

# Lecture 4

## Consumption, Saving, Interest Rates

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Macroeconomics EC2B1

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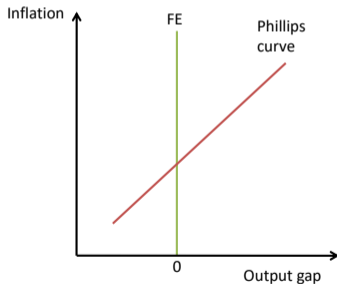
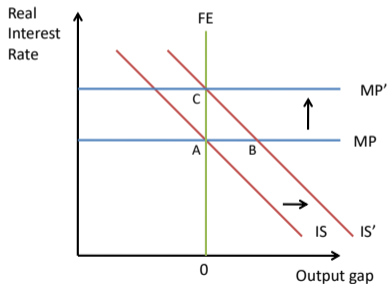
# Today: your first **dynamic** modern macro model

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- So far: static models = models without time
- But lots of questions in macro are inherently **dynamic** = they involve time
  - Why do economies **grow**? Why  $\approx$  constant long-run **growth rates**?
  - Why **booms, busts**? Should policy **stabilize** business **cycles**? How?
  - In typical recession, why does  $I$  **decline** more than  $C$ ?
  - even true for questions we've already covered, e.g. Germany without Russian gas  $\Rightarrow$  importance of time horizon for (le Chatelier)
  - ... but so far we discussed such issues “outside of the model”
- **Today and rest of course:** there is time  $t = 0, 1, 2, \dots$  (typically discrete)
- Have already seen a dynamic model = Solow model, but no microfoundations (so  $\neq$  modern macro)
- Also recall Keynesian IS-MP-PC model from EC1B1:  $t$ -subscripts but not really a proper dynamic model. And definitely not microfounded.

# IS-MP-PC model from EC1B1: words tell dynamic story, but then why are we just shifting around these curves?

## Stabilisation of IS Curve Shock



**In response to IS curve shocks (demand shocks), monetary policy can (in principle) stabilise both output and inflation**

# Plan

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1. The Keynesian view of consumption
2. A two-period model of consumption
3. The permanent income hypothesis
4. Consumption, saving and interest rates in general equilibrium

# Background Readings

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- Follow closely Kurlat Chapter 6, except part 4 on previous slide
- EC1A1 lecture notes on consumption and saving (intertemporal choice)
- EC1B1 notes on Keynesian consumption function and Keynesian cross

# The Keynesian view of consumption

## Keynes on consumption (also see EC1B1 notes)

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- Keynes in his 1936 “General Theory of Interest, Employment & Money”:  
“The fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that **men are disposed**, as a rule and on the average, **to increase their consumption as their income increases but not by as much as the increase in the income.**”

- In equations, consumption  $C$  depends on income  $Y$  as

$$C = c(Y) \quad \text{with} \quad c'(Y) > 0 \quad \text{and} \quad c'(Y) < 1$$

- $c(Y)$  = consumption function. Question for you: name for  $c'(Y)$ ?
- Depending on how interpret language, he may also mean

$$\frac{c'(Y)Y}{c(Y)} = \frac{\partial \log c(Y)}{\partial \log Y} < 1$$

i.e. elasticity of  $C$  with respect to  $Y < 1$

# Importance of MPC for multipliers (Keynesian cross)

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From EC1B1 notes:  $MPC = \gamma$ , consumption function  $C = \alpha + \gamma(Y - T)$

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## Keynesian Cross

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- Equilibrium condition in simple Keynesian Cross model
  - Output must equal planned expenditures:

$$Y = \alpha + \gamma(Y - T) + I + G + NX$$

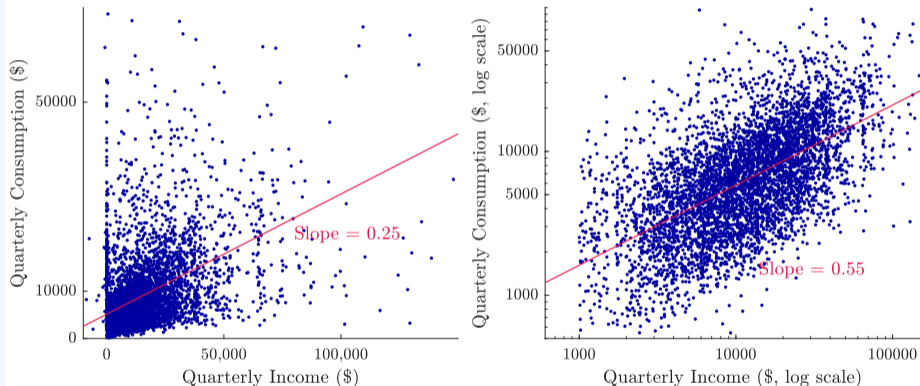
- A little bit of algebra then gives:

$$Y = \frac{1}{1 - \gamma} [\alpha - \gamma T + I + G + NX]$$

- Government purchases multiplier =  $\frac{1}{1 - \gamma}$
- Tax cut multiplier =  $\frac{\gamma}{1 - \gamma}$



