CHAPTER VI

Distribution and Economic Progress

I

The subject of this chapter is one of the most venerable of economic problems. The effect of progress upon distribution was a question inevitably raised by the Ricardian theory of rent, and naturally it often engaged the attention of the classical economists. But we do not now need to go back to the classical economists; for we possess today, in the marginal productivity theory, a much superior line of approach to it. The marginal productivity theory is simply an extension of the Ricardian law of rent; and it suggests the problem as infallibly as its predecessor did.

Nevertheless, none of the modern treatments of the problem seem wholly satisfactory. The best account in English is undoubtedly that of Professor Pigou, in the *Economics of Welfare*.[1] Almost everything which is there said seems to be beyond criticism; but it must be remembered that his account does not profess to give a oomplete examination of the problem. He is simply concerned with one special question—whether anything which is to the advantage of the National Dividend as a whole is likely at the same time to be to the disadvantage of the poorer members of society. He concludes—rightly, it appears— that while it is possible for economic progress sometimes

[1] 2nd ed., bk. iv., chs. ii. and iii.

to make the poor poorer, while it makes the rich richer, this is highly unlikely.

So far as this goes, it is satisfactory; but this is not the only question to which a theory of distribution and progress ought to provide an answer. For example, there is the question of relative shares which was raised by Professor Cannan.[1] Is economic progress likely to raise or lower the proportion of the National Dividend which goes to labour? A complete theory ought to answer this question too.[2]

Before setting out a positive solution, it is necessary to make clear two assumptions on which the following argument rests. For one thing, although we are really dealing with a community in constant change, and comparing two stages of that change, we are obliged to assume that in each case the system is in equilibrium. The use of the marginal productivity method implies this.[3] But although this assumption is a grave weakness, it need not deprive our results of all usefulness. For some purposes, it is the equilibrium position which we want to know about; and for the rest, although we should have to introduce large qualifications if we sought to apply our results to the distribution of the National Dividend in two years quite close together, the error from this source will generally be quite small if we are comparing two fairly long periods separated by a considerable span of time.

The other assumption is more recondite, and at

[1] "The Division of Income" in *The Economic Outlook*, p. 215.

[2] Professor Cannan's aversion from the more abstract and rigorous methods of economic analysis probably prevented him from giving a final solution. An attempt at a solution on more abstract lines is, however, to be found in Dalton, *The Inequality of Incomes*, pp. 185-220. If it were possible to accept Dr. Dalton's argument, much of the discussion in this chapter would be unnecessary. But it appears to contain a flaw.

[3] See above, p. 21.

the same time its significance is much more doubtful. We have to ignore the possibility of increasing returns, using that ambiguous expression in the sense of economies of mere size, arising from an increase in the quantity of resources in general at the disposal of the community, independently of any variation in the proportions between the quantities of different kinds of resources available. Clearly the possibility of such economies has an enormous importance in the theory of Production and Economic Progress. It is not impossible that they have a bearing on distribution. This could conceivably be allowed for to some extent, but only at the cost of wrecking completely any simplicity which it has been possible to import into the following arguments. And it could probably be shown that the conclusions would be substantially unaffected.[1]

II

The kinds of "progress" which have to be dealt with in economic theory are four in number:

1. Increase in population.
2. Increase in the ability or willingness to work of a constant population.
3. Increase in capital.
4. Inventions and improvements.

To these there should perhaps be added changes in the tastes of consumers, as a fundamental cause of secular economic change, very similar, as we shall see, to invention, as far as their effects on distribution

[1] See Appendix, section (ii).

are concerned; but they cannot, by any stretch of the imagination, be classified as "progress."

From a purely analytical point of view, 1, 2, and 3 are the same problem. The consequences of a change in the quantity of labourers, of labour, or of capital, can all be treated as special cases of the general question of the effect on distribution of a change in the supply of one factor of production.

The answer to this question can be stated in the form of three rules, of which one is scarcely more than a definition, but is put in for completeness; the second is a generally accepted, but less obvious, proposition; the third appears to be new. Much the most satisfactory way of proving the validity of the second and third rules is to use the mathematical method set out in the Appendix to this book;[1] but an attempt at non-mathematical proof can be made, and will be set out here.

The three propositions are:

1. *An increase in the supply of any factor of production will increase the absolute share* (i.e., *the real income*) *accruing to that factor if the elasticity of demand for that factor is greater than unity.*

2. *An increase in the supply of any factor will always increase the absolute share of all other factors taken together.* If the increase in the variable factor is small, then the return to the additional units will approximately equal the addition which they have made to the whole product. But since the marginal product of the variable factor is now reduced, the units previously present will get a smaller return than they got before, so that the old total product will be divided between these units and the other factors in a ratio more

[1] See Appendix, sections (iii.) and (iv.)

favourable to the latter. The return to the other factors will therefore be increased.[1]

It is possible, however, that the increased return to the other factors may affect their supply. But in whatever way their supply is affected, whether it increases or diminishes, it is inconceivable that it should diminish to such an extent as to leave the total return to them smaller than it was before. The most extreme case conceivable is that in which the providers of these other factors have a completely inelastic demand for income in terms of the factor they supply; in this case the return to these other factors will of course be unchanged.[2]

Although the absolute share of all other factors taken together cannot diminish, this is not necessarily true of any particular other factor. For example, if the demand for bakers' services is inelastic, but bakers are easily transmuted into confectioners, then an increase in the supply of bakers will probably not increase the real income of confectioners. But we need not trouble ourselves with this difficulty so long as we are talking about groups which are reasonably distinct. In nearly any application which we are likely to want to make, it will be true that an increase in the supply of any factor will increase the real income of any other factor.[3]

[1] This is seen at once if we use the rent diagram, continually used by Clark in *The Distribution of Wealth* (*e.g.* on p. 366).

[2] See above, p. 98, note.

