. Then optimal taxes change such that*

$$\sum_{t=0}^{T} \overline{R}_{0\to t}^{-1} \Delta T_t(\theta) = [k_0(\theta) - \Omega(\theta) K_0] \, (\Delta D_0 + \Delta p_0)$$

In words, the change in the present value of taxes in this special case is given by the accrued gains in period 0, precisely like in Corollary 2 in the two-period model. One particular implementation is $\Delta T_0(\theta) = [k_0(\theta) - \Omega(\theta)K_0] \, (\Delta D_0 + \Delta p_0)$ and $\Delta T_t = 0, t \geq 1$, i.e. a one-time accrual-based capital gains tax at $t = 0$.

[32] Corollary 5 expresses the change in the present value of taxes using sales $x_t(\theta)$ and asset holdings $k_t(\theta)$ under the new prices and dividends, but the rates of return $\overline{R}_{0\to t}$ under the old prices and dividends. Analogously to Proposition 1, it is also possible to write the change in taxes in the opposite way, namely

$$\sum_{t=0}^{T} R_{0\to t}^{-1} \Delta T_t(\theta) = \sum_{t=0}^{T} R_{0\to t}^{-1} [\overline{x}_t(\theta)\Delta p_t + \overline{k}_t(\theta)\Delta D_t - \Omega(\theta)(\overline{X}_t \Delta p_t + \overline{K}_t \Delta D_t)].$$

[33] Note also that Corollary 5 fixes the present value of taxes, consistent with a requirement of Vickrey (1939) for desirable tax systems: *"The discounted value of the series of tax payments made by any taxpayer should be independent of the way in which his income is allocated to the various income years."*

Another useful perspective on Corollary 6 which explains why only initial asset holdings $k_0(\theta)$ show up is via the alternative tax implementation discussed in Section 3.4 that taxes changes in total capital income due to changes in returns $\{\Delta R_t\}$. To this end, Proposition 10 in Appendix D.6 spells out the multi-period analogue of Proposition 2. Analogous to (29), a natural per-period implementation is

$$\Delta T_t(\theta) = a_t(\theta)\Delta R_t - \Omega(\theta) A_t \Delta R_t \quad \text{for all } t.$$

The key observation for understanding why only initial asset holdings $k_0(\theta)$ show up in Corollary 6 is that Special Case 2 with purely dividend-driven asset prices means that, while the initial return changes $\Delta R_0 = (\Delta D_0 + \Delta p_0)/p_{-1} \neq 0$, all returns going forward are unchanged $\Delta R_t = 0$ for all $t \geq 1$. Hence only the time-zero terms $a_0(\theta)\Delta R_0$ show up in the tax formula. But, since $a_0(\theta) = p_{-1}k_0(\theta)$, this exactly equals the term $k_0(\theta)(\Delta D_0 + \Delta p_0)$ in Corollary 6. In contrast, in all other cases except Special Case 2, rising asset prices affect returns going forward so that $\Delta R_t \neq 0$ for $t \geq 1$ and hence there are extra terms in the optimal tax formula – see Section 3.4 for the intuition.

However, even in Special Case 2 with a purely dividend-driven asset price change, this Haig-Simons tax only works once in the initial period, not each period. Indeed, from (9) we have $\Delta p_0 = \sum_{t=1}^{T} \overline{R}_{0\to t}^{-1} \Delta D_t$ and hence another way of writing the tax change is

$$\sum_{t=0}^{T} \overline{R}_{0\to t}^{-1} \Delta T_t(\theta) = [k_0(\theta) - \Omega(\theta)K_0] \sum_{t=0}^{T} \overline{R}_{0\to t}^{-1} \Delta D_t.$$

Hence, a period-by-period implementation would set $\Delta T_t(\theta) = k_0(\theta)\Delta D_t - \Omega(\theta)K_0\Delta D_t$ for all t, which does *not* correspond to a tax on the accrued gains (nor dividend income) in each period.

6 Extensions

In this section, we show how the results derived in the benchmark setting extend to richer environments, namely a closed economy general equilibrium model, heterogeneous returns, and intergenerational transfers. For simplicity, we return to the two-period case.

6.1 General equilibrium

Our baseline model features a small open economy with an exogenously given asset price and dividend. Instead, we now consider a closed economy with the asset in fixed supply, so

$$\int k_0(\theta)dF(\theta) = \int k_1(\theta)dF(\theta) = K. \tag{30}$$

The asset price p must adjust to satisfy the market clearing condition (30). For example, if preferences are given by (27), the equilibrium asset price p^* can be solved in closed form:

$$p^* = \beta D \left(\frac{Y_0}{Y_1 + DK} \right)^{\frac{1}{\sigma}}. \tag{31}$$

This illustrates the various potential drivers of asset price changes in general equilibrium. A particularly natural one is an increase in the discount factor β, which increases the asset price p^* proportionally (holding dividends fixed). More generally, regardless of what causes the change in the equilibrium price p^*, we obtain the following result:

Proposition 7. *Suppose the equilibrium asset price changes from $\overline{p}^*$ to $p^* = \overline{p}^* + \Delta p^*$, holding dividends D and the aggregate endowment (Y_0, Y_1) fixed. Then the optimal tax $T_0(\theta)$ is given by*

$$T_0(\theta) = \overline{T}_0(\theta) + \overline{x}(\theta)\Delta p^* = \overline{T}_0(\theta) + x(\theta)\Delta p^*$$

where $\overline{T}_0(\theta)$ is the optimal tax at the initial price $\overline{p}^*$ and $\overline{x}(\theta) = x(\theta)$ are investor θ's asset sales at the initial and old prices $\overline{p}^*$ and p^*, respectively.

Hence, we obtain the same result as in Corollary 1 except that, since aggregate asset sales X must be zero in the closed economy, the intercept term vanishes. Moreover, individual asset sales in fact remain unchanged in response to the asset price change, so $x(\theta) = \overline{x}(\theta)$ for all θ. Intuitively, in the closed economy, total resources do not change when dividends and the aggregate endowment are held fixed. Hence, the planner aims to get each investor back to its original consumption bundle after the asset price change. The tax reform in Proposition 7 achieves this via Slutsky compensation as in Lemma 2.

We can also consider a change in dividends D in general equilibrium. Equation (31) reveals that an increase in D has a less than proportional effect on the asset price p^* due to the indirect effect on the aggregate endowment. Hence, a change in dividends will simultaneously increase the equilibrium rate of return $R^* = D/p^*$. As a result, the knife-edge result in Corollary 2, which lended support to a Haig-Simons accrual-based tax in the special case of a purely dividend-driven asset price change, does not extend to general equilibrium.

6.2 Heterogeneous returns

So far, we have assumed that investors are heterogeneous in their initial endowments $k_0(\theta)$ and incomes $y_0(\theta)$ and $y_1(\theta)$, but they all achieve the same dividends D per unit of their asset holdings $k_1(\theta)$ in period 1. We next show how our results extend to the case with heterogeneous dividends $D(\theta)$ which implies that different investors earn different returns $R(\theta) \equiv D(\theta)/p$.[34]

Just introducing this additional heterogeneity into our baseline model does not change our results on Pareto optimal tax policy. The reason is that, in the absence of further frictions, it is efficient to allocate all asset holdings to the individual with the highest dividends $D^{\max} \equiv \max_\theta D(\theta)$. Hence, the planner can transfer resources at rate of return $R = D^{\max}/p$, effectively

[34] See, for example, Gerritsen et al. (2020), Schulz (2021) and Guvenen et al. (2023, 2024).

returning us to the case without return heterogeneity. Those individuals with lower dividends will not hold the asset, but the government saves for them (using the highest-return individual) through taxes and transfers $T_0(\theta)$ and $T_1(\theta)$. Hence, Proposition 1 goes through, with the only twist that almost all investors will be sellers with $x(\theta) = k_0(\theta)$ and $k_1(\theta) = 0$.

Trading with adjustment costs. To prevent this trivial outcome, we build on our analysis in Section 4.4 and re-introduce a bond and some trading friction. Using the same notation as there, an investor's budget constraints are:

$$\begin{aligned} c_0(\theta) + qb(\theta) &= px(\theta) - \chi(x(\theta)) + y_0(\theta) - T_0(\theta) \\ c_1(\theta) &= D(\theta)(k_0(\theta) - x(\theta)) + b(\theta) + y_1(\theta). \end{aligned}$$

The adjustment cost $\chi(x)$ ensures that it is no longer efficient to allocate all capital to the individual with the highest return. The present-value budget constraint is:

$$c_0(\theta) + qc_1(\theta) = qD(\theta)(k_0(\theta) - x(\theta)) + px(\theta) - \chi(x(\theta)) + y_0(\theta) + qy_1(\theta) - T_0(\theta),$$

which immediately implies that an investor's optimal asset sales $x(\theta)$ satisfy

$$qD(\theta) + \chi'(x(\theta)) = p. \tag{32}$$

The left-hand side captures the marginal cost of selling more assets: the investor will have less dividend income and will need to pay the additional trading cost. On the other hand, the asset price on the right-hand side is the additional revenue from the sale. Due to the convex adjustment cost, investors with higher returns $D(\theta)$ will sell less and hold more of the asset. Also note that the presence of adjustment costs in (32) implies that

$$R(\theta) \equiv \frac{D(\theta)}{p} \geq \frac{1}{q}, \tag{33}$$

so that (i) the usual no-arbitrage condition equalizing the return on the asset $R(\theta)$ to that on the bond $1/q$ may not hold and, therefore, (ii) different investors θ may obtain different asset returns $R(\theta)$ in equilibrium. This opens up the door to different investors' returns $R(\theta)$ changing differentially in response to heterogeneous cash flow changes.

The aggregate resource constraint can be written as

$$\int c_0(\theta)dF(\theta) + q\int c_1(\theta)dF(\theta) = Y \tag{34}$$

where

$$Y = Y_0 + qY_1 + \max_{\{x(\theta)\}} \int \left[px(\theta) + qD(\theta)(k_0(\theta) - x(\theta)) - \chi(x(\theta))\right] dF(\theta)$$

Thus, the first-best Pareto problem takes the same form as in Section 3. Normalizing $T_1(\theta) = 0$, we have:

Proposition 8. *Suppose the equilibrium asset price changes from $\overline{p}$ to $p = \overline{p} + \Delta p$, holding dividends $D(\theta)$ and the bond price q fixed. Then the optimal tax $T_0(\theta)$ satisfies*

$$T_0(\theta) = \overline{T}_0(\theta) + x(\theta)\Delta p - \Omega(\theta)X\Delta p + \mathcal{O}\left(\Delta x(\theta)^2\right) \quad \textit{where} \;\; \Delta x(\theta) \equiv x(\theta) - \overline{x}(\theta).$$

Hence, even with heterogeneous returns and trading frictions, Corollary 1 goes through to first order, and an additional second-order term emerges that captures the change in adjustment costs due to the asset price change.

General equilibrium. This result is particularly useful when combining it with our previous general-equilibrium analysis. Suppose the asset is in fixed supply, as in the preceding subsection, and assume, for simplicity, a quadratic adjustment cost $\chi(x) = \kappa x^2$. Then the optimality condition (32) together with the market clearing condition $X = 0$ immediately implies

$$p^* = q \int D(\theta) dF(\theta).$$

The equilibrium price equals the discounted average dividends in the economy. This is intuitive, since even investors with a low dividend can sell their asset to other investors with higher dividends, so the asset price must reflect the average dividend. As already anticipated above, in equilibrium, different investors experience differential returns given by

$$R^*(\theta) \equiv \frac{D(\theta)}{p^*} = \frac{D(\theta)}{q \int D(\tilde{\theta}) dF(\tilde{\theta})}.$$

Consider now an increase in dividends $D(\theta)$ for *some subset* of the investors in the economy. This induces an increase in the equilibrium asset price p^* for *all* investors, including for those whose dividends did not change.[35] Put differently, those investors whose dividends $D(\theta)$ increase experience an *increase* in their asset return $R^*(\theta)$; however those investors whose dividends $D(\theta)$ remain unchanged experience a *decline* in their asset return $R^*(\theta)$. Since these latter investors face a pure asset price increase without a simultaneous dividend change, their optimal tax change satisfies $\Delta T_0(\theta) \approx x(\theta)\Delta p^*$ as in Propositions 7 and 8.

Importantly, this is true even though the asset price change is ultimately driven by a dividend change. Hence, our knife-edge result from Corollary 2, which found that a Haig-Simons accrual-based tax can implement the optimum in the special case of a purely dividend-driven asset price change, does not survive in this richer model. The reason is that Corollary 2 relied on the fact that the effect of the asset price change and the dividend change happened to cancel in the baseline model where everyone achieves the same rate of return. With heterogeneous dividends, these effects no longer cancels (not even for the investors whose dividends change) so the Haig-Simons tax never applies. By contrast, a tax on both realized capital gains and dividends as in Proposition 1 continues to work.