In micro data both  $c'(Y)$  and  $\frac{\partial \log c(Y)}{\partial \log Y}$  are considerably  $< 1$



**Fig. 6.1.1:** Evidence on the Keynesian consumption function. Each dot represents a household. Source: Consumer Expenditure Survey, 2014.

# Implications for aggregate consumption and income

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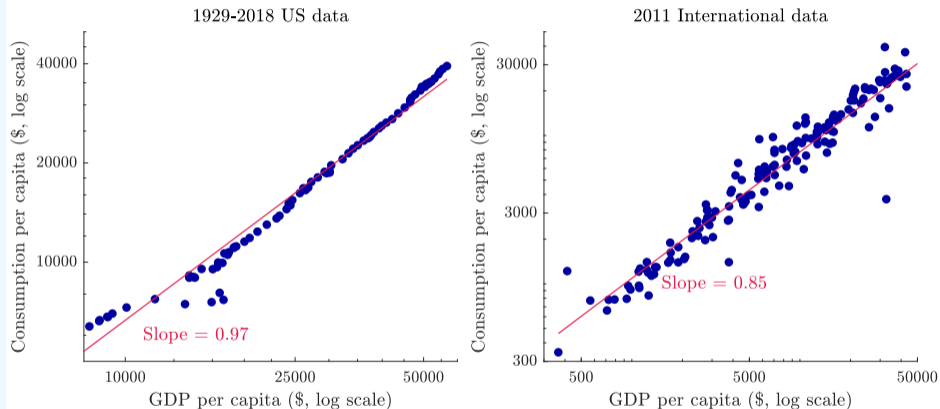
- Recall: in micro data both  $c'(Y)$  and  $\frac{\partial \log c(Y)}{\partial \log Y}$  are considerably  $< 1$
- Around 1950s: seen as conclusive evidence in favor of Keynesian consumption function
- But then what happens as countries grow richer over time and  $Y$  grows?
- If  $\frac{\partial \log c(Y)}{\partial \log Y} < 1$  then we should have

$$\frac{c(Y)}{Y} \quad \text{declining with } Y$$

so people consume proportionately less and less as country grows richer

- Do we see this? No.

In macro data  $\frac{\partial \log c(Y)}{\partial \log Y}$  is much larger,  $\frac{C}{Y}$  stable over time



*Fig. 6.1.2: Evidence on the Keynesian consumption function from aggregate data. The left panel is US time-series evidence; the right panel is cross-country evidence. Sources: NIPA and Feenstra et al. (2015)*

# Summary

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- In micro data both  $c'(Y)$  and  $\frac{\partial \log c(Y)}{\partial \log Y}$  are considerably  $< 1$
- but in macro data,  $\frac{\partial \log c(Y)}{\partial \log Y}$  is much larger (perhaps even  $\approx 1$ ),  $\frac{C}{Y}$  stable over time
- Seems puzzling, what's going on?
- Next: use economic theory to resolve puzzle

# A two-period model of consumption

# A two-period model of consumption

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- For now: study problem of individual in isolation taking as given prices (partial equilibrium)
- In a bit: general equilibrium
- Two time periods  $t = 1$  and  $t = 2$
- Consumption  $c_1$  and  $c_2$ , income  $y_1$  and  $y_2$
- Utility function

$$u(c_1) + \beta u(c_2)$$

with  $u$  strictly increasing, concave, discount factor  $0 < \beta < 1$

- Vocabulary that sometimes comes up: discount **factor** vs discount **rate**
  - discount **factor**:  $\beta$ , typically a bit below one
  - discount **rate**:  $\rho$ , typically a bit above zero
  - link:  $\rho = 1/\beta - 1 \Leftrightarrow \beta = \frac{1}{1+\rho}$ , e.g.  $\beta = 0.95, \rho = 1/0.95 - 1 = 5.26\%$

# A two-period model of consumption

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Household solves

$$\max_{c_1, c_2, a} u(c_1) + \beta u(c_2) \quad \text{s.t.}$$

$$c_1 + a = y_1 \quad (1)$$

$$c_2 = y_2 + (1 + r)a \quad (2)$$

Notation:

- $c_1, c_2$ : consumption at  $t = 1$  and  $t = 2$
- $y_1, y_2$ : income at  $t = 1$  and  $t = 2$
- $r$ : interest rate (for now exogenously given)
- $a$ : saving
- Note:  $a$  can be negative,  $a < 0$  means household is borrowing
- From (1)  $a < 0 \Rightarrow c_1 > y_1$ , i.e. consume more than income by borrowing

## Formulation in terms of present-value budget constraint

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- Implicit assumption: can borrow and save as much you want at rate  $r$
- then can combine (1) and (2) into present-value budget constraint

$$c_1 + a = y_1 \quad \text{and} \quad a = \frac{c_2}{1+r} - \frac{y_2}{1+r} \quad \Rightarrow \quad \underbrace{c_1 + \frac{c_2}{1+r}}_{\text{PV of consumption}} = \underbrace{y_1 + \frac{y_2}{1+r}}_{\text{PV of income}}$$

where PV stands for present value

- Hence households maximize utility s.t. present-value budget constraint

$$\begin{aligned} \max_{c_1, c_2} \quad & u(c_1) + \beta u(c_2) \quad \text{s.t.} \\ & c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \end{aligned}$$



# Euler equation

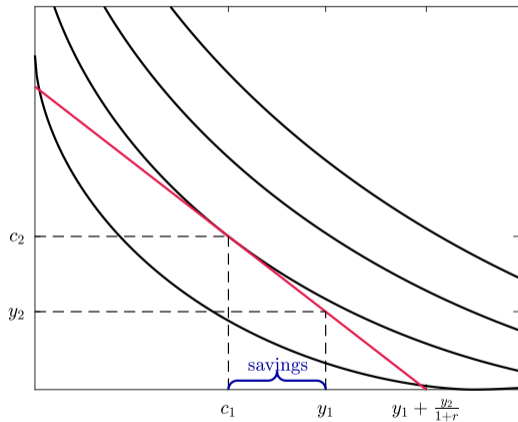
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- Optimality condition

$$u'(c_1) = \beta(1+r)u'(c_2) \quad (*)$$

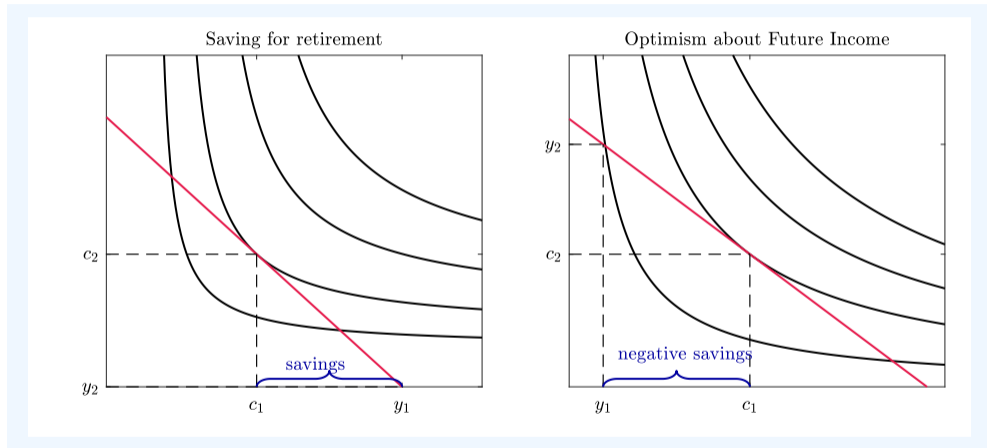
- Intuition: consume \$1 now vs save and consume later
  - consume \$1 now: utility increases by  $u'(c_1)$
  - save \$1: have  $\$(1+r)$  at  $t=2$ , utility by  $\beta u'(c_2)$  for each \$
- (\*) is called “Euler equation” after Leonard Euler, plays an important role in macroeconomics
- Note: “Euler equation” sounds fancy but simply means “intertemporal optimality condition”

# Graphical representation



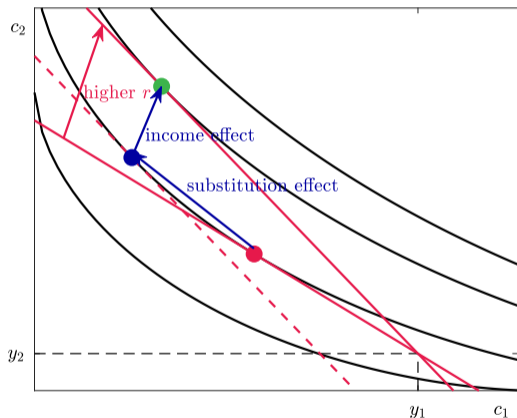
*Fig. 6.2.1: The consumption-savings decision as a two-good consumption problem.*