[3] Some of the conclusions which follow from this are very far-reaching and illuminating. It is always to the interest of a particular man that other people in the same trade as himself should not work too hard; for if he works with the same intensity as before, and they work harder, his wages will tend to fall. But it is nevertheless to his interest that people in other trades (at any rate in those which do not compete very directly with his own) should work as hard as possible, for by doing so they raise his real wages. Similarly, it is nearly always to his interest that as much as possible of the national

3. *An increase in the supply of any factor will increase its relative share* (i.e., *its proportion of the National Dividend*) *if its "elasticity of substitution" is greater than unity.* This is the new rule, involving a new definition. The "elasticity of substitution" is a measure of the ease with which the varying factor can be substituted for others. If the same quantity of the factor is required to give a unit of the product, in any circumstances whatever, then its elasticity of substitution is zero.[1] If all the factors employed are for practical purposes identical, so that the varying factor can be substituted for any co-operating factor without any trouble at all, then the elasticity of substitution is infinite. The case where the elasticity of substitution is unity can only be defined in words by saying that in this case (initially, before any consequential changes in the supply of other factors takes place) the increase in one factor will raise the marginal product of all other factors taken together in the same proportion as the total product is raised.

The proposition can thus be expressed in another way. In so far as the direction of change in the relative sharing of the National Dividend is concerned, secon-

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income should be saved. In the short run, particular men may be displaced by an increase in saving; but in the long run, the accumulation of capital is always favourable to the interests of labour.

The following special case is particularly worth noting. Although it may well be to the interest of working men to work for shorter hours as their economic position improves (even if this involves a sacrifice in wages), it is definitely against the interest of the employing and capitalist classes that they should do so. And, looking at the same thing the other way round: if we seek for an economic policy designed to serve the long-run interests of the working class, it ought to be one which discourages the rich from taking out their privileged economic position in consumption and in leisure, but encourages them to work and to save. One cannot help feeling that the obvious change in this respect between the nineteenth and twentieth centuries is a sad comment on the success of progressive policy.

[1] In the terminology of Walras, this is the case where the "coefficient of production" of the varying factor is constant.

dary and consequential changes in the supply of the other factors *do not matter*. If the conditions of technique and consumers' demand (which determine the elasticity of substitution) are such that an increase in the supply of a particular factor would increase its relative share with constant supplies of the other factors, its relative share will still be increased in whatever way the providers of the other factors react to the change in their fortunes. It is not too difficult to show this—at least with some degree of plausibility. If the elasticity of substitution is greater than unity, the initial effect of an increase in the supply of one factor will be to increase that factor's relative share. But at the same time the real return to the other factors will be increased, so that the supply of the other factors is likely to change to some extent, upwards or downwards. If the supply of the other factors falls, the relative supply of the first factor is greater than ever, and thus its relative share (under the present assumption) is likely to rise still further. There is thus no danger of our proposition breaking down in this case. The dangerous case is the other one, where the supply of the other factors *increases*. In order to prove that this does not disturb the rule, it is best to take the most extreme case. Suppose the elasticity of supply of the other factors to be infinite, so that their supply increases, as a result of their now more favourable position, to such a point that their real return per unit is unchanged. It cannot increase so far as to lower their real return per unit, since otherwise the first situation would not have been one of equilibrium. If the real return per unit to the other factors (or their marginal product) is unchanged, this must mean that the relation between the supplies of the factors is the same as before; for we are ruling out the possibility of increasing returns to all the factors taken altogether, and diminishing returns to all the factors taken together is obviously impossible. If the proportion between the supplies of the factors is the same as before, and their marginal products the same as before (which evidently follows), the relative shares of the factors in the distribution of the National Dividend must also be the same.

Thus in the most extreme case conceivable, the increase in the supply of the other factors can only just cancel out the effect of the primary change. In any less extreme case, the direction of the change in relative shares must be the same as if there were no secondary effect through the supply of the factors. And this could be proved in a similar fashion for an elasticity of substitution less than unity.

Another important consequence of our third proposition is that the condition for an increase in supply increasing a factor's relative share is symmetrical. If we classify all our factors of production into two groups—whether we label them "work" and "property" with Dr. Dalton, or "labour" and "capital" "supposing that land can be neglected" with Professor Pigou, the elasticity of substitution of labour for capital is the same as the elasticity of substitution of capital for labour. If the conditions of technique and consumers' demand are such that an increase in the supply of capital will increase capital's relative share, then an increase in the supply of labour will increase labour's relative share. And *vice versa*.[1]

[1] The startling conclusion put forward by Dr. Dalton (*Inequality of Incomes*, p. 204), that "the relative share of property will increase, as the result of increases in the supply of work and property, or in the amount of either alone", is therefore untenable. Some remarks on the detail of Dr. Dalton's argument will be found below (see Appendix, p. 247).

We may now proceed to examine more closely the things upon which the elasticity of substitution depends. Substitution, in the sense in which we are using it, may take any of three forms:

1. The change in the relative prices of the factors may lead simply to a shift over from the production of things requiring little of the increasing factor to things requiring more. If capital increases, the commodities in whose production capital had already been used to an extent above the average will become cheaper relatively to others, and presumably, therefore, more of them will be made.

2. Methods of production already known, but which did not pay previously, may come into use. This form will include, possibly as its most important case, the mere extension of the use of instruments and methods of production from firms where they were previously employed to firms which could not previously afford them.

3. The changed relative prices will stimulate the search for new methods of production which will use more of the now cheaper factor and less of the expensive one.

Partly, therefore, substitution takes place by a change in the proportions in which productive resources are distributed among existing types of production. But partly it takes place by affording a stimulus to the invention of new types. We cannot really separate, in consequence, our analysis of the effects of changes in the supply of capital and labour from our analysis of the effects of invention. To the theory of invention we must now turn.