[35] If bonds are in fixed supply as well, then the bond price q also adjusts, but in general this does not undo the change in the average dividends.

6.3 Bequests and Suboptimality of Step-Up in Basis at Death

We finally consider a version of our model with multiple generations in which parents bequeath to their children. We use this version to consider a peculiarity of the tax system in the U.S. and many other advanced economies: step-up in basis at death for inherited assets, a tax rule that eliminates the taxable capital gain that occurred between the original purchase of the asset and the heir's acquisition, thereby reducing the heir's tax liability.[36]

To keep things simple, we model dynasties of non-overlapping generations that are altruistic toward their offspring (Barro and Becker, 1989). A new generation of investors is born every τ years and lives for $\tau - 1$ periods. An investor of dynasty θ born at time t has lifetime utility

$$V_t(\theta) = U(c_t(\theta), ..., c_{t+\tau-1}(\theta)) + \alpha\beta^\tau V_{t+\tau}(\theta), \tag{35}$$

where $0 \leq \alpha \leq 1$ measures altruism toward the next generation and $U(c_t, ..., c_{t+\tau-1})$ is homothetic. The sequential budget constraint is still given by (10) but now with the convention that $k_\tau(\theta), k_{2\tau}(\theta), k_{3\tau}(\theta)$, and so on denote bequests left by investors in their last year of life toward their offspring in the next generation. Because investors are altruistic, these bequests will generally be positive. As is standard, the Barro-Becker assumption implies that we can work with the preferences of dynasties.[37] The Pareto problem is to maximize $\int \omega(\theta)V_0(\theta)dF(\theta)$ subject to (3) where $\omega(\theta)$ is the Pareto weight on dynasty θ. Hence, everything collapses to the multi-period model from Section 5 and Corollary 5 applies with the only modification that, in general, it includes a time-varying consumption allocation rule $\Omega_t(\theta)$ in place of the constant rule $\Omega(\theta)$ (i.e. the planner now allocates consumption $c_t(\theta) = \Omega_t(\theta)C_t$ to dynasty θ).

Suboptimality of Step-Up of Basis on Death. Because (a modified) Corollary 5 still applies, so does the discussion in Section 3.3 about the baseline relative to which capital gains are calculated. As discussed there, a natural benchmark is the price path on an initial BGP on which dividends and hence prices grow at a constant rate $\overline{p}_t = G^t\overline{p}_0$. Step-up of basis at death would instead correspond to a case in which the baseline price $\overline{p}_t$ resets to the current market price p_t every τ years, i.e. whenever a generation dies. From the point of a view of a dynasty or the social planner, there is nothing special about the dates at which one generation passes the baton to the next and therefore also no argument for resetting the basis in this way. Instead, a natural approach is the "carry-over basis" already used by a number of countries including Germany, Italy, and Japan (OECD, 2021).

[36] Many good explanations of step-up in basis can be found on the internet, particularly by financial and estate planning services. Some of these are explicit that they consider the rule to be a loophole, for example Trust and Will (2024) which begins the discussion thus: "Loopholes – you may not always use them, but when you do need them, you're sure glad they're there. [...] The Step-Up in Basis loophole is used to circumvent capital gains taxes, or to pay the least amount of this type of inheritance tax as is legally possible."

[37] Repeated substitution of (35) implies that the dynasty θ's utility at time 0 is given by

$$V_0(\theta) = U(c_0(\theta), ..., c_{\tau-1}(\theta)) + \alpha\beta^\tau U(c_\tau(\theta), ..., c_{2\tau-1}(\theta)) + \alpha^2\beta^{2\tau} U(c_{2\tau}(\theta), ..., c_{3\tau-1}(\theta)) + ...$$

Buy, Borrow, Die. A tax avoidance strategy of wealthy families known as "buy, borrow, die" has received attention in recent years (e.g. Ensign and Rubin, 2021; The Economist, 2024).[38] The idea is to borrow against appreciating assets rather than selling them and then taking advantage of the stepped-up basis at death, thereby avoiding capital gains taxes altogether. In combination with Section 5.2, our results suggests that the stepped-up basis loophole should be eliminated. Absent stepped-up basis, the wealthy would still benefit from borrowing against high-return assets with lower-interest loans but this is just like any other levered investment and should not be considered a tax avoidance strategy.

7 Conclusion

We "put the 'finance' into 'public finance'," meaning that we study optimal redistributive taxation with changing asset prices. Importantly, we adopt the modern finance view that asset prices change not only because of changing cash flows but also due to changes in discount rates, risk premia, or subjective beliefs.

It is useful to juxtapose our results with the following naïve intuition implicit in proposals for wealth taxes or taxes on unrealized capital gains: when the value of Jeff Bezos' Amazon stocks doubles so should his tax liability. We show that this intuition is, in general, incorrect. Optimal taxes instead generally depend on (i) whether Bezos sells his Amazon shares and (ii) whether and by how much cash flows, here Amazon's profits, increase. In our baseline model these are, in fact, the only determinants of optimal taxes. Generalizations of the type considered in Section 6 complicate the optimal tax formulas in some cases but it remains true that taxes that target only asset holdings rather than asset transactions are generally suboptimal.

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[38] Fox and Liscow (2025) question this strategy's prevalence and argue that "buy, *save*, die" is more widespread.

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Appendix

A Appendix for Section 1

A.1 Mapping the Neoclassical Growth Model into Our Setup

As is standard, a representative consumer has preferences $\sum_{t=0}^{\infty} \beta^t U(C_t)$ where C_t is consumption, the consumption good is produced according to a constant-returns technology $\mathcal{Y}_t = f(K_t, A_t L_t)$ where K_t is capital, A_t is productivity and L_t is labor, labor is supplied inelastically $L_t = 1$, and the resource constraint is

$$C_t + I_t = \mathcal{Y}_t, \quad K_{t+1} = I_t + (1-\delta)K_t, \tag{36}$$

where I_t is investment. Importantly, the fact that the consumption good $\mathcal{Y}_t$ can be converted into investment one-for-one immediately pins down the unit price of capital (relative to consumption) at one. We discuss this property in more detail momentarily where we also discuss how to break it.

A.1.1 The Price of Capital in Variants of the Growth Model

To understand why the unit price of capital is pinned down at one in the growth model and to see how to break this result, it is useful to consider a more general model in which the result does not necessarily hold: a two-sector growth model with a separate investment goods production sectors. The model is the same as in Section 1.5 except that the resource constraint is

$$C_t + \iota_t = \mathcal{Y}_t, \qquad I_t = G_t(\iota_t), \qquad K_{t+1} = I_t + (1-\delta)K_t. \tag{37}$$

Here ι_t units of the consumption produce I_t units of investment according to a production function G_t which is increasing but which may be concave $G_t'' \leq 0$ or may vary over time. Profit maximization of investment goods producer is

$$\max_{\iota_t} \; p_t G_t(\iota_t) - \iota_t.$$

As long as the marginal product $G'(\iota_t)$ is positive, producers choose ι_t to satisfy the optimality condition

$$p_t G_t'(\iota_t) = 1. \tag{38}$$

This model has two interesting polar special cases:

1. Neoclassical growth model: $I_t = G(\iota_t) = \iota_t$ so that $G'(\iota_t) = 1$. In this case the resource constraints (37) become $C_t + I_t = \mathcal{Y}_t$ and the optimality condition (38) implies $p_t = 1$.
2. Capital in fixed supply (Section 6.1): $G(\iota_t) = 0$ for all ι_t and $\delta = 0$. In this case $I_t = \iota_t = 0$ so that the resource constraints (37) become $C_t = \mathcal{Y}_t$. The price of capital p_t is pinned down by market clearing $I_t = 0$ rather than the optimality condition (38) because there is no optimization problem for investment goods production.

A.1.2 Growth Model with a Stock Market

This appendix spells out in more detail a decentralization of the growth model in which households trade shares in the representative firm which are in unit fixed supply. The budget constraint of the representative household is

$$p_t(S_{t+1} - S_t) + C_t = Y_t + D_t S_t.$$

Here, each share S_t is a claim on the profits of the representative firm. In equilibrium, shares are in unit fixed supply, so $S_t = 1$, and enoting the wage by W_t, the firm's cash flows are $D_t = \mathcal{Y}_t - W_t L_t - I_t$. Using that labor is paid its marginal product $W_t = f_L(K_t, A_t L_t) A_t$, that $f(K_t, A_t L_t) = f_K(K_t, A_t L_t) K_t + f_L(K_t, A_t L_t) A_t L_t$ because of constant returns, and $L_t = 1$:

$$D_t = f_K(K_t, A_t) K_t + (1-\delta) K_t - K_{t+1}. \tag{39}$$

The discount rate is still given by (12) and hence $R_{t+1} = f_K(K_{t+1}, A_{t+1}) + 1 - \delta$ in equilibrium.

We next show that the share price equals the value of the capital stock $p_t = K_{t+1}$. First, as usual, the share price equals the present-discounted value of dividends, i.e. it satisfies (7) with $T = \infty$.

Lemma 4. *The share price equals the value of the capital stock: $p_t = K_{t+1}$.*

Proof. Because the share price satisfies (7) with $T = \infty$ it equivalently satisfies

$$p_t = R_{t+1}^{-1}(D_{t+1} + p_{t+1}).$$

Using $R_{t+1} = f_K(K_{t+1}, A_{t+1}) + 1 - \delta$ and (39)

$$p_t(f_K(K_{t+1}, A_{t+1}) + 1 - \delta) = (f_K(K_{t+1}, A_{t+1})K_{t+1} + (1-\delta)K_{t+1} - K_{t+2}) + p_{t+1}.$$

The p_t sequence satisfying this equation is $p_t = K_{t+1}$ as claimed. $\square$

Remark on Lemma 4. Note that it is still true that the price per unit of capital (rather than the price of the entire capital stock $p_t = K_{t+1}$) equals one.

Balanced Growth Path (BGP). Assume that productivity A_t grows at a constant rate

$$A_{t+1} = GA_t, \quad G > 1 \quad \Rightarrow \quad A_t = G^t A_0$$

and that households have isoelastic preferences

$$U(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma}.$$

Then the economy has a BGP on which the capital stock, output, and consumption all grow at the same rate G. On this BGP, the asset return is constant

$$R_{t+1} = f_K(K_{t+1}, A_{t+1}) + 1 - \delta = f_K(K_0^*, A_0) + 1 - \delta = \overline{R}$$

where we have used that $f_K(K_t, A_t)$ is homogeneous of degree zero in (K_t, A_t). The initial location of the BGP K_0^* is pinned down by the discount rate

$$\overline{R} = \frac{1}{\beta} G^{\sigma}.$$

On the BGP, the asset price grows at a constant rate resulting in capital gains

$$\frac{p_{t+1}}{p_t} = \frac{K_{t+2}}{K_{t+1}} = G.$$

From (39), the dividend yield is given by

$$\frac{D_{t+1}}{p_t} = \frac{f_K(K_{t+1}, A_{t+1})K_{t+1} + (1-\delta)K_{t+1} - K_{t+2}}{K_{t+1}} = f_K(K_{t+1}, A_{t+1}) + 1 - \delta - G = \overline{R} - G, \tag{40}$$

so that

$$\frac{D_{t+1}}{p_t} + \frac{p_{t+1}}{p_t} = \overline{R}$$

as expected. Also note that from (40) we have

$$p_t = \frac{D_{t+1}}{\overline{R} - G}.$$

Therefore, all capital gains are driven entirely by growing cash flows and the price-dividend ratio is constant as in the Gordon growth model (Gordon and Shapiro, 1956). Also note that all capital gains are, in fact, unrealized. This is because, in equilibrium, the representative household does not buy or sell any shares (which are in fixed supply).

A.2 Proof of Lemma 1

Since preferences are assumed to be homothetic, we can write them as $U(\{c(s^t)\}) = G(H(\{c(s^t)\}))$, where H is homogenous of degree ρ and G is a monotonic function. We assume that G and H are such that $G(H(\cdot))$ is differentiable, strictly increasing, and strictly concave. The first-order condition for person θ in history s^t from problem (13) is

$$\omega(\theta)G'(H)\frac{\partial H(\{c(s^t,\theta)\})}{\partial c(s^t)} = \lambda(s^t),$$

where $\lambda(s^t)$ is the multiplier on the resource constraint in history s^t. The solution to the first-order conditions and resource constraints is the unique optimal allocation.

