Online Appendix

Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy

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A Calibration Details

A.1 Summary of External Calibration

	Description	Value	Target / Source
Preferences			
$\frac{1}{\gamma}$	Intertemporal Elasticity Parameter	$\frac{1}{2}$	Literature
ϕ	Procrastination Decay Rate	$-\log(0.5)$	Andersen et al. (2020)
Income			
y_t	Transitory Income	$\{0.75, 0.98, 1.28\}$	Guerrieri and Lorenzoni (2017)
$\mathbf{A}^y$	Income Transition Matrix	(see text)	Guerrieri and Lorenzoni (2017)
Interest Rates			
r_t	Interest Rate	$\{-1\%, 0\%, 1\%, 2\%\}$	10-Year TIPS
$\mathbf{A}^r$	Interest Rate Transition Matrix	(see text)	10-Year TIPS
ω^{cc}	Credit Card Wedge	10.3%	Credit Card - 10-Yr Treasury Spread
ω^m	Mortgage Wedge	1.7%	30-Yr FRM - 10-Yr Treasury Spread
Assets and Liabilities			
h	House Value	3.29	2016 SCF
θ	Max LTV	0.8	Greenwald (2018)
ξ	Mortgage Paydown	0.035	20 Year Half-Life
κ^{prepay}	Prepayment Fixed Cost	0.002	Numerical Stability
κ^{refi}	Refinancing Fixed Cost	0.05	FRB Documentation
$\underline{b}$	Credit Limit	$-\frac{1}{3}$	2016 SCF
Other Structural Assumptions			
λ^F	Rate of Forced Adjustment	$\frac{1}{15}$	2016 CPS Avg. Moving Rate
λ^R	Retirement Rate	$\frac{1}{30}$	Average Working Life
y^R	Retirement Fixed Income	y_L	Retirement Replacement Rate
-	New-Homeowner Distribution	$m_0 = \theta h, b_0 \sim U(0, \frac{y_L}{2})$	Lifecycle Dynamics

Table 4: Externally Calibrated Parameters.

Notes: This table presents the model's externally calibrated parameters. See Section 4.1 for details.

A.2 SCF Details

Many of our calibrated parameters rely on data from the 2016 SCF. To construct a sample of households that is consistent with our model we impose the following data filters. The household must own a home, possess a credit card, have a head or spouse in the labor force, and have a head aged 25-61. In order to limit both measurement error and extreme heterogeneity in home values and income, we also restrict our analysis to households with after-tax permanent income between the 1st and 99th percentile, and a home value to permanent income ratio that is below the 95th percentile. Our sample is broadly representative of working-age homeowners, and captures 74% of homeowners aged 25-61.[51]

All of our variables are scaled relative to permanent income. Following Kennickell (1995), Kennickell and Lusardi (2004), and Fulford (2015) we use the SCF's question about "normal income" to measure each household's permanent income.[52] Though this is an imperfect proxy for the household's permanent income, it has the benefit of being both straightforward and respecting the household's information set. We adjust each household's normal income for 2015 federal taxes, and deduct an additional 5% for state taxes.

We use the 2016 SCF to estimate six moments that are used in our calibration: (i) permanent income; (ii) average home value to permanent income; (iii) average LTV; (iv) average credit card debt to permanent income; (v) share of households with revolving credit card debt; and (vi) average credit limit to permanent income. Moments (ii) – (v) are reported in the main text. The average after-tax permanent income for our sample of homeowners is $95,718. The average credit limit to permanent income is 0.35.

A.3 Calibration of Income and Interest Rate Processes

To calibrate our income and interest rate processes, we assume that these processes are discretized versions of continuous-time Ornstein-Uhlenbeck (OU) processes.

Using Discrete-Time Estimates to Calibrate Continuous-Time Process. Consider a generic mean-zero OU process $u(t) = \int_0^t e^{-\kappa(t-s)}\sigma dZ_s$. Process $u(t)$ has the conditional distribution $u(t+\tau)|u(t) = \mathcal{N}\left(u(t)e^{-\kappa\tau}, \frac{\sigma^2}{2\kappa}(1-e^{-2\kappa\tau})\right)$.

Assume that $u(t)$ is only observed in snapshots every Δ years. Let $d_s = u(s\Delta)$ denote the

[51]Working-age homeowners represent 61% of all households aged 25-61 in the 2016 SCF.

[52]SCF respondents are asked whether or not their 2015 income was normal. If not, they are asked to report what their total income would be if it had been normal.

s'th snapshot of process $u(t)$. The discrete process d_s can be modeled as an AR(1) process:

$$\begin{aligned} d_{s+1} &= \rho d_s + \sigma_d \varepsilon_{s+1}, \text{ where} \\ \rho &= e^{-\kappa\Delta} \\ \sigma_d^2 &= \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa\Delta}). \end{aligned}$$

Given any discrete-time AR(1) estimate, we can use the above formulas to back out the parameters of the underlying OU process: κ and σ. We discretize the OU process using finite difference methods. For details, see the Numerical Appendix of Achdou et al. (2022).

Implementation for Income Process. As already stated in the main text our income process is calibrated following Guerrieri and Lorenzoni (2017) who in turn use data from Floden and Lindé (2001). Specifically, Guerrieri and Lorenzoni assume that the logarithm of income follows an AR(1) process at a quarterly frequency, and calibrate this process with persistence of $\rho = 0.967$ and variance of $\sigma_d^2 = 0.017$. From the formulas we just discussed, we get the parameters for our OU process of $\kappa = 0.134$ and $\sigma = 0.265$.

We set the three income states $y_t \in \{y_L, y_M, y_H\}$ as follows. We set the low and high income states equal to -1 and $+1$ annualized standard deviations of the log income process, more precisely $y_L = y_M e^{-\sigma}$ and $y_H = y_M e^{\sigma}$ where $\sigma = 0.265$. We then set the middle income state y_M to normalize mean income to 1 which yields $y_M = 0.98$ and hence $\{y_L, y_M, y_H\} = \{0.75, 0.98, 1.28\}$. In the stationary distribution of the income process, 31% of households are low income, 39% are middle income, and 31% are high income. The expected persistence of the low, middle, and high income states are 1.6 years, 1 year, and 1.6 years, respectively.

B Naive Present Bias: Passing to Continuous Time

Here we present a heuristic derivation of naive IG preferences as the continuous-time limit of a model where some of the decisions are made discretely. This heuristic approach is designed to capture the intuition of the more rigorous derivation in Harris and Laibson (2013). We begin by assuming a constant effort cost, as in Sections 2.1 and 2.2. The full setup with a stochastic effort cost, as introduced in Section 2.3, is presented in Appendix B.3.

B.1 Naive IG Current-Value Function

Assume that the current self lives for a discrete length of time, denoted Δ. After this time has elapsed, starting with the next self, time progresses continuously again.[53] Since the naive present-biased household incorrectly perceives that all future selves will discount exponentially, continuation-value function $v(x)$ characterizes the equilibrium starting with the next self at time Δ. The current self discounts all future selves by β, so the current-value function for the naive present-biased household is given by:

$$\begin{aligned} w(x) &= \max\left\{ \max_{c}\ u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\ w^*(x) - \bar{\varepsilon} \right\} \quad \text{with} \\ w^*(x) &= \max\left\{ w^{prepay}(x), w^{refi}(x) \right\} \\ w^{prepay}(x) &= \max_{b',m'} w(b', m', y, r^m, r) \quad \text{ s.t. prepayment constraint (4) holds} \\ w^{refi}(x) &= \max_{b',m'} w(b', m', y, r + \omega^m, r) \quad \text{ s.t. refinancing constraint (5) holds} \end{aligned} \tag{14}$$

and where x_Δ denotes the vector of household states after time interval Δ has elapsed, for example $b_\Delta = b + (y + rb + \omega^{cc} b^- - (r^m + \xi)m - c)\Delta$.

Equation (14) captures the consumption/adjustment decisions made by the current self. In the left branch of the first line the household does not adjust, and chooses consumption rate c over the next Δ units of time to maximize the current-value function. In the right branch of the first line the household pays effort cost $\bar{\varepsilon}$ and fixed monetary cost κ^i to discretely adjust its mortgage. Importantly, this discrete-time value function is written such that there is no delay to refinancing (i.e., the current self benefits from refinancing).[54] Though this is unrealistic – there are time delays in refinancing – we write the Bellman equation in this

[53] This mixed discrete- and continuous-time setup is of course slightly non-standard. Alternatively, we could have assumed that future selves also make decisions in discrete time, as done in Laibson and Maxted (2023). In this case the continuation-value function $v(x)$ would be the discrete-time analogue of the continuous-time $v(x)$ that we use below.

[54] To see how the value function is written in this way, note that refinancing gives the current self the current-value function of w^*. As the first line of equation (14) shows, this value function consists of an undiscounted utility flow earned for the current self, $u(c)\Delta$.

way to emphasize that our results do not rely on assumptions about temporal delays.

Discrete-time Bellman equation (14) can be used to derive the current-value function in continuous time. Taking the time-step Δ to its continuous-time limit, we see that the term $u(c)\Delta$ drops out of the current-value function, leaving:

$$w(x) = \max\left\{\beta v(x),\ w^*(x) - \bar{\varepsilon}\right\}.$$

This recovers equation (9) in the main text.

B.2 Continuous Control: Consumption (Proof of Lemma 1)

We now derive the continuous-time first-order condition for consumption stated in Lemma 1. As shown in equation (14), the household makes a consumption choice in every period. For the consumption decision, equation (14) implies that consumption is given by the following first-order condition:[55]

$$u'(c(x)) = \beta e^{-\rho\Delta}\ \frac{\partial}{\partial b}\mathbb{E}[v(x_\Delta)|x].$$

Taking $\Delta \to 0$ at the points where $\frac{\partial v(x)}{\partial b}$ exists yields

$$u'(c(x)) = \beta\frac{\partial v(x)}{\partial b},$$

which is equation (11) in Lemma 1. This derivation continues to hold in the full setup with a stochastic effort cost presented in Appendix B.3 below.

B.3 Full Setup with a Stochastic Effort Cost (Section 2.3)

Here we briefly spell out the full set of equations for the generalization with a stochastic effort cost that evolves according to the two-state process in Assumption 1. In what follows, we will denote value and policy functions in the normal high-cost state by the same functions as in the baseline model with a constant effort cost, e.g. $v(x)$ or $\mathfrak{R}(x)$. Alternatively, we will denote the corresponding value and policy functions in the temporary low-cost state with underlines, e.g. $\underline{v}(x)$ or $\underline{\mathfrak{R}}(x)$.

We first show how to generalize equation (8′), the HJBQVI equation for the value function

[55] We ignore difficulties such as kinks when taking this first-order condition.

$v(x)$ of a $\beta = 1$ household:

$$\rho v(x) = \max \left\{ \max_c \left\{ u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) \right\} \right. \tag{8''}$$
$$- \frac{\partial v(x)}{\partial m}(\xi m)$$
$$+ \sum_{y' \neq y} \lambda^{y \to y'} \left[v(b, m, y', r^m, r) - v(b, m, y, r^m, r) \right]$$
$$+ \sum_{r' \neq r} \lambda^{r \to r'} \left[v(b, m, y, r^m, r') - v(b, m, y, r^m, r) \right]$$
$$+ \lambda^R \left[v^R(x) - v(x) \right]$$
$$+ \lambda^F \left[(v^*(x) - \bar{\varepsilon}) - v(x) \right]$$
$$+ \phi \left[\underline{v}(x) - v(x) \right],$$
$$\left. \rho(v^*(x) - \bar{\varepsilon}) \right\}.$$

Relative to (8′), there is a new entry $\phi\big[\underline{v}(x) - v(x)\big]$. Parameter ϕ is the arrival rate of the low-effort-cost state, and $\underline{v}(x)$ is the household's value in this state. This value is given by

$$\underline{v}(x) = \max\{v(x), v^*(x) - \underline{\varepsilon}\}. \tag{15}$$

Intuitively, since the low-cost state only lasts for an instant (Assumption 1), the household either takes advantage of refinancing at the lower effort cost $\underline{\varepsilon}$ or it loses the opportunity in the next instant in which case its value reverts back to $v(x)$.

We next show how to generalize (9), the equation for the current-value function $w(x)$:

$$\begin{aligned}
w(x) &= \max \left\{ \beta v(x),\ w^*(x) - \bar{\varepsilon} \right\} \quad \text{and} \\
\underline{w}(x) &= \max \left\{ \beta v(x),\ w^*(x) - \underline{\varepsilon} \right\} \quad \text{with} \\
w^*(x) &= \max \left\{ w^{prepay}(x), w^{refi}(x) \right\} \\
w^{prepay}(x) &= \max_{b', m'}\ w(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b', m'}\ w(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned} \tag{16}$$

Relative to (9), there is a new line $\underline{w}(x) = \max \left\{ \beta v(x),\ w^*(x) - \underline{\varepsilon} \right\}$ that captures the current-value of a household that has the opportunity to refinance at the low-effort-cost $\underline{\varepsilon}$.

Like in Appendix B.1, the current-value function in (16) can be derived from a setup in which the current self lives for a discrete length of time Δ:

$$\begin{aligned}
w(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1-e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\},\\
\underline{w}(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\underline{\phi}\Delta}\mathbb{E}[\underline{v}(x_\Delta)|x] + \left(1-e^{-\underline{\phi}\Delta}\right)\mathbb{E}[v(x_\Delta)|x]\right],\ \underline{w}^*(x) - \underline{\varepsilon}\right\},\\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\}\\
w^{prepay}(x) &= \max_{b',m'} w(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds}\\
w^{refi}(x) &= \max_{b',m'} w(b',m',y,r+\omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}\\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\}\\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds}\\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b',m',y,r+\omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}$$

where ϕ and $\underline{\phi}$ denote the Poisson switching rates between the two effort-cost states. As stated in Assumption 1 we assume that $\underline{\phi} \to \infty$. Therefore $e^{-\underline{\phi}\Delta} \to 0$ and

$$\begin{aligned}
w(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\left[e^{-\phi\Delta}\mathbb{E}[v(x_\Delta)|x] + \left(1-e^{-\phi\Delta}\right)\mathbb{E}[\underline{v}(x_\Delta)|x]\right],\ w^*(x) - \bar{\varepsilon}\right\},\\
\underline{w}(x) &= \max\left\{ \max_c\ u(c)\Delta + \beta e^{-\rho\Delta}\mathbb{E}[v(x_\Delta)|x],\ \underline{w}^*(x) - \underline{\varepsilon}\right\},\\
w^*(x) &= \max\left\{w^{prepay}(x), w^{refi}(x)\right\}\\
w^{prepay}(x) &= \max_{b',m'} w(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds}\\
w^{refi}(x) &= \max_{b',m'} w(b',m',y,r+\omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}\\
\underline{w}^*(x) &= \max\left\{\underline{w}^{prepay}(x), \underline{w}^{refi}(x)\right\}\\
\underline{w}^{prepay}(x) &= \max_{b',m'} \underline{w}(b',m',y,r^m,r) \quad \text{s.t. prepayment constraint (4) holds}\\
\underline{w}^{refi}(x) &= \max_{b',m'} \underline{w}(b',m',y,r+\omega^m,r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}$$

Finally, we take the limit as $\Delta \to 0$. Using the property that $\underline{w}^*(x) = w^*(x)$ in the limit as $\Delta \to 0$ – which one can see by inspection since the left branch of the first line converges to the left branch of the second line – we recover equation (16).

C Proofs

C.1 Proof of Corollary 1

Recall that, with naiveté, the perceived continuation-value function of a $\beta < 1$ household equals the value function of an exponential $\beta = 1$ household and solves (8″). Assume that the household is not in an adjustment region at time t so that the perceived continuation-value function $v(x_t)$ is characterized by a standard HJB equation. This HJB equation is given by the left branch of (8″), which we write here as

$$\rho v(x) = \max_c \; u(c) + \frac{\partial v(x)}{\partial b}(y + rb + \omega^{cc} b^- - (r^m + \xi)m - c) + (\mathcal{B}v)(x) \tag{17}$$

where the operator $(\mathcal{B}v)(x)$ is short-hand notation for lines two to seven of (8″). Recall that we use hat-notation to denote the policy functions that naive households perceive for future selves, and denote by $\widehat{c}(x)$ and $\widehat{s}(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - \widehat{c}(x))$ the corresponding perceived consumption and liquid saving policy functions. In contrast, denote by $c(x)$ (from Proposition 1) and $s(x) = (y + rb + \omega^{cc} b^- - (r^m + \xi)m - c(x))$ the *actual* policy functions.