III

Under the assumption of competition, it inevitably follows that an invention can only be profitably adopted if its ultimate effect is to increase the National Dividend. For if it is to raise the profits of the entrepreneur who adopts it, it must lower his costs of production—that is to say, it must enable him to get the same product with a smaller amount of resources. On balance, therefore, resources are set free by the invention; and they can be used, either to increase the supply of the commodity in whose production the invention is used (if the demand for it is elastic), or to increase the supply of other commodities (if the demand for the first is inelastic). In either case, the total Dividend must be increased, as soon as the liberated resources can be effectively transferred to new uses.[1]

But although an invention must increase the total Dividend, it is unlikely at the same time to increase the marginal products of all factors of production in the same ratio. In most cases, it will select particular factors and increase the demand for those factors to a special extent. If we concentrate on two groups of factors, "labour" and "capital," and suppose them to exhaust the list, then we can classify inventions according as their initial effects are to increase, leave unchanged, or diminish the ratio of the marginal product of capital to that of labour. We may call these inventions "labour-saving," "neutral," and "capital-saving" respectively. "Labour-saving" inventions increase the

[1] For a fuller elaboration of this argument, see Wicksell, *Vorlesungen*, vol. i., pp. 195-207. Also Kaldor, "A Case against Technical Progress?" (*Economica*, May, 1932).

marginal product of capital more than they increase the marginal product of labour; "capital-saving" inventions increase the marginal product of labour more than that of capital; "neutral" inventions increase both in the same proportion.

A labour-saving invention, according to this definition, need not actually diminish the marginal product of labour, and consequently labour's absolute share in the Dividend. It may do so, if it is very labour-saving; there is nothing to prevent the ratio of marginal products being changed to such an extent as to make the absolute size of one lower than it was before. But equally it may not. In every case, however, a labour-saving invention will diminish the relative share of labour. Exactly the same holds, *mutatis mutandis*, of a capital-saving invention.

It may be observed that the definition of a labour-saving invention just given is not identical with that given by Professor Pigou.[1] He supposes the technical change to take place in an industry which produces no wage-goods—*i.e.* none of whose products are bought by labourers. (This is, of course, a very unreal assumption if we interpret labour in the very wide sense which it has to be given in this discussion. The Attorney-General is a labourer.) However, taking this special case, he defines a labour-saving invention as one which diminishes the ratio of capital to labour employed in the rest of industry. Now if the ratio of capital to labour in the rest of industry is diminished, the marginal product of labour in terms of the products of the rest of industry (which is all that matters to labour) must be diminished. An extension of Professor Pigou's definition—and it cries out to be extended—would thus

[1] *Op. cit.*, p. 632.

make a labour-saving invention one which diminished the *absolute* marginal product of labour. Professor Pigou's case then becomes a useful illustration of this definition, but it is too limited to serve as a definition itself.

But even the extended Pigou definition appears on reflection rather unsatisfactory for our purposes. For if we were to call "labour-saving" inventions those which diminished the absolute marginal product of labour, and "capital-saving" inventions those which diminished the marginal product of capital, there would be a wide range of neutral inventions between—quite possibly including the great bulk of those inventions in which we are actually interested. But some of these "neutral" inventions would be more favourable to capital than labour and some the contrary. They would all increase both marginal products, but some would increase that of capital more than that of labour, and some the reverse. If we have any interest in relative shares, we do not want to leave this distinction in the dark. Thus it seems best to make the definition hinge upon relative shares; but it must of course be realised that any invention which is *very* labour-saving may diminish the absolute marginal product of labour; and similarly for capital.

Although this amendment of Professor Pigou's definition appears desirable, the definitions are still fairly close, and most of the things which he says about inventions can be perfectly well applied with the definition just given. In particular, there is no reason to question his view that inventions have a decided bias in the labour-saving direction. It is indeed difficult to find clear cases of important capital-saving inventions—wireless is, of course, the standard case, but beyond that, although there can be little doubt that capital-saving inventions occur, they are not easily identified. Obvious labour-saving inventions, on the other hand, are frequent. Not all those inventions popularly called labour-saving are labour-saving in the strict sense, but there can be little doubt that the great majority are.

This predominance of labour-saving inventions strikes one as curious. It may conceivably be the case that it is a mere "optical illusion"; labour-saving inventions cause more social friction than others, and so force themselves on the attention of the observer. There is probably some truth in this, but it hardly seems a sufficient explanation. It is also possible that the utilisation of fixed capital has a close relation to the particular kind of scientific knowledge which has been available for industry during the last two centuries: that it is to be connected with the special growth of mechanical and physical science. But this again does not seem very probable. For after all, wireless is the result of physics; and there seems no reason in the nature of physical enquiry why the growing complexity of industrial technique should not have been kept in check through the constant supersession of complex methods by simpler methods requiring less capital.

The real reason for the predominance of labour-saving inventions is surely that which was hinted at in our discussion of substitution. A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economising the use of a factor which has become relatively expensive. The general tendency to a more rapid increase of capital than labour which has marked European history during the last few centuries has naturally provided a stimulus to labour-saving invention.

If, therefore, we are properly to appreciate the place of invention in economic progress, we need to distinguish two sorts of inventions. We must put on one side those inventions which are the result of a change in the relative prices of the factors; let us call these "induced" inventions. The rest we may call "autonomous" inventions. We shall expect, in practice, all or nearly all induced inventions to be labour-saving; but there is no reason why autonomous inventions should be predominantly labour-saving. There is no obvious reason why autonomous inventions should incline, on balance, to one side more than to the other. In the absence of special knowledge we may reasonably assume a random dispersion. Then, since induced inventions are mainly labour-saving, both kinds taken together will give us a predominance of labour-saving inventions—precisely what we appear to find in practice. There is nothing therefore in observed fact inconsistent with the hypothesis that autonomous inventions are evenly distributed. But of course, this even distribution will, at the most, be a long-run affair; it is quite conceivable that scientific discovery may tend to produce inventions with a bias in one direction over quite long periods.

In order to complete this classification, one further distinction must be drawn—within the field of induced inventions. An induced invention is made as the result of a change in relative prices; but it may be such that its adoption depends upon the change in prices, or it may not. Capital increases, let us say, and in consequence a labour-saving invention is made and adopted. But either this invention would have paid before capital increased—and would therefore have been adopted if it had been known—or not. If it would not have paid under the old circumstances, then it is simply a cause increasing the facility of adjustment to a change in circumstances—*i.e.* increasing the elasticity of substitution. The elasticity of substitution is greater than it would have been in the absence of such an invention; consequently the possibility of capital increasing its relative share in the Dividend is greater. But so long as the invention is of this type the second rule about absolute shares still holds; it is quite certain that as a result of the whole change the absolute share of labour will be increased.