Given the unique optimum, we guess and verify that $c(s^t,\theta) = \Omega(\theta)C(s^t)$ satisfies the first-order conditions and resource constraints for some $\Omega(\theta)$. Let $\bar{H} \equiv H(\{C(s^t)\})$, which is pinned down by the aggregate resource conditions in (13). This implies that $H(\{c(s^t,\theta)\}) = H(\{\Omega(\theta)C(s^t)\}) = \Omega(\theta)^\rho \bar{H}$, where the last equality uses the homogeneity of H. The first-order condition for θ in state s^t becomes

$$\begin{aligned}
\lambda(s^t) &= \omega(\theta)G'(H)\frac{\partial H(\{c(s^t,\theta)\})}{\partial c(s^t)} \\
&= \omega(\theta)G'(\Omega(\theta)^\rho \bar{H})\frac{\partial H(\Omega(\theta)\{C(s^t,\theta)\})}{\partial c(s^t)} \\
&= \omega(\theta)G'(\Omega(\theta)^\rho \bar{H})\Omega(\theta)^{\rho-1}\frac{\partial H(\{C(s^t,\theta)\})}{\partial c(s^t)} \\
&= \omega(\theta)G'(\Omega(\theta)^\rho \bar{H})\Omega(\theta)^{\rho-1}\frac{\partial \bar{H}}{\partial c(s^t)},
\end{aligned}$$

where the second equality uses our conjectured consumption rule, the third uses the homogeneity of H, and the last uses the definition of $\bar{H}$. This condition implies that $\omega(\theta)G'(\Omega(\theta)^\rho \bar{H})\Omega(\theta)^{\rho-1}$ is constant across θ and s^t. That is, there is an $\bar{\Omega}$ such that

$$\omega(\theta)G'(\Omega(\theta)^\rho \bar{H})\Omega(\theta)^{\rho-1} = \bar{\Omega} \quad \forall\theta.$$

Since $G(H(\cdot))$ is strictly concave, the left-hand side is strictly decreasing in $\Omega(\theta)$ and we can invert it to solve for $\Omega(\theta)$ given $\bar{\Omega}$. That is, $\Omega(\theta) = f(\omega(\theta), \bar{\Omega})$, where f is strictly increasing in its first argument. We can solve for $\bar{\Omega}$ as the solution to

$$\int f(\omega(\theta), \bar{\Omega}) dF(\theta) = 1,$$

which ensures that all resource conditions are satisfied.

B Appendix for Section 3

B.1 Proof of Proposition 1

We begin with proving the first equation in Proposition 1. From the budget constraint (22) we have

$$\overline{T}_0(\theta) = y_0(\theta) - \overline{c}_0(\theta) + \overline{p}\,\overline{x}(\theta)$$
$$T_0(\theta) = y_0(\theta) - c_0(\theta) + px(\theta),$$

where we denote by $\overline{c}_t(\theta)$, $t = 0, 1$, consumption at the old prices and dividends. Subtracting the former from the latter, we obtain

$$\Delta T_0(\theta) = T_0(\theta) - \overline{T}_0(\theta) = \overline{x}(\theta)\Delta p + p(x(\theta) - \overline{x}(\theta)) - (c_0(\theta) - \overline{c}_0(\theta)). \tag{41}$$

By the second-period budget constraint (23) and the normalization that $T_1(\theta)$ is held fixed, we have

$$c_1(\theta) - \overline{c}_1(\theta) = \overline{k}_1(\theta)\Delta D - D(x(\theta) - \overline{x}(\theta)) \tag{42}$$

and thus

$$p(x(\theta) - \overline{x}(\theta)) = \frac{p}{D}\left[\overline{k}_1(\theta)\Delta D - (c_1(\theta) - \overline{c}_1(\theta))\right].$$

Substituting in (41), we obtain

$$\Delta T_0(\theta) = \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D - \left[(c_0(\theta) - \overline{c}_0(\theta)) + \frac{p}{D}(c_1(\theta) - \overline{c}_1(\theta))\right].$$

Next, by Lemma 1 we have

$$\Delta T_0(\theta) = \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D - \Omega(\theta)\left[C_0 - \overline{C}_0 + \frac{p}{D}(C_1 - \overline{C}_1)\right]. \tag{43}$$

To rewrite the expression in square brackets, we work with the aggregate resource constraints. Since $\int T_0(\theta)dF(\theta) = \int T_1(\theta)dF(\theta) = 0$ we have $C_0 = pX + Y_0$ and $C_1 = D(K_0 - X) + Y_1$. Therefore

$$C_0 - \overline{C}_0 = pX - \overline{p}\overline{X} = \overline{X}\Delta p + p\Delta X,$$
$$C_1 - \overline{C}_1 = D(K_0 - X) - \overline{D}(K_0 - \overline{X}) = K_0\Delta D - D\Delta X - \overline{X}\Delta D = \overline{K}_1\Delta D - D\Delta X$$

where $\Delta X = X - \overline{X}$. Combining yields

$$C_0 - \overline{C}_0 + \frac{p}{D}(C_1 - \overline{C}_1) = \overline{X}\Delta p + \frac{p}{D}\overline{K}_1\Delta D.$$

Substituting in (43) delivers the final result:

$$\Delta T_0(\theta) = \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D - \Omega(\theta)\left[\overline{X}\Delta p + \frac{p}{D}\overline{K}_1\Delta D\right].$$

The proof of the second equation in Proposition 1 follows the same steps. Equation (41) can equivalently be written as

$$\Delta T_0(\theta) = x(\theta)\Delta p + \overline{p}(x(\theta) - \overline{x}(\theta)) - (c_0(\theta) - \overline{c}_0(\theta)) \tag{44}$$

and equation (42) as

$$c_1(\theta) - \overline{c}_1(\theta) = k_1(\theta)\Delta D - \overline{D}(x(\theta) - \overline{x}(\theta))$$

and thus

$$\overline{p}(x(\theta) - \overline{x}(\theta)) = \frac{\overline{p}}{\overline{D}}\left[k_1(\theta)\Delta D - (c_1(\theta) - \overline{c}_1(\theta))\right].$$

Substituting in (44), we obtain

$$\begin{aligned}\Delta T_0(\theta) &= x(\theta)\Delta p + \frac{\overline{p}}{\overline{D}}k_1(\theta)\Delta D - \left[(c_0(\theta) - \overline{c}_0(\theta)) + \frac{\overline{p}}{\overline{D}}(c_1(\theta) - \overline{c}_1(\theta))\right] \\ &= x(\theta)\Delta p + \frac{\overline{p}}{\overline{D}}k_1(\theta)\Delta D - \Omega(\theta)\left[C_0 - \overline{C}_0 + \frac{\overline{p}}{\overline{D}}(C_1 - \overline{C}_1)\right].\end{aligned} \tag{45}$$

By the aggregate resource constraints

$$\begin{aligned}C_0 - \overline{C}_0 &= pX - \overline{p}\overline{X} = X\Delta p + \overline{p}\Delta X \\ C_1 - \overline{C}_1 &= D(K_0 - X) - \overline{D}(K_0 - \overline{X}) = K_0\Delta D - \overline{D}\Delta X - X\Delta D = K_1\Delta D - \overline{D}\Delta X\end{aligned}$$

Combining yields

$$C_0 - \overline{C}_0 + \frac{\overline{p}}{\overline{D}}(C_1 - \overline{C}_1) = X\Delta p + \frac{\overline{p}}{\overline{D}}K_1\Delta D.$$

Substituting in (45) delivers the final result:

$$\Delta T_0(\theta) = x(\theta)\Delta p + \frac{\overline{p}}{\overline{D}}k_1(\theta)\Delta D - \Omega(\theta)\left[X\Delta p + \frac{\overline{p}}{\overline{D}}K_1\Delta D\right].$$

B.2 Proof of Lemma 2

As in the Lemma, denote the old price by $\overline{p}$ and the new price by $p = \overline{p} + \Delta p$. Similarly, denote the old dividend by $\overline{D}$ and the new dividend by $D = \overline{D} + \Delta D$. Denote the original consumption bundle by $(\overline{c}_0(\theta), \overline{c}_1(\theta))$. Slutsky compensation is defined as the change in the investor's total budget $y_0(\theta)$ that keeps the original consumption bundle $(\overline{c}_0(\theta), \overline{c}_1(\theta))$ affordable at the new asset price p and dividend D. In the remainder of the proof, we suppress the dependence of variables on θ for notational simplicity.

The lifetime budget line at the original price is the set of points (c_0, c_1) such that

$$c_0 + \frac{\overline{p}}{\overline{D}}c_1 = y_0 + \frac{\overline{p}}{\overline{D}}y_1 + \overline{p}k_0 \tag{46}$$

The Slutsky-compensated budget line at the new price is the set of points (c_0, c_1) such that

$$c_0 + \frac{p}{D}c_1 = y_0 + \frac{p}{D}y_1 + pk_0 + \Delta y_0, \tag{47}$$

where Δy_0 is the Slutsky compensation term. The aim is to solve for Δy_0 such that the two budget lines intersect at the point $(c_0, c_1) = (\overline{c}_0, \overline{c}_1)$, i.e. so that the original consumption bundle remains affordable at the new prices. To this end, evaluate (46) and (47) at $(\overline{c}_0, \overline{c}_1)$ and subtract the old budget constraint (46) from the new budget constraint (47)

$$\left(\frac{p}{D} - \frac{\overline{p}}{\overline{D}}\right)\overline{c}_1 = \left(\frac{p}{D} - \frac{\overline{p}}{\overline{D}}\right)y_1 + k_0\Delta p + \Delta y_0$$

Rearranging, we have

$$\Delta y_0 = \left(\frac{p}{D} - \frac{\overline{p}}{\overline{D}}\right)(\overline{c}_1 - y_1) - k_0\Delta p = \left(\frac{p}{D} - \frac{p}{\overline{D}} + \frac{\Delta p}{\overline{D}}\right)(\overline{c}_1 - y_1) - k_0\Delta p$$

where the second equality used $\overline{p} = p - \Delta p$. Using the second-period budget constraint (15), which implies $\overline{c}_1 - y_1 = \overline{D}\overline{k}_1$, yields

$$\Delta y_0 = \left(\frac{p}{D} - \frac{p}{\overline{D}}\right)\overline{D}\overline{k}_1 + (\overline{k}_1 - k_0)\Delta p = p\left(\frac{\overline{D}}{D} - 1\right)\overline{k}_1 + (\overline{k}_1 - k_0)\Delta p = -\frac{p}{D}\overline{k}_1\Delta D - \overline{x}\Delta p$$

where the last equality uses $\overline{x} = k_0 - \overline{k}_1$. Reintroducing the explicit dependence on θ yields the in the lemma.

B.3 Endogenous payout policy and share repurchases

The capital-structure neutral reformulation of our setup is easiest to explain in the multi-period model of Section 1. Consider a firm that produces an income stream (i.e. earnings minus investment) $\{\Pi_t\}_{t=0}^T$ from its fundamental (e.g., non-financial) operations. Investors have budget constraint (2) and we assume for simplicity that the only asset at their disposal is firm shares so that $k_t(\theta)$ denotes share holdings, p_t denotes the share price, and D_t denotes the business dividends per share. The firm's cash flows are distributed to shareholders through both dividends and share repurchases:

$$\Pi_t = \mathcal{K}_t D_t + (\mathcal{K}_t - \mathcal{K}_{t+1})p_t \tag{48}$$

where and $\mathcal{K}_t = \int k_t(\theta)dF(\theta)$ denotes the total amount of outstanding shares. When $\mathcal{K}_{t+1} < \mathcal{K}_t$ the business is repurchasing its own shares. From this equation it is already apparent that share repurchases and dividend payments are equivalent means of distributing cash flows $\{\Pi_t\}_{t=0}^T$ to shareholders as a whole. When the business repurchases its shares (i.e., $\mathcal{K}_{t+1} < \mathcal{K}_t$) this results in an income stream $(k_t(\theta) - k_{t+1}(\theta))p_t$ for those individual selling their shares to the business.

Denoting by $s_t(\theta) \equiv k_t(\theta)/\mathcal{K}_t$ the individual's ownership share of the business and by $V_t \equiv \mathcal{K}_t p_t$ the market value of the business, we can combine the individual and business budget constraints, (2) and (48), to obtain:

$$c_t(\theta) + V_t(s_{t+1}(\theta) - s_t(\theta)) = y_t(\theta) + \Pi_t s_t(\theta) \tag{49}$$

This budget constraint has the same form as (2), except that (i) the dividend per share D_t is replaced by the income stream from operations Π_t, (ii) the price per share p_t is replaced by the market value of the firm V_t, and (iii) the number of shares held by the individuals $k_t(\theta)$ is replaced by the ownership share in the business $s_t(\theta)$. An alternative viewpoint on this consolidated budget constraint is to consider the return to investing in the business. See Fagereng et al. (2023) for more discussion on this capital-structure neutral reformulation.