The following observation is important in the proof below: the HJB equation for the perceived continuation-value function (17) features the *perceived* policy functions $\widehat{c}(x), \widehat{s}(x)$, rather than the *actual* policy functions. But what determines the evolution of liquid wealth b are the *actual* policy functions.

Differentiate (17) with respect to b and use the envelope theorem:

$$(\rho - r(b))\frac{\partial v(x)}{\partial b} = \frac{\partial^2 v(x)}{\partial b^2}\widehat{s}(x) + \frac{\partial}{\partial b}(\mathcal{B}v)(x). \tag{18}$$

Define the marginal continuation-value of wealth $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$. From (18) it satisfies

$$(\rho - r(b))\,\eta(x) = \frac{\partial \eta(x)}{\partial b}\widehat{s}(x) + (\mathcal{B}\eta)(x). \tag{19}$$

If $\beta = 1$, from Itô's formula, the right-hand side of (19) also governs the expected change in the marginal value of wealth: $\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}\widehat{s}(x_t) + (\mathcal{B}\eta)(x_t)\right] dt$. But with $\beta < 1$ this is no longer true: the evolution of b is governed by the *actual* drift $s(x)$ rather than the *perceived* drift $\widehat{s}(x)$ and so

$$\mathbb{E}_t[d\eta(x_t)] = \left[\frac{\partial \eta(x_t)}{\partial b}s(x_t) + (\mathcal{B}\eta)(x_t)\right] dt. \tag{20}$$

Therefore, evaluating (19) along a particular trajectory x_t, we have

$$(\rho - r(b_t))\,\eta(x_t) = \frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] - \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)).$$

Rearranging

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[d\eta(x_t)] &= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(s(x_t) - \widehat{s}(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}(\widehat{c}(x_t) - c(x_t)) \\
&= (\rho - r(b_t))\,\eta(x_t) + \frac{\partial \eta(x_t)}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t)
\end{aligned}$$

Finally, recalling that $\eta(x) \equiv \frac{\partial v(x)}{\partial b}$, the first-order condition is $u'(c(x)) = \beta\eta(x)$ and therefore

$$\begin{aligned}
\frac{1}{dt}\mathbb{E}_t[du'(c(x_t))] &= (\rho - r(b_t))\,u'(c(x_t)) + \frac{\partial u'(c(x_t))}{\partial b}\left(\beta^{\frac{1}{\gamma}} - 1\right) c(x_t) \\
&= (\rho - r(b_t))\,u'(c(x_t)) - u''(c(x_t))c(x_t)\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} \\
&= \left[\rho + \gamma\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b} - r(b_t)\right] u'(c(x_t)),
\end{aligned}$$

where going from the second line to the third line uses that, with CRRA utility, the coefficient of relative risk aversion is $\gamma = \frac{-u''(c(x_t))c(x_t)}{u'(c(x_t))}$. Dividing by $u'(c(x_t))$, we have (12). ■

C.2 Proof of Proposition 2

When proving Proposition 2, we refer to Appendix B.3 which spells out the full set of equations for the model with a stochastic effort cost satisfying Assumption 1.

We also note that clauses 1, 2a, and 2b do not rely on the instantaneous low-effort-cost period used in Assumption 1. The purpose of Assumption 1 is to create the sorts of deadlines that incentivize present-biased agents to complete effortful tasks (clause 2c).

C.2.1 Proof of Proposition 2, Clause 1

The proof of clause 1 follows from equation (16). Equation (16) shows that we can rewrite w^{prepay} and w^{refi} as:

$$\begin{aligned}
w^{prepay}(x) &= \max_{b',m'} \ \beta v(b', m', y, r^m, r) \quad \text{s.t. prepayment constraint (4) holds} \\
w^{refi}(x) &= \max_{b',m'} \ \beta v(b', m', y, r + \omega^m, r) \quad \text{s.t. refinancing constraint (5) holds}
\end{aligned}$$

These are exactly the same formulas as for v^{prepay} and v^{refi} in (7), except that there is an additional β discount factor. Since the additional β discount factor has no effect on the optimal choice of (b', m'), we recover clause 1 of Proposition 2 — the choice of (b', m') is independent of β. ■

Since $v^*(x) = \max\left\{v^{prepay}(x), v^{refi}(x)\right\}$ and $w^*(x) = \max\left\{w^{prepay}(x), w^{refi}(x)\right\}$, the above proof also implies that:

$$w^*(x) = \beta v^*(x). \tag{21}$$

This property will be used in the proof of clause 2 of Proposition 2.

C.2.2 Proof of Proposition 2, Clause 2

For clause 2a, when $\beta = 1$ the assumption that $\bar{\varepsilon}$ and $\underline{\varepsilon}$ are vanishingly small (Assumption 2) implies that $v(x)$ is arbitrarily close to $\underline{v}(x)$. Accordingly, policy function $\mathfrak{R}(x)$ converges pointwise to $\underline{\mathfrak{R}}(x)$ as the effort cost vanishes.

To prove clause 2b (procrastination when $\beta < 1$ and $\varepsilon = \bar{\varepsilon}$), consider the self in control at point x in the state space. Recall from (16) that the current-value function is given by $w(x) = \max\{\beta v(x), w^*(x) - \bar{\varepsilon}\}$. Therefore the current self will not adjust their mortgage when the value from not adjusting, $\beta v(x)$, is larger than the value from adjusting, $w^*(x) - \bar{\varepsilon}$.

The value of not adjusting is given by

$$\beta v(x) \geq \beta(v^*(x) - \bar{\varepsilon}), \tag{22}$$

where the inequality $v(x) \geq v^*(x) - \bar{\varepsilon}$ follows directly from equation (8′′).

Alternatively, adjusting requires the household to incur the effort cost $\bar{\varepsilon}$ in the current period and the value of adjusting is given by

$$w^*(x) - \bar{\varepsilon} = \beta v^*(x) - \bar{\varepsilon}, \tag{23}$$

where the equality follows from equation (21).

Comparing the two alternatives (22) and (23) shows that the $\beta < 1$ household will always prefer to procrastinate whenever $\varepsilon_t = \bar{\varepsilon}$, since

$$\beta(v^*(x) - \bar{\varepsilon}) > \beta v^*(x) - \bar{\varepsilon}.$$

Procrastination enables the effort cost $\bar{\varepsilon}$ to be discounted by β, while there is at most an infinitesimal cost to delaying refinancing for an instant.

To prove clause 2c (no procrastination when $\beta < 1$ and $\varepsilon = \underline{\varepsilon}$), consider the self in control at point x in the state space. Following the second line of equation (16), it will be (weakly) optimal for the current self to adjust their mortgage if and only if:

$$w^*(x) - \underline{\varepsilon} \geq \beta v(x).$$

Above, the left-hand side is the current-value from refinancing at effort cost $\underline{\varepsilon}$, and the right-hand side is the current-value from not refinancing and having the effort cost reset immediately to $\bar{\varepsilon}$. Since $w^*(x) = \beta v^*(x)$ (see equation (21)), this can be rewritten as

$$\beta v^*(x) - \underline{\varepsilon} \geq \beta v(x). \tag{24}$$

First, consider the case in which the next self is expected to adjust the mortgage if the current self procrastinates.[56] Since the next self is expected to have $\beta = 1$, this means $\widehat{\mathfrak{R}}(x) > 0$. In this case, equation ($8''$) implies that $v(x) = v^*(x) - \bar{\varepsilon}$. Plugging this into (24) shows that the current self will adjust their mortgage whenever $\beta v^*(x) - \underline{\varepsilon} \geq \beta v^*(x) - \beta\bar{\varepsilon}$ or

$$\underline{\varepsilon} \leq \beta\bar{\varepsilon},$$

which is satisfied because Assumption 1 imposes that $\underline{\varepsilon} < \beta\bar{\varepsilon}$. Intuitively, this says that the current self will adjust their mortgage now if the cost of doing so, $\underline{\varepsilon}$, is less than the discounted cost of adjusting next period, $\beta\bar{\varepsilon}$. Thus, if $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$, meaning that the household does not procrastinate.

Next, consider the case in which a $\beta = 1$ household would not refinance at point x, even in the low-effort-cost state $\varepsilon_t = \underline{\varepsilon}$, i.e. $\widehat{\underline{\mathfrak{R}}}(x) = 0$. In that case, equation (15) implies that $v(x) \geq v^*(x) - \underline{\varepsilon}$. Multiplying by β, this also implies that $\beta v(x) \geq \beta v^*(x) - \beta\underline{\varepsilon}$, and therefore

$$\beta v(x) > \beta v^*(x) - \underline{\varepsilon}.$$

Comparing this to equation (24) shows that it will not be optimal for the naive present-biased household to refinance. This is intuitive — if it is not optimal for a $\beta = 1$ household to refinance, there is no reason for it to be optimal for a naive $\beta < 1$ household to refinance. Thus, if $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$.

Tying these two cases together, we have shown that:

1. If $\widehat{\mathfrak{R}}(x) > 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\mathfrak{R}}(x)$
2. If $\widehat{\underline{\mathfrak{R}}}(x) = 0$ then $\underline{\mathfrak{R}}(x) = \widehat{\underline{\mathfrak{R}}}(x)$

Since clause 2a of Proposition 2 implies that $\widehat{\underline{\mathfrak{R}}}(x)$ converges pointwise to $\widehat{\mathfrak{R}}(x)$ as the effort

[56] Note that the next self will face the high-effort-cost $\bar{\varepsilon}$ if the current self procrastinates when $\varepsilon_t = \underline{\varepsilon}$.

cost vanishes, the first bullet above can be rewritten as: if $\widehat{\underline{\mathfrak{R}}}(x) > 0$ then $\underline{\mathfrak{R}}(x)$ converges pointwise to $\widehat{\underline{\mathfrak{R}}}(x)$. This completes the proof of clause 2c of Proposition 2. ■

D Model Extensions and Robustness

D.1 The Distributional Effects of Policy

The top row of Figure 6 breaks down the consumption response to fiscal policy for our two benchmark cases. Each panel plots the consumption response to fiscal stimulus on impact as a function of pre-shock consumption. In the Exponential Benchmark (left) the consumption response is relatively evenly spread across the consumption distribution. In the Present-Bias Benchmark (right) this is not the case — the lowest consumption households experience a drastic consumption boom from fiscal policy. These households are borrowing-constrained, and sharply increase consumption following the liquidity shock.

The bottom row of Figure 6 breaks down the consumption response to monetary policy on impact. In the Exponential Benchmark, the largest consumption response comes from low-consumption households. These households are near $\underline{b}$, and implement a cash-out refinance following the rate cut. Thus, in the Exponential Benchmark the refinancing channel of monetary policy endogenously targets itself to constrained households.

Alternatively, in the Present-Bias Benchmark the low-consumption households respond very little to monetary policy on impact. The largest response now comes from households with intermediate levels of pre-shock consumption. Low-consumption households are constrained on impact, and because they procrastinate on refinancing they cannot immediately adjust consumption. Households with intermediate consumption are not liquidity constrained on impact. These households will typically be in either a refinancing region following the rate cut (in which case they expect to refinance in the next instant), or near a refinancing region (in which case they expect to refinance soon). In both cases, consumption smoothing implies that these households will increase consumption today in expectation of the cash-out refinance that they plan to conduct in the near future. This ability to smooth consumption relies on pre-existing liquidity at the time of the monetary policy shock, which households at $\underline{b}$ do not have.

As discussed in the main text, the key takeaway from Figure 6 is that present bias reverses the distributional consequences of fiscal versus monetary policy. In the Exponential Benchmark, monetary policy is an effective way to stimulate the consumption of low-consumption households. In the Present-Bias Benchmark, procrastination hampers the ability for monetary policy to stimulate the short-run consumption of constrained households. However, fiscal policy instead becomes highly effective at increasing these households' consumption.

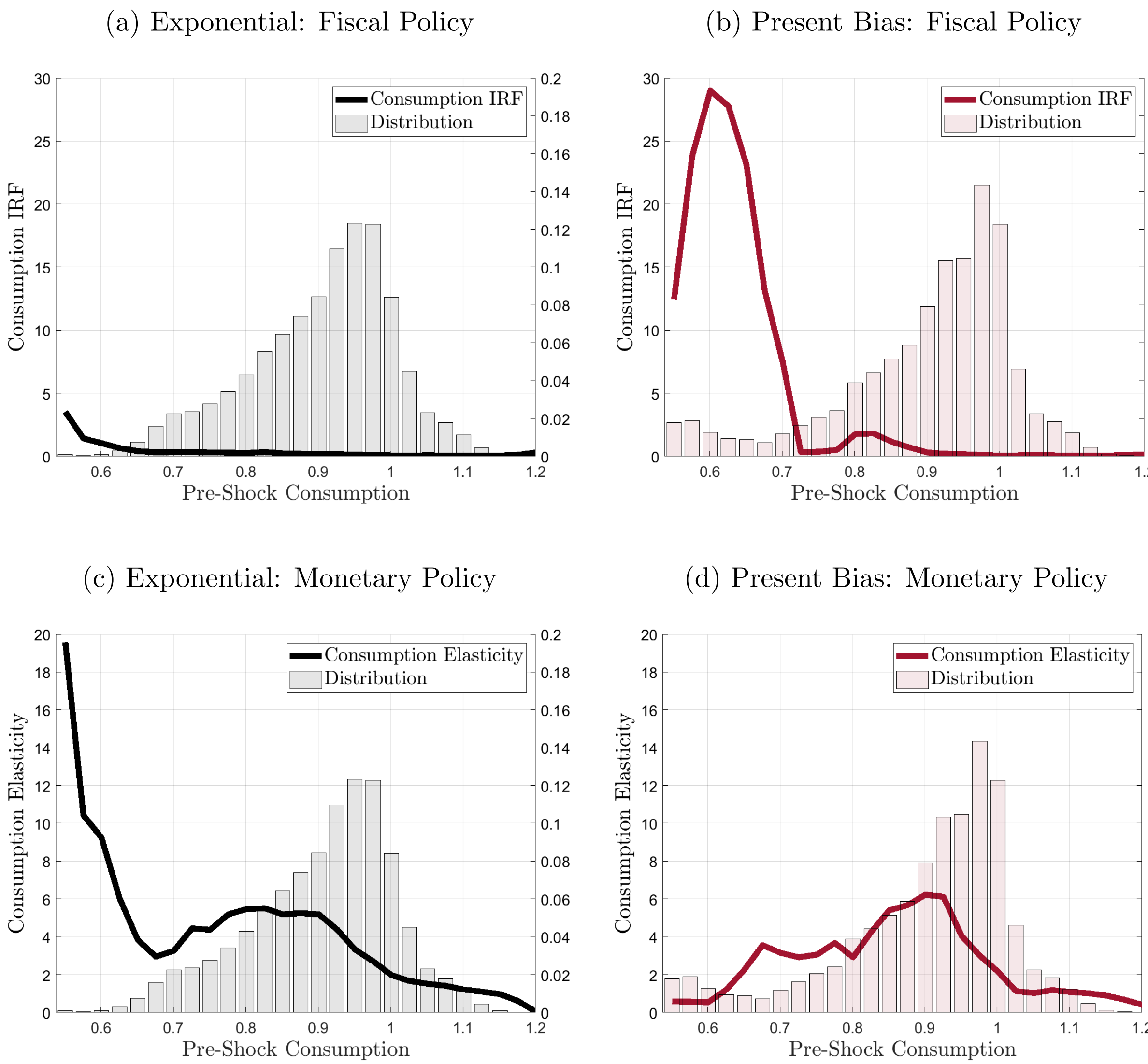


Figure 6: Heterogeneity Analysis.