But it is certainly quite conceivable that a change in relative prices will stimulate invention to do more than this—to discover methods which, if they had been known, would have paid even before prices changed. Now induced inventions of this type (if they are labour-saving, as we may suppose generally to be the case) may reduce not only the relative share of labour, but also its absolute share. Such inventions as these are perhaps not very common, but there is little reason to doubt their occurrence; they are the only kind which are really dangerous to the real income of labour.

The classification of invention just made is a purely economic classification; there is no reason to suppose that it corresponds to any kind of scientific or technical division. At times when scientific and technical activity is great it will probably manifest itself in a large crop both of autonomous and induced inventions. In the dark ages of science, both autonomous and induced inventions will be rare. Further, although the kind of induced inventions just referred to (those which are induced by a change in prices, but do more than adjust technical methods to the new economic conditions) may occur at any stage of development, they are perhaps most likely to be important when the accumulation of capital has been proceeding for a long while, but many kinds of production have retained conservative methods, and have not benefited by technical progress.

IV

The significance of this theoretical analysis can perhaps best be illustrated if we examine its working in two extreme cases. In both we shall assume population constant and capital increasing; but in one technical progress is very lethargic, in the other very rapid.

In the first case, where inventions of all kinds are almost wholly absent, substitution is practically confined to the first two lines mentioned above—the increased use of those commodities requiring much capital, and the more extensive use of known capitalistic methods. It is conceivable that in an early stage these may be sufficient to keep the elasticity of substitution greater than unity. In that case, the relative share of capital will increase, even though the absolute share of labour increases simultaneously. But as capital continues to grow, it is certain that the more advantageous applications will be used up; the elasticity of substitution must fall, and ultimately the relative share of capital must fall and that of labour rise. It is impossible to say how soon this stage will set in, but it must set in sooner or later. But of course this process involves a fall in the marginal product of capital and therefore of the rate of interest. Eventually the fall in interest will check saving, and the community whose technique does not progress will approach the "stationary state" of the classical economists.

In the other case, where invention is very active, the elasticity of substitution will be high and will remain high. Thus the relative share of capital will tend to increase, and that of labour to fall. But not only will induced inventions be active, autonomous inventions will be active too. If we are right in assuming that autonomous inventions have no particular tendency to stimulate a special demand for either factor, then the initial effect of autonomous inventions will be to increase the marginal products of both labour and capital in much the same proportions, and so leave the relative distribution of the Dividend unchanged. However, since an enlarged absolute return is more likely to stimulate an increase in the supply of capital than an increase in the supply of labour, autonomous inventions may have a secondary effect in encouraging the accumulation of capital. But under the supposed conditions, an increase in the supply of capital will increase capital's relative share, and thus activity in autonomous inventions will, indirectly, have a similar effect to activity in induced invention.

But although for both these reasons the relative share of labour will diminish, neither a great activity in autonomous invention, nor a high elasticity of substitution, has any tendency to reduce the real income of labour. The only kind of invention which is likely to have this effect is that which has already been mentioned—that which is inspired by a change in relative prices, but which would have been profitable to apply even before prices changed.

Some inventions of this kind doubtless occur fairly frequently, but if they are—as is probably usual—merely a small part of general inventive activity, then it is most unlikely that their influence will be dominant. For if they tend to reduce labour's marginal product, there are simultaneously at work other forces, derived from the increase of capital and the expansion of autonomous invention, tending to increase the marginal product of labour. There can be no doubt that these latter forces are usually far more powerful.

It may be suggested, very tentatively, that a fall in the general level of real wages is really likely to occur as the result of invention only on those rare occasions when invention breaks into a new and extensive field of industry that has long been conservative in its methods. Such "economic revolutions" always cause maladjustment, and social unrest arising from the maladjustment; but it may be useful to point out that in such times the malaise may go deeper. A fall in the equilibrium level of real wages is here a real possibility.

But it is difficult to feel that this danger is a very pressing one today. The generalised character of technical change is a considerable safeguard against it. Inventive activity usually makes itself felt quickly enough, so that a prolonged failure to adjust technical methods to new circumstances is unlikely on a large scale. Our continuous "industrial revolution" protects us from the discontinuous revolutions of the past.

Thus, so far as the absolute share of labour is concerned, a rather different line of enquiry does not lead us to modify in any way the optimism of Professor

Pigou. It is possible, but extremely improbable, that economic progress may cause a decline in the equilibrium level of real wages. And further, it should be remembered, even if this unlikely event should materialise, it would be temporary; enlarged profits would mean new saving; increased capital would raise the level of real wages again.

But it is difficult to feel the same degree of optimism in the matter of relative shares. For the chance of an elasticity of substitution greater than unity stands in an altogether different order of probability. Increasing capital, accompanied by stagnant invention, may very well raise labour's relative share in the Dividend; but increasing capital, with active invention, is very likely to do the contrary. And since the activity of invention is definitely favourable to the growth of the Dividend—and with few exceptions also favourable to growth in the real income of labour—it is highly probable that periods of most rapidly rising real wages will also be periods of a falling relative share to labour. It is clear that we have here a divergence of no small significance.

V

The application of these conclusions to historical fact is no easy matter; and what follows must be largely guess-work. But it seems worth while to state the most probable interpretation, if only to serve as a basis for future discussion. According to Professor Bowley,[1] the share of property in the National Income of Britain just before the war was about one-third;

[1] *The Change in the Distribution of the National Income*, 1880-1913, p. 25.

and it would seem to follow from this one ascertained fact that there must have been periods in English history when the elasticity of substitution between labour and property was greater than unity. For it is practically inconceivable that a few centuries ago the share of property can have been anywhere near this figure.[1] In the Middle Ages, capital was scarce; but not only was the supply small, the demand was undoubtedly small too, so that it cannot have made up to any appreciable extent for its lack of quantity by a high rate of remuneration. Nor is it possible that the smaller share of capital can have been made up by a larger share of land; for (if we exclude predatory and monopolistic gains, as we are entitled to do, for all the large part which they played in a pre-capitalist economy) we cannot escape the evident fact that land was far more plentiful relatively to the population than it is today. Thus it seems clear that the equilibrium relative share of property must have been much smaller than it was in 1913; at some stage it must have risen considerably.

On the other hand, it seems clear from Professor Bowley's figures that it was not rising in the period immediately before the war. He gives 37½ per cent. as the proportion of the National Income going to property both in 1913 and in 1880, though these percentages require some correction for our purposes. Clearly income from property held abroad ought not to be included; but when it is omitted, the results are even more striking. For the proportion of home-produced income going to property in 1880 was about 34 per cent.; in 1913 it was only about 31 per cent.

[1] See Cannan, "The Changed Outlook in Regard to Population" (*Econ. Jour.*, December, 1931, p. 528).

On the whole this period seems to be long enough for us to be able to neglect disturbances arising from the fact that it is really unjustifiable to regard the situation of the economic system at these dates as being one of equilibrium—although it would be much more satisfactory if we had figures for an average of several years round about each date instead of figures for a single year. If we accept these figures, then it is clear that the elasticity of substitution must at this time have been rather less than unity. Not necessarily very much less; quite a small difference would be sufficient to give the observed result.

These facts, if they are correct, do not upset our theoretical conclusions; but the theory does suggest a clear interpretation of them. If capital is increasing more rapidly than the supply of labour (and it may be fairly supposed that this has generally been the case in modern English history[1]), a tendency towards a diminished elasticity of substitution will generally set in as capital grows. This diminution may be counteracted by invention—it is conceivable that it might be counteracted indefinitely—but clearly invention has a progressively harder task as the process goes on. Invention has generally been increasing in activity, but it is quite possible that this increase has failed to set off the fall due to the first cause. But because it failed to do so in the period under consideration, because in this period it is probable that the elasticity of substitution tended to fall, we should not be over-confident that in the future it may not rise again. And in many ways it would be good for us if it did

[1] This is indeed less certain than usual for the years which immediately preceded the War, in view of the extraordinary export of capital in that period, and its natural consequence, a great retardation in the rate of increase of real wages. (*Cf.* Taussig, *International Trade*, ch. 21.)

so; for it would probably be a mark of national prosperity.

Changes in the distribution of the Dividend since 1914 are harder to interpret; and it seems most unlikely that we can hope to do so if we leave out of account the regulation of wages.

VI

The theoretical conclusions of this chapter have considerable interest in relation to the question of the causes governing inequality of incomes; but there are other implications of hardly less importance. These are in connection with the theory of money wages. If we assume a monetary policy designed to stabilise the price-level of consumers' goods, and successful in that end, then, of course, no theory of money wages is necessary, for money wages and real wages are always directly proportionate. Recent investigations, however, have thrown doubt upon the feasibility of such a policy in a community where the fundamental determinants of economic wealth are in process of change; they suggest rather, that the price-level ought to fall with rising productivity, and rise with falling productivity; if it does not do so, there will be in the one case a boom in trade, leading to dangerous over-expansion, in the other case there will be monetary causes making for a depression.[1] Examination of this contention would be out of place here; but if we accept it provisionally, we can draw from it some consequences which do seem to belong to the theory of wages.

[1] See Haberler, *Der Sinn der Indexzahlen*, p. 112 *ff*. Hayek, *Prices and Production*, p. 23. Also Robertson in *The International Gold Problem*, pp. 21-24 and 45.

If stabilisation of the price-level is ruled out, as being in normal times more or less inflationary, our thoughts naturally turn to other less ambitious forms of stabilisation. One of these is stabilisation of the "money earnings of the factors of production" or of the money value of the Social Dividend. If we assume a monetary policy of this character, the conclusions about relative shares reached in this chapter begin to have some practical significance. If population is increasing, then it is true that this monetary policy must lead to a fall in the level of money wages—under all circumstances; while the level of money wages would rise with diminishing population. But if population is constant and capital increasing, then the trend of money wages depends upon the elasticity of substitution. If the elasticity of substitution is less than unity, the average level of money wages will rise; but in the contrary case it will fall. And as we have seen, it is this latter case which is likely to be associated with the most rapid rise in general economic prosperity, in the level of real wages.

Even if the elasticity of substitution is less than unity, it is unlikely, in any community that can genuinely be called progressive, to be much less than unity. If this is the case, it cannot be expected that the average level of money wages would rise much. But this would mean, in a world where men are specialised to particular trades, and do not move easily, that frequent cases of reductions of money wages in particular trades would be unavoidable. And it is useless to minimise the gravity of this conclusion.

For the raising of real wages through falling money wages with prices of consumption goods falling more rapidly could not be a smooth and painless process.

The reductions in wages would almost inevitably take place at intervals, which would not correspond exactly in time with equivalent falls in prices. There would thus certainly be temporary reductions in real wages; the trend of real wages might be upward, but there would be sharp fluctuations about the trend. It would thus not be in the least surprising if the reductions in money wages were strongly resisted. We shall see at a later stage what would be the probable effects of this.

There is no doubt that these unpleasant results could be avoided, initially at any rate, by a more elastic monetary policy. But whether this would be a real cure, or whether it would only put off the evil day, is one of the major unsettled questions of economics. It is possible that there is some third alternative, intermediate between stabilisation of prices and stabilisation of the social income, which would avoid intense fluctuations of industry and also avoid a downward pressure on money wages. But it seems improbable that in a period of increasing productivity, all, or nearly all, money wages could be exempted from such pressure.[1] Further consideration of this problem lies outside the scope of this book.

[1] *Cf.* Robertson, *op. cit.*, p. 24.

APPENDIX

THE principal object of this appendix is the construction of a mathematical proof of the conclusions about absolute and relative shares in the Social Dividend put forward in Chapter VI; but since the chief value of such a mathematical proof must lie in the disclosure of the exact assumptions and the precise limitations under which the propositions are true, it is convenient to begin with a consideration of certain problems whose connection with these propositions may appear at first sight a little remote.

(i.) THE CO-ORDINATION OF THE LAWS OF DISTRIBUTION

Ever since the early days of the marginal productivity theory in the eighteen-nineties, the mathematical application of the theory has been greatly hampered by the difficulty which was raised by P. H. Wicksteed, in his essay, "The Co-ordination of the Laws of Distribution" (1894). If each factor is paid according to its marginal product, is the total product exhausted, or is there a surplus or deficit? Clearly it is most consonant with the conditions of equilibrium that each factor should be remunerated according to its marginal product, including the factor which "employs" the others, and takes the surplus for its share. But will there be enough residue to pay the employing factor its marginal product?

The solution which Wicksteed himself offered to his own problem is unsatisfactory, as, indeed, he admitted on subsequent occasions.[1] But it is not true, as most English and American economists seem still to imagine, that the problem remained unsolved. Within a few months of the publication of

[1] *Common Sense of Political Economy*, p. 373. The argument in the text of the *Common Sense*, while perfectly valid, does not meet the mathematical difficulty. See also Robbins, "The Economic Works of Philip Wicksteed" (*Economica*, November, 1930).

Wicksteed's Essay, Léon Walras put forward a solution which is altogether free from the objections to which Wicksteed's own solution is liable.[1] But, unfortunately, Walras expressed himself in so crabbed and obscure a manner that it is doubtful if he conveyed his point to anyone who did not possess some further assistance. Anyone who knows the answer can see that Walras has got it; but anyone who does not must find it almost impossible to get it from Walras.

A perfectly intelligible solution did, however, appear a few years later in the *Vorlesungen* of Knut Wicksell.[2] With Wicksell's aid it is not difficult to clear up this matter; after which we shall be in a position to proceed with our principal enquiry.

The first thing on which we have to be clear, if we want to see our way towards a solution of this question, is that we are concerned solely with the internal coherence of the conditions of economic equilibrium. Our problem is purely one of the conditions of equilibrium, and therefore it is extremely unwise to complicate our discussions with the consideration of phenomena which only arise in the real world because the economic system is not in equilibrium; and among these fall the greater part of the activities of enterprise and management. If we persist in thinking of the factor which receives the residue as the "entrepreneur", we shall get into endless difficulties; but fortunately, without any serious departure from reality, we can think of our typical firm as a Joint Stock Company, and suppose the residue to fall to the capitalist as capitalist, management (so far as management is required) being hired like labour of other grades. Or, alternatively, we can follow Wicksell's example, and suppose the landlord or the labourer to take the residue, hiring other factors.

Once we adopt this assumption, the most ordinary non-mathematical analysis shows that every factor must get its marginal product. For every *hired* factor must get its marginal

[1] "Note sur la réfutation de la Théorie anglaise du fermage de M. Wicksteed." This was republished as an appendix to the third edition of Walras' *Eléments* (1896). It is omitted in subsequent editions.

[2] Vol. i., pp. 186-191.

product, since otherwise the demand for it would expand or contract; and every *unhired* factor (which is "acting as entrepreneur") must get its marginal product, since if it got less, its owners would prefer to hire it out; and if it got more, some would be transferred from the hired to the unhired class.

This is a perfectly satisfactory line of argument, and it is evidently reasoning of this kind which has generally persuaded non-mathematical economists (for example, J. B. Clark and his followers) that the "adding-up" difficulty is a delusion. And we shall see that they are right.

The trouble is that the alternative mathematical line of approach did not appear to lead to the same conclusion.

Let $x=$ the amount of product, and $a, b, c. \ldots$ the quantities of factors required to make that product x. In order that the marginal productivity law should be fulfilled, the share of the product which goes to the factor a must be $a \frac{\partial x}{\partial a}$, and similarly for the other factors. If the product is to be exactly divided among the factors, leaving no residue, positive or negative, then

$$x=a\frac{\partial x}{\partial a}+b\frac{\partial x}{\partial b}+\ldots.$$

Wicksteed's explanation was based upon the well-known mathematical proposition, due to Euler, that if x is a homogeneous function of the first degree in $a, b, c \ldots$ so that it can be written

$$af\left(\frac{b}{a}, \frac{c}{a}, \ldots\right)$$

this relation

$$x=a\frac{\partial x}{\partial a}+b\frac{\partial x}{\partial b}+\ldots.$$

will always be satisfied.

It was this that drew the scathing remark of Edgeworth: "There is a magnificence in this generalisation which recalls the youth of philosophy. Justice is a perfect cube, said the ancient sage; and rational conduct is a homogeneous function, adds the modern savant."[1]

[1] "Theory of Distribution," in *Papers*, vol. i., p. 31.

But when it is expressed in economic language, the Wicksteed-Euler proposition appears much less ridiculous than it seems to have appeared to Edgeworth. It means simply that there will be no residue, positive or negative, if the commodity in question is produced under conditions of "constant returns" —using that ill-treated expression in yet another unfamiliar, but nevertheless highly convenient, sense. The production function will have the requisite form if a proportional increase in *all* the quantities of factors employed will increase the quantity of product in the same proportion in which the factors were increased; that is to say, if the amounts of factors required per unit of product (the "coefficients of production") are independent of the amount of product.

Put in this way, the condition appears much less startling; yet it is doubtful if it can be considered to be generally satisfied. So long as all the factors are increased in the same proportion, the general condition of diminishing returns—the disproportionate increase of some factors—is absent. But the condition of increasing returns—economies of specialisation and co-operation due to size—may be present. It does seem possible that "increasing returns" (used here in a special sense, but one that has many of the implications of the ordinary meaning) may come in to upset the marginal productivity theory, as they are inclined to upset, unless we are very careful, so many economic generalisations.

We may now turn to the solution of Walras and Wicksell.

We are concerned here solely with one part of the general equilibrium system, the conditions that a particular firm should be in equilibrium. We assume perfect competition, both in the market where the firm sells its products, and in the market where it buys its factors. Thus, so far as the action of this particular firm is concerned, we can assume all the prices with which it deals to be given; for the influence of its individual action on prices, whether of product or of factors, will be negligible. In order that the firm should be in equilibrium, two conditions have to be satisfied: (1) the unit cost of production of its product must be a minimum; (2) that unit cost must equal the selling price of the product. The first condition must be fulfilled, since otherwise the owners of that factor which is "acting as entrepreneur" could increase their profits by a change in methods. The second condition must be fulfilled, since otherwise the owners of that factor would be receiving a return either higher or lower than was being earned by similar services elsewhere in the market, and someone would therefore have an incentive to act differently. In order to minimise its costs of production, the firm can vary indefinitely the quantities of factors which it uses, and therefore, of course, the quantity of product it turns out. The production function (the relation between the quantities of factors and the quantity of product) is naturally given by technical considerations.[1] The coefficients of production do not only have to be chosen so that the unit cost of production for a given output is a minimum; the output has also to be chosen so that the unit cost of production is a minimum.

We have then

$$x = f(a, b, c. \ . \ . \ .) \quad \text{(production function).}$$

Total cost of production $= ap_a + bp_b + \ . \ . \ . \ .$

where p_a, p_b are the prices of the factors.

$$\text{Cost of production per unit} = \pi_x = \frac{1}{x}(ap_a + bp_b + . \ . \ . \ .) \text{——(1)}$$

$$\pi_x = p_x, \ \textit{i.e.} \text{ cost of production} = \text{selling price.}$$

In order that π_x should be a minimum

$$\frac{\partial \pi_x}{\partial a}, \frac{\partial \pi_x}{\partial b}, \ . \ . \ . \ . \text{ must all} = 0.$$

Now $$\frac{\partial \pi_x}{\partial a} = \frac{\partial}{\partial a}\left\{\frac{1}{x}(ap_a + bp_b + \ . \ . \ . \ .)\right\}$$

$$= \frac{1}{x}p_a - \frac{1}{x^2}\frac{\partial x}{\partial a}(ap_a + bp_b + \ . \ . \ . \ .)$$

[1] Once we grant the universality of substitution, as we have seen cause to do, as a result of the discussions of Chapter I., the existence of a production function follows necessarily.

$$= \frac{1}{x} p_a - \frac{1}{x^2} \frac{\partial x}{\partial a} . x\pi_x$$

$$= \frac{1}{x}\left(p_a - \pi_x \frac{\partial x}{\partial a}\right).$$

Then, since $\frac{\partial \pi_x}{\partial a} = 0$, $p_a = \pi_x \frac{\partial x}{\partial a} = p_x \frac{\partial x}{\partial a}$, and similarly for the other factors.

This is the marginal productivity law, and by substituting in (1) we have

$$x = a \frac{\partial x}{\partial a} + b \frac{\partial x}{\partial b} + \ldots .$$

proved independently of any assumption about "constant returns".

The explanation which lies behind this proof lies in the essential hypothesis that each firm is producing at that scale of output which makes its unit cost a minimum. If, as before, we assume that the prices of the factors are constant, and if we assume further that the proportions in which the factors are employed remain unchanged as output varies, we can construct a (very specialised) cost curve for the firm, giving the cost per unit of producing various outputs. Wicksteed thought he had proved that it was a necessary condition for the truth of the marginal productivity theory that this curve should be a horizontal straight line. Walras and Wicksell showed that it was only necessary that the curve should have a minimum point, and that in equilibrium output must be at that point.

Now it is clear that in the neighbourhood of the minimum point, where the tangent to the curve must be horizontal, the curve will approximate very closely to the straight line. It is not surprising that, at this point, Wicksteed's condition should be satisfied. Where Wicksteed went wrong was in his assumption that he could argue from the shape of the curve at one particular point to the general shape of the curve.

Wicksteed's difficulty can therefore be overcome by substituting for his untenable condition of "constant returns" the condition of "minimum cost" which appears, on the surface at least, more in keeping with the fundamental assumptions on which it is reasonable to base an equilibrium theory. But, as Mr. Sraffa has pointed out,[1] the condition of minimum cost is not without its difficulties. We are excluded from the assumption of diminishing returns in the usual sense; but if we assume no tendency to diminishing returns—that a simultaneous increase in all the factors in the same proportion will never increase the product less than proportionately—then either competitive equilibrium is impossible (which will be the case if increasing returns go on indefinitely) or alternatively the distribution output among the different firms in an industry will be altogether indeterminate (if increasing returns give way to constant returns). Neither of these conclusions is welcome; but if we are to avoid them, we are driven to assume that "technical diseconomies" will, after a certain point, induce diminishing returns. There can be little question that in fact there is generally a limit to the extent to which any firm can grow under given conditions, independently of the limitation of the market. But a doubt must remain how far the limitations which we do find in experience have not been assumed away on the level of abstraction on which we are now working.

Further consideration of this point would lead us too far into the more arid regions of higher general theory; its relevance to the theory of distribution is remote.

(ii.) Increasing Returns

The marginal product which measures the actual return which a factor of production must get in a state of equilibrium, is the addition which is made to the product of a firm when a small unit is added to the supply of the factor available to that firm, when the organisation of the firm is adjusted to the new supply (so that it is used in the most economical way), but when the rest of the organisation of industry, including the general system of prices, remains unchanged. Now there is no

[1] "The Laws of Returns under Competitive Conditions" (*Econ. Jour.*, 1926).

reason why this increment should be the same as the increment of production which would accrue if the additional unit were made available to the whole of industry, and the whole organisation of industry, including the general price-system, were adjusted to the new supply.

If all the firms were operating in accordance with Wicksteed's law, under conditions of "constant cost"; and if we leave out of account the fact that the allocation of the increase in resources to one firm only would mean an uneconomic distribution of production; then there can be no question that these two "marginal products" would be equal. But in fact an increase in the supply of one factor generally involves a complicated redistribution of production between firms and between industries, and in consequence of these changes it is quite likely that the marginal product of a factor in the second sense will be greater than the marginal product in the first sense. The division of labour progresses as the supply of the factors increases, and the advantages of the division of labour are gained as much, or more, through an increase in specialisation between firms and between industries, as through an increase in the size of firms.[1]

Thus we have to distinguish between the "private" marginal product, which does, in equilibrium, equal the wage of labour; and the "social" marginal product, which results from an increase in the supply of labour, when we suppose that increase to have worked out its full effect. And in general it is safe to assume that the latter will exceed the former.

This divergence has awkward consequences for the application of the general marginal productivity theory. If we can assume "constant returns" and a consequent equality of "social" and "private" marginal products, it is possible to deduce certain not uninteresting results about the effect of increases in the factors on the distribution of the product. But in so far as we have to allow for increasing returns, these re-

[1] *Cf.* Allyn Young, "Increasing Returns and Economic Progress" (*Econ. Jour.*, 1928); Shove, "Varying Costs and Marginal Net Products" (*Econ. Jour.*, 1928).

sults are surrounded by a margin of doubt. Yet it does not seem probable that the divergence would be very great.

Nevertheless, the reader is asked to bear in mind that the exact conclusions of the following pages depend for their strict validity upon the assumption of "constant returns" in the Wicksteed-Wicksell sense; and thus upon the identity of "private" and "social" marginal products.[1]

(iii.) The Elasticity of Derived Demand

In examining the effects on Distribution of changes in the supply of the factors of production, it is convenient to begin with the special case of a change in the supply of a factor which is specialised to some particular purpose, and can only be used in one industry. The problem which is then raised within that industry is then simply a problem of the elasticity of derived demand—the problem which was studied by Marshall in his well-known example of plasterers' wages. Marshall gave four rules for the things on which the elasticity of derived demand depends; and in their discussions of this matter, economists have generally been content to use Marshall's rules, without making them the subject of any further investigation. These rules are an excellent example of the convenience of the elasticity concept, in enabling essentially mathematical notions to be used in formally non-mathematical arguments. But such procedure, although convenient, is dangerous; it will enable us to proceed more securely, if, instead of merely accepting Marshall's conclusions, we examine their mathematical foundation.

Marshall himself no doubt derived his rules from mathematics; Note XV. in the mathematical appendix to the *Prin-*

[1] Of the two rules about absolute and relative shares in the Dividend put forward in Chapter VI. and to whose consideration this discussion is ultimately leading, it seems extremely improbable that the rule about absolute shares could possibly be affected by increasing returns. The rule about relative shares, on the other hand, almost certainly must be affected to some extent, although it is unlikely that the difference would be very serious unless it could be shown that an increase in one particular factor would be much more likely to call forth a strong development of those tendencies making for increasing returns than an increase in the other.

ciples is enough to assure us of that. But he does not there give the full mathematical derivation; he confines himself to a simplified case, that in which the proportions of factors employed (the "coefficients of production") remain constant. A more extended enquiry, he assures us, would lead to "substantially the same results." But we may as well see for ourselves.

The four rules (in Professor Pigou's more convenient formulation) are:

I. "The demand for anything is likely to be more elastic, the more readily substitutes for that thing can be obtained."

II. "The demand for anything is likely to be less elastic, the less important is the part played by the cost of that thing in the total cost of some other thing, in the production of which it is employed."

III. "The demand for anything is likely to be more elastic, the more elastic is the supply of co-operant agents of production."

IV. "The demand for anything is likely to be more elastic, the more elastic is the demand for any further thing which it contributes to produce."[1]

We may now proceed to our mathematical enquiry.

A product is being made by the co-operation of two factors, a and b, which are remunerated according to the value of their marginal products. Let x be the quantity of product (x is thus a function of a and b), p_x its price; p_a and p_b the prices of the factors a and b respectively. If η is the elasticity of demand for the product, and e the elasticity of supply of b, how is λ, the elasticity of demand for a, determined?

We have $p_a = p_x \frac{\partial x}{\partial a}$, $p_b = p_x \frac{\partial x}{\partial b}$ (marginal products).

Also
$$\eta = -\frac{p_x}{x\frac{dp_x}{dx}},\quad e = \frac{p_b}{b\frac{dp_b}{db}},\quad \lambda = -\frac{p_a}{a\frac{dp_a}{da}}.$$

[1] Marshall, *Principles*, bk. v., ch. vi.; Pigou, *Economics of Welfare*, bk. iv., ch. v.

Since the total expenditure of the firm equals total receipts,

$$p_x x = p_a a + p_b b.$$

This can also be written

$$x = a\frac{\partial x}{\partial a} + b\frac{\partial x}{\partial b}.$$

Since we are assuming "constant returns" we can treat this last equation as an identity, and differentiate it partially with respect to b,

$$\frac{\partial x}{\partial b} = a\frac{\partial^2 x}{\partial a \partial b} + b\frac{\partial^2 x}{\partial b^2} + \frac{\partial x}{\partial b}.$$

$$\therefore \quad b\frac{\partial x^2}{\partial b^2} = -a\frac{\partial^2 x}{\partial a \partial b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Further, the total differential of x,

$$dx = \frac{\partial x}{\partial a}da + \frac{\partial x}{\partial b}db$$

$$\therefore \quad p_x dx = p_a da + p_b db \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Since the condition of equality of receipts and expenditure must still be satisfied after we have made our small change in a,

$$p_x dx + x dp_x = p_a da + a dp_a + p_b db + b dp_b.$$

But from (2) this becomes

$$x dp_x = a dp_a + b dp_b.$$

And by the elasticity formulæ,

$$\frac{p_x dx}{\eta} = \frac{p_a da}{\lambda} - \frac{p_b db}{e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Now the change in b, which results from the change in a as independent variable,

$$= db = \frac{be}{p_b}dp_b = \frac{be}{p_b}d\left(p_x\frac{\partial x}{\partial b}\right).$$

By expansion and application of (1), this becomes

$$db = \frac{be}{p_b}\left\{-\frac{p_b dx}{x\eta} + p_x\frac{\partial^2 x}{\partial a \partial b}\left(da - \frac{a}{b}db\right)\right\}.$$

Now write $\sigma = \dfrac{p_a p_b}{p_x{}^2 x \dfrac{\partial^2 x}{\partial a \partial b}}$ and $\kappa = \dfrac{p_a a}{p_x x}$, and simplify.

Then $$\frac{p_x dx}{\eta} = \frac{p_a da}{\sigma} - \frac{p_b db}{1-\kappa}\left(\frac{1}{e} + \frac{\kappa}{\sigma}\right) \quad . \quