Lecture 5
Investment and Capital Accumulation

Macroeconomics EC2B1

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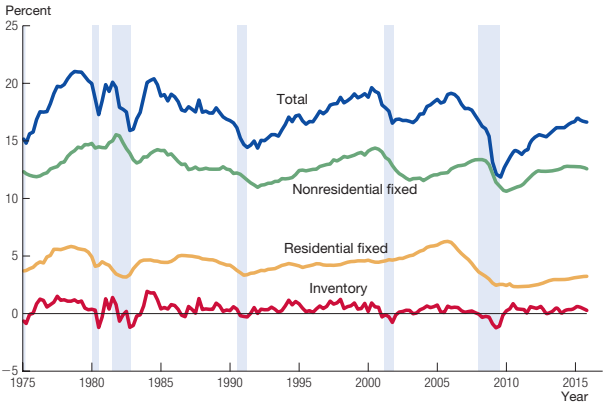
Investment over the business cycle

FIGURE 17.4

Components of Investment (as a Share of GDP)



Source: The FRED database.

Investment consists of three main components: nonresidential fixed investment, which includes equipment and structures purchased by businesses as well as intellectual property products; residential investment (housing); and the change in inventories held by businesses.

Source: Jones, Macroeconomics

Plan

1. Investment and capital accumulation in partial equilibrium
2. Dynamic general equilibrium with capital accumulation
3. Half-way mark for this course: taking stock and looking ahead

Readings and supplementary materials

1. Supplement on moodle: write-up of model including all the derivations
    - I will provide such write-ups going forward, partly so we can skip the derivations during lectures
2. Chapters 8 and 9 of Kurlat
    - general equilibrium model in his chapter 9 is similar but more complicated
    - also includes labor demand and supply which we abstract from

Investment and capital accumulation in partial equilibrium

Investment and capital accumulation in partial equilibrium

- Two time periods $t = 1, 2$, world ends afterwards
- Representative firm
- Firms produce using capital K_t, $t = 1, 2$

$$Y_t = A_t F(K_t), \quad F' > 0, F'' < 0$$

- K_1 is fixed
- K_2 can be changed through investment in period 1, I_1

$$K_2 = I_1 + (1 - d)K_1, \quad 0 < d \leq 1$$

where d is depreciation (same equation as in Solow model)

Firm profit maximization

- Firm profits per period

$$\Pi_1 = A_1 F(K_1) - I_1, \quad \Pi_2 = A_2 F(K_2) + (1-d)K_2$$

- Why $(1-d)K_2$ in Π_2?
- Firms maximize present discounted value (PDV) of profits

$$W = \Pi_1 + \frac{\Pi_2}{1+r_1}$$

  where r_1 is interest rate between $t=1$ and $t=2$

- Why would firms maximize this present value? Why discounted at r_1?
- See supplement. In a nutshell because
  (a) firms are owned by households who can save and borrow at r_1
  (b) also firms can save and borrow at r_1

  (a)+(b) $\Rightarrow$ households instruct firms to maximize $W = \Pi_1 + \frac{\Pi_2}{1+r_1}$

Firm profit maximization

- Combining, firm problem is

$$W = \max_{K_2} \left\{ A_1 F(K_1) + (1-d)K_1 - K_2 + \frac{A_2 F(K_2) + (1-d)K_2}{1+r_1} \right\}$$