# A saver (left panel) and a borrower (right panel)



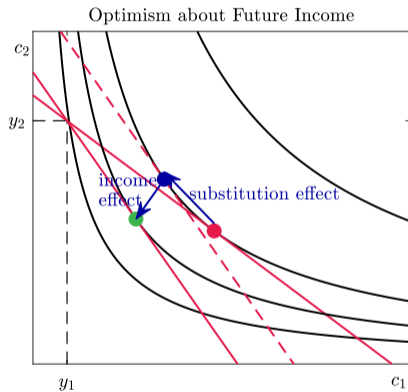
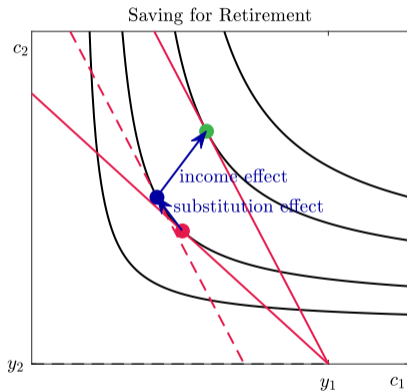
Left panel:  $y_2 = 0$  and  $y_1 > 0 \Rightarrow$  save. Right panel:  $y_1 << y_2 \Rightarrow$  borrow.

# What happens when $r$ changes?



*Fig. 6.2.3: Consumption response to higher interest rates.*

# Effect of change in $r$ for saver (left) and borrower (right)



# The permanent income hypothesis

# An important observation about our model

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- Can write two-period model as

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \quad \text{s.t.} \quad c_1 + \frac{c_2}{1+r} = W$$

$$\text{where } W = y_1 + \frac{y_2}{1+r} = \text{PV of income}$$

sometimes also called “lifetime income” or “permanent income”

- Can immediately see:  $c_1$  will depend on  $y_1, y_2$  only through  $W$ , i.e.

$$c_1 = c(W), \quad W = y_1 + \frac{y_2}{1+r}$$

- It's not current income that matters, but PV of current + future income
- This way of thinking: “permanent income hypothesis” (Friedman, 1957)
- Contrast with Keynes quote, captured w consumption function  $c_1 = c(y_1)$
- Can already see: our model is really very different from Keynes view
- Next: flesh this out with parametric example

## A parametric example

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$$u(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \quad (**)$$

- $\sigma$  is called “intertemporal elasticity of substitution (IES)”
- Sometimes  $1/\sigma$  is called “coefficient of relative risk aversion (RRA)” and (\*\*\*) is called “CRRA utility”
- Can show: log utility = special case with  $\sigma = 1$  (use l’Hopital’s rule)

$$u(c) = \lim_{\sigma \rightarrow 1} \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} = \log c$$

- Note: weird  $-1$  in numerator only there because we want to take this limit
- Important for those reading Kurlat: he, many others use  $\sigma$  for coeff of RRA

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \text{or} \quad u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

- Same but  $\sigma$  in place of  $\frac{1}{\sigma}$ . Unfortunate notation, but don’t get confused! 23



## Euler equation in parametric example

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- With functional form (\*\*) have

$$u'(c) = c^{-1/\sigma}$$

- Euler equation

$$u'(c_1) = \beta(1+r)u'(c_2) \Rightarrow c_1^{-1/\sigma} = \beta(1+r)c_2^{-1/\sigma}$$

or

$$\frac{c_2}{c_1} = [\beta(1+r)]^\sigma$$

- IES  $\sigma$  governs responsiveness of growth rate of consumption  $c_2/c_1$  to changes in  $r$  and  $\beta$ , e.g.

$$\frac{\partial \log(c_2/c_1)}{\partial \log(1+r)} = \sigma \Rightarrow \text{hence the name IES}$$

- A low IES  $\sigma$  means households dislike intertemporal substitution (want to smooth  $c$ )  $\Rightarrow$  low responsiveness of  $c_2/c_1$  to changes in  $r$  and  $\beta$

## Analytic solution in parametric example

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- Not hard to show (you should be able to!): solution to problem is

$$c_1 = \frac{\left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}{1 + \left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)} W, \quad c_2 = \frac{1+r}{1 + \left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)} W$$

where

$$W = y_1 + \frac{y_2}{1+r}$$

- Useful special case (see next slide why):  $\beta(1+r) = 1$

$$c_1 = c_2 = \frac{1+r}{2+r} W$$

- If further  $r = 0$ , then  $c_1 = c_2 = \frac{W}{2}$

# Reasonable parameterizations

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Reasonable parameterizations have two features

1.  $\beta(1 + r)$  not too far from 1, equivalently discount rate  $\rho = 1/\beta - 1 \approx r$ 
  - e.g.  $\rho = 0.05, r = 0.01$  but not  $\rho = 0.5, r = 0.01$  or  $\rho = 0.05, r = 0.5$
  - in general equilibrium model of part 4,  $1 + r^* \approx 1/\beta$
  - market discounts future at roughly the same rate as individuals
2.  $\sigma$  not too large
  - e.g.  $\sigma = 1/2$ , probably  $\leq 1$ , definitely not  $\sigma = 5$  or  $10$
  - empirical evidence: people do not massively substitute intertemporally, i.e. they do not massively increase spending when  $r \downarrow$

## MPC out of transitory income shock

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- Question: suppose  $y_1$  increases by \$1 but  $y_2$  is unchanged. By how much does  $c_1$  increase? What is MPC out of **transitory** income shock?
- Answer: the marginal propensity to consume (MPC) out of a transitory income shock is

$$\frac{\partial c_1}{\partial y_1} = \frac{\left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}{1 + \left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}$$

- **Contrast with Keynesian cross:**  $C = \alpha + \gamma(Y - T)$ ,  $\gamma$  = exogenously given
- In current model, MPC is instead endogenous and depends on preferences  $(\beta, \sigma)$  and prices  $(r)$ !
- This is precisely what we mean when we say “Keynesian cross is not microfounded but modern macro models are”

## MPC out of transitory income shock

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- Question: suppose  $y_1$  increases by \$1 but  $y_2$  is unchanged. By how much does  $c_1$  increase? What is MPC out of **transitory** income shock?
- Answer: the marginal propensity to consume (MPC) out of a transitory income shock is

$$\frac{\partial c_1}{\partial y_1} = \frac{\left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}{1 + \left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}$$

- Also note: **this MPC is a number  $< 1$**  and  $\ll 1$  for reasonable parameters
- In fact, in this 2-period model,  $\text{MPC} \approx 1/2$ , e.g. with  $\beta(1+r) = 1$

$$\frac{\partial c_1}{\partial y_1} = \frac{1+r}{2+r}$$

and if  $r = 0$  then  $c_1 = W/2$  and

$$\frac{\partial c_1}{\partial y_1} = \frac{1}{2}$$

## MPC out of permanent income shock

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- Question: suppose both  $y_1$  and  $y_2$  increase by \$1. By how much does  $c_1$  increase? What is MPC out of **permanent** income shock?
- Assumption:  $y_1 = \bar{y}_1 + \Delta$ ,  $y_2 = \bar{y}_2 + \Delta$  and  $\Delta$  increases
- From  $W = y_1 + \frac{y_2}{1+r}$ : when both  $y_1, y_2 \uparrow$  by \$1,  $W \uparrow$  by  $\approx$  \$2

$$\frac{\partial W}{\partial \Delta} = 1 + \frac{1}{1+r} = \frac{2+r}{1+r} \approx 2$$

- Similarly

$$\frac{\partial c_1}{\partial \Delta} = \frac{\left(\frac{1}{\beta(1+r)}\right)^\sigma (2+r)}{1 + \left(\frac{1}{\beta(1+r)}\right)^\sigma (1+r)}$$

- For reasonable parameters **this is a number  $\approx$  1**. Exact when  $\beta(1+r) = 1$

$$\frac{\partial c_1}{\partial \Delta} = 1$$

## Summary: main prediction of permanent income hypothesis

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- Main prediction of permanent income hypothesis (PIH):

MPC out of transitory income  $\ll$  MPC out of permanent income

- Intuition: save large fraction of transitory income change so as to smooth consumption over time

## Extension to many time periods

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- Macro models used for research, policy etc: not just two time periods  $t = 1, 2$ . Instead: many time periods  $t = 1, 2, 3, \dots, T$ 
  - consumption  $c_1, c_2, c_3, \dots, c_T$ , income  $y_1, y_2, y_3, \dots, y_T$
  - relatively straightforward extension – see Kurlat chapter 6.3
- **Takeaway: more time periods make PIH prediction even starker**
- Key: PV budget constraint

$$c_1 + \frac{c_2}{1+r} + \frac{c_3}{(1+r)^2} + \dots + \frac{c_T}{(1+r)^{T-1}} = W$$

$$\text{where } W = y_1 + \frac{y_2}{1+r} + \frac{y_3}{(1+r)^2} + \dots + \frac{y_T}{(1+r)^{T-1}}$$

- **Makes PIH prediction even starker:** effect of transitory change in  $y_1$  (say) has negligible effect on permanent income  $W$ , hence  $\partial c_1 / \partial y_1$  very small
- Example: if  $\beta(1+r) = 1$  and  $r = 0$ , then  $c_1 = c_2 = \dots = W/T$  and
$$\frac{\partial c_1}{\partial y_1} = \frac{1}{T} = \text{small number}$$



# Friedman's example: paydays staggered throughout week

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## CHAPTER IX

### Summary and Conclusion

THE central theme of this monograph can be illustrated by a simple hypothetical example. Consider a large number of men all earning \$100 a week and spending \$100 a week on current consumption. Let them receive their pay once a week, the pay days being staggered, so that one-seventh are paid on Sunday, one-seventh on Monday, and so on. Suppose we collected budget data for a sample of these men for one day chosen at random, defined income as cash receipts on that day, and defined consumption as cash expenditures. One-seventh of the men would be recorded as having an income of \$100, six-sevenths as having an income of zero. It may well be that the men would spend more on pay day than on other days but they would also make expenditures on other days, so we would record the one-seventh with an income of \$100 as having positive savings, the other six-sevenths as having negative savings. Consumption might appear to rise with income, but, if so, not as much as income, so that the fraction of income saved would rise with income. These results tell us nothing meaningful about consumption behavior; they simply reflect the use of inappropriate concepts of income and consumption.

Source: Friedman (1957) "A Theory of the Consumption Function"

<https://www.nber.org/books-and-chapters/theory-consumption-function>

This excerpt from conclusion <https://www.nber.org/system/files/chapters/c4411/c4411.pdf>

# The PIH predicts that multipliers are small

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## Keynesian Cross

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- Equilibrium condition in simple Keynesian Cross model
  - Output must equal planned expenditures:

$$Y = \alpha + \gamma(Y - T) + I + G + NX$$

- A little bit of algebra then gives:

$$Y = \frac{1}{1 - \gamma} [\alpha - \gamma T + I + G + NX]$$

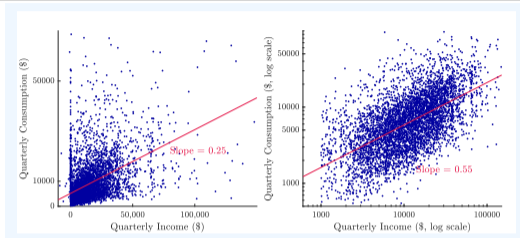
- Government purchases multiplier =  $\frac{1}{1 - \gamma}$
- Tax cut multiplier =  $\frac{\gamma}{1 - \gamma}$

PIH predicts that aggregate MPC out of current income is small, say  $\gamma = 0.05$

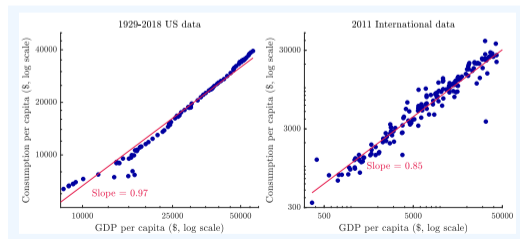
⇒ Multipliers are small

- government purchases multiplier  $\approx 1$ , say  $= \frac{1}{1 - 0.05} = 1.053$
- tax cut multiplier close  $\approx 0$ , say  $= \frac{0.05}{1 - 0.05} = 0.053$

# Resolving our puzzle with the permanent income hypothesis



*Fig. 6.1.1: Evidence on the Keynesian consumption function. Each dot represents a household. Source: Consumer Expenditure Survey, 2014.*



*Fig. 6.1.2: Evidence on the Keynesian consumption function from aggregate data. The left panel is US time-series evidence; the right panel is cross-country evidence. Sources: NIPA and Feenstra et al. (2015)*

# Resolving our puzzle with the permanent income hypothesis

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## Micro data:

- Transitory income shocks are important, some people have good year, others have bad year. Get mixed together with permanent income shocks.
- Coefficient in regression of  $C$  on  $Y$  reflects MPCs out of both permanent and transitory income shocks
- Estimate  $c'(Y)$  and  $\frac{\partial \log c(Y)}{\partial \log Y}$  considerably  $< 1$

## Macro data:

- Transitory income shocks mostly wash out in the aggregate
- Coefficient in regression of  $C$  on  $Y$  mostly reflects MPCs out of permanent income changes (e.g. country as a whole is richer on average)
- Estimate  $c'(Y)$  and  $\frac{\partial \log c(Y)}{\partial \log Y}$  much closer to 1

# Empirical failure of the PIH

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- Multi-period versions of the PIH generate MPCs out of transitory income shocks that are **way too small** relative to data
- Can show: model MPC  $\approx r$  so something like 0.02 or 0.05 per year
- In contrast, empirical MPCs are around 0.3 per year on average (i.e. people consume 30 cents for every \$1)
- ... lots of heterogeneity in MPCs
- ... and this heterogeneity depends on things like liquid wealth, i.e. more cash-strapped households have higher MPCs
- See evidence on next slides
- So perhaps Keynes wasn't as wrong as we thought?

## Measuring the MPC

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- Parker, Souleles, Johnson, McClelland (2012):
  - Look at 2008 Economic Stimulus Payments
  - People got checks from the government (~1000 USD)
  - Timing was random (based on last digits of SS#)
  - Compare those that got a check at time  $t$  with those that didn't (treatment vs. control)
  - How much more did treated people spend over a 3-month period
- **... Another natural experiment!**
  - Randomisation means on average the treated and control people are the same

# Empirical failure of the PIH (from EC1B1 notes)

## Evidence of MPC Levels

TABLE 3—THE RESPONSE TO ESP RECEIPT AMONG HOUSEHOLDS RECEIVING PAYMENTS

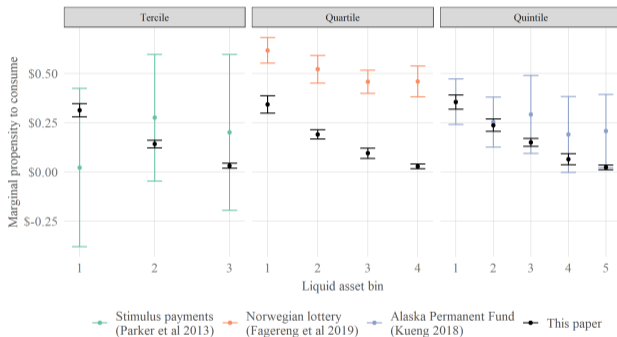
	Dollar change in		Percent change in		Dollar change in	
	Nondurable spending OLS	All CE goods and services OLS	Nondurable spending OLS	All CE goods and services OLS	Nondurable spending 2SLS	All CE goods and services 2SLS
<i>Panel A. Sample of all households (N = 17,478)</i>						
ESP	0.117 (0.060)	0.507 (0.196)			0.123 (0.081)	0.509 (0.253)
I(ESP)			2.63 (1.07)	3.97 (1.34)		
$I(ESP_{it} > 0 \text{ for any } t)_i$	9.58 (36.07)	21.21 (104.00)	-0.88 (0.50)	-1.17 (0.63)	8.23 (38.79)	20.77 (112.18)
<i>Panel B. Sample of households receiving ESPs (N = 11,239)</i>						
ESP	0.185 (0.066)	0.683 (0.219)			0.252 (0.103)	0.866 (0.329)
I(ESP)			3.91 (1.33)	5.63 (1.69)		
<i>Panel C. Sample of households receiving only on-time ESPs (N = 10,488)</i>						
ESP	0.214 (0.070)	0.590 (0.217)			0.308 (0.112)	0.911 (0.342)
I(ESP)			4.52 (1.50)	6.05 (1.89)		

- Technical point:
  - These estimates come from regressions
  - Cf. your metrics class!

Source: Parker, Souleles, Johnson, McClelland (2012)

# Empirical failure of the PIH

Figure 6: Marginal Propensity to Consume by Asset Buffer



Note: This figure compares the estimates of heterogeneity by assets in the passthrough of income shocks to consumption. Parker et al. (2013), Fagereng, Holm and Natvik (2018) and Kueng (2018) use terciles, quartiles, and quintiles respectively. To enable comparability with these prior papers, we calculate the marginal propensity to consume (instead of the elasticity of consumption to income) using their respective bin cutoffs. Our paper, Parker et al. (2013), and Kueng (2018) measure the MPC on nondurables. Fagereng, Holm and Natvik (2018) measures the MPC on total consumption. See Section 3.5 for details.



# How to fix the PIH prediction?

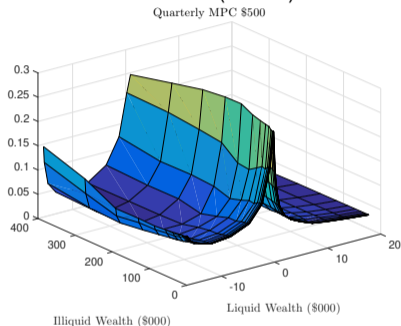
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Huge literature. Most common fixes:

- Credit constraints, i.e. drop assumption that can borrow as much as you want at rate  $r$
- Behavioral theories
  - mental accounting
  - present bias or “salience of the present”  $\Rightarrow$  dynamic inconsistency and procrastination, see EC1P1 lecture notes
  - ...

# This is exactly what the HANK literature is all about

- Graph from Kaplan, Moll and Violante (2018)



- When PIH is fixed, something interesting happens: model behaviour resembles Keynesian cross again, in particular sizable multipliers
- But important difference: micro founded model, makes precise predictions about behavior as well as inequality, can use it to think about welfare

# Consumption, saving and interest rates in general equilibrium

# Consumption, saving, interest rates in general equilibrium

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- So far: exogenously given interest rate  $r$
- Now: equilibrium determination of  $r$
- Continue to work with two-period model
- Economy without production, i.e. endowment economy (like in EC2A1)
- Main application: representative household model
- But start with environment with multiple households  $i = 1, \dots, I$  for reason that will become clear shortly

# Borrowing and saving in equilibrium

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- $I$  households  $i = 1, \dots, I$
- Recall:  $a_i > 0 =$  saving,  $a_i < 0 =$  borrowing
- Claim: in a closed system, e.g. a closed economy or this classroom

$$\sum_{i=1}^I a_i = 0$$

- Logic: there is a saver for every borrower, i.e. you can only borrow if someone takes the other side of this trade and lends to you
- What if all households are alike, i.e.  $a_i = a$  for all  $i = 1, \dots, I$ ? Then clearly

$$a_i = 0 \quad \text{all } i = 1, \dots, I$$

i.e. in equilibrium noone will borrow and noone will save

- Next: competitive equilibrium with rep household  $\Rightarrow$  exactly what happens<sub>44</sub>

# Competitive equilibrium with representative household

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**Definition:** a competitive equilibrium are quantities  $(c_1, c_2, a)$  and an interest rate  $r$  such that

1. Utility maximization: taking as given  $r$ , the representative household chooses  $(c_1, c_2, a)$  to solve

$$\max_{c_1, c_2, a} u(c_1) + \beta u(c_2) \quad \text{s.t.} \quad c_1 + a = y_1, \quad c_2 = y_2 + (1 + r)a$$

2. Profit maximization: none because no production
3. Market clearing: demand = supply for goods and credit market

$$\text{goods in period 1:} \quad c_1 = y_1$$

$$\text{goods in period 2:} \quad c_2 = y_2$$

$$\text{credit market:} \quad a = 0$$

Note on goods market: if no borrowing, can only consume your income

## What's the point of this equilibrium model?

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- We usually solve for equilibrium quantities like  $(c_1, c_2, a)$  because we're interested in their behavior, how they change with parameters etc
- But here we already know that  $(c_1, c_2, a) = (y_1, y_2, 0)$  so what's even the point?
- Answer: the model makes interesting predictions about the equilibrium interest rate

## Solving for the equilibrium interest rate

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- Household maximization  $\Rightarrow$  Euler equation

$$u'(c_1) = \beta(1+r)u'(c_2)$$

- But we know that  $c_1 = y_1$  and  $c_2 = y_2$ . Therefore

$$1 + r^* = \frac{1}{\beta} \frac{u'(y_1)}{u'(y_2)}$$

- Parametric example from earlier:  $u'(c) = c^{-1/\sigma}$

$$1 + r^* = \frac{1}{\beta} \left( \frac{y_2}{y_1} \right)^{1/\sigma}$$

- Note: for entire economy, reasonable to assume  $y_1 \approx y_2 \Rightarrow 1 + r^* \approx 1/\beta$



## Comparative statics: how does $r^*$ depend on parameters?

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- Recall

$$1 + r^* = \frac{1}{\beta} \left( \frac{y_2}{y_1} \right)^{1/\sigma}$$

- What happens to equilibrium interest rate  $r^*$  when
  - $\beta \uparrow$ ?
  - $y_1 \uparrow$ ?
  - $y_2 \uparrow$ ?

and what is the intuition?

- Note: questions like these are excellent exam questions!

## Next few lectures

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- More interesting dynamic macro models with non-degenerate predictions for the quantities?
- $\Rightarrow$  add production and capital accumulation
- Use them to think about business cycles and macroeconomic stabilization policies, in particular monetary and fiscal policy
  - microfounded analogue of Keynesian IS-MP-PC model from EC1B1