B.4 Proof of Proposition 2

In a similar manner to Proposition 1, we use the budget constraint in the first period to get

$$\overline{T}_0(\theta) = y_0(\theta) - \overline{c}_0(\theta) - \overline{a}_1(\theta) + \overline{R}_0 a_0(\theta)$$
$$T_0(\theta) = y_0(\theta) - c_0(\theta) - a_1(\theta) + R_0 a_0(\theta).$$

Subtracting the former from the latter, we obtain

$$\Delta T_0(\theta) = T_0(\theta) - \overline{T}_0(\theta) = (\bar{c}_0(\theta) - c_0(\theta)) + (\bar{a}_1(\theta) - a_1(\theta)) + a_0(\theta)\Delta R_0. \tag{50}$$

From the second-period budget constraint we have

$$c_1(\theta) - \bar{c}_1(\theta) = R_1 a_1(\theta) - \overline{R}_1 \bar{a}_1(\theta). \tag{51}$$

Note that we can write it as

$$c_1(\theta) - \bar{c}_1(\theta) = \bar{a}_1(\theta)(R_1 - \overline{R}_1) + R_1(a_1(\theta) - \bar{a}_1(\theta)) \tag{52}$$

Thus we have

$$\bar{a}_1(\theta) - a_1(\theta) = \frac{1}{R_1}\left(\bar{c}_1(\theta) - c_1(\theta)\right) + \frac{1}{R_1}\bar{a}_1(\theta)\Delta R_1$$

Replacing in (50) we get

$$\Delta T_0(\theta) = a_0(\theta)\Delta R_0 + \frac{1}{R_1}\bar{a}_1(\theta)\Delta R_1 - \left[(c_0(\theta) - \bar{c}_0(\theta)) + \frac{1}{R_1}\left(c_1(\theta) - \bar{c}_1(\theta)\right)\right].$$

Again, by Lemma 1, we know $c_t(\theta) = \Omega(\theta)C_t$. So we have

$$\Delta T_0(\theta) = a_0(\theta)\Delta R_0 + \frac{1}{R_1}\bar{a}_1(\theta)\Delta R_1 - \Omega(\theta)\left[C_0 - \overline{C}_0 + \frac{1}{R_1}(C_1 - \overline{C}_1)\right]. \tag{53}$$

Similar to Proposition 1, working with the aggregate resource constraints we have

$$C_0 + A_1 = Y_0 + R_0 A_0,$$

$$C_1 = Y_1 + R_1 A_1.$$

Therefore

$$C_0 - \overline{C}_0 = -\Delta A_1 + A_0 \Delta R_0,$$

$$C_1 - \overline{C}_1 = R_1 A_1 - \overline{R}_1 \overline{A}_1 = R_1 \Delta A_1 + \overline{A}_1 \Delta R_1.$$

Replacing in equation (53) we get the final result

$$T_0(\theta) = \overline{T}_0(\theta) + a_0(\theta)\Delta R_0 + \frac{1}{R_1}\bar{a}_1(\theta)\Delta R_1 - \Omega(\theta)\left[A_0 \Delta R_0 + \frac{1}{R_1}\overline{A}_1 \Delta R_1\right]. \tag{54}$$

The proof of the second equation in Proposition 2 follows the same steps.

B.5 An expenditure tax

This appendix contains the details for Section 3.4.2, in particular Proposition 9. Denote by $\{\bar{c}_t(\theta)\}$, $t = 0, 1$, the optimal consumption allocation at the old prices and dividends $\bar{p}$ and $\overline{D}$ (i.e., the solution to the Pareto problem (21)), and by $\{c_t(\theta)\}$ at the new prices and dividends $p = \bar{p} + \Delta p$ and $D = \overline{D} + \Delta D$. Let $\{\overline{T}_t(\theta)\}$, $t = 0, 1$, be some lump-sum taxes that implement the optimum at the old prices and dividends. Finally, let $\hat{c}_t(\theta)$, $t = 0, 1$, denote investor θ's *individually* optimal consumption allocation under the new prices and dividends but when taxes are held fixed at the old level. Formally, $\hat{c}_t(\theta)$

solves

$$\max_{c_0(\theta),c_1(\theta),x(\theta)} U(c_0(\theta),c_1(\theta)) \quad \text{s.t.} \quad (22) \text{ and } (23)$$

when taxes are given by $\{\overline{T}_t(\theta)\}$. Then we have the following result:

Proposition 9. *Suppose asset prices increase from $\overline{p}$ to $p = \overline{p} + \Delta p$ and dividends from $\overline{D}$ to $D = \overline{D} + \Delta D$. The optimal consumption allocation at the new prices and dividends $\{c_t(\theta)\}$, $t = 0, 1$, can be implemented with taxes given by*

$$T_t(\theta) = \overline{T}_t(\theta) + \Delta\widehat{c}_t(\theta) - \Omega(\theta)\Delta C_t, \quad t = 0,1$$

where $\Delta\widehat{c}_t(\theta) \equiv \widehat{c}_t(\theta) - \overline{c}_t(\theta)$ and $\Delta C_t = \int c_t(\theta)dF(\theta) - \int \overline{c}_t(\theta)dF(\theta)$.

Hence, the new optimum can be implemented with a combination of a lump-sum tax equal to $\Delta\widehat{c}_t(\theta)$, which is the amount by which individuals would have changed their consumption after the price and dividend change if taxes had stayed at their old level $\overline{T}_t(\theta)$, and a transfer equal to the difference between the old and new desired consumption $\Delta c_t(\theta) = \Omega(\theta)\Delta C_t$. Notably, if the parameter changes Δp and ΔD are "zero-sum," so that optimal aggregate consumption C_t does not change, then the tax equals $\Delta\widehat{c}_t(\theta)$ meaning that optimal redistributive taxation simply taxes away any increase in consumption from the asset-price and dividend changes (or compensates the corresponding reduction in consumption), i.e. this is a "pure" expenditure tax without an additional transfer. In line with Kaldor's logic, just like Proposition 1, this works for any combination of asset price and dividend changes, i.e. regardless of the source of capital gains. Indeed, we show below that

$$\Delta\widehat{c}_0(\theta) + \frac{p}{D}\Delta\widehat{c}_1(\theta) = \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D, \tag{55}$$

so the present value of the consumption change (holding taxes fixed) is directly linked to the capital gains and change in dividend income when p and D change. For instance, the investors who would increase their consumption in response to a pure asset price increase (in the absence of a further tax change) are precisely those who sell the asset, and vice versa.

Proof of Proposition 9. An investor's present-value budget constraint at the new prices and dividends (p, D) and new taxes $T_t(\theta)$ is

$$c_0(\theta) + \frac{p}{D}c_1(\theta) + T_0(\theta) + \frac{p}{D}T_1(\theta) = y_0(\theta) + \frac{p}{D}y_1(\theta) + pk_0(\theta) \tag{56}$$

The present-value budget constraint at the new prices and dividends (p, D) but *old* taxes $\overline{T}_t(\theta)$ is

$$\widehat{c}_0(\theta) + \frac{p}{D}\widehat{c}_1(\theta) + \overline{T}_0(\theta) + \frac{p}{D}\overline{T}_1(\theta) = y_0(\theta) + \frac{p}{D}y_1(\theta) + pk_0(\theta) \tag{57}$$

Subtracting (57) from (56) yields

$$c_0(\theta) - \widehat{c}_0(\theta) + \frac{p}{D}(c_1(\theta) - \widehat{c}_1(\theta)) + T_0(\theta) - \overline{T}_0(\theta) + \frac{p}{D}(T_1(\theta) - \overline{T}_1(\theta)) = 0$$

which we can rewrite as

$$\Delta T_0(\theta) + \frac{p}{D}\Delta T_1(\theta) = \widehat{c}_0(\theta) - c_0(\theta) + \frac{p}{D}(\widehat{c}_1(\theta) - c_1(\theta)) \tag{58}$$

Next, observe that, for $t = 1, 2$,

$$\widehat{c}_t(\theta) - c_t(\theta) = \widehat{c}_t(\theta) - \overline{c}_t(\theta) - (c_t(\theta) - \overline{c}_t(\theta)) = \Delta\widehat{c}_t(\theta) - \Delta c_t(\theta) = \Delta\widehat{c}_t(\theta) - \Omega(\theta)\Delta C_t,$$

where the last step uses Lemma 1. Substituting back in equation (58) yields

$$\Delta T_0(\theta) + \frac{p}{D}\Delta T_1(\theta) = \Delta\widehat{c}_0(\theta) - \Omega(\theta)\Delta C_0 + \frac{p}{D}(\Delta\widehat{c}_1(\theta) - \Omega(\theta)\Delta C_1).$$

One way of implementing this is to set, in each period $t = 0, 1$,

$$\Delta T_t(\theta) = \Delta\widehat{c}_t(\theta) - \Omega(\theta)\Delta C_t$$

as in Proposition 9.

Proof of Equation (55). Start with the first-period budget constraint (22) holding fixed the old taxes when prices and dividends change

$$\overline{T}_0(\theta) = y_0(\theta) - \overline{c}_0(\theta) + \overline{p}\,\overline{x}(\theta)$$
$$\overline{T}_0(\theta) = y_0(\theta) - \widehat{c}_0(\theta) + p\widehat{x}(\theta)$$

where we denote by $\widehat{x}(\theta)$ the investor's optimal asset sales at the old taxes but new prices and dividends. Subtracting the former from the latter, we obtain

$$0 = \overline{x}(\theta)\Delta p + p(\widehat{x}(\theta) - \overline{x}(\theta)) - (\widehat{c}_0(\theta) - \overline{c}_0(\theta)). \tag{59}$$

By the second-period budget constraint (23) and holding the old taxes $\overline{T}_1(\theta)$ fixed, we have

$$\widehat{c}_1(\theta) - \overline{c}_1(\theta) = \overline{k}_1(\theta)\Delta D - D(\widehat{x}(\theta) - \overline{x}(\theta))$$

and thus

$$p(\widehat{x}(\theta) - \overline{x}(\theta)) = \frac{p}{D}\left[\overline{k}_1(\theta)\Delta D - (\widehat{c}_1(\theta) - \overline{c}_1(\theta))\right].$$

Substituting in (59), we obtain

$$0 = \overline{x}(\theta)\Delta p + \frac{p}{D}\overline{k}_1(\theta)\Delta D - \left[\Delta\widehat{c}_0(\theta) + \frac{p}{D}\Delta\widehat{c}_1(\theta)\right].$$

C Appendix for Section 4

C.1 Second-best problem for alternative tax instruments

A tax on c_0. Consider first a tax on period-0 consumption. This means that pre-tax consumption in period 0, given by $z_0(\theta) \equiv px(\theta) + y_0(\theta)$, is observable, so after-tax consumption is $c_0(\theta) = z_0(\theta) - T_0(z_0(\theta))$, where $T_0(z_0)$ is the nonlinear consumption tax in $t = 0$. Hence,

$$x(\theta) = \frac{z_0(\theta) - y_0(\theta)}{p}$$

and we can write the global incentive constraints as

$$\begin{aligned}\mathcal{U}(\theta) \equiv U\left(c_0(\theta), D\left(k_0(\theta) - \frac{z_0(\theta) - y_0(\theta)}{p}\right) + y_1(\theta)\right)\\ \geq U\left(c_0(\hat{\theta}), D\left(k_0(\theta) - \frac{z_0(\hat{\theta}) - y_0(\theta)}{p}\right) + y_1(\theta)\right) \quad \forall\theta, \hat{\theta}.\end{aligned}$$

The local incentive constraints are therefore given by (26) with $A(\theta) = 0$ and

$$B(\theta) = Dk_0'(\theta) + \frac{D}{p}y_0'(\theta) + y_1'(\theta).$$

A tax on c_1. Consider next a tax on period-1 consumption. Pre-tax consumption in period 1 is $z_1(\theta) \equiv D(k_0(\theta) - x(\theta)) + y_1(\theta)$ and after-tax consumption is $c_1(\theta) = z_1(\theta) - T_1(z_1(\theta))$, where $T_1(z_1)$ is the nonlinear consumption tax in $t = 1$. Hence,

$$x(\theta) = \frac{y_1(\theta) - z_1(\theta)}{D} + k_0(\theta)$$

and we can write the global incentive constraints as

$$\begin{aligned} \mathcal{U}(\theta) &\equiv U\left(\frac{p}{D}(y_1(\theta) - z_1(\theta)) + pk_0(\theta) + y_0(\theta), c_1(\theta)\right) \\ &\geq U\left(\frac{p}{D}(y_1(\theta) - z_1(\hat{\theta})) + pk_0(\theta) + y_0(\theta), c_1(\hat{\theta})\right) \quad \forall \theta, \hat{\theta}. \end{aligned}$$

The local incentive constraints are therefore again given by (26) but with

$$A(\theta) = pk_0'(\theta) + y_0'(\theta) + \frac{p}{D}y_1'(\theta)$$

and $B(\theta) = 0$.

Further tax instruments. More generally, for any tax instrument conditioning on some observable choices, we can decompose consumption in each period $t = 0, 1$ into its observable and its unobservable components: $c_t(\theta) = c_t^o(\theta) + c_t^u(\theta)$. For instance, with an assets sales tax, the observable components are $c_0^o(\theta) = z_x(\theta)$ in period 0 and $c_1^o(\theta) = -Dx(\theta)$ in period 1, whereas the unobservable components are $c_0^u(\theta) = y_0(\theta)$ and $c_1^u(\theta) = Dk_0(\theta) + y_1(\theta)$. Hence, the general incentive constraint can be written as

$$\begin{aligned} \mathcal{U}(\theta) &\equiv U\left(c_0^o(\theta) + c_0^u(\theta), c_1^o(\theta) + c_1^u(\theta)\right) \\ &\geq U\left(c_0^o(\hat{\theta}) + c_0^u(\theta), c_1^o(\hat{\theta}) + c_1^u(\theta)\right) \quad \forall \theta, \hat{\theta}. \end{aligned}$$

The local incentive constraints are therefore always given by (26) with $A(\theta) = c_0^{u\prime}(\theta)$ and $B(\theta) = c_1^{u\prime}(\theta)$. Note that this general approach also allows for combinations of the tax instruments considered so far. For example, suppose there is both an asset sales tax in period 0 and a wealth tax in period 1. Then $c_0^o(\theta) = z_x(\theta)$ and $c_1^o(\theta) = D(k_0(\theta) - x(\theta))$ while $c_0^u(\theta) = y_0(\theta)$ and $c_1^u(\theta) = y_1(\theta)$, so we obtain $A(\theta) = y_0'(\theta)$ and $B(\theta) = y_1'(\theta)$.[39]

C.2 Solving the general second-best problem

For any preferences $U(c_0, c_1) = G(\mathcal{C}(c_0, c_1))$ and any of the tax instruments considered, we can write the second-best Pareto problem as

$$\max_{\{c_0(\theta), c_1(\theta), V(\theta)\}} \int \omega(\theta) G(V(\theta)) dF(\theta)$$

subject to the incentive constraints

$$V'(\theta) = \mathcal{C}_{c_0}(c_0(\theta), c_1(\theta)) A(\theta) + \mathcal{C}_{c_1}(c_0(\theta), c_1(\theta)) B(\theta) \quad \forall \theta \tag{60}$$

[39] In this case, the asset sales tax and the wealth tax are not separately determined, but the optimal consumption allocation is.

where $\mathcal{C}_{c_t} \equiv \partial \mathcal{C}/\partial c_t$, the resource constraint

$$Y \geq \int \left(c_0(\theta) + \frac{p}{D} c_1(\theta) \right) dF(\theta) \tag{61}$$

with

$$Y \equiv pK_0 + Y_0 + \frac{p}{D} Y_1,$$

and

$$V(\theta) = \mathcal{C}(c_0(\theta), c_1(\theta)) \ \ \forall \theta.$$

It is useful to substitute out $c_0(\theta) = \Phi(V(\theta), c_1(\theta))$ where $\Phi(., c_1)$ is the inverse function of $\mathcal{C}(., c_1)$ with respect to its first argument. This allows us to write the maximization problem in terms of $V(\theta)$ and $c_1(\theta)$ only. Attaching multipliers $\mu(\theta)$ to the incentive constraint for type θ and η to the resource constraint, the corresponding Lagrangian becomes, after integrating by parts,

$$\begin{aligned}
\mathcal{L} &= \int \omega(\theta) G(V(\theta)) dF(\theta) - \int \mu'(\theta) V(\theta) d\theta \\
&\quad - \int \mu(\theta) \left[\mathcal{C}_0(\Phi(V(\theta), c_1(\theta)), c_1(\theta)) A(\theta) + \mathcal{C}_1(\Phi(V(\theta), c_1(\theta)), c_1(\theta)) B(\theta) \right] d\theta \\
&\quad - \eta \int \left[\Phi(V(\theta), c_1(\theta)) + \frac{p}{D} c_1(\theta) \right] dF(\theta).
\end{aligned}$$

Using the fact that

$$\frac{\partial \Phi}{\partial V} = \frac{1}{\mathcal{C}_0} \quad \text{and} \quad \frac{\partial \Phi}{\partial c_1} = -\frac{\mathcal{C}_1}{\mathcal{C}_0}$$

and dropping arguments to simplify notation, the first-order condition for $c_1(\theta)$ is

$$\mu \left[\left(\frac{\mathcal{C}_{c_0 c_0} \mathcal{C}_{c_1}}{\mathcal{C}_{c_0}} - \mathcal{C}_{c_0 c_1} \right) A + \left(\frac{\mathcal{C}_{c_0 c_1} \mathcal{C}_{c_1}}{\mathcal{C}_{c_0}} - \mathcal{C}_{c_1 c_1} \right) B \right] = \eta f \left[\frac{p}{D} - \frac{\mathcal{C}_{c_1}}{\mathcal{C}_{c_0}} \right] \tag{62}$$

and for $V(\theta)$

$$\omega f G'(V) = \mu' + \mu \left[\frac{\mathcal{C}_{c_0 c_0}}{\mathcal{C}_{c_0}} A + \frac{\mathcal{C}_{c_0 c_1}}{\mathcal{C}_{c_0}} B \right] + \frac{\eta f}{\mathcal{C}_{c_0}} \tag{63}$$

where $\mathcal{C}_{c_s c_t}$ denotes the second derivates $\partial^2 \mathcal{C}/(\partial c_s \partial c_t)$. Together with the incentive constraints (60), the resource constraint (61) and the boundary conditions $\mu(\underline{\theta}) = \mu(\bar{\theta}) = 0$, equations (62) and (63) determine the optimal solution $\{V(\theta), c_1(\theta), \mu(\theta), \eta\}$.

C.3 CES utility and numerical algorithm

Under the CES preferences given in equation (27), it turns out to be convenient to work with

$$\xi(\theta) \equiv \frac{c_0(\theta)}{c_1(\theta)}.$$

Then the first-order conditions (62) and (63) can be written as

$$\frac{\mu(\theta)}{\sigma c_1(\theta)} \left(\xi(\theta)^{\frac{\sigma-1}{\sigma}} + \beta \right)^{\frac{1}{\sigma-1}} \left(B(\theta) - A(\theta)/\xi(\theta) \right) = \eta f(\theta) \left(\frac{p}{\beta D} - \xi(\theta)^{\frac{1}{\sigma}} \right) \tag{64}$$

$$\omega(\theta) f(\theta) G'(V(\theta)) = \mu'(\theta) + \frac{\beta \mu(\theta)}{\sigma c_1(\theta)} \frac{B(\theta) - A(\theta)/\xi(\theta)}{\xi(\theta)^{\frac{\sigma-1}{\sigma}} + \beta} + \eta f(\theta) \left(1 + \beta \xi(\theta)^{\frac{1-\sigma}{\sigma}} \right)^{\frac{1}{1-\sigma}}. \tag{65}$$

Moreover, the incentive constraints (60) become

$$V'(\theta) = \left(\xi(\theta)^{\frac{\sigma-1}{\sigma}} + \beta\right)^{\frac{1}{\sigma-1}} \left(\xi(\theta)^{-\frac{1}{\sigma}} A(\theta) + \beta B(\theta)\right) \tag{66}$$

and, by definition of CES utility,

$$V(\theta) = c_1(\theta) \left(\xi(\theta)^{\frac{\sigma-1}{\sigma}} + \beta\right)^{\frac{\sigma}{\sigma-1}}. \tag{67}$$

We first use (64) together with (67) to numerically solve for $\xi(\theta)$ as a function of $\mu(\theta)$, $V(\theta)$ and η. Substituting this in (65) and (66) delivers a system of two ordinary differential equations in $\mu(\theta)$ and $V(\theta)$ that we can solve, given any η, using the boundary conditions $\mu(\underline{\theta}) = \mu(\overline{\theta}) = 0$. Finally, we find η such that the resource constraint (61) is satisfied, noting that $c_0(\theta) = \xi(\theta)c_1(\theta)$.

C.4 Proof of Proposition 3

We establish the result in a series of steps. We first characterize the first-best allocation $\Gamma^*(\sigma)$ under preferences (27).

Lemma 5. *With preferences (27), we have*

$$\Omega(\theta) = \frac{\omega(\theta)^{\frac{1}{\gamma}}}{\int \omega(\theta')^{\frac{1}{\gamma}} dF(\theta')}.$$

Proof. This is a special case of Lemma 1 with $G'(x) = x^{-\gamma}$ and $\rho = 1$. We then have

$$\omega(\theta)\Omega(\theta)^{-\gamma}\bar{H}^{-\gamma} = \bar{\Omega}.$$

Solving this for $\Omega(\theta)$ yields

$$\Omega(\theta) = f(\omega(\theta), \bar{\Omega}) = \left(\frac{\omega(\theta)}{\bar{\Omega}}\right)^{\frac{1}{\gamma}} \bar{H}^{-1}.$$

Integrating, we have

$$\bar{\Omega} = \left(\int \omega(\theta')^{\frac{1}{\gamma}} dF(\theta')\right)^{\gamma} \bar{H}^{-\gamma},$$

which delivers the desired expression. □

The next lemma characterizes the first-best allocation for any $\sigma \geq 0$:

Lemma 6. *The first-best allocation $\Gamma^*(\sigma)$ for $\sigma \geq 0$ is*

$$c_0^*(\theta, \sigma) = \Omega(\theta) \left(1 + R^{-1}(\beta R)^{\sigma}\right)^{-1} \left(Y_0 + R^{-1}Y_1 + pK_0\right)$$
$$c_1^*(\theta, \sigma) = (\beta R)^{\sigma} c_0^*(\theta, \sigma),$$

where $R \equiv D/p$ is the interest rate.

Proof. For $\sigma > 0$, the first-order conditions for the first-best allocation are (dropping σ for notational simplicity):

$$\omega(\theta)U_{c_0}(\theta) = \omega(\theta)V(\theta)^{-\gamma}\left(\frac{c_0(\theta)}{V(\theta)}\right)^{-\frac{1}{\sigma}} = \eta$$

$$\omega(\theta)U_{c_1}(\theta) = \omega(\theta)\beta V(\theta)^{-\gamma}\left(\frac{c_1(\theta)}{V(\theta)}\right)^{-\frac{1}{\sigma}} = R^{-1}\eta,$$

where $\eta > 0$ is the multiplier on the resource constraint and $V(\theta) = \mathcal{C}(c_0(\theta), c_1(\theta))$. Taking the ratio and rearranging, we have:

$$c_1(\theta) = (\beta R)^{\sigma}\, c_0(\theta).$$

Substituting into $\mathcal{C}$, we obtain:

$$\begin{aligned} V(\theta) &= \left(c_0(\theta)^{\frac{\sigma-1}{\sigma}} + \beta\left[(\beta R)^{\sigma} c_0(\theta)\right]^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(1+\beta\,(\beta R)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} c_0(\theta). \end{aligned}$$

The first-order condition for c_0 then becomes (after rearranging):

$$c_0(\theta) = \eta^{-\frac{1}{\gamma}}\omega(\theta)^{\frac{1}{\gamma}}\left(1+\beta\,(\beta R)^{\sigma-1}\right)^{\frac{1/\gamma-\sigma}{\sigma-1}}.$$

We can eliminate $\eta^{-\frac{1}{\gamma}}$ using the resource condition (19), Lemma 1 and Lemma 5 to obtain:

$$c_0(\theta) = \Omega(\theta)\left(1+R^{-1}(\beta R)^{\sigma}\right)^{-1}\left(Y_0 + R^{-1}Y_1 + pK_0\right).$$

For $\sigma = 0$, we have $\mathcal{C}(c_0, c_1) = \min\{c_0, c_1\}$. The first-best will set $c_0 = c_1$, and hence solves the problem

$$\max_{c(\theta)} \int \omega(\theta)\frac{c(\theta)^{1-\gamma}}{1-\gamma}dF(\theta),$$

subject to

$$\int c(\theta)dF(\theta) = (1+R^{-1})^{-1}\left(Y_0 + R^{-1}Y_1 + pK_0\right).$$

The first-order condition is $\omega(\theta)c(\theta)^{-\gamma} = \eta$, which, after substituting in the resource constraint, yields the proposed outcome with σ set to zero. □

Lemma 6 implies that the first-best allocation $\Gamma^*(\sigma)$ is continuous in σ. Hence, in the neighborhood of $\sigma = 0$, the associated first-best allocations $\Gamma^*(\sigma)$ are also in the neighborhood of the $\sigma = 0$ first-best allocation $\Gamma^*(0)$. If our second-best allocation $\Gamma^M(\sigma)$ converges to $\Gamma^*(0)$ as $\sigma \to 0$, this means that it also converges to $\Gamma^*(\sigma)$.

The second-best allocation $\Gamma^M(\sigma)$ also needs to satisfy the incentive constraints (26) (in addition to the resource constraint (19)). With preferences (27), the incentive constraints simplify to (66). The next lemma shows that Assumption 1(i) is needed for the first-best allocation $\Gamma^*(0)$ to be incentive compatible.