- Optimality condition

$$1 = \frac{A_2 F'(K_2) + 1 - d}{1 + r_1}$$

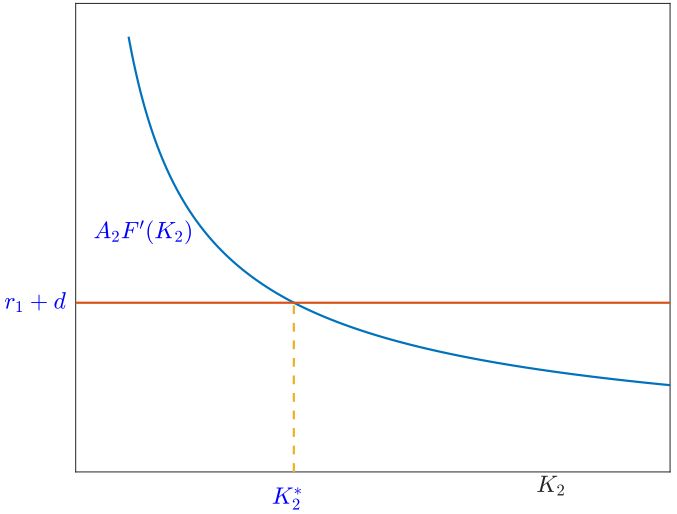
or

$$A_2 F'(K_2) = r_1 + d \qquad (*)$$

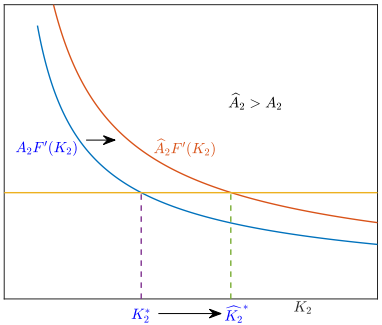
- $(*)$ pins down optimal capital K_2^* and therefore optimal investment

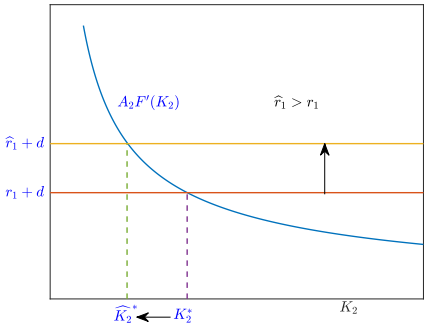
$$I_1^* = K_2^* - (1-d)K_1$$

Graphical representation of optimality condition



How does investment respond to changes in A_2 and r_1?





- What is the intuition for these comparative statics?

Capital and investment demand: parametric example

- Example: $F(K) = K^{\alpha}$ with $0 < \alpha < 1$
- Optimality condition

$$\alpha A_2 K_2^{\alpha - 1} = r_1 + d$$

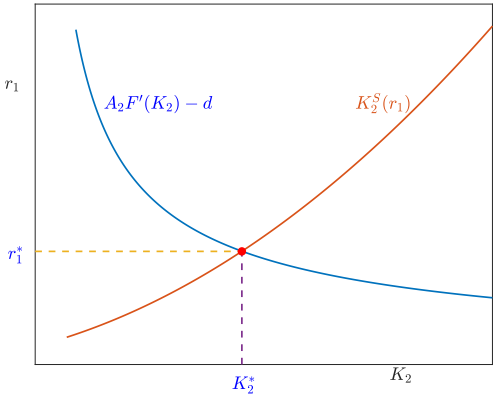
- $\Rightarrow$ optimal capital demand and investment

$$K_2^* = \left(\frac{\alpha A_2}{r_1 + d} \right)^{\frac{1}{1-\alpha}}$$

$$I_1^* = \left(\frac{\alpha A_2}{r_1 + d} \right)^{\frac{1}{1-\alpha}} - (1 - d) K_1$$

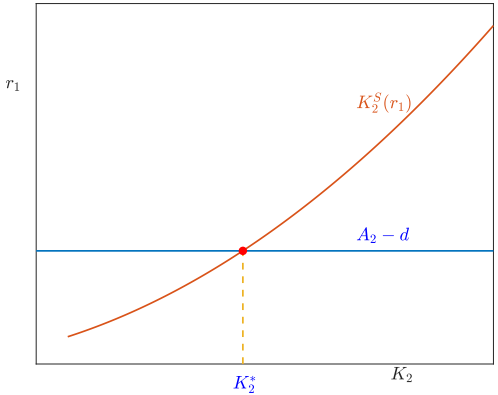
Warmup for part 2: equilibrium in the capital market

- Optimality condition $A_2F'(K_2) = r_1 + d$ traces out a capital demand curve
- Plot of $r_1 = A_2F'(K_2) - d$ vs K_2 is capital demand curve in (K_2, r_1)-space
- Warmup for part 2: can put this together w model of capital supply $K_2^S(r_1)$



Warmup: equilibrium with infinitely elastic capital demand

- Interesting special case: linear production $Y_2 = A_2 K_2$
- Optimality: $A_2 \gtrless r_1 + d \Rightarrow$ capital demand infinitely elastic at $r_1 = A_2 - d$
- Equilibrium interest rate $r_1 = A_2 - d$, quantity pinned down by supply



Dynamic general equilibrium with capital accumulation

Overview

- Lecture 4: household saving
- Lecture 5 so far: firm capital accumulation and production
- Now put these together in general equilibrium
- To make progress, work with special functional forms
  - no labor, linear production: $Y_t = A_t K_t$
  - full depreciation $d = 1$
  - utility with constant intertemporal elasticity of substitution
- Later: heavy use of these in our version of New Keynesian model
- Special assumptions but should be clear: construction of equilibrium follows steps that would also be valid with more general functions

Primitives of economy with capital accumulation

- **Preferences:** households have utility function

$$U(C_1) + \beta U(C_2) \quad \text{with} \quad U(C) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$$

- **Technology:** firms have production function

$$Y_t = A_t K_t, \quad t = 1, 2$$

and capital accumulates according to $K_2 = I_1 + (1 - d)K_1$ with $d = 1$, i.e.

$$K_2 = I_1$$

- **Resource constraints (feasibility):**

$$\begin{aligned} &\text{goods in period 1:} \quad C_1 + I_1 = Y_1 \\ &\text{goods in period 2:} \quad C_2 = Y_2 \end{aligned}$$

Competitive equilibrium with capital accumulation

Definition: a competitive equilibrium are quantities $(C_1, C_2, I_1, K_2, Y_1, Y_2)$ and an interest rate r_1 such that

1. Utility maximization: taking as given r_1 and W, households choose (C_1, C_2) to solve

$$\max_{C_1, C_2} \; U(C_1) + \beta U(C_2) \quad \text{s.t.} \quad C_1 + \frac{C_2}{1 + r_1} = W$$

   where W is the PDV of firm profits (because households own firms)

2. Profit maximization: firms maximize $W = \Pi_1 + \frac{\Pi_2}{1+r_1}$ or equivalently

$$W = \max_{K_2} \left\{ A_1 K_1 - I_1 + \frac{A_2 K_2}{1 + r_1} \right\}, \quad K_2 = I_1, \; Y_1 = A_1 K_1, \; Y_2 = A_2 K_2$$

3. Market clearing: demand = supply for goods

$$\text{goods in period 1:} \quad C_1 + I_1 = Y_1$$
$$\text{goods in period 2:} \quad C_2 = Y_2$$

Comment on market clearing conditions: credit market

- There really is a third market and correponding market clearing condition
  - credit market in which households and firms borrow/lend from/to each other at interest rate r_1 (recall that's why firms maximize PDV)
  - but can drop this due to Walras' Law
- Can write this as

$$b + B = 0$$

  where
  - b: household saving with $b < 0$ denoting borrowing
  - B: firm saving with $B < 0$ denoting borrowing
- Typical situation $b > 0$ and $B = -b < 0$, i.e. firms borrow from households to finance their investment
- See supplement. Can forget about this from now, no future appearances.

Solving for the competitive equilibrium allocation

- What is a convenient strategy to solve for the equilibrium allocation?
- Supplement: alternative strategy

Result: competitive equilibrium allocation (see supplement)

$$C_1 = \frac{\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1$$

$$C_2 = \frac{A_2}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1$$

$$K_2 = I_1 = \frac{1}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1$$

$$Y_1 = A_1 K_1$$

$$Y_2 = \frac{A_2}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1$$

$$1 + r_1 = A_2$$

Derivation: see supplement (one of the derivations, there are at least two)

Investment in the competitive equilibrium

- Focus on investment which is most interesting decision

$$I_1 = \frac{1}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1$$

- Note: can write this as saving rate $s(A_2)$ out of current output, Y_1:

$$I_1 = s(A_2)Y_1, \quad C_1 = [1 - s(A_2)]Y_1, \quad s(A_2) = \frac{1}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}$$

- Contrast this with Solow model: $I_t = sY_t$ where s = exogenously given
- In current model, saving rate is instead endogenous and depends on preferences (β, σ) and technology (A_2)!
- This is precisely what we mean when we say "Solow model is not microfounded but modern macro models are"

A recession due to a drop in A_1

- Assume current productivity $A_1 \downarrow$. What happens to $(C_1, C_2, I_1, Y_1, Y_2)$?
  - momentarily: what on earth is a drop in productivity?
- Recall expressions for $(C_1, C_2, I_1, Y_1, Y_2)$ a few slides ago, e.g.

$$Y_1 = A_1 K_1, \qquad Y_2 = \frac{A_2}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2} A_1 K_1, \quad \ldots$$

- Clearly all of $(C_1, C_2, I_1, Y_1, Y_2)$ fall when A_1 falls
- Intuition: economy is less productive, people are poorer $\Rightarrow$ cut consumption in both periods as well as investment
- Also note: both C_1 and I_1 fall. Reason: total resources Y_1 available for consuming or investing fall

$$C_1 + I_1 = Y_1, \quad Y_1 = A_1 K_1$$

A recession due to a drop in A_2

- Assume future productivity $A_2 \downarrow$. What happens to $(C_1, C_2, I_1, Y_1, Y_2)$?
- Y_2 unambiguously decreases when A_2 falls

$$Y_2 = A_2 K_2 = \frac{1}{\frac{1}{A_2} + \left(\frac{1}{\beta A_2}\right)^{\sigma}} A_1 K_1 \downarrow \quad \text{when} \quad A_2 \downarrow.$$

- But: effect on I_1 and C_1 is ambiguous, one falls the other rises

$$I_1 = s(A_2) Y_1, \quad C_1 = [1 - s(A_2)] Y_1, \quad s(A_2) = \frac{1}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}$$

- What happens to I_1 and C_1 depends on σ = IES

$$\sigma < 1 \quad \Rightarrow \quad s'(A_2) < 0 \quad \Rightarrow \frac{\partial I_1}{\partial A_2} < 0, \quad \frac{\partial C_1}{\partial A_2} > 0 \quad \Rightarrow \quad I_1 \uparrow, \; C_1 \downarrow$$

- Why? And what is the intuition?
- From now on always assume $\sigma < 1$ (= also the empirically realistic case)

A recession due to a drop in both A_1 and A_2

Can also consider combination: both A_1 and A_2 fall at same time

- For example A_1 falls and A_2 is correlated with A_1

$$\log A_2 = \rho \log A_1 + \varepsilon_2$$

  where ρ captures persistence, ε_2 the innovation at $t = 2$

- Economic effect is combination of effects on two previous slides
- Such correlated productivity movements will be important in next lecture

Oil shocks as productivity shocks (or gas shocks)

- What on earth is a drop in productivity?
  - do we think people forget how to make stuff? Not really
  - hard to come up with sensible justifications
- One possible justification: oil shocks or energy (gas etc) shocks
- Technology: firms use oil to produce

$$\tilde{Y}_t = \tilde{A}_t K_t^{\alpha} O_t^{1-\alpha}.$$

- Firms maximize output net of oil expenditure

$$Y_t = \max_{O_t} \tilde{A}_t K_t^{\alpha} O_t^{1-\alpha} - p_t O_t \quad \text{where } p_t = \text{oil price}$$

$$\Rightarrow \quad Y_t = A_t K_t \text{ where } A_t = \text{effective productivity} = \alpha \tilde{A}_t^{1/\alpha} \left(\frac{1-\alpha}{p_t} \right)^{(1-\alpha)/\alpha}$$

so an increase in p_t causes a drop in effective productivity

Half-way mark for this course:
taking stock and looking ahead

What we have done in first half of course

- Learned the basics of modern macroeconomics
- What is a model? “The map is not the territory” and so on
- Reminded ourselves of basic microeconomics
  - intratemporal choice, e.g. labor supply
  - intertemporal choice, e.g. consumption, saving, investment
- Learned the following key concepts
  1. Competitive equilibrium
  2. Pareto effiency
  3. Welfare theorems

What we have done in first half of course

- Worked with some key parameters, corresponding functional forms
  - elasticity of substitution in production
  - intertemporal elasticity of substitution
- Reminded ourselves that income and substitution effects are everywhere!
- Applied these tools to think about some key topics
  1. Labor supply and labor demand, long-run trends in hours worked
  2. Substitution in production, e.g. in response to cut-off of Russian gas
  3. Consumption and saving decisions, permanent income hypothesis & its shortcomings, MPCs out of transitory & permanent income shocks
  4. Investment and capital accumulation

What we will do in second half of course

- Put your basic training to use to think about more and more applied topics

Section 6	Business Cycle Macro and Lucas Critique
Section 7	New Keynesian Model I
Section 8	New Keynesian Model II
Section 9	The Financial Crisis, Asset Bubbles
Section 10	Unemployment (Pissarides), Inequality in Macro

- Looking for something to read during reading week? Read Mankiw and Weinzierl (2011) "An Exploration of Optimal Stabilization Policy"

  https://www.brookings.edu/wp-content/uploads/2011/03/2011a_bpea_mankiw.pdf

  - Two-period version of New Keynesian model I will teach is based on this paper
  - Model from this lecture but with "nominal rigidities", i.e. sticky prices
  - You have basic tools to read this but may still lack some vocabulary
  - Read sections I-IV and IX-X (skip sections V-VIII)