Notes: This figure plots the on-impact consumption response to fiscal (top row) and monetary (bottom row) policy as a function of households' pre-shock consumption. The solid line plots the consumption response, and the bars show the distribution of households over pre-shock consumption.

D.2 Adding Aggregate House Price and Income Shocks

Macroeconomic stabilization policy typically responds to shocks hitting the economy. In particular, expansionary monetary and fiscal policy are often used in recessions. Recessionary shocks, by definition, correspond to a temporary decline in aggregate income. Recessions can also coincide with declining house prices. This section examines the ways in which shocks to house prices and aggregate income affect our results.

D.2.1 House Price Shocks

Present bias amplifies monetary policy by producing a consumption boom driven by home-equity extractions. However, this cash-out channel of monetary policy is limited to households with enough home equity to actually conduct a cash-out refinance. This makes monetary policy sensitive to house price shocks, which can quickly create or destroy home equity.

Understanding the effect of house price shocks on macroeconomic policy is particularly important when considering the three most recent recessions: the COVID-19 Recession, the Great Recession, and the Early 2000s Recession. Home prices collapsed during the Great Recession, but boomed throughout the Early 2000s Recession and the COVID-19 Recession.

Our baseline analysis in Section 5 corresponds to the case where home prices are stable before the cut to interest rates. To examine the effect of house price shocks we exogenously shock the home value h by $\pm$ 25%.[57] The negative 25% shock corresponds to the Great Recession. The positive 25% shock corresponds to the early 2000s, where house prices boomed while the Federal Reserve adopted a multi-year path of low interest rates. Second, policymakers immediately respond to this house price shock with either monetary or fiscal policy. As in Section 5, the monetary policy experiment is a rate cut from 1% to 0%, and the fiscal policy experiment is a $1,000 liquid transfer.

Figure 7 plots the consumption response to monetary policy after a negative (left panel) or positive (right panel) 25% shock to house prices. The solid curves plot the consumption response to monetary policy in the shocked economy. For reference, the transparent lines mark the baseline case in Figure 5. Though the magnitude of the consumption response is sensitive to house price shocks, our main result that present bias amplifies the consumption response to monetary policy holds in both cases.

The left panel of Figure 7 shows that monetary policy is significantly weakened by the collapse in house prices. The negative shock wipes out home equity and prevents many homeowners from refinancing. This result is consistent with recent research documenting that negative house price shocks undermined monetary policy following the Great Recession (e.g., Beraja et al., 2019).

The right panel of Figure 7 plots the positive 25% shock case. Now, the consumption boom generated by the rate cut is even larger than in the baseline case. The positive shock generates additional home equity, strengthening the cash-out channel of monetary policy. This is consistent with the boom in home-equity extractions that was observed in the mid-2000s (Khandani et al., 2013; Bhutta and Keys, 2016).

It is also important to explore whether house price shocks affect fiscal policy. Figure 8 plots the consumption response to fiscal stimulus in the negative (left) and positive (right)

[57] The economy starts in the "steady state" before the shock to h. For simplicity we assume that the shock is permanent. However, we only study the short-run consumption response to monetary and fiscal policy.

(a) -25% House Price Shock (b) +25% House Price Shock

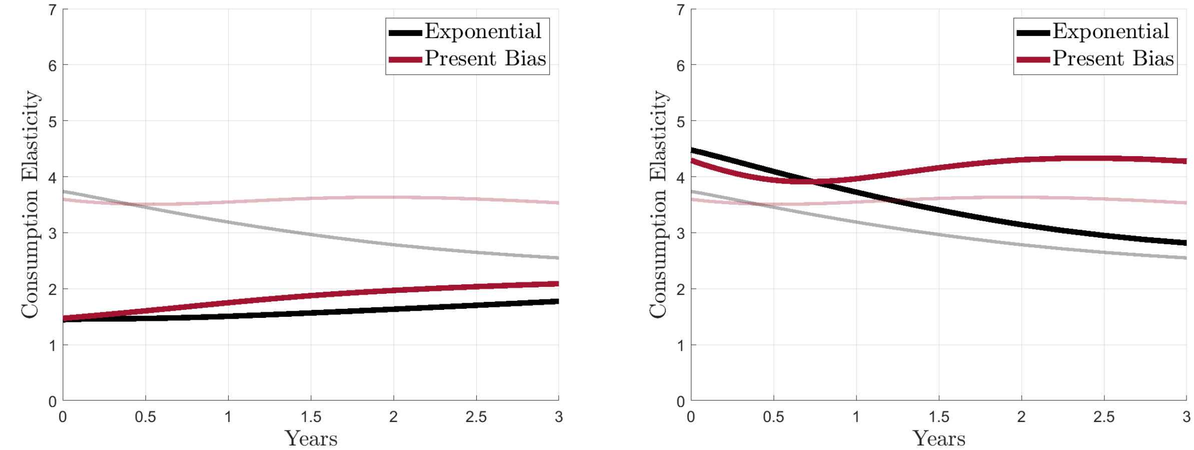


Figure 7: Monetary Policy and House Price Shocks.

Notes: This figure plots the consumption response to an interest rate cut that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot the baseline case in Figure 5, and are included for reference.

shock cases. For both positive and negative house price shocks, we find that present bias continues to strongly amplify the consumption response to fiscal policy.

(a) -25% House Price Shock (b) +25% House Price Shock

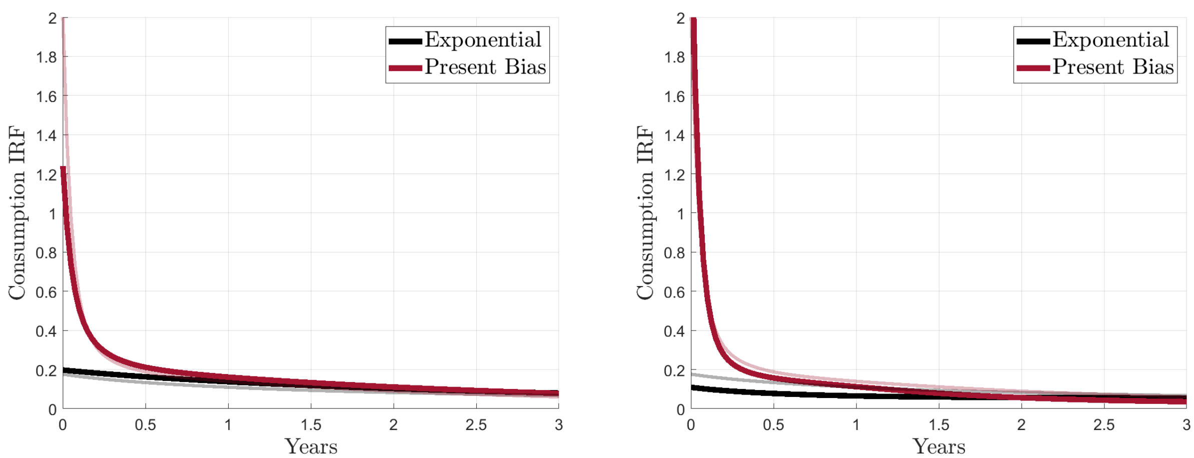


Figure 8: Fiscal Policy and House Price Shocks.

Notes: This figure plots the IRF of aggregate consumption to a $1,000 fiscal transfer that immediately follows a house price shock of -25% (left) or +25% (right). The transparent lines plot the baseline case in Figure 4, and are included for reference.

D.2.2 Income Shocks

We also evaluate the effect of recessionary income shocks on monetary and fiscal policy. We generate a temporary 5% fall in aggregate income by shifting a share of high-income households to the middle-income state, and a share of middle-income households to the low-income state.[58] Policymakers immediately respond to this recessionary income shock with either monetary or fiscal policy.

The left panel of Figure 9 plots the consumption response to monetary policy and the right panel plots the consumption response to fiscal policy. Though this recessionary income shock leads to an immediate decline in aggregate consumption, Figure 9 shows that the subsequent consumption response to monetary and fiscal policy is almost identical to the baseline results in Section 5. This is because liquidity, not income, is the key driver of the consumption response to these policies.

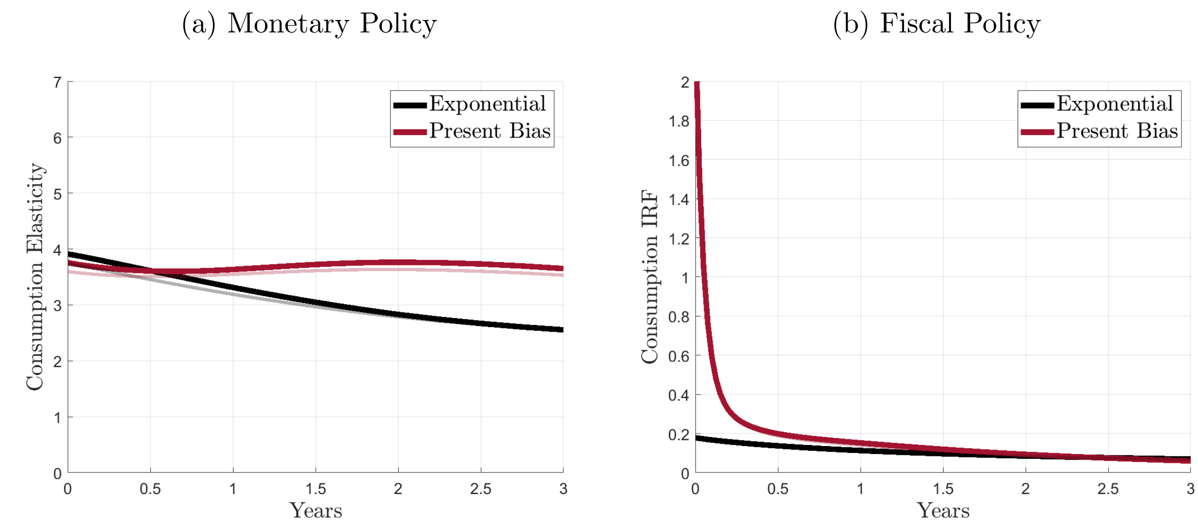


Figure 9: Fiscal and Monetary Policy Following a Negative Income Shock.
Notes: This figure plots the consumption response to monetary (left) and fiscal (right) policy that is implemented immediately following a transitory 5% decline in aggregate income.

D.3 A Call to ARMs?

In order to reflect the typical features of the U.S. mortgage market our paper studies macroeconomic stabilization policy under the assumption that households have fixed-rate mortgages (FRMs). Since the 2007-08 Financial Crisis, many economists have argued that downwardly

[58]We shift 9.5 percentage points of high-income households to middle income, and 9.5 percentage points of middle-income households to low income. The share of households across low, middle, and high income goes from 31%, 39%, 31%, respectively, to 40%, 39%, 21%.

flexible mortgages, such as adjustable-rate mortgages (ARMs), improve macroeconomic stability (e.g., Eberly and Krishnamurthy, 2014; Andersen et al., 2020; Campbell et al., 2021; Guren et al., 2021). If the monetary authority cuts interest rates in a recession, ARM payments automatically adjust downward, thereby increasing households' disposable income. This creates a fast and direct transmission of monetary policy to household balance sheets.

Section 5.2 shows that refinancing procrastination slows down the transmission of monetary policy. A natural policy question is whether such procrastination implies that downwardly flexible mortgages would improve the potency of monetary policy in our model with present bias.

To study this question, we re-solve our Present-Bias Benchmark under the assumption that all households have ARMs instead of FRMs. The model from Section 2 remains the same, except that mortgage rate r_t^m automatically adjusts with interest rate r_t. We also recalibrate the mortgage wedge from 1.7% to 0.9%.[59] This corresponds to the average difference between a 5/1 hybrid ARM and the 10-year treasury yield from 2015 – 2017.

In our monetary policy experiment with ARMs we cut r_t from 1% down to -1%. This doubles the magnitude of the rate cut from our earlier FRM analysis, where interest rates were reduced from 1% to 0%. This change ensures comparability across the two experiments, since ARMs are more sensitive to monetary policy than long-duration FRMs.[60]

We find that the consumption response to monetary policy is almost identical with ARMs versus FRMs (see Appendix Figure 17 for details). This result highlights that there is a tradeoff between ARMs and FRMs that arises when households are present biased. On the one hand, ARMs produce a fast pass-through of monetary policy that applies to all mortgage holders. This is particularly important for constrained households who procrastinate on refinancing. On the other hand, ARMs reduce the liquidity injection features of monetary policy because ARMs imply that households no longer need to refinance when the interest rate is cut. Present bias generates a powerful cash-out channel of monetary policy, but this channel is stifled by ARMs. Overall, the stimulative effect of ARMs accrues quickly and to all households, but is small. The stimulative effect of FRMs accrues slowly and only to households who plan to refinance, but is large. These two effects are of similar magnitude in our model.

An important factor explaining these offsetting effects is the large size difference between ARM payment adjustments versus home-equity extractions. Recall that the home value is calibrated to 3.29 times permanent income, and the average LTV ratio is roughly 0.5. With ARMs, a 2% reduction in mortgage rates is therefore equivalent to roughly a 3.5% increase

[59]FRMs are typically more expensive than ARMs because FRMs lead to lower payments if interest rates rise, and come with the option to refinance if rates fall. The borrower has to pay ex-ante for this insurance.

[60]Empirically there is roughly a 50% pass-through from monetary policy to the 30-year mortgage interest rate. See Gertler and Karadi (2015), Gilchrist et al. (2015), and Eichenbaum et al. (2022) for details.

in income for the average household (roughly \$3,500 per year). Alternatively, with FRMs the typical cash-out is about 35% of permanent income (\$35,000) – an order of magnitude larger than the typical ARM payment reduction – and present-biased households have large MPCs out of these liquidity injections. Such large cash-outs are consistent with the data. The average cash-out amount from 1999–2010 was \$40,000 (Bhutta and Keys, 2016), and there is little evidence that the majority of these home-equity extractions are kept as savings (Greenspan and Kennedy, 2008; Bhutta and Keys, 2016). However, ARMs prevent this large stock of dry powder from ever being ignited.

While we do not find that ARMs increase the power of monetary policy, we note that our model is too stylized to make rigorous quantitative claims. Our analysis also assumes that house prices are fixed. Negative house price shocks, such as those observed following the financial crisis, can significantly reduce the cash-out channel of FRMs (see Section D.2.1). Our results nevertheless highlight a new tradeoff between FRMs and ARMs that policymakers should be aware of when considering different mortgage contract designs. Our results also suggest that monetary policy is most powerful if mortgage contracts feature a fast pass-through (like ARMs) while simultaneously allowing for cash-outs (like FRMs). Appendix Figure 18 shows just how powerful monetary policy can be in a FRM environment if policymakers are able to reduce procrastination concurrently with a monetary expansion.

In addition to being stylized this section ignores important welfare considerations. For example, FRMs produce a consumption boom by encouraging overconsumption out of home equity. Monetary policy also appears to be more equitable under ARMs than FRMs. In Section D.1 we showed that low-consumption households procrastinate on refinancing a FRM, whereas ARMs provide immediate payment relief to low-consumption households.[61]

D.4 Alternate Calibrations: Intermediate Cases

As mentioned in Section 4.2, we also examine various Intermediate Cases that differentially allow for present bias and/or refinancing inertia. We introduce four additional cases here which, when combined with the two Benchmark calibrations in the main text, mean that we study six calibration cases overall. These six calibration cases are summarized in Table 5 below, and we describe them more fully in the next paragraph. Overall, we consider three types of refinancing inertia (no inertia, rational inertia, and procrastination), and two types of time preferences (exponential preferences and present-biased preferences). In Table 5, the cases marked Benchmark are the cases that we already studied in the main text, and the cases marked IC are additional intermediate cases that we present below.

[61] As discussed in Harris and Laibson (2013) and Maxted (2023), the IG model is well-suited for studying welfare because it features a single welfare criterion despite preferences being dynamically inconsistent. As mentioned in the conclusion, we view welfare analyses as an interesting pathway for future research.

	Exponential	Present Bias
No Refinancing Inertia	**Benchmark**	IC: $\varepsilon_t \equiv 0$
Rational Inertia	IC: $\bar{\varepsilon} \to \infty$	IC: $\bar{\varepsilon} \to \infty$
Procrastination	IC: $\beta \uparrow 1$	**Benchmark**

Table 5: Summary of Calibration Cases.

Notes: The case with exponential time preferences and no refinancing inertia is our Exponential Benchmark. The case with present-biased time preferences and procrastination is our Present-Bias Benchmark. The other four cases marked IC are intermediate cases that we examine here.

Starting with the first row, the Exponential Benchmark studied in the main text has exponential time preferences and no refinancing inertia. Moving to the second column of that row, our first intermediate case is a model with present bias ($\beta < 1$) but *without* refinancing inertia, which we achieve by setting all effort costs equal to zero ($\varepsilon_t \equiv 0$). Note that present bias without refinancing inertia can also be viewed as present bias under full sophistication (further details in Appendix D.5).

In the second row, we study an alternate setup that generates what we refer to as rational inertia. Specifically, we break Assumption 2 that effort costs are vanishingly small, and instead take the typical effort cost $\bar{\varepsilon} \to \infty$ so that households *optimally* do not refinance in the high-cost state (since the effort cost of doing so is exceedingly onerous). Below, we study this alternate setup of rational inertia both for exponential and present-biased households.

In the third row, we return to our original assumption that effort costs are vanishingly small (Assumption 2). While we cannot generate slow refinancing in this case when $\beta = 1$, any amount of naive present bias is sufficient to generate procrastination.[62] Hence, the case in column one of the third row is not a true "exponential" case. Instead, we numerically set $\beta = 0.999$: by setting $\beta < 1$ we introduce procrastination, but this tiny amount of present bias also has essentially no effect on households' consumption choices. Finally, our Present-Bias Benchmark studied in the main text features both present bias and procrastination.

In all four of the intermediate cases presented below, we recalibrate the discount function to fit the same steady state targets as the Benchmark calibrations. Specifically, for all cases we recalibrate ρ to fit the LTV moment, and for the present-bias cases we also recalibrate β to fit the credit card borrowing moment. The purpose of these intermediate cases is not necessarily to be realistic, but rather to provide various stepping stones that help the reader traverse between the Exponential Benchmark and the Present-Bias Benchmark. Additionally, the comparison between the Rational Inertia cases and the Procrastination cases highlights the extent to which naive present bias can generate *unexpected* refinancing inertia, which differentiates present-bias-driven procrastination from the sorts of "rational"

[62] I.e., there is a discontinuity in the limit as $\beta \to 1$; with any amount of naive present bias, the effort costs will always be small enough under Assumption 2 that households choose to procrastinate.

refinancing inertia that could arise under exponential time preferences.

D.4.1 Intermediate Cases: Exponential Discounting

We start by presenting the intermediate cases with exponential discounting. The left panel of Figure 10 plots the consumption response to fiscal policy, and the right panel plots the consumption response to monetary policy. Both panels plot the two Benchmark calibrations (opaque lines), and the two new intermediate cases (transparent lines) in order to provide stepping stones between the two benchmarks. Solid lines correspond to No Refinancing Inertia, dashed lines to Rational Inertia, and dotted lines to Procrastination.

(a) Fiscal Policy: Exponential Cases (b) Monetary Policy: Exponential Cases

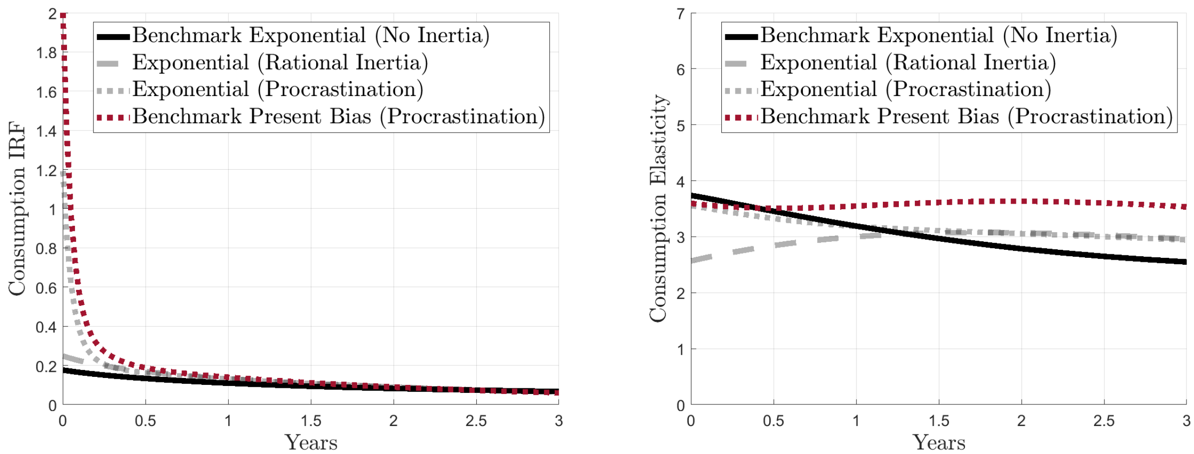


Figure 10: Fiscal and Monetary Policy with Exponential Intermediate Cases.
Notes: This figure plots the consumption response to fiscal (left) and monetary (right) policy for the two Benchmark calibrations and the two Intermediate Cases with exponential preferences (see Table 5).

Starting with fiscal policy, the consumption response is comparable in the Exponential Benchmark and the exponential case with rational inertia, but is much larger in the cases with procrastination. This relates to the share of borrowing-constrained households. In the Exponential Benchmark, there are very few households at $\underline{b}$ because they refinance as soon as they hit the constraint. In the case with rational inertia, households know that they will be slow to refinance, so they endogenously refinance before hitting $\underline{b}$ if they are in a stochastic low-cost state. Only in the case where households *unexpectedly* procrastinate on refinancing do we get a buildup of households at $\underline{b}$ that generates a sizable short-run consumption response.

Turning to monetary policy, the notable feature is that we see less of an on-impact consumption response when households are rationally inertial. In this case, households are fully aware that they will be slow to refinance following the rate cut, which makes them more cautious about increasing consumption before that refinance is actually enacted.

D.4.2 Intermediate Cases: Present-Biased Discounting

Next, Figure 11 presents the intermediate cases with present-biased discounting.

(a) Fiscal Policy: Present-Bias Cases

(b) Monetary Policy: Present-Bias Cases

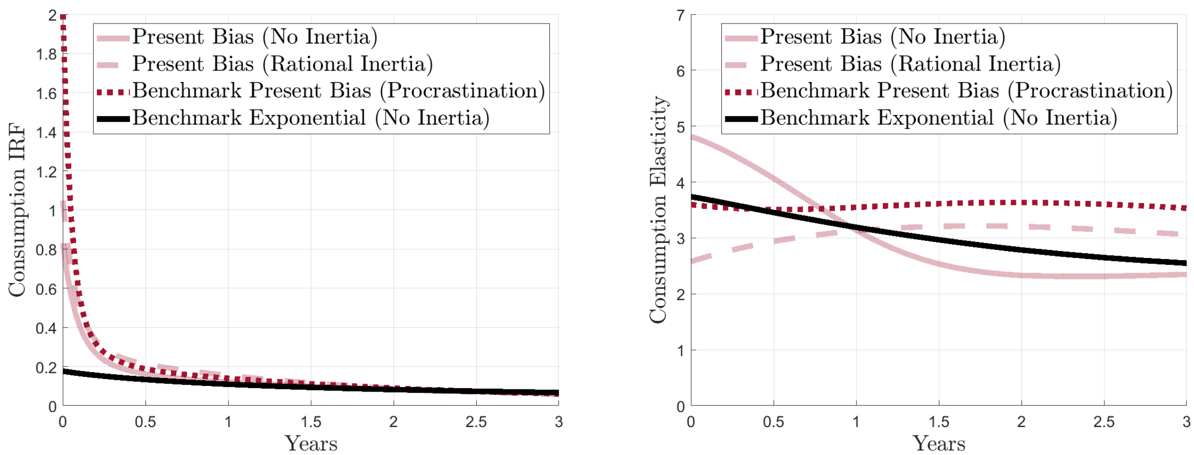


Figure 11: Fiscal and Monetary Policy with Present-Bias Intermediate Cases.
Notes: This figure plots the consumption response to fiscal (left) and monetary (right) policy for the two Benchmark calibrations and the two Intermediate Cases with present-biased preferences (see Table 5).

Starting with fiscal policy, we first see that present bias generates a larger consumption response than the Exponential Benchmark in all cases. Second, we again see that the consumption response is broadly comparable for the two intermediate cases of no inertia and rational inertia, but jumps up in the Present-Bias Benchmark with procrastination. The intuition here is broadly similar to our discussion of the exponential cases, where unexpected procrastination is key to getting a large short-run consumption response.

Turning to monetary policy, we again see a smaller on-impact consumption response in the case of rational inertia, similar to the exponential case with rational inertia described above. For the present-bias case with no inertia, we now see a larger short-run consumption response, but also one that decays more quickly. The intuition for this no-inertia case is largely similar to the "Present Bias 1Q No Proc." case in Figure 5, except that the initial response here is somewhat more mild because there are fewer constrained households ex-ante (since households never procrastinate on refinancing).

D.5 Generalization to (Partial and Full) Sophistication

To this point we have assumed that households are fully naive about their present bias. One benefit of our naiveté assumption is that it is theoretically and computationally easy to handle — starting from a model without present bias, it is relatively simple to back out the behavior of households with naive present bias (see Propositions 1 and 2). As such, naive present bias provides a bridge for the broader macroeconomics literature.

The case with (partial or full) sophistication is a much more complicated theoretical object. Let β^E denote the short-run discount factor that the current self expects all future selves to have. Full naiveté means that $\beta^E \equiv 1$, whereas (partial) sophistication sets $\beta^E \in [\beta, 1)$ so that the current self is at least partially aware that future selves will also face a self-control problem. Sophistication therefore implies that the current self is aware that preferences are dynamically inconsistent, thus making behavior the equilibrium outcome of a dynamic game. Despite being more complicated theoretically, the technical advances of Harris and Laibson (2013) and Maxted (2023) imply that solutions to models with sophistication are still available. We now utilize these advances to extend our analysis to the case of sophistication.

In this appendix we ask how our results vary with sophistication, and we provide two perspectives. On the one hand, we show that a model with partial sophistication can be recalibrated with a different β so that it produces household-level behavior that is analogous to that of full naiveté. That is, while we assumed full naiveté in the main text because it is theoretically and computationally easy to handle, our analysis is robust to all but the limiting case of complete sophistication ($\beta^E = \beta$).[63]

On the other hand, one could also ask about comparative statics with respect to partial sophistication (while not recalibrating any other model parameters). In this case, we show that households' propensity to procrastinate is (weakly) decreasing in their sophistication.

D.5.1 The Household Balance Sheet: A Slight Modification

Formalizing this analysis is, admittedly, complex, and we provide only a heuristic analysis here. The material presented below likely requires familiarity with Harris and Laibson (2013) and Maxted (2023), whose equilibrium construction techniques we adopt herein.

We begin by slightly modifying the model of the household balance sheet in Section 2. Note that this modification is not necessary. However, we make it so that we can use the theoretical results in Maxted (2023) to study the effects of sophistication in closed form, which we believe provides clearer economic insights.[64]

To utilize the results in Maxted (2023), we need to respecify the model so that households always remain in the interior of the liquid wealth space rather than occasionally facing a

[63] As we detail further below, the case of full sophistication does not generate procrastination under Assumption 2 that effort costs are vanishingly small. Intuitively, without some scope for incorrect expectations, we cannot use vanishingly small effort costs to generate non-vanishing spells of procrastination.

[64] Without this modification one could still numerically solve the model under sophistication by using the $\hat{u}$ technology developed in Harris and Laibson (2013), though numerical implementation could be difficult.

binding hard borrowing constraint at $\underline{b}$ (see the "Key Assumption" of Maxted (2023)).[65] To do so we make two (realistic) changes, as presented below.

First, we now allow households to borrow beyond the ad-hoc limit of $\underline{b}$ to a lower limit of $\underline{\underline{b}} < \underline{b}$. For all debt beyond $\underline{b}$, we assume that the household must incur a very large borrowing wedge of $\omega^{\uparrow} \gg \omega^{cc}$. For example, $\omega^{\uparrow} \approx 400\%$ is a typical payday loan interest rate (Lee and Maxted, 2023), but one could imagine $\omega^{\uparrow}$ to be taken even larger. Following the "Key Assumption" of Maxted (2023), we impose in Assumption 3 below that $\omega^{\uparrow}$ is made large enough that households always keep their liquidity strictly above $\underline{\underline{b}}$ in equilibrium.[66]

Second, we introduce a small delay to refinancing. In the model in the main text we assumed for simplicity that there are no delays to refinancing; refinancing occurs instantly after a household completes its application. Without refinancing delays, households that expect to refinance in the next instant will not care about borrowing at interest rate $\omega^{\uparrow}$. So long as households only expect their borrowing to persist for one instant, the rate at which they borrow will not affect their overall wealth. Thus, we additionally assume that, after a household fills out its refinancing paperwork, the new mortgage only closes at a Poisson rate denoted λ^C.[67] Such delays are realistic, as mortgage underwriting often takes over a month.

With these updates, we now have a model that can be calibrated so that households will remain in the interior of their liquid wealth space. That is, we assume following Maxted (2023):

Assumption 3 *The model is calibrated such that borrowing limit $\underline{\underline{b}}$ never binds along the equilibrium path. Formally, if $b_0 > \underline{\underline{b}}$ then $b_t > \underline{\underline{b}}$ for all $t \geq 0$.*

We maintain Assumption 3 throughout the remainder of Appendix D.5, which will allow us to utilize the equilibrium construction results of Maxted (2023) in this modified model (while we focus here on an equilibrium construction, uniqueness is not guaranteed).

We also briefly mention one other complexity, which is that a full-sophisticate's refinancing policy function takes the form of a mixed-strategy equilibrium where the household refinances probabilistically (e.g., O'Donoghue and Rabin, 2001). However, this added complexity effectively drops out under Assumption 2 that $\bar{\varepsilon}$ is vanishingly small, since in this case

[65] The basic issue which these modifications aim to address is that sophisticated present bias interacts with binding hard borrowing constraints like $\underline{b}$, because present-biased agents value such constraints as a commitment device of sorts that limits the overconsumption of future selves (see equation (3)). This new interaction – which does not arise under naiveté – limits our ability to directly map between sophistication and naiveté (see Maxted (2023) for a fuller discussion). Note too that had we instead kept this interaction effect, it would have led to *larger* consumption discontinuities at $\underline{b}$ and hence *larger* MPCs at $\underline{b}$ under sophistication. In other words, this interaction effect would have led to *larger* differences relative to exponential discounting.

[66] Note that this will always be possible once we introduce refinancing delays (next paragraph). The basic argument here is that as $\omega^{\uparrow}$ increases, then households must keep their debt closer and closer to $\underline{b}$ in order to avoid the possibility of a zero-consumption state (and hence $-\infty$ utility).

[67] Specifically, the household pays the effort cost up front, but does not finalize its new mortgage terms nor receive the cash out (or pay the cash in) until the mortgage closes (details in Appendix D.5.3).

the probability of refinancing over any discrete interval of time converges to one. Further details are left to Appendix D.5.4.

D.5.2 The Effect of Sophistication on Policy Functions and Results

We can now generalize Propositions 1 and 2 to allow for sophistication. To avoid complicating the exposition with additional technicalities that arise when households are sophisticated, we state the main points here and then add details later in Appendices D.5.3 and D.5.4.

We continue to use hat-notation to denote the policy functions of the "comparable household" that has $\beta = 1$. However, in this more general setup that allows for sophistication, that comparable household is an otherwise-identical household that has $\beta = 1$ *and* faces a refinancing effort cost of $H^E \times \varepsilon_t$ (rather than just ε_t), where $H^E = \left(\frac{\gamma-(1-\beta^E)}{\gamma}\right)^{-\gamma}$. The intuition behind this rescaling is that, once agents are at least partially sophisticated ($\beta^E < 1$), they are aware that their self-control problems will cause them to act as if they face a higher hurdle rate to refinance.[68] Further details are provided in Appendix D.5.3 below. Note that this additional step of rescaling ε_t was not needed in the main text since we assumed that $\beta^E \equiv 1$ (full naiveté) so that $H^E = 1$. Nonetheless, this rescaling effect remains trivial here, due to Assumption 2 that effort costs are vanishingly small.

Using this agent with $\beta = 1$ and refinancing effort costs of $H^E \times \varepsilon_t$ as a point of comparison, we can start by characterizing the consumption function of present-biased agents using a result from Maxted (2023):

Proposition 3 (Continuous Control) *For all $b > \underline{\underline{b}}$, the household sets*

$$c(x) = \left(\frac{\beta^E}{\beta}\right)^{\frac{1}{\gamma}} \frac{\gamma}{\gamma - (1-\beta^E)} \widehat{c}(x),$$

where $\widehat{c}(x)$ is the consumption policy function of an exponential $\beta = 1$ household but with effort costs of $H^E \times \varepsilon_t$ in place of ε_t where $H^E = \left(\frac{\gamma-(1-\beta^E)}{\gamma}\right)^{-\gamma}$.

Proof. See Maxted (2023), with additional details in Appendices D.5.3 and D.5.4. ■

Proposition 3 is just like Proposition 1 in the main text, except that the consumption scaling factor is generalized to allow for sophistication. For our purposes, the key implication of Proposition 3 is the following corollary, which implies that there is a limiting observational equivalence between the consumption decisions of sophisticates and naifs:

[68] Accordingly, this perceived hurdle rate increases as the agent gets more sophisticated, i.e. as β^E decreases.

Corollary 2 *The consumption function of a naif with short-run discount factor β converges pointwise (as the effort cost vanishes) to the consumption function of a (partial) sophisticate with perceived $\beta^E \in [\beta, 1)$ and a true short-run discount factor of $\beta' = \beta\beta^E \left(\frac{\gamma}{\gamma-(1-\beta^E)}\right)^{\gamma}$. This means that there is an observational equivalence between the consumption policy of naifs and (partial) sophisticates in the limit as $\bar{\varepsilon} \to 0$.*

Proof. This corollary would follow directly from Proposition 3, except that the consumption function $\widehat{c}(x)$ changes with β^E (since we use hat-notation to denote an otherwise-identical household that has $\beta = 1$ *and* faces a refinancing effort cost of $H^E \times \varepsilon_t$, where $H^E = \left(\frac{\gamma-(1-\beta^E)}{\gamma}\right)^{-\gamma}$ depends on β^E). However, Assumption 2 that $\bar{\varepsilon} \to 0$ means that the effect of β^E on $\widehat{c}(x)$ becomes arbitrarily small. ■

Next, we show that our results on refinancing procrastination in Proposition 2 also continue to hold for all but the limit case of full sophistication.

Proposition 4 (Optimal Stopping)

1. *Adjustment targets m' and b' are independent of β. Thus, $m'(x) = \widehat{m}'(x)$, $b'(x) = \widehat{b}'(x)$, $\underline{m}'(x) = \widehat{\underline{m}}'(x)$, and $\underline{b}'(x) = \widehat{\underline{b}}'(x)$ for all x. These adjustment-target functions may still vary with β^E, though this effect vanishes under Assumption 2. This effectively means that neither β nor β^E affects the adjustment targets.*

2. (a-1) *For $\beta = 1$, the refinancing policy function $\mathfrak{R}(x)$ converges pointwise to $\underline{\mathfrak{R}}(x)$ as the effort cost vanishes. This effectively means that the $\beta = 1$ household's mortgage adjustment behavior does not depend on the state of the effort cost.*

   (a-2) *For $\beta < 1$ and $\beta^E = \beta$ (full sophistication), it is effectively the case that the fully sophisticated household's mortgage adjustment behavior does not depend on the state of the effort cost (details are provided in Appendix D.5.4).*[69]

   (b) *For $\beta < 1$, $\beta^E \in (\beta, 1]$, and $\varepsilon = \bar{\varepsilon}$, $\mathfrak{R}(x) = 0$ for all x. This means that for all but the limit of full sophistication, the present-biased household procrastinates and will not adjust its mortgage when $\varepsilon = \bar{\varepsilon}$.*

   (c) *For $\beta < 1$, $\beta^E \in (\beta, 1]$, and $\varepsilon = \underline{\varepsilon}$, $\underline{\mathfrak{R}}(x)$ converges pointwise to $\widehat{\underline{\mathfrak{R}}}(x)$ as the effort cost vanishes. This effectively means that the present-biased household does not procrastinate when $\varepsilon = \underline{\varepsilon}$.*

Proof. The first step of this proof is to show that full sophisticates do not procrastinate (clause 2a-2). This is an important first step, since partial sophisticates believe themselves

[69] For now, a formal statement of this result is complicated by the fact that a sophisticate's refinancing policy function takes the form of a mixed-strategy equilibrium (details in Appendix D.5.4).

to be fully sophisticated (with a perceived short-run discount factor of β^E) starting next instant. Though we leave details to Appendix D.5.4, the intuition is fairly straightforward. Because a full sophisticate is aware of future selves' behavior, the current self would rather refinance now at cost $\bar{\varepsilon}$ than let future selves procrastinate for long enough that the cost of such procrastination amounts to more than $\bar{\varepsilon}$ of effort. Under Assumption 2 that $\bar{\varepsilon}$ is vanishingly small, this effectively implies that a fully sophisticated household will never procrastinate (conversely, this argument also implies that if $\bar{\varepsilon}$ was not vanishingly small then there would still be scope for a fully sophisticated household to procrastinate).

Next, we turn to clause 1 that the adjustment targets are independent of β and (effectively) β^E. The proof here is in some sense much more difficult than under full naiveté in Appendix C.2, because it is no longer the case that the continuation-value function of a (partially) sophisticated household is identical to that of a $\beta = 1$ household. But, in Appendix D.5.3 we show that the continuation-value function of a (partially) sophisticated household is still an affine transformation of the continuation-value function of a $\beta = 1$ household, building on arguments from Harris and Laibson (2013) and Maxted (2023). Thus, it is still the case that the m' and b' that maximize the value function of a present-biased household will maximize that of the corresponding $\beta = 1$ household, and vice versa. The one slight complication to this proof is that the corresponding $\beta = 1$ household denoted by the hat-notation faces a refinancing effort cost of $H^E \times \varepsilon_t$ instead of just ε_t. However, this effect is trivial under Assumption 2 that ε_t is vanishingly small.

Third, we discuss clause 2b regarding the procrastination decision of a less-than-fully-sophisticated household when $\varepsilon = \bar{\varepsilon}$. The basic intuition follows from the fact that so long as a household is at least partially naive, then they expect to be less present biased next instant than they are now (i.e., $\beta^E > \beta$). Accordingly, any time that a household with a short-run discount factor of β would want to refinance, a less-present-biased household with a short-run discount factor of $\beta^E > \beta$ would *also* want to refinance. But then, the current self will always choose to procrastinate (again for one instant in expectation) in order to push the effort cost off into the future. In short, we do not need full naiveté to generate procrastination. Rather, all that is needed to generate procrastination is for the current self to think that the next self is less present biased than they are, which is the case for all but full sophistication.

More formally, a partially naive household will consider refinancing only if $\beta v(x) \leq \beta v^*(x) - \bar{\varepsilon}$, or equivalently $v(x) \leq v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$. However, recall that the household also expects that next instant it will have a short-run discount factor of $\beta^E > \beta$. And, a sophisticated household with a short-run discount factor of β^E will refinance whenever $v(x) < v^*(x) - \frac{1}{\beta^E}\bar{\varepsilon}$. Thus, the household perceives that $v(x) \geq v^*(x) - \frac{1}{\beta^E}\bar{\varepsilon}$, which also implies that the current self will never consider refinancing now (since $v^*(x) - \frac{1}{\beta^E}\bar{\varepsilon} > v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$).

Finally, the proofs of clauses 2a-1 and 2c are similar to those in Appendix C.2. ■

Robustness of Results to Partial and Full Sophistication. In the modified model described above in Section D.5.1, we are now prepared to present our main results on the effects of sophistication. Namely, as the effort cost vanishes: *(i) there is a limiting observational equivalence between the behavior of partial sophisticates and naifs, and (ii) in the case of full sophistication ($\beta^E = \beta$), there is a limiting observational equivalence between the behavior of full sophisticates and naifs without any refinancing inertia.* These two statements follow from Corollary 2 and Proposition 4.

Explaining clause 1 in more detail, Corollary 2 implies that the consumption policy function generated by discount-function parameters β and ρ under full naiveté is analogous to that produced by discount-function parameters β' and ρ under partial sophistication. Similarly, Proposition 4 implies that the refinancing policy functions generated by discount-function parameters β and ρ under full naiveté are analogous to those produced by discount-function parameters β' and ρ under partial sophistication. What this tells us is that, if we were to assume that households were partially sophisticated instead of fully naive, then by calibrating the discount function with parameters β' and ρ we would still hit the same LTV moment, the same credit card borrowing moment, and generate the same responses to fiscal and monetary policy as in the full-naiveté case with discount-function parameters β and ρ.

The argument for clause 2 is similar, except that full sophisticates do not procrastinate. Thus, the full-sophistication case can be calibrated to produce behavior that is analogous to the case of naive present bias with no refinancing inertia.

As a final step, we note that the two clauses just discussed only apply in the modified model of Section D.5.1, not the main-text model of Section 2. However, we conjecture that there exist calibrations of the modified model studied here in which the naif's equilibrium behavior in the modified model is comparable to their equilibrium behavior in our main-text model. In particular, by taking $\omega^{\uparrow} \to \infty$ we can approximate the main-text model with a hard constraint at $\underline{b}$. Then, by taking $\lambda^C \to \infty$ we minimize refinancing delays, again as in the main-text model. Under such calibrations, our analysis would then imply that a model with partial sophistication can provide comparable predictions about LTVs, credit card borrowing, and consumption responses to fiscal and monetary policy as those in the full-naiveté case studied in the main text. Similarly, the full-sophistication case can be viewed as an alternate motivation for the intermediate case of naive present bias without refinancing inertia that was already presented in Appendix D.4.

Generalization: Comparative Statics for β^E. Proposition 4 implies that procrastination is unaffected by β^E except for the limit case of full sophistication. We emphasize, however, that this is partially due to our simple two-state effort cost in Assumption 1, which can be generalized for additional richness. We summarize one possible extension in the following remark:

Remark 2 *In Proposition 4 above, we get the stark result that households procrastinate homogeneously for all but the limit case of full sophistication. This result can be modified for additional richness by generalizing the structure of effort costs in Assumption 1.*

For example, consider the case where the low-cost state $\underline{\varepsilon}$ is stochastic, such that conditional on drawing an instantaneous low-cost state, the effort cost in that state is given by $\tilde{\underline{\varepsilon}} \sim Uniform[0, \bar{\varepsilon}]$. In this case, households will procrastinate until they draw a low-cost state in which $\tilde{\underline{\varepsilon}} < \frac{\beta}{\beta^E}\bar{\varepsilon}$.[70] *Since $\frac{\beta}{\beta^E}\bar{\varepsilon}$ is increasing as households become more sophisticated, this implies that procrastination decreases as agents become more sophisticated.*

D.5.3 Additional Details: Value Functions with Sophistication

Turning to adding further technical details to the arguments above, we now present the fuller system of value functions for fully and partially sophisticated households. Similar to the main text, we will use $v(x)$ to denote a household's (perceived) continuation-value function, and $w(x)$ to denote its (perceived) current-value function.

Step 1: Defining Continuation-Value Function $v(x)$ for Full Sophisticates. To implement the equilibrium techniques in Harris and Laibson (2013) and Maxted (2023), we start by expressing the continuation-value function $v(x)$ for a fully sophisticated household. As will be discussed in Step 2, this also provides the (perceived) continuation-value function for a partially sophisticated household, since a partial sophisticate perceives themselves to be fully sophisticated in the next instant with a short-run discount factor of β^E.

Step 1a: Defining $v^{refi}(x)$ for Full Sophisticates. To begin, we need to pin down the continuation-value function from choosing to incur the refinancing effort cost in order to enter the "intermediate" state where the household has filled out its refinancing paperwork, but is waiting for the new mortgage to close. We denote the household's continuation-value

[70] Because a partially sophisticated agent expects their future selves to be fully sophisticated (with short-run discount factor β^E), the current self expects future selves to refinance such that $v(x) = v^*(x) - \frac{1}{\beta^E}\bar{\varepsilon}$. So, by not refinancing the current-value is $w(x) = \beta v^*(x) - \frac{\beta}{\beta^E}\bar{\varepsilon}$. Alternatively, by refinancing the current-value is $w(x) = \beta v^*(x) - \tilde{\underline{\varepsilon}}$. Thus, the current self will refinance whenever $\tilde{\underline{\varepsilon}} < \frac{\beta}{\beta^E}\bar{\varepsilon}$.

function in this intermediate state by $v^{refi}(x)$, and it is expressed as follows:

$$\begin{aligned}\rho v^{refi}(x) = u(c(x)) &+ \frac{\partial v^{refi}(x)}{\partial b}(y + rb + \omega^{cc}\left(\max\{b^-, \underline{b}\}\right) + \omega^{\uparrow}(b - \underline{b})^- - (r^m + \xi)m - c(x)) \\ &- \frac{\partial v^{refi}(x)}{\partial m}(\xi m) \\ &+ \sum_{y' \neq y} \lambda^{y \to y'}\left[v^{refi}(b, m, y', r^m, r) - v^{refi}(x)\right] \\ &+ \sum_{r' \neq r} \lambda^{r \to r'}\left[v^{refi}(b, m, y, r^m, r') - v^{refi}(x)\right] \\ &+ \lambda^R\left[v^R(x) - v^{refi}(x)\right] \\ &+ \lambda^C\left[\max_{b', m'}\left\{v(b', m', y, r + \omega^m, r)\right\} - v^{refi}(x)\right], \text{ s.t. refi. constraint (5) holds,}\end{aligned}$$

subject to the optimality condition $u'(c(x)) = \beta\frac{\partial v^{refi}(x)}{\partial b}$. In the first line of this equation, we use $(b - \underline{b})^- = \min\{b - \underline{b}, 0\}$ to denote that the consumer pays the borrowing wedge of ω^{cc} up to $\underline{b}$, and then any debt beyond $\underline{b}$ incurs the higher borrowing wedge of $\omega^{\uparrow}$.

This equation for v^{refi} is similar to equation (8′), but with three main changes. First, in the equation above for v^{refi}, consumption is pinned down by the condition $u'(c(x)) = \beta\frac{\partial v^{refi}(x)}{\partial b}$.[71] This is in contrast to equation (8′), where the time-consistent household with $\beta = 1$ chooses consumption optimally to maximize v.

Second, in the equation for v^{refi} we remove the outer maximization that exists in equation (8′) over whether or not to adjust the mortgage, since the household in this intermediate state is *already* in the process of refinancing. Instead, the equation for v^{refi} adds a new line relative to (8′), which is line six: $\lambda^C\left[\max_{b', m'}\left\{v(b', m', y, r + \omega^m, r)\right\} - v^{refi}(x)\right]$. This line captures the household's continuation-value conditional on having its mortgage close, which occurs at Poisson rate λ^C.[72] Conditional on closing, the household chooses b' and m' to maximize current-value function $w(x)$. However, since $w(x) = \beta v(x)$, we work directly with $v(x)$ in line six for notational simplicity.

Third and relatedly, the equation for v^{refi} removes the possibility of forced refinancing. We make this simplification since the household is already in the process of adjusting its mortgage.

Step 1b: Continuation-Value Function for Full Sophisticates. Next, we characterize when a sophisticated agent will choose to adjust their mortgage in the typical high-effort-

[71]For further details on the Bellman equation for the continuation-value function of households with present bias, see Harris and Laibson (2013) and also Maxted (2023).

[72]As discussed above, by taking $\lambda^C \to \infty$ we effectively remove refinancing delays and get back to $v^{refi}(x) \to \max_{b', m'} v(b', m', y, r + \omega^m, r)$.

cost state of $\bar{\varepsilon}$. Let $v^*(x) = \frac{1}{\beta} w^*(x)$ denote the continuation-value from adjusting.

In the typical high-cost state, a sophisticated agent with present bias will adjust their mortgage whenever $v(x) < v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$, and will not adjust whenever $v(x) > v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$.[73] Using this property, we are now prepared to define the continuation-value function $v(x)$ for full sophisticates, following similar methods as in Harris and Laibson (2013) and Maxted (2023). Specifically, the sophisticate's continuation-value function v can be expressed as an HJBQVI, as follows:

$$\rho v(x) = \max \left\{ u(c(x)) + (\mathcal{A}'' v)(x) \ , \ \rho \left(v^*(x) - \frac{1}{\beta}\bar{\varepsilon} \right) \right\}, \tag{25}$$

subject to the optimality condition $u'(c(x)) = \beta \frac{\partial v(x)}{\partial b}$. For notational compactness, equation (25) uses similar infinitesimal generator notation as in (8), where $\mathcal{A}''$ is the generator defined by the right-hand side of (8″) but with $\underline{v}(x) = \max\{v(x), v^*(x) - \frac{1}{\beta}\underline{\varepsilon}\}$ instead of (15).

Step 2: Continuation-Value Function for Partial Sophisticates. A partially sophisticated agent perceives themselves to be fully sophisticated in the next instant, with a short-run discount factor of β^E. Thus, a partial sophisticate's continuation-value function again follows equation (25), except that β is replaced with β^E. That is, the partial sophisticate's continuation-value function v can be expressed as $\rho v(x) = \max \Big\{ u(c(x)) + (\mathcal{A}'' v)(x) \ , \ \rho \left(v^*(x) - \frac{1}{\beta^E}\bar{\varepsilon} \right) \Big\}$, subject to the optimality condition $u'(c(x)) = \beta^E \frac{\partial v(x)}{\partial b}$.

Step 3: Current-Value Function. We now express the current-value function $w(x)$. In this modified model, current-value function $w(x)$ remains similar to equation (16) in Appendix B.3 (which is itself just the generalized version of equation (9) in the main text), but with one key modification: we update the definition of $w^{refi}(x)$ (i.e., the current-value function from refinancing) because the modified model presented in Section D.5.1 imposes delays between the start of a refinance and its close. Fully, $w(x)$ is now given by:

$$\begin{aligned} w(x) &= \max \left\{ \beta v(x), \ w^*(x) - \bar{\varepsilon} \right\} \quad \text{and} \\ \underline{w}(x) &= \max \left\{ \beta v(x), \ w^*(x) - \underline{\varepsilon} \right\} \quad \text{with} \\ w^*(x) &= \max \left\{ w^{prepay}(x), w^{refi}(x) \right\} \\ w^{prepay}(x) &= \max_{b',m'} \ w(b', m', y, r^m, r) \qquad \text{s.t. prepayment constraint (4) holds} \\ w^{refi}(x) &= \beta v^{refi}(x) \qquad \text{where } v^{refi}(x) \text{ is defined above} \end{aligned} \tag{26}$$

[73] Note the rescaling of the effort cost by $\frac{1}{\beta}$, which we explained intuitively in Appendix D.5.2 above and which follows from the first line of equation (26) in Step 3 below.

Step 4: Defining the Comparable $\beta = 1$ Household. We end this subsection by adding further details on mapping the (partially or fully sophisticated) present-biased household to a corresponding household with $\beta = 1$ *and* a refinancing effort cost of $H^E \times \varepsilon_t$. Specifically, equation (25) above helps us to see that we can make use of the following result from Harris and Laibson (2013).

Remark 3 (Value Function Equivalence) *Continuation-value function $v(x)$ of a fully sophisticated present-biased agent is an affine transformation of the value function of an otherwise-identical agent with $\beta = 1$ and a refinancing effort cost of $H \times \varepsilon_t$, where $H = \left(\frac{\gamma-(1-\beta)}{\gamma}\right)^{-\gamma}$.*

Remark 4 (Extension to Partial Sophistication) *Continuation-value function $v(x)$ of a partially sophisticated present-biased agent is an affine transformation of the value function of an otherwise-identical agent with $\beta = 1$ and a refinancing effort cost of $H^E \times \varepsilon_t$, where $H^E = \left(\frac{\gamma-(1-\beta^E)}{\gamma}\right)^{-\gamma}$.*

These remarks are informal, and we direct the reader to Harris and Laibson (2013) and Maxted (2023) for further details regarding the value function equivalence in Remark 3. Remark 4 follows from Remark 3, since a partial sophisticate perceives themselves to be fully sophisticated in the next instant with a short-run discount factor of β^E.

Briefly sketching out the value function equivalence in Remark 3, the methods presented in Harris and Laibson (2013) and Maxted (2023) show that the continuation-value function $v(x)$ of the fully sophisticated present-biased agent in equation (25) can be constructed from the value function $\hat{v}(x)$ of a time-consistent (i.e., $\beta = 1$) "$\hat{u}$ agent" with a refinancing effort cost of $\frac{1}{\beta}\varepsilon_t$. In particular, this time-consistent $\hat{u}$ agent has a modified utility function of $\hat{u}(\hat{c}) = \frac{\psi}{\beta} u\left(\frac{1}{\psi}\hat{c}\right) + \frac{\psi-1}{\beta}$, where $u(c)$ is the standard CRRA utility function and $\psi = \frac{\gamma-(1-\beta)}{\gamma}$. From inspection one can see that $\hat{u}(\hat{c})$ is an affine transformation of standard CRRA utility, such that $u(c) = \frac{\beta}{\psi^\gamma}\hat{u}(c) + (\text{other constants})$. Thus, by applying this affine transformation to both the $\hat{u}$ utility function and to the $\hat{u}$ agent's refinancing effort costs, we see that the behavior of this $\hat{u}$ agent will be equivalent to the behavior of a "standard exponential agent" with $\beta = 1$, standard CRRA utility $u(c)$, and a refinancing effort cost of $\frac{\beta}{\psi^\gamma} \times \frac{1}{\beta}\varepsilon_t = H \times \varepsilon_t$, exactly as stated in the Remark.

D.5.4 Additional Details: Policy Functions with Sophistication

As a final step, we provide further details on the policy functions of households with partially or fully sophisticated present bias.

Consumption. Proposition 3 follows similar arguments as in Maxted (2023).

Mortgage Adjustment (Full Sophistication). The mortgage-adjustment decision of partially sophisticated households was described above in Proposition 4. However, the mortgage-adjustment decision of fully sophisticated households features the added technical complexity that the refinancing policy function becomes a mixed-strategy equilibrium, which we discuss further here.

Specifically, so long as $\varepsilon_t = \bar{\varepsilon}$ then in equilibrium it cannot be the case that a sophisticate adjusts their mortgage with probability-one at any point x. As with naiveté, if the current self believes with certainty that the next self will adjust, then the current self will simply procrastinate. Thus, in equilibrium we instead assume that sophisticates adjust their mortgage at a Poisson rate. Let $\lambda^{adjust} : x \to [0, \infty)$ denote this rate.

The fact that sophisticates adjust their mortgage probabilistically also means that there is something "going on behind the scenes" in the right branch of equation (25). While it is the case that $v(x) = v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$ when the household is in the adjustment region, it is not necessarily the case that this adjustment happens immediately. Rather, in an adjustment region the agent's continuation-value function can be expressed as:

$$\rho v(x) = u(c(x)) + (\mathcal{A}''v)(x) + \lambda^{adjust}(x) \left(v^*(x) - \bar{\varepsilon} - v(x)\right), \tag{27}$$

where $\lambda^{adjust}(x)$ is determined in equilibrium precisely to ensure that $v(x) = v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$.[74] Rearranging this equation using the property that $v(x) = v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$ gives:

$$\lambda^{adjust}(x) = \frac{\rho v(x) - u(c(x)) - (\mathcal{A}''v)(x)}{\bar{\varepsilon}\left(\frac{1}{\beta} - 1\right)}.$$

On the one hand, when $\bar{\varepsilon}$ is large then one can see how λ^{adjust} could be relatively low and hence that slow refinancing can arise under full sophistication. On the other hand, under Assumption 2 that $\bar{\varepsilon}$ is vanishingly small, λ^{adjust} gets arbitrarily large and hence refinancing occurs arbitrarily quickly.

[74] Alternatively, $\lambda^{adjust}(x) = 0$ whenever $v(x) > v^*(x) - \frac{1}{\beta}\bar{\varepsilon}$ because the household does not want to adjust.

E Supplements to Section 4

E.1 MPCs and MPXs out of Discrete Wealth Shocks

In Section 4 the MPC and the MPX are defined over infinitesimal wealth shocks. Following Achdou et al. (2022), this section extends these definitions to discrete wealth shocks.

Let $C_\tau(x) = \mathbb{E}\left[\int_0^\tau c(x_t)dt \mid x_0 = x\right]$ denote total expected consumption from time 0 to time τ. Recall that $x = (b, m, y, r^m, r)$. Let $x + \chi$ be shorthand for the vector $(b + \chi, m, y, r^m, r)$, i.e. $x + \chi$ is point x plus a liquid wealth shock of size χ.

For a discrete liquidity shock of size χ the MPC is defined as:

$$MPC_\tau^\chi(x) = \frac{C_\tau(x+\chi) - C_\tau(x)}{\chi}.$$

The MPX is defined as (see Laibson et al. (2022) for details):

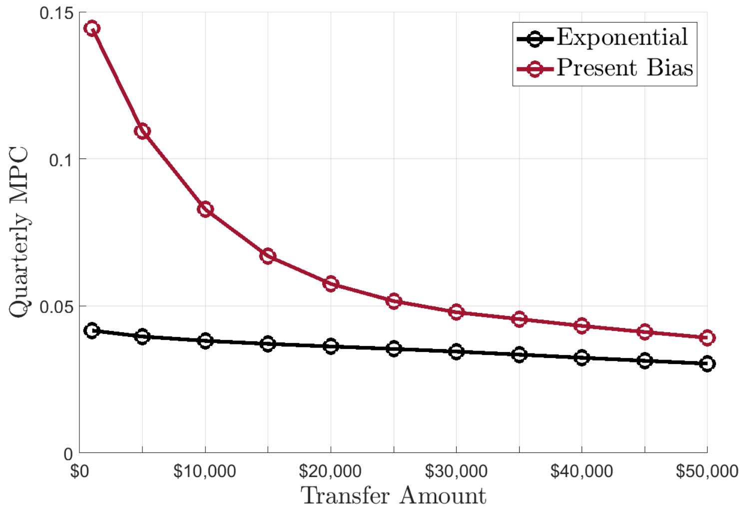
$$MPX_\tau^\chi(x) = \left(1 - s + \frac{\nu s}{r_0 + \nu}\right) MPC_\tau^\chi(x) + \frac{s}{\nu + r_0}\left(\frac{\mathbb{E}\left[c(x_\tau) \mid x_0 = x + \chi\right] - \mathbb{E}\left[c(x_\tau) \mid x_0 = x\right]}{\chi}\right).$$

Total consumption $C_\tau(x)$, which is used in the MPC calculation, can be calculated numerically using a Feynman-Kac formula (see Lemma 2 of Achdou et al. (2022) for details). To calculate the MPX we also need to solve for the expected consumption rate at time τ, $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$. Again, a Feynman-Kac formula can be used to solve for this directly.[75] Numerically, we solve the Feynman-Kac formula for the sample path $r_t = 1\%$ for all t (i.e., no aggregate interest rate shocks) since these calculations are conducted in the steady state.

[75] The Feynman-Kac formula for $C_\tau(x)$ is provided in Achdou et al. (2022). The Feynman-Kac formula for $\mathbb{E}\left[c(x_\tau)|x_0 = x\right]$ is specified slightly differently, but is again a direct application of the formula.

E.2 Model Solution Details: MPCs

(a) Quarterly MPCs Across Transfer Amounts



(b) Present-Bias Benchmark: MPCs over Liquid Wealth

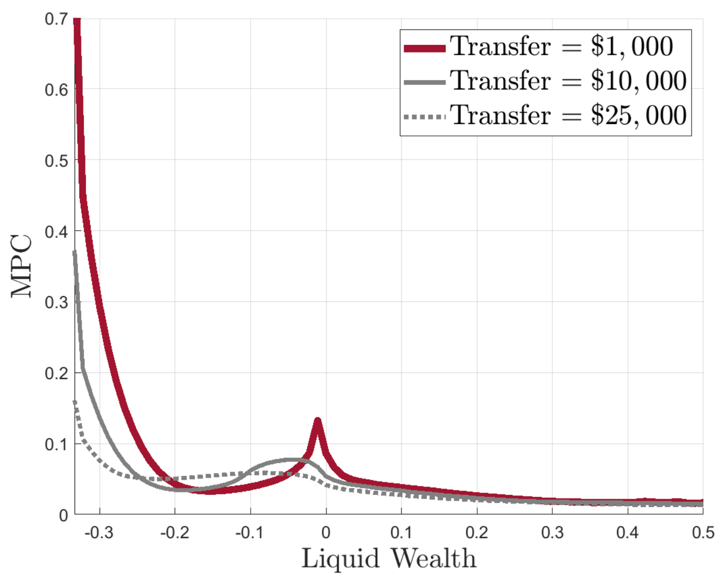


Figure 12: MPCs Across Transfer Amounts.

Notes: The top panel plots quarterly MPCs out of transfers ranging from $1,000 to $50,000 for the two calibration cases. The bottom panel replicates the MPC analysis in Figure 3 for the Present-Bias Benchmark calibration across transfer amounts of $1,000 (benchmark), $10,000, and $25,000.

E.3 Model Solution Details: Steady State Distributions

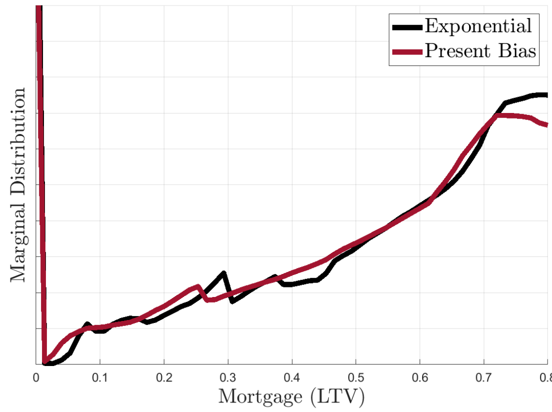
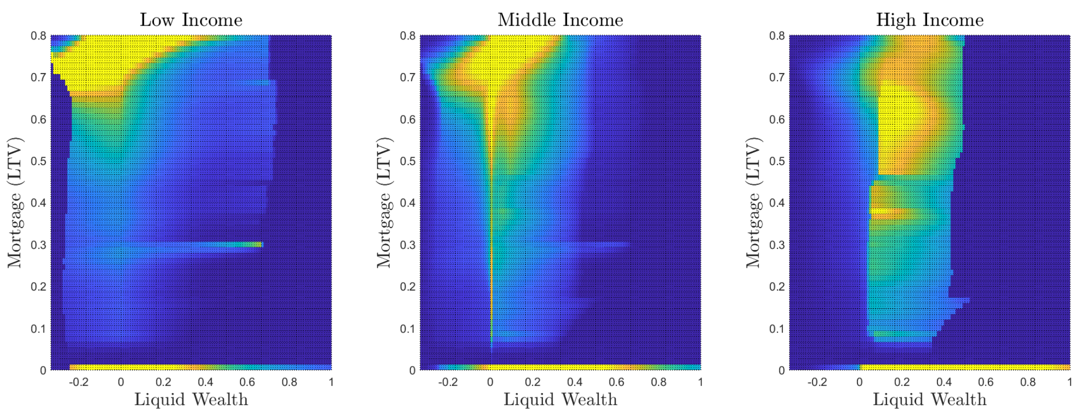


Figure 13: LTV Distribution.

Notes: This figure shows the steady state distribution of households over the LTV ratio.

(a) Exponential Benchmark



(b) Present-Bias Benchmark

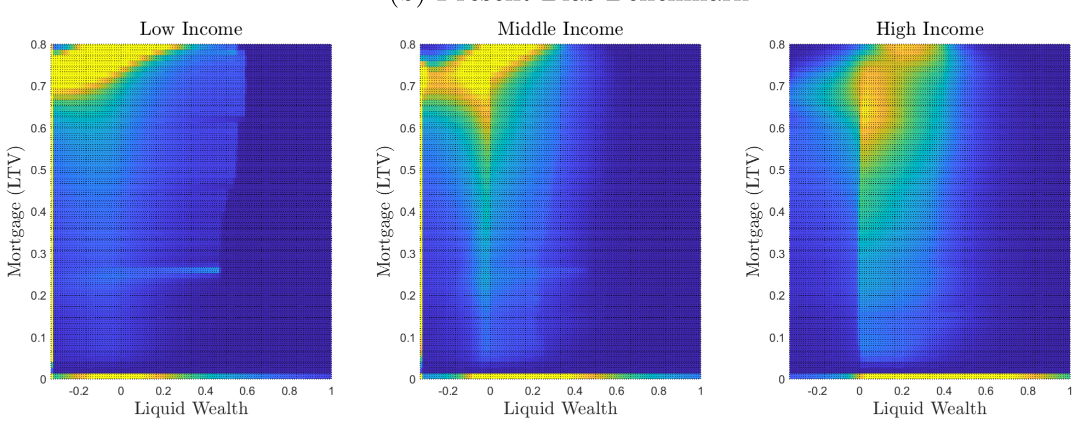


Figure 14: Steady State Distribution.

Notes: This figure presents the full steady state distribution over income, liquid wealth, and mortgage debt. Dark blue regions are rarely encountered, while light yellow regions feature large masses of households.

F Supplements to Section 5

F.1 Fiscal Policy: Financing of Fiscal Stimulus with Future Taxes

As stated in Section 5.1, the government finances the initial \$1,000 stimulus payment with a flow income tax on all households in perpetuity that is chosen so as to satisfy the government budget constraint. This appendix spells out the details of the fiscal rule the government uses to achieve this in our environment with a stochastic interest rate on government debt, building on work by Bohn (1998) and Blanchard (2023).

Before spelling out this fiscal rule, we spell out the government budget constraint. To this end, B_t denotes real government debt in per-capita terms which the government can issue at interest rate r_t, the same stochastic interest rate as that on households' liquid savings. In our continuous-time model, we allow for fiscal stimulus payments not just in the form of continuous flow payments but also as lumpy wealth transfers. To this end, denote by T_t the government's *cumulative* fiscal stimulus from time 0 to time t and denote by dT_t the fiscal stimulus at time t (so that $T_t = \int_0^t dT_s$). Finally, denote by τ_t the tax revenues from a flow income tax that the government uses to pay for such stimulus (more on this below).

With this notation in hand, the flow government budget constraint is:

$$dB_t = (r_t B_t - \tau_t)dt + dT_t. \tag{28}$$

This differential equation needs to hold for all t and for *any* realization of interest rates, including the possible negative interest-rate realizations (recall from Section 4.1 that interest rates in our calibration follow a Markov process with values $r_t \in \{-1\%, 0\%, 1\%, 2\%\}$). The fact that interest rates can go negative introduces a difficulty with writing the government budget constraint in present-value form or, equivalently, with writing the appropriate no-Ponzi condition. To resolve this difficulty we adopt the approach developed by Reis (2021) to analyze present-value budget constraints when $r < g$ (here with $g = 0$). Because this difficulty is not central to our choice of fiscal rule satisfying the government budget constraint, we postpone its discussion until the end of this appendix section.

Starting from the steady state at time $t = 0$, the government unexpectedly pays each household a lumpy "helicopter drop" fiscal stimulus payment of \$1,000, i.e. $T_{0^+} = \$1,000$. Thereafter, there are no more fiscal stimulus payments, i.e. $dT_t = 0$ for all $t > 0$ and so also $T_t = \$1,000$ for all $t > 0$. We assume that, in the initial steady state, government debt is zero, $B_{0^-} = \bar{B}$. Given the time path for fiscal policy, government debt initially jumps up to $B_{0^+} = \bar{B} + T_{0^+} = \$1,000$.

Going forward, we then assume that, at each time $t > 0$, the government levies a stochastic tax of τ_t given by

$$\tau_t = r_t B_t + \kappa, \quad \text{with } \kappa > 0 \text{ but } \kappa \downarrow 0 \text{ (until the debt is fully repaid).} \tag{29}$$

This simple fiscal rule ensures that the government budget constraint is satisfied and that government debt eventually reverts to its initial steady state level. The intuition is as follows.

Suppose first that $\kappa = 0$ so that tax revenues are $\tau_t = r_t B_t$. Examining (28) and recalling that $dT_t = 0$ for all $t > 0$, we then have $dB_t = 0$ for all $t > 0$ and all interest-rate realizations so that the real value of the government's debt stays constant in perpetuity and hence $B_t = B_{0^+} = \$1,000$ all $t > 0$. If all interest-rate realizations were strictly positive, then this would be enough to satisfy a standard no-Ponzi condition and hence to satisfy the present-value government budget constraint. However, as just discussed, they are not; in particular r_t can take the values -1% and 0%.

The alternative fiscal rule with $\kappa \approx 0$ as in (29) ensures that the present-value government budget constraint is satisfied even with the possibility of zero or negative interest rates. Intuitively, substituting (29) into (28) and recalling $dT_t = 0$ for all $t > 0$, we have

$$dB_t = -\kappa dt < 0 \quad \text{for all } t > 0 \text{ and all interest-rate realizations until the debt is repaid.}$$

That is, at each point in time, the government raises "just a little bit more" tax revenue than the interest payments resulting from the initial stimulus and thus repays a little bit of the initial debt at each point in time, i.e. $dB_t < 0$ for all $t > 0$ and all interest-rate realizations until the debt is fully repaid (when $r_t < 0$ we have $\tau_t < 0$, i.e. the government makes a transfer to households but one that is a little bit less than the revenues from the negative interest rate on its debt). Therefore $B_T \to 0$ as $T \to \infty$, i.e. government debt eventually reverts to its initial steady state level $\bar{B} = 0$ regardless of the time path of interest-rate realizations.

The fiscal rule (29) is similar to that proposed by Blanchard (2023), who suggests "making the primary balance a function of debt service [...] with one-to-one pass-through" and points out that the rule is a natural extension of the "Bohn rule" (Bohn, 1998) in which the primary balance is an increasing function of the level of debt (rather than debt service).

Given that our model features heterogeneous households, there are some degrees of freedom in specifying how exactly to raise the tax revenues τ_t satisfying (29). In practice, we do this by levying a proportional tax $\bar{\tau}_t \times y_{it}$ on households' inelastically supplied labor income y_{it} so that higher-income households pay higher taxes in dollar terms. Tax revenues are then given by

$$\tau_t = \bar{\tau}_t \times \bar{y},$$

where $\bar{y} = \int y_{it} di$ is average household income (which is constant because we assume a stationary income process).

Finally, it is worth emphasizing just how small the per-period taxes can be while still satisfying the government budget constraint. Since households' average income of \$95,718 has been normalized to 1, $\bar{\tau}_t$ is given by:

$$\bar{\tau}_t \approx r_t \times \frac{B_t}{\bar{y}} \approx r_t \times \frac{\$1,000}{\$95,718},$$

where the approximations use that $\kappa \approx 0$ and $B_t \approx \$1,000$ because the government pays its debt down only slowly. That is, even in time periods with the highest interest-rate realization $r_t = 2\%$, the tax rate equals only about $\bar{\tau}_t \approx 2\% \times \frac{\$1,000}{\$95,718} \approx 0.02\%$ of labor income. This explains why the initial short-run consumption response with our fiscal rule (29) is very similar in size to the consumption response to the same fiscal stimulus but without imposing a government budget constraint at all.

Present-Value Budget Constraint with Negative Interest Rates (Reis, 2021). As already noted, a difficulty is that our model allows for the possibility of negative interest rates on government debt. We here show how to write an appropriate present-value government budget constraint corresponding to (28), and then show that the simple tax rule (29) satisfies this present-value constraint.

To write the present-value budget constraint, we follow Reis (2021) and use a strictly positive discount rate $\delta > 0$ in place of interest-rate realizations $\{r_t\}_{t\geq 0}$ to compute present values. Imposing the no-Ponzi condition

$$\lim_{T\to\infty} e^{-\delta T} B_T = 0, \tag{30}$$

the present-value budget constraint becomes[76]

$$\int_0^\infty e^{-\delta t} dT_t + B_0 = \int_0^\infty e^{-\delta t}\tau_t dt + \int_0^\infty e^{-\delta t}(\delta - r_t)B_t dt. \tag{31}$$

As discussed by Reis (2021), in principle, *any* discount rate $\delta > 0$ can be used, but a sensible choice is the private return from investing in productive capital (as opposed to government bonds) which, in the data, has historically exceeded the economy's growth rate. In this case (31) has the interpretation of the present-value government budget constraint but discounted at the return of private investors δ. The equation then states that the government's present value of spending dT_t (discounted at δ) plus its initial debt B_0 must not exceed the present value of future taxes τ_t (discounted at δ) plus a non-standard term that Reis names the "bubble premium revenue term" which is the present value of the implicit government revenues that arise from paying an interest rate r_t on its debt that is below the private return δ (the convenience yield of government debt).

The only thing that remains is to show that the fiscal rule (29) satisfies the no-Ponzi condition (30) and the present-value budget constraint (31). This follows immediately from the fact that this rule implies that government debt converges to zero in the long run, $B_T \to 0$ as $T \to \infty$. In particular, the no-Ponzi condition is immediately satisfied. Similarly, in (31), the bubble-premium revenue term is bounded for any sequence of interest-rate realizations $\{r_t\}_{t\geq 0}$ because $(\delta - r_t)B_t \to 0$ as $t \to \infty$ for any $\{r_t\}_{t\geq 0}$.

Full General Equilibrium Analysis. Finally, we again note that our model is not a general equilibrium model and therefore leaves out a number of considerations that may be important in practice, in particular "Keynesian" multiplier effects of fiscal stimulus that work by stimulating aggregate demand. Future research should explore the effects of fiscal stimulus in full-blown general equilibrium models with present-biased households.

[76]The advantage of using $\delta > 0$ rather than $\{r_t\}_{t\geq 0}$ when writing the no-Ponzi condition and government budget constraint is that it sidesteps the issue that these conditions are ill-defined when r_t can go negative. For example, the standard approach of integrating the flow budget constraint while discounting at $\{r_t\}_{t\geq 0}$ involves the term $\lim_{T\to\infty} e^{-\int_0^t r_s ds} B_T$, which may converge to infinity for negative interest-rate realizations (in particular, one possible history is $r_t = -1\%$ for all t).

To derive (31), write the flow budget constraint (28) as

$$dB_t - \delta B_t dt = ((r_t - \delta)B_t - \tau_t)dt + dT_t.$$

Multiplying by $e^{-\delta t}$ and integrating between 0 and T we have

$$B_T e^{-\delta T} - B_0 = \int_0^T e^{-\delta t} dT_t - \int_0^T e^{-\delta t}\tau_t dt + \int_0^T e^{-\delta t}(r_t - \delta)B_t dt.$$

Taking $T \to \infty$ and using the no-Ponzi condition (30) yields the present-value budget constraint (31).

F.2 Fiscal Policy: Implementation Details

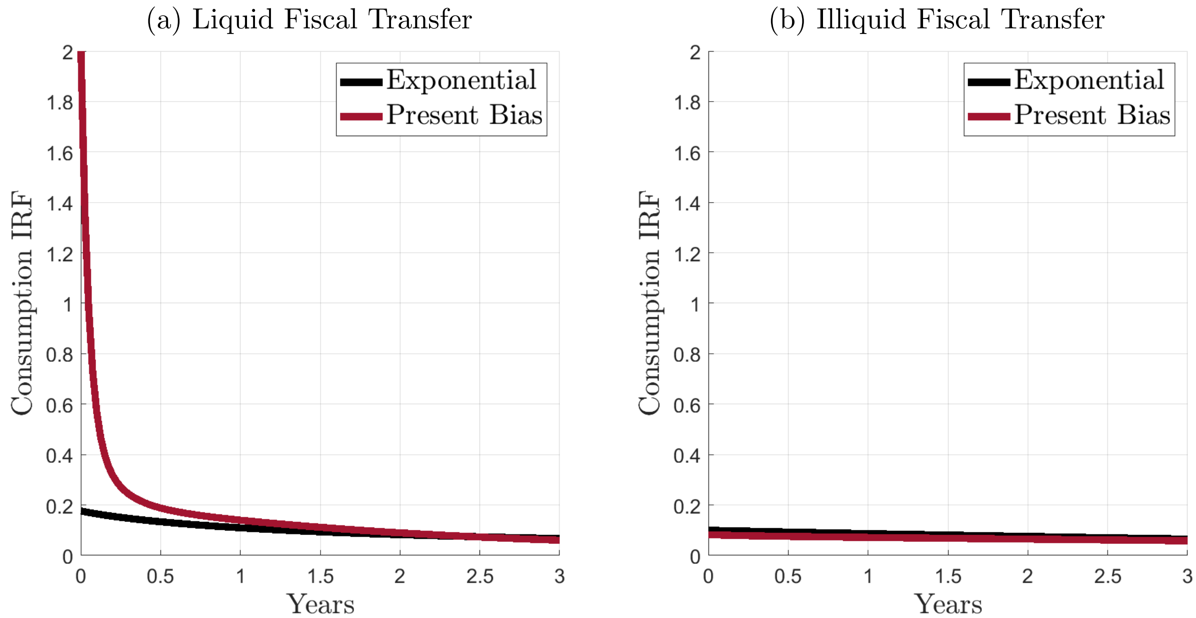


Figure 15: Liquidity of Fiscal Policy.

Notes: The left panel reproduces the benchmark fiscal policy analysis of Figure 4. The right panel plots the IRF of aggregate consumption to a $1,000 mortgage principal reduction.

F.3 Monetary Policy: Refinancing Dynamics

Figure 16 plots the adjustment regions following an interest rate cut from 1% to 0%. This figure replicates the phase diagrams in Figure 1, but now for the case of $r_t = 0\%$ and $r_t^m = 1\% + \omega^m$. Thus, Figure 16 plots the adjustment regions for households with a mortgage rate that is above the rate they can refinance into.

As in the main text, the red regions mark where households take a cash-out refinance and the blue regions mark where households prepay their mortgage. The gray regions indicate where households conduct a rate refinance, defined as the household increasing its mortgage balance by less than 5% during the refinance. Relative to the steady state adjustment regions, the interest rate cut causes the red/gray refinancing regions to expand drastically. In particular, households with larger LTVs are more likely to refinance, since households with larger mortgages have more to gain by reducing their mortgage interest payments.

Table 6 presents details of the refinancing decision. The first row lists the share of households who find themselves in a refinancing region at the time of the interest rate cut. Conditional on refinancing, the second row lists the share of households who extract equity when refinancing. The next four rows list the share of households who have actually refinanced within 1 quarter, 1 year, 2 years, and 3 years following the interest rate cut. While refinancing is instant in the Exponential Benchmark, procrastination means that refinancing occurs slowly in the Present-Bias Benchmark.

	Exponential	Present Bias
Share Refi Region (On Impact)	70.6%	72.3%
(Share Cash Out)	81.2%	77.7%
$\frac{1}{4}$ Year Realized Refi	72.5%	13.2%
1 Year Realized Refi	76.9%	40.9%
2 Year Realized Refi	81.3%	61.2%
3 Year Realized Refi	84.5%	72.0%
Average Refi Amount	0.37	0.35

Table 6: Refinancing Details.

Notes: This table summarizes details of household refinancing following an interest rate cut from 1% to 0%.

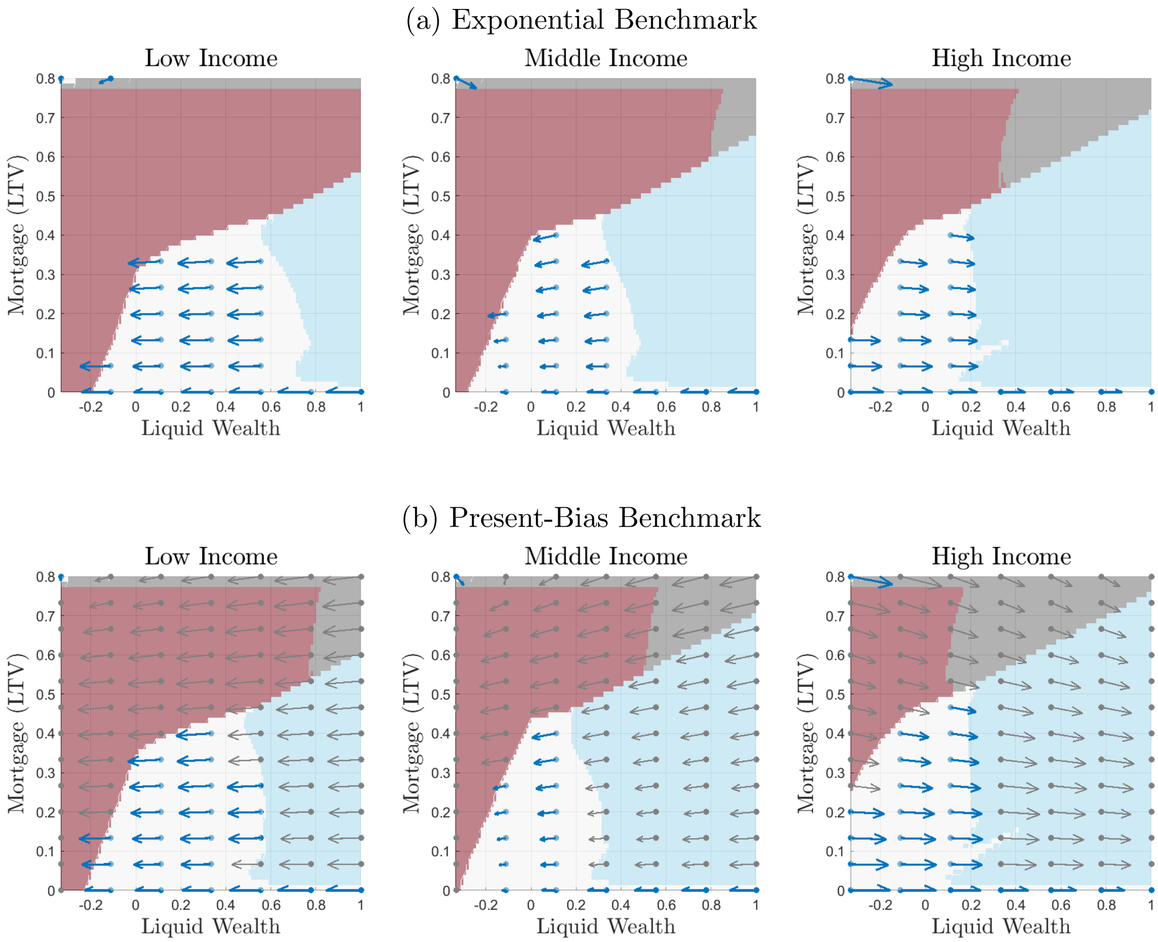


Figure 16: Rate-Cut Phase Diagrams.

Notes: This figure presents the phase diagrams for households who can refinance into a lower mortgage rate following an interest rate cut from 1% to 0% (see Figure 1 for phase diagram details).

F.4 Monetary Policy and Refinancing Procrastination

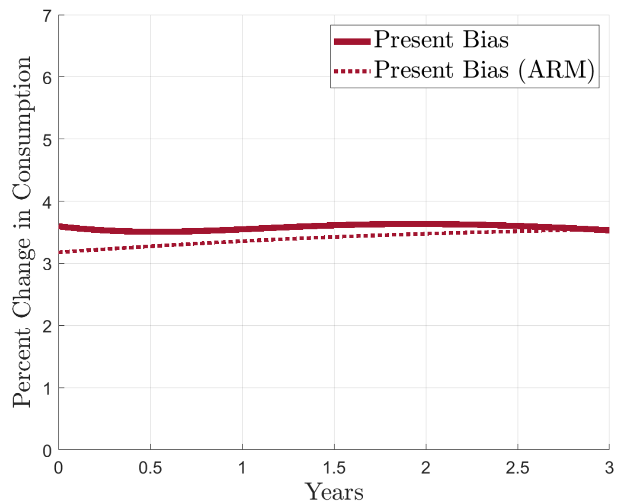


Figure 17: Monetary Policy Under FRMs Versus ARMs.

Notes: For the Present-Bias Benchmark calibration, this figure compares the consumption response to monetary policy under FRMs (solid line) versus ARMs (dotted line). The interest rate is cut by 2% in the ARM experiment, compared to 1% in the FRM experiment, since monetary policy produces larger movements in ARM rates than long-duration FRM rates.

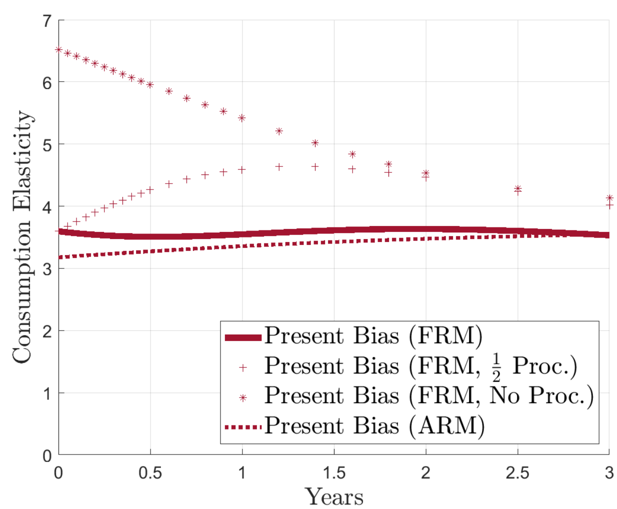


Figure 18: Monetary Policy with Procrastination Reduction.

Notes: This figure presents the consumption response to monetary policy in the Present-Bias Benchmark across varying levels of refinancing procrastination. The +'s assume that policymakers are able to halve the expected duration of procrastination at the time of the rate cut. The $*$'s make refinancing immediate at the time of the rate cut. The baseline consumption response under FRMs (solid red line) and ARMs (dotted red line) are presented for comparison.

G Discussion: Implications of Present Bias for Macroeconomic Policy in Fully-Fledged Macro Models

As we have already discussed, our model is set in partial equilibrium because abstracting from general equilibrium considerations allows for a richer, and more straightforward, investigation of the household problem. We also omit a number of model elements that could be important in principle, such as modeling residential investment. Finally, our calibration focuses on a specific subset of the population, namely homeowners. This raises the question of how present bias would affect the transmission of monetary and fiscal policy in a full general equilibrium analysis that relaxes these assumptions and models the entire population including non-homeowners.

G.1 Omissions from the Analysis and Restriction to Homeowners

We start by discussing certain omissions from the analysis other than general equilibrium effects. In many cases, while filling these gaps would clearly affect our monetary and fiscal policy results, it is less clear as to how doing so would affect our main results about present bias amplifying the impact of these policies (i.e., the comparison of the Exponential and Present-Bias Benchmarks). In others, the omission may materially affect our main results.

Residential Investment. As discussed in Section 2.1, we assume that each household is endowed with a home of fixed value h. That is, the housing size is completely fixed and cannot be adjusted (in contrast, housing *equity* can be adjusted via mortgage balances).

The omission of residential investment means that our model provides an incomplete picture of the effects of monetary policy, particularly as it relates to the spending response to cash-outs. While households often report using extracted home equity for residential investment (e.g., Greenspan and Kennedy, 2008), this channel is broadly missing from our model with fixed housing h. This channel could interact with present bias, thus affecting our main results.[77]

Yet another possibility that we assume away with our fixed-h assumption is that present-biased agents would buy different-sized houses in the first place. On the one hand, present-biased households may struggle to accumulate the liquid wealth required to make a down

[77]That said, the reduced-form MPX tool from Laibson et al. (2022) that we used in Section 4 does capture household spending on home improvements (which differs from residential investment as we explain momentarily). Specifically, we calibrate our mapping from MPCs to MPXs such that the MPX includes consumer spending on "furnishings and durable household equipment" like furniture, household appliances, and gardening equipment. In other words, while our MPX measure excludes residential investment, it does include other types of home improvements (like a new washing machine) that are included as Personal Consumption Expenditures by the BEA.

payment and hence will hold less housing. On the other hand, (partial) sophistication of the type modeled in Appendix D.5 in combination with binding financial constraints may lead present-biased households to buy an illiquid asset like housing as a commitment device (Laibson, 1997; Maxted, 2023), and may hence result in them buying larger houses. If present-biased and exponential households make different housing choices, they may of course also respond differently to monetary policy.

Supply Side of the Credit Card Market. While we calibrate the credit card wedge ω^{cc} to 10.3% in order to match the data on the commercial bank interest rate charged on credit cards, we do not actually model the supply side of the credit card market. An important question for future research is why such high interest rates arise in equilibrium. Default risk is certainly part of the story, though Dempsey and Ionescu (2023) suggest that interest rate spreads far exceed the risk of default. While default *levels* alone may not explain credit card interest rates, it also seems likely that issuers' profits covary positively with the business cycle, since loan loss provisions will generally peak during downturns. This suggests that credit cards are a "high-beta" product for credit card issuers, which would provide a risk-premium explanation for why credit card debt commands an elevated interest rate.

In light of this discussion, another simplification of our model is that it abstracts from credit card default. It is likely that a model with present-biased households will feature a different level of defaults, and perhaps also a different covariance of defaults with the business cycle, than a model with exponential households. Given that this will affect the equilibrium interest rates and borrowing limits that households face, it is again likely that modeling such considerations explicitly will affect our main results.

Supply Side of the Mortgage Market. We also abstract from the supply side of the mortgage market, though many of the questions above still apply. Indeed, we have already shown that present bias affects households' mortgage choices, so it is likely that present bias also influences both the product menus and equilibrium mortgage rates that households face.

Restriction to Homeowners. The subpopulation of homeowners that we calibrated our model to – and hence the individuals that populate our model – differs from the full U.S. population in a number of ways. Perhaps most importantly, our model population overrepresents debtors. More precisely, while our model does feature some households with substantial liquid savings (e.g., $100,000), the number of such households is relatively small (see Appendix Figure 14 and recall that we normalized average income of $95,718 to 1). Instead, most households in our model have substantial mortgage debt, credit card debt, or both.

As a result of these modeling and calibration choices, our results paint only a partial picture of the transmission of monetary and fiscal policy, and of how present bias affects this transmission. That is, we leave out a number of offsetting or amplifying effects of such policy changes. For example, given that our model is mostly populated by borrowers, our discussion of monetary policy in Section 5.2 shows primarily the effects on this subgroup of the population. This likely matters because the consumption of borrowers may respond more to such interest rate cuts than that of lenders due to standard income effects (similarly, borrowers may benefit more in welfare terms). A more fully-fledged macro analysis would also model the other side of the mortgage and credit card markets, in particular the households who lend as well as the financial institutions that facilitate this lending. While fully modeling the entire population would therefore clearly affect the economy's overall consumption response to monetary policy, it is again less clear as to how doing so would affect our main results about the impact of present bias on policy transmission.

G.2 General Equilibrium Effects

We next turn to the question of how present bias would affect the transmission of monetary and fiscal policy in a full general equilibrium analysis. Here we briefly discuss this question through the lens of the literature on Heterogeneous Agent New Keynesian (HANK) models. That is, we ask what the effect of present bias would be on the consumption response to monetary and fiscal policy in a general equilibrium version of the model with nominal rigidities (i.e., as before we focus on the comparison between the Exponential and Present-Bias Benchmarks, but now take into account general equilibrium considerations).[78]

In HANK models, macroeconomic stabilization policy can trigger a number of different indirect general equilibrium effects, particularly effects working through household labor income, asset prices, and returns (see e.g. Werning, 2015; Kaplan et al., 2018; Auclert, 2019; Alves et al., 2020; Slacalek et al., 2020). The size of these indirect effects depends on the size of these variables' movements as well as households' responsiveness to these changes, e.g. MPCs (and MPXs) out of labor income and asset price changes. In heterogeneous-agent models with idiosyncratic income risk and borrowing constraints of the type analyzed here these indirect effects can be important because such models often generate sizable MPCs (e.g., Kaplan et al., 2018).

As we have shown above, present bias increases both households' average MPC and the direct consumption effect of an interest rate cut. The likely implications for the transmission

[78]One could also imagine studying the impact of present bias on the consumption response to macroeconomic policy in models without nominal rigidities, and this may overturn our result that present bias amplifies this response. For example, in a model with a classical dichotomy, changes in nominal interest rates would have no effect on real consumer spending regardless of whether the economy features present bias. Similarly, one may be able to construct a general equilibrium version of our model in which an extreme form of Ricardian equivalence holds so that fiscal stimulus has no effect on consumer spending, again regardless of present bias. We view such exercises as less interesting and instead discuss environments in which policy affects consumer spending also in the absence of present bias.

of monetary and fiscal policy in a full general equilibrium analysis are as follows.

Fiscal Policy. We conjecture that, also in a general equilibrium HANK version of our model, present bias would continue to amplify households' spending response to fiscal policy. This follows from a simple "Keynesian cross" logic which takes as its starting point that the most potent general equilibrium effect triggered by fiscal policy is likely the one working through households' labor incomes: a fiscal transfer increases aggregate consumption demand (the impulse or direct effect); in equilibrium, firms hire more which increases households' labor incomes and leads to additional spending (the multiplier or indirect effect).[79] The key ingredient determining the size of both this impulse and multiplier are households' MPCs, which increase with present bias. Present bias therefore likely amplifies not only the direct effects but also the indirect general equilibrium effects of fiscal policy.

Monetary Policy. We conjecture that the situation is similar for monetary policy, namely that present bias would increase not only direct but also indirect effects and therefore the overall consumption response. Just like fiscal policy, monetary policy triggers indirect effects working through labor income and present bias would amplify these via higher MPCs. Monetary policy can also trigger indirect effects working through asset prices and returns (Gornemann et al., 2016; Kaplan et al., 2018; Alves et al., 2020; Slacalek et al., 2020).[80] However, since present bias does not significantly affect MPCs out of liquid wealth for high-liquidity households (see Figure 3), nor MPCs out of illiquid wealth (see Figure 15), it is natural to conjecture that present bias does little to impact the indirect effects working through asset prices and returns. Taken together, this discussion suggests that, in a HANK-version of our model, present bias would continue to amplify the effects of monetary policy once general equilibrium effects are taken into account.

Fully evaluating the impact of present bias on the economy's response to monetary and fiscal policy in a general equilibrium model is an important task for future work.

[79] For modern macro versions of this mechanism, see for example Auclert et al. (2018) and Wolf (2023).

[80] Since we study the effect of monetary policy on consumption at relatively high frequencies, we are mostly interested in asset price changes at those same frequencies. In this regard, empirical evidence usually points to interest rate cuts as increasing stock prices (e.g., Bernanke and Kuttner, 2005; Gürkaynak et al., 2005). There is also evidence that loose monetary policy increases house prices (e.g., Jordà et al., 2015), but this mechanism seems to operate at a lower frequency than we study.

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