Lemma 7. *If $\Gamma^*(0)$ is incentive compatible for $\sigma = 0$, it satisfies Assumption 1 (i).*

Proof. As $\sigma \to 0$, (66) can be written as

$$V'(\theta) = \frac{\xi(\theta)^{-\frac{1}{\sigma}}A(\theta) + \beta B(\theta)}{\xi(\theta)^{-\frac{1}{\sigma}} + \beta}.$$

Hence, $V'(\theta)$ must be a convex combination of $A(\theta)$ and $B(\theta)$. Moreover, when $\sigma = 0$, $V(\theta) = c_0(\theta) = c_1(\theta) \equiv c(\theta)$. $\square$

For general $\sigma \geq 0$, consider allocations that take the following form:

$$\begin{aligned} c_0(\theta, \sigma) &= e^{\sigma g(\theta)} c(\theta, \sigma) \\ c_1(\theta, \sigma) &= c(\theta, \sigma), \end{aligned} \tag{68}$$

where $c(\theta, \sigma)$ is the solution to the following linear ODE:

$$c'(\theta, \sigma) = q(\theta, \sigma) - \sigma p(\theta, \sigma) c(\theta, \sigma), \tag{69}$$

where

$$\begin{aligned} q(\theta, \sigma) &\equiv \frac{A(\theta) + \beta e^{g(\theta)} B(\theta)}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}} \\ p(\theta, \sigma) &\equiv \frac{g'(\theta) e^{\sigma g(\theta)}}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}}. \end{aligned}$$

Note that Assumption 1(ii) ensures p exists and is bounded. Also note that

$$q(\theta, \sigma) = \left(\frac{1 + \beta e^{g(\theta)}}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}} \right) c^{*\prime}(\theta). \tag{70}$$

We next establish that the proposed allocation is incentive compatible:

Lemma 8. *The proposed allocation (68) where $c(\theta, \sigma)$ solves (69) satisfies (66).*

Proof. An allocation $\{c_0(\theta), c_1(\theta)\}$ satisfies (66) iff

$$c_0'(\theta) - A(\theta) + \beta \left(\frac{c_0(\theta)}{c_1(\theta)} \right)^{\frac{1}{\sigma}} \left(c_1'(\theta) - B(\theta) \right) = 0. \tag{71}$$

To see that the proposed allocation satisfies (71), note that

$$\begin{aligned} c_0'(\theta, \sigma) &= e^{\sigma g(\theta)} \left[\sigma g'(\theta) c(\theta) + c'(\theta, \sigma) \right] \\ c_1'(\theta, \sigma) &= c'(\theta, \sigma). \end{aligned}$$

Substituting into (71), we have

$$\begin{aligned} & e^{\sigma g(\theta)} \left[\sigma g'(\theta) c(\theta, \sigma) + c'(\theta, \sigma) \right] - A(\theta) + \beta e^{g(\theta)} \left(c'(\theta, \sigma) - B(\theta) \right) \\ &= (e^{\sigma g(\theta)} + \beta e^{g(\theta)}) c'(\theta, \sigma) - A(\theta) - \beta e^{g(\theta)} B(\theta) + \sigma g'(\theta) e^{\sigma g(\theta)} c(\theta, \sigma) \\ &= (e^{\sigma g(\theta)} + \beta e^{g(\theta)}) \left[c'(\theta, \sigma) - q(\theta, \sigma) + \sigma p(\theta, \sigma) \right]. \end{aligned}$$

Setting this equal to zero yields (69). $\square$

The proposed allocation satisfies the resource constraint if

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(e^{\sigma g(\theta)} + R^{-1} \right) c(\theta, \sigma) dF(\theta) = Y_0 + R^{-1} Y_1 + p K_0. \tag{72}$$

Note that (69) is a linear first-order ODE. Hence, it can be solved in closed form up to a boundary condition $c(\underline{\theta}, \sigma)$. In particular, let $P(\theta; \sigma) \equiv \int_{\underline{\theta}}^{\theta} p(\hat{\theta}, \sigma) d\hat{\theta}$. From Assumption 1(ii), P is bounded. Then,

$$c(\theta, \sigma) = e^{-\sigma P(\theta, \sigma)} \left(\int_{\underline{\theta}}^{\theta} e^{\sigma P(\hat{\theta}, \sigma)} q(\hat{\theta}, \sigma) d\hat{\theta} + c(\underline{\theta}, \sigma) \right). \tag{73}$$

Substituting (73) into the above, the resource condition uniquely pins down $c(\underline{\theta}, \sigma)$.

The final restriction on the proposed allocation is that it is weakly positive. We will verify this in the neighborhood of $\sigma = 0$ below.

Thus a weakly positive solution to (69) that satisfies (72) is feasible and incentive compatible. We show that such a sequence of solutions converges uniformly to the first-best.

We start with:

Lemma 9. *Let c solve (69). Then $c'(\theta, \sigma)$ converges uniformly to $c^{*\prime}(\theta)$ as $\sigma \to 0$.*

Proof. From (70), we have

$$c'(\theta, \sigma) - c^{*\prime}(\theta) = \left[\left(\frac{1 + \beta e^{g(\theta)}}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}} \right) - 1 \right] c^{*\prime}(\theta).$$

Letting $\| \|$ denote the sup norm over θ, this implies

$$\|c'(\theta, \sigma) - c^{*\prime}(\theta)\| = \left\| \frac{1 - e^{\sigma g(\theta)}}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}} c^{*\prime}(\theta) \right\| \leq \left\| 1 - e^{\sigma g(\theta)} \right\| * \left\| \frac{c^{*\prime}(\theta)}{e^{\sigma g(\theta)} + \beta e^{g(\theta)}} \right\|.$$

As the final term on the far right is bounded, we need to show the first term in the far right expression converges to zero. To see this, note that

$$\left\| 1 - e^{\sigma g(\theta)} \right\| \leq e^{\sigma \|g(\theta)\|} \left(1 - e^{-\sigma \|g(\theta)\|} \right) \to 0.$$

$\square$

A corollary of this lemma is that

$$\begin{aligned}
\|c(\theta, \sigma) - c^*(\theta)\| &= \left\| \int_{\underline{\theta}}^{\theta} (c'(\hat{\theta}, \sigma) - c^{*\prime}(\hat{\theta})) d\hat{\theta} + c(\underline{\theta}, \sigma) - c^*(\underline{\theta}) \right\| \\
&\leq \left\| \int_{\underline{\theta}}^{\theta} (c'(\hat{\theta}, \sigma) - c^{*\prime}(\hat{\theta})) d\hat{\theta} \right\| + |c(\underline{\theta}, \sigma) - c^*(\underline{\theta})| \\
&\leq \|c'(\theta, \sigma) - c^{*\prime}(\theta, \sigma)\| (\overline{\theta} - \underline{\theta}) + |c(\underline{c}, \sigma) - c^*(\underline{\theta})| \to |c(\underline{\theta}, \sigma) - c^*(\underline{\theta})|.
\end{aligned}$$

Hence to show uniform convergence of $c(\theta, \sigma)$ to $c^*(\theta)$ we need to show that $c(\underline{\theta}, \sigma) \to c^*(\underline{\theta})$. This follows from the fact that the resource condition requires that

$$\int_{\underline{\theta}}^{\overline{\theta}} c(\theta, \sigma) dF = \int_{\underline{\theta}}^{\overline{\theta}} c^*(\theta) dF.$$

This is true for all σ. Using the Fundamental Theorem of Calculus, we can write $c(\theta, \sigma) = c(\underline{\theta}, \sigma) + \int_{\underline{\theta}}^{\theta} c'(\theta', \sigma) d\theta'$ and similarly for $c^*(\theta)$. Thus, the resource constraint can be written as

$$c(\underline{\theta}, \sigma) = c^*(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} (c^{*\prime}(\theta') - c(\theta', \sigma)) d\theta' dF.$$

We showed above that the last term on the right converges to zero as $\sigma \to 0$. Hence, $c(\underline{\theta}, \sigma) \to c^*(\underline{\theta})$. As $c(\theta, \sigma) \to c^*(\theta)$ uniformly, and $c^*(\theta) > 0$ for all θ, this also establishes that $c(\theta, \sigma) \geq 0$ in a neighborhood of $\sigma = 0$.

In sum, we have shown that there exists a sequence of feasible, incentive compatible allocations that converge uniformly to $\Gamma^*(0)$ as $\sigma \to 0$. Continuity of $\Gamma^*(\sigma)$ shown in Lemma 6 implies that this sequence of allocations also converges uniformly to $\Gamma^*(\sigma)$ as $\sigma \to 0$. Finally, the second-best optimum $\Gamma^M(\sigma)$ must also be feasible and incentive compatible but achieve at least weakly higher welfare than the proposed allocation. Since $\Gamma^*(\sigma)$ uniquely achieves the highest welfare among all feasible allocations, we must also have $\Gamma^M(\sigma) \to \Gamma^*(\sigma)$ as $\sigma \to 0$, which is the result in Proposition 3.

C.5 Proof of Proposition 4

We first characterize the efficient portfolio choice. Without taxes, the investor solves

$$\max_{x,b} U(px - qb - \chi(x) + y_0, b + D(k_0 - x) + y_1)$$

with FOCs

$$U_0(p - \chi') = U_1 D$$

$$U_0 q = U_1.$$

Dividing yields

$$p - \chi'(x) = qD. \tag{74}$$

It is straightforward to see that the same efficiency condition would apply in the case of lump-sum taxes, a tax on net trades $z = px - qb - \chi(x)$ in period 0, or on net trades $b - Dx$ in period 1.

Turning to the Mirrlees problem with two assets, suppose both x and b are observable, so a tax $T(x, b)$ is feasible. This means that $px(\theta) - qb(\theta) - \chi(x(\theta))$ is observable. Like in Appendix C.1, we can write the allocation in terms of the observable and unobservable components of consumption, with

$$c_0^o(\theta) = px(\theta) - qb(\theta) - \chi(x(\theta)) - T(x(\theta), b(\theta))$$

$$c_1^o(\theta) = b(\theta) - Dx(\theta)$$

and $c_0^u(\theta) = y_0(\theta)$, $c_1^u(\theta) = Dk_0(\theta) + y_1(\theta)$. Thus, the incentive constraint is

$$V(\theta) \equiv U(c_0^o(\theta) + c_0^u(\theta), c_1^o(\theta) + c_1^u(\theta)) \geq U(c_0^o(\theta') + c_0^u(\theta), c_1^o(\theta') + c_1^u(\theta)) \;\; \forall \theta, \theta'$$

with the usual envelope condition

$$V'(\theta) = U_0(c_0(\theta), c_1(\theta)) c_0^{u\prime}(\theta) + U_1(c_0(\theta), c_1(\theta)) c_1^{u\prime}(\theta). \tag{75}$$

We can thus write the planning problem as

$$\max_{\{c_0(\theta), c_1(\theta), V(\theta)\}} \int \omega(\theta) V(\theta) dF(\theta)$$

subject to

$$V(\theta) = U(c_0(\theta), c_1(\theta)),$$

the local incentive constraint (75), and the resource constraint

$$\int c_0(\theta)dF(\theta) + q\int c_1(\theta)dF(\theta) = Y$$

with

$$Y \equiv \int y_0(\theta)dF(\theta) + q\int [Dk_0(\theta) + y_1(\theta)]dF(\theta) + \max_{\{x(\theta)\}} \int [px(\theta) - \chi(x(\theta)) - qDx(\theta)]dF(\theta)$$

It is clear from this formulation that any second-best optimum also prescribes the efficient portfolio choice given by (74) for each investor.

Now suppose that what we can observe is only the net trade $z \equiv px - qb - \chi(x)$ in period 0, and thus we impose a tax $T(z)$ (a tax on the "net trade" in period 1, $b - Dx$, is equivalent). We know from above that this implements the efficient portfolio choice, which pins down $x(\theta)$. Since we know $x(\theta)$ and $z(\theta)$, we then also know $b(\theta)$. Thus, a tax $T(z)$ is equivalent to a tax $T(x, b)$.

D Appendix for Section 5

D.1 Proof of Lemma 3

In the decentralized equilibrium, the individual's problem is

$$\begin{aligned} \max U(\{c_0(\theta, s_0), c_1(\theta, s^1)\}) \quad \text{s.t.} \\ c_0(\theta, s_0) = y_0(\theta) + p(s_0)x(\theta, s_0) - q(s_0)b(\theta, s_0) - T_0(\theta, s_0) \quad \forall s_0 \\ c_1(\theta, s^1) = b(s_0) + D(s^1)(k_0(\theta) - x(\theta, s_0)) - T_1(\theta, s^1) \quad \forall s^1. \end{aligned}$$

Eliminating b, we can write the budget set as present-value constraints (suppressing θ):

$$c_0(s_0) + q(s_0)c_1(s^1) = y_0 + q(s_0)y_1 + p(s_0)x(s_0) + q(s_0)D(s^1)(k_0 - x(s_0)) - T_0(s_0) - q(s_0)T_1(s^1)\ \forall s^1.$$

The first-order conditions for this problem take the same form as the planning problem, so the first-best allocation satisfies the individual's problem as long as it satisfies the budget set. For this, we need to find a tax scheme $\{T_0(\theta, s_0), T_1(\theta, s^1)\}$ and asset positions $\{b(s_0), x(s_0)\}$ such that for all θ and s^1:

$$\begin{aligned} \Omega(\theta)C_0^*(s_0) = y_0(\theta) + p(s_0)x(\theta, s_0) - q(s_0)b(\theta, s_0) - T_0(\theta, s_0) \\ \Omega(\theta)C_1^*(s^1) = y_1(\theta) + D(s^1)(k_0(\theta) - x(\theta, s_0)) + b(\theta, s_0) - T_1(\theta, s^1) \end{aligned}$$

where we used Lemma 1. Using $T_1(\theta, s^1) = \alpha(\theta, s_0) + \gamma(\theta, s_0)D(s^1)$ and the aggregate resource constraint in history s^1, we have:

$$\Omega(\theta)\left(D(s^1)K_1^*(s_0) + Y_1 + B^*(s_0)\right) = y_1(\theta) + D(s^1)(k_0(\theta) - x(\theta, s_0)) + b(\theta, s_0) - \alpha(\theta, s_0) - \gamma(\theta, s_0)D(s^1).$$

Set

$$\begin{aligned} x(\theta, s_0) &= -\gamma(\theta, s_0) - \Omega(\theta)K_1^*(s_0) + k_0(\theta) \\ b(\theta, s_0) &= \Omega(\theta)(Y_1 + B^*(s_0)) - y_1(\theta) + \alpha(\theta, s_0), \end{aligned}$$

and the second-period budget constraint is satisfied for all s^1. Setting

$$T_0(\theta, s_0) = -\Omega(\theta)C_0^*(s_0) + y_0(\theta) + p(s_0)x(\theta, s_0) - q(s_0)b(\theta, s_0),$$

the first-period budget constraint is satisfied, as well. Moreover, we have $\int T_0(\theta, s_0) dF(\theta) = 0$ for all s_0. Hence, the tax system along with the proposed policies $\{b, x\}$ ensures that the household's necessary and sufficient conditions are satisfied evaluated at the first-best allocation.

D.2 Proof of Proposition 5

To simplify notation, we write the prices associated with the reference state $\bar{s}_0$ as $\overline{p}$ and $\overline{q}$ and analogously for the respective allocations. Similary, let p and q denote the prices under the new shock, suppressing s_0, and again we use the same convention for the allocations. From the aggregate resource condition:

$$\Delta C_0 = pX - \overline{p}\overline{X} - qB + \overline{q}\overline{B} = X\Delta p + \overline{p}\Delta X - B\Delta q - \overline{q}\Delta B,$$

and

$$\Delta C_1(s^1) = D(s^1)(K_0 - X) + B - \overline{D}(s^1)(K_0 - \overline{X}) - \overline{B} = (K_0 - X)\Delta D(s^1) + \overline{D}(s^1)\Delta X + \Delta B$$

where $\overline{D}(s^1)$ stands short for $D(\bar{s}_0, s_1)$. By Lemma 1, the change in the first-best allocation is

$$\Delta c_0(\theta) = \Omega(\theta)\Delta C_0$$
$$\Delta c_1(\theta, s^1) = \Omega(\theta)\Delta C_1(s^1).$$

We now show that the first-best allocation is affordable for each θ. Let individual θ alter first-period saving by:

$$\Delta b(\theta) = \Omega(\theta)\Delta B,$$

and capital sales:

$$\Delta x(\theta) = \Omega(\theta)\Delta X.$$

If the first-best allocation is attainable with these portfolio choices and the proposed lump-sum transfers, they are consistent with individual optimization. This follows from the fact that the first-best allocation optimizes the distribution of consumption across time and states for each θ given the budget set.

The proposed tax change for period zero is:

$$\Delta T_0(\theta) = x(\theta)\Delta p - b(\theta)\Delta q - \Omega(\theta)(X\Delta p - B\Delta q).$$

Then, under the proposed tax policy:

$$\begin{aligned}
\Delta c_0(\theta) &= px(\theta) - \overline{px}(\theta) - qb + \overline{q}\overline{b} - \Delta T_0(\theta) \\
&= x(\theta)\Delta p + \overline{p}\Delta x(\theta) - b(\theta)\Delta q - \overline{q}\Delta b(\theta) - \Delta T_0(\theta) \\
&= x(\theta)\Delta p - b(\theta)\Delta q + \overline{p}\Omega(\theta)\Delta X - \overline{q}\Omega(\theta)\Delta B - [x(\theta)\Delta p - b(\theta)\Delta q - \Omega(\theta)(X\Delta p - B\Delta q)] \\
&= \Omega(\theta)\left(X\Delta p - B\Delta q - \overline{q}\Delta B + \overline{p}\Delta X\right) \\
&= \Omega(\theta)\Delta C_0.
\end{aligned}$$

For the second period in state s^1, the proposed tax change is:

$$\Delta T_1(\theta, s^1) = (k_0(\theta) - x(\theta))\Delta D(s^1) - \Omega(\theta)((K_0 - X)\Delta D(s^1)).$$

The change in the budget constraint in the second period becomes:

$$
\begin{aligned}
\Delta c_1(\theta, s^1) &= D(s^1)(k_0(\theta) - x(\theta)) - \overline{D}(s^1)(k_0(\theta) - \overline{x}(\theta)) + \Delta b - \Delta T_1(\theta, s^1) \\
&= (k_0(\theta) - x(\theta))\Delta D(s^1) - \overline{D}(s^1)\Delta x(\theta) + \Delta b - \Delta T_1(\theta, s^1) \\
&= (k_0(\theta) - x(\theta))\Delta D(s^1) - \overline{D}(s^1)\Omega(\theta)\Delta X + \Omega(\theta)\Delta B \\
&\quad - \Big[(k_0(\theta) - x(\theta))\Delta D(s^1) - \Omega(\theta)((K_0 - X)\Delta D(s^1))\Big] \\
&= \Omega(\theta)\left((K_0 - X)\Delta D(s^1) - \overline{D}(s^1)\Delta X + \Delta B\right) \\
&= \Omega(\theta)C_1(\theta, s^1).
\end{aligned}
$$

Hence, the proposed taxes allow each θ to afford the change in the first-best allocation.

D.3 Proof of Corollary 4

$\Delta m(s^1) = 0, \forall s^1$ implies that

$$
\Delta p = \sum_{s_1} \pi(s_1)\overline{m}(s^1)\Delta D(s^1) \quad \text{and} \quad \Delta q = 0
$$

where $\overline{m}(s^1)$ stands short for $m(\overline{s}_0, s_1)$. Substituting into Proposition 5, we have

$$
\begin{aligned}
T_0(\theta) &= \overline{T}_0(\theta) + x(\theta)\Delta p - \Omega(\theta)X\Delta p \\
T_1(\theta, s^1) &= \overline{T}_1(\theta, s^1) + (k_0(\theta) - x(\theta))\Delta D(s^1) - \Omega(\theta)\left(K_0 - X\right)\Delta D(s^1).
\end{aligned}
$$

We can exploit Lemma 3 and consider an alternative tax system, as stated in the corollary:

$$
\begin{aligned}
\widehat{T}_0(\theta) &= \overline{T}_0(\theta) + k_0(\theta)\Delta p - \Omega(\theta)K_0\Delta p \\
\widehat{T}_1(\theta, s^1) &= \overline{T}_1(\theta, s^1).
\end{aligned}
$$

In the latter, relative to the former, investor θ pays $(k_1(\theta) - \Omega(\theta)K_1)\Delta p$ more in taxes in the first-period. Instead, she pays $(k_1(\theta) - \Omega(\theta)K_1)\Delta D(s^1)$ less in taxes in period 1 in every state s. Reducing its investment in risky capital by the amount of extra taxes in period 0, the investor perfectly undoes the tax changes in both periods and each state of the world. Hence, the budget sets are invariant to this transformation, and either allows the investor to afford the first-best allocation.

D.4 Proof of Proposition 6

An investor's sequential budget constraint under history s^t is

$$
T_t(\theta, s^t) = y_t(\theta) + \left(D_t(s^t) + p_t(s^t)\right)k_t(\theta, s^{t-1}) - p_t(s^t)k_{t+1}(\theta, s^t) + b_t(\theta, s^{t-1}) - q_t(s^t)b_{t+1}(\theta, s^t) - c_t(\theta, s^t)
$$

and under the reference history $\overline{s}^t$

$$
T_t(\theta, \overline{s}^t) = y_t(\theta) + \left(D_t(\overline{s}^t) + p_t(\overline{s}^t)\right)k_t(\theta, \overline{s}^{t-1}) - p_t(\overline{s}^t)k_{t+1}(\theta, \overline{s}^t) + b_t(\theta, \overline{s}^{t-1}) - q_t(\overline{s}^t)b_{t+1}(\theta, \overline{s}^t) - c_t(\theta, \overline{s}^t).
$$

Subtracting one from the other yields:

$$\begin{aligned}\Delta T_t(\theta, s^t, \bar{s}^t) &= k_t(\theta, s^{t-1})\Delta D_t(s^t, \bar{s}^t) + x_t(\theta, s^t)\Delta p_t(s^t, \bar{s}^t) - b_{t+1}(\theta, s^t)\Delta q_t(s^t, \bar{s}^t) - \Delta c_t(\theta, s^t, \bar{s}^t) \\ &\quad + \left(D_t(\bar{s}^t) + p_t(\bar{s}^t)\right)\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + \Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1}) \\ &\quad - p_t(\bar{s}^t)\Delta k_{t+1}(\theta, s^t, \bar{s}^t) - q_t(\bar{s}^t)\Delta b_{t+1}(\theta, s^t, \bar{s}^t).\end{aligned} \tag{76}$$

Multiplying the second line by $\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)$ and summing over t and $\bar{s}^t$ yields

$$\begin{aligned}&\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\left[\left(D_t(\bar{s}^t) + p_t(\bar{s}^t)\right)\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + \Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right] \\ =\ &\sum_{t=1}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\left[\left(D_t(\bar{s}^t) + p_t(\bar{s}^t)\right)\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + \Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right] \\ =\ &\sum_{t=1}^{T}\sum_{\bar{s}^{t-1}}\sum_{\bar{s}_t}\pi(\bar{s}^{t-1}, \bar{s}_t)m_{0\to t}(\bar{s}^{t-1}, \bar{s}_t)\left[\left(D_t(\bar{s}^{t-1}, \bar{s}_t) + p_t(\bar{s}^{t-1}, \bar{s}_t)\right)\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + \Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right] \\ =\ &\sum_{t=1}^{T}\sum_{\bar{s}^{t-1}}\pi(\bar{s}^{t-1})m_{0\to t-1}(\bar{s}^{t-1})\sum_{\bar{s}_t}\pi(\bar{s}_t|\bar{s}^{t-1})m_t(\bar{s}_t|\bar{s}^{t-1})\left[\left(D_t(\bar{s}^{t-1}, \bar{s}_t) + p_t(\bar{s}^{t-1}, \bar{s}_t)\right)\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1})\right. \\ &\qquad \left. + \Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right] \\ =\ &\sum_{t=1}^{T}\sum_{\bar{s}^{t-1}}\pi(\bar{s}^{t-1})m_{0\to t-1}(\bar{s}^{t-1})\left[p_{t-1}(\bar{s}^{t-1})\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + q_{t-1}(\bar{s}^{t-1})\Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right]\end{aligned}$$

where the first equation uses $\Delta k_0(\theta) = \Delta b_0(\theta) = 0$ and the last one uses the pricing conditions (5). Similarly, multiplying the third line in (76) by $\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)$ and summing over t and $\bar{s}^t$ yields

$$\begin{aligned}&-\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\left[p_t(\bar{s}^t)\Delta k_{t+1}(\theta, s^t, \bar{s}^t) + q_t(\bar{s}^t)\Delta b_{t+1}(\theta, s^t, \bar{s}^t)\right] \\ =\ &-\sum_{t=1}^{T}\sum_{\bar{s}^{t-1}}\pi(\bar{s}^{t-1})m_{0\to t-1}(\bar{s}^{t-1})\left[p_{t-1}(\bar{s}^{t-1})\Delta k_t(\theta, s^{t-1}, \bar{s}^{t-1}) + q_{t-1}(\bar{s}^{t-1})\Delta b_t(\theta, s^{t-1}, \bar{s}^{t-1})\right]\end{aligned}$$

where we adjusted the initial date of summation and used $k_{T+1}(\theta, s^T) = b_{T+1}(\theta, s^T) = 0$ for all s^T.[40] Hence, the second and third line cancel and (76) becomes

$$\begin{aligned}&\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\Delta T_t(\theta, s^t, \bar{s}^t) \\ =\ &\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\left[k_t(\theta, s^{t-1})\Delta D_t(s^t, \bar{s}^t) + x_t(\theta, s^t)\Delta p_t(s^t, \bar{s}^t) - b_{t+1}(\theta, s^t)\Delta q_t(s^t, \bar{s}^t) - \Omega(\theta)\Delta C_t(s^t, \bar{s}^t)\right]\end{aligned} \tag{77}$$

where we used Lemma 1. Subtracting the sequential resource constraints under histories s^t and $\bar{s}^t$ yields

$$\begin{aligned}\Delta C_t(s^t, \bar{s}^t) &= X_t(s^t)\Delta p_t(s^t, \bar{s}^t) + K_t(s^{t-1})\Delta D_t(s^t, \bar{s}^t) - B_{t+1}(s^t)\Delta q_t(s^t, \bar{s}^t) \\ &\quad + (D_t(\bar{s}^t) + p_t(\bar{s}^t))\Delta K_t(s^{t-1}, \bar{s}^{t-1}) - p_t(\bar{s}^t)\Delta K_{t+1}(s^t, \bar{s}^t) + \Delta B_t(s^{t-1}, \bar{s}^{t-1}) - q_t(\bar{s}^t)\Delta B_{t+1}(s^t, \bar{s}^t).\end{aligned}$$

[40] When $T = \infty$ we impose the analogous no-Ponzi and transversality conditions.

Following the same steps as before yields

$$\begin{aligned}
&\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\Delta C_t(\theta,s^t,\bar{s}^t)\\
=\;&\sum_{t=0}^{T}\sum_{\bar{s}^t}\pi(\bar{s}^t)m_{0\to t}(\bar{s}^t)\left[X_t(s^t)\Delta p_t(s^t,\bar{s}^t)+K_t(s^{t-1})\Delta D_t(s^t,\bar{s}^t)-B_{t+1}(s^t)\Delta q_t(s^t,\bar{s}^t)\right].
\end{aligned}$$

Substituting back in (77), we obtain Proposition 6.

D.5 Proof of Corollary 6

Using (1), returns remains unchanged when $R_{t+1}=(D_{t+1}+p_{t+1})/p_t=(\overline{D}_{t+1}+\overline{p}_{t+1})/\overline{p}_t=\overline{R}_{t+1}$. Using that $D_t=\overline{D}_t+\Delta D_t$ and $p_t=\overline{p}_t+\Delta p_t$, this happens when:

$$\frac{\Delta D_{t+1}+\Delta p_{t+1}}{\Delta p_t}=\frac{\overline{D}_{t+1}+\overline{p}_{t+1}}{\overline{p}_t}\quad\text{for all } t. \tag{78}$$

Under condition (78), we have

$$\begin{aligned}
\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}\Delta T_t(\theta)&=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}[k_t(\theta)(\Delta p_t+\Delta D_t)-k_{t+1}(\theta)\Delta p_t-\Omega(\theta)(K_t(\Delta p_t+\Delta D_t)-K_{t+1}\Delta p_t)]\\
&=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}[k_t(\theta)-\Omega(\theta)K_t](\Delta p_t+\Delta D_t)-\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}[k_{t+1}(\theta)-\Omega(\theta)K_{t+1}]\Delta p_t\\
&=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}\left[k_t(\theta)-\Omega(\theta)K_t\right]\left(\Delta p_t+\Delta D_t\right)\\
&\quad-\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}\left[k_{t+1}(\theta)-\Omega(\theta)K_{t+1}\right]\frac{\overline{p}_t}{\overline{D}_{t+1}+\overline{p}_{t+1}}(\Delta p_{t+1}+\Delta D_{t+1})\quad\text{by (78)}\\
&=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}\left[k_t(\theta)-\Omega(\theta)K_t\right]\left(\Delta p_t+\Delta D_t\right)\\
&\quad-\sum_{t=0}^{T}\overline{R}_{0\to t+1}^{-1}\left[k_{t+1}(\theta)-\Omega(\theta)K_{t+1}\right]\left(\Delta p_{t+1}+\Delta D_{t+1}\right)\\
&=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}\left[k_t(\theta)-\Omega(\theta)K_t\right]\left(\Delta p_t+\Delta D_t\right)-\sum_{t=1}^{T+1}\overline{R}_{0\to t}^{-1}\left[k_t(\theta)-\Omega(\theta)K_t\right]\left(\Delta p_t+\Delta D_t\right)\\
&=\left[k_0(\theta)-\Omega(\theta)K_0\right]\left(\Delta p_0+\Delta D_0\right)-\overline{R}_{0\to T+1}^{-1}\left[k_{T+1}(\theta)-\Omega(\theta)K_{T+1}\right]\left(\Delta p_{T+1}+\Delta D_{T+1}\right)\\
&=\left[k_0(\theta)-\Omega(\theta)K_0\right]\Delta p_0
\end{aligned}$$

since the last term vanishes and $\Delta D_0=0$.

D.6 Multi-period version of Proposition 2 (taxing total capital income)

Proposition 10. *Suppose asset prices change by $\{\Delta p_t\}_{t=0}^{T}$ and dividends by $\{\Delta D_t\}_{t=0}^{T}$ resulting in return changes $\{\Delta R_t\}_{t=0}^{T}$. Then optimal taxes $\{T_t(\theta)\}_{t=0}^{T}$ are such that*

$$\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}T_t(\theta)=\sum_{t=0}^{T}\overline{R}_{0\to t}^{-1}[\overline{T}_t(\theta)+a_t(\theta)\Delta R_t-\Omega(\theta)A_t\Delta R_t]$$

Proof. The proof follows exactly analogous steps to the proof of Proposition 6. □

E Appendix for Section 6

E.1 Proof of Equation (31)

The investor's Euler equation under preferences (27) is:

$$c_0(\theta)^{-1/\sigma} = \beta R c_1(\theta)^{-1/\sigma}$$

with $R = D/p$. Since it holds for all investors, it aggregates to

$$C_1 = \left(\beta \frac{D}{p}\right)^{\sigma} C_0.$$

Moreover, integrating the budget constraints (14) and (15) across investors and using the market clearing condition (30), we obtain $C_0 = Y_0$ and $C_1 = Y_1 + DK$ in the closed economy. Substituting back in the aggregate Euler equation, the equilibrium asset price p^* must satisfy

$$Y_1 + DK = \left(\beta \frac{D}{p^*}\right)^{\sigma} Y_0,$$

which can be rearranged to deliver equation (31).

E.2 Proof of Proposition 7

First observe that, by Lemma 1, the optimal consumption allocation still satisfies $c_t(\theta) = \Omega(\theta) C_t$, $t = 0, 1$. Since $C_0 = Y_0$ and $C_1 = Y_1 + DK$ in the closed economy and we hold both dividends and the aggregate endowment fixed, this immediately implies that no investor's consumption is changing in response to the asset price change Δp^*, so $c_t(\theta) = \bar{c}_t(\theta)$ for all θ, $t = 0, 1$. By the second-period budget constraint (23) and the normalization $T_1(\theta) = 0$, this implies in turn that $x(\theta) = \bar{x}(\theta)$ for all θ. The result then follows from Proposition 1 and the fact that $\Delta D = 0$ and $X = K_0 - K_1 = 0$.

E.3 Proof of Proposition 8

Subtract an investor's budget constraints under the old and new prices in period 0:

$$\begin{aligned} c_0(\theta) - \bar{c}_0(\theta) + q(b(\theta) - \bar{b}(\theta)) &= p x(\theta) - \bar{p}\,\bar{x}(\theta) - (\chi(x(\theta)) - \chi(\bar{x}(\theta))) - (T_0(\theta) - \overline{T}_0(\theta)) \\ &= (p - \bar{p}) x(\theta) + \bar{p}(x(\theta) - \bar{x}(\theta)) - (\chi(x(\theta)) - \chi(\bar{x}(\theta))) - (T_0(\theta) - \overline{T}_0(\theta)) \end{aligned}$$

and in period 1:

$$c_1(\theta) - \bar{c}_1(\theta) = D(\theta)(x(\theta) - \bar{x}(\theta)) + b(\theta) - \bar{b}(\theta)$$

We eliminate $b(\theta) - \bar{b}(\theta)$ by substituting the latter into the former:

$$\begin{aligned} &c_0(\theta) - \bar{c}_0(\theta) + q(c_1(\theta) - \bar{c}_1(\theta)) + qD(\theta)(x(\theta) - \bar{x}(\theta)) \\ &= (p - \bar{p}) x(\theta) + \bar{p}(x(\theta) - \bar{x}(\theta)) - (\chi(x(\theta)) - \chi(\bar{x}(\theta))) - (T_0(\theta) - \overline{T}_0(\theta)) \end{aligned}$$

Rearranging and using (32) yields

$$\begin{aligned} & c_0(\theta) - \bar{c}_0(\theta) + q(c_1(\theta) - \bar{c}_1(\theta)) - \chi'(\bar{x}(\theta))(x(\theta) - \bar{x}(\theta)) \\ &= (p - \bar{p})x(\theta) - (\chi(x(\theta)) - \chi(\bar{x}(\theta))) - (T_0(\theta) - \bar{T}_0(\theta)) \end{aligned}$$

The second-order Taylor approximation for $\chi(x)$ around the point $\bar{x}(\theta)$ is:

$$\chi(x(\theta)) - \chi(\bar{x}(\theta)) \approx \chi'(\bar{x}(\theta))(x(\theta) - \bar{x}(\theta)) + \frac{1}{2}\chi''(\bar{x}(\theta))(x(\theta) - \bar{x}(\theta))^2$$

Substituting this, we obtain

$$c_0(\theta) - \bar{c}_0(\theta) + q(c_1(\theta) - \bar{c}_1(\theta)) = x(\theta)\Delta p - \frac{1}{2}\chi''(\bar{x}(\theta))(\Delta x(\theta))^2 - (T_0(\theta) - \bar{T}_0(\theta)) \tag{79}$$

where $\Delta x(\theta) \equiv x(\theta) - \bar{x}(\theta)$.

Since the aggregate resource constraint (34) takes the same form as (19), the Pareto problem (21) subject to (34) still implies $c_t(\theta) = \Omega(\theta)C_t$, $t = 0, 1$ by Lemma 1. Hence, (79) can be written as

$$\Delta T_0(\theta) = x(\theta)\Delta p - \frac{1}{2}\chi''(\bar{x}(\theta))(\Delta x(\theta))^2 - \Omega(\theta)\left[C_0 - \bar{C}_0 + q(C_1 - \bar{C}_1)\right]$$

Integrating (79) across all investors implies

$$C_0 - \bar{C}_0 + q(C_1 - \bar{C}_1) = X\Delta p - \frac{1}{2}\int \chi''(\bar{x}(\theta))\Delta x(\theta)^2 dF(\theta)$$

and substituting this back delivers Proposition 8.

F Wealth taxes as taxes on presumptive income

This appendix shows why taxing fluctuating wealth market values based on an analogy to a tax on "presumptive income" is problematic. The following simple numerical example illustrates that actual and presumptive income diverge whenever asset valuations are not exclusively driven by cash flows.

Consider an investor with an asset (e.g. a private business) initially worth \$100m which generates a dividend income of \$5m and which is subject to a 2% wealth tax of \$2m. In the notation of equation (1), D_t and p_t are initially fixed at $\bar{D}$ and $\bar{p}$ with an asset return $\bar{R} - 1 = \bar{D}/\bar{p} = 5\%$. The asset value then jumps up permanently by a factor two to $p = 2\bar{p} = \$200m$ so that also the investor's wealth tax liability doubles to \$4m. The key question is what happens to the investor's presumptive versus actual income.

Suppose first that the increased asset value is exclusively due to higher cashflows, i.e. dividend income also doubles to $D = \$10m$ (Special Case 2). From (1), the asset return remains constant at $D/p = \$10m/\$200m = 5\%$ and therefore the increase in presumptive income exactly matches the increase in actual income. However, in all other cases in which dividends increase by less than a factor of two, this is no longer true: actual income increases by less than presumptive income. The problem is that it is incorrect to apply the same constant 5% presumed return *to the new valuation* of p =\$200m because the true return to wealth D/p *falls*. In the extreme case in which dividend income remains fixed (Special Case 1), presumptive income doubles to $5\% \times \$200m = \$10m$ while actual income is unchanged at \$5m. The unchanged dividend income corresponds to a lower return to wealth of only $\bar{R} - 1 = \bar{D}/p = 2.5\%$ so the correct income calculation would have been $2.5\% \times \$200m = \$5m$ rather than the (incorrect) presumptive income calculation of $5\% \times \$200m = \$10m$. Thus "presumptive income" is overestimated and wealth taxes redistribute suboptimally away from Special Case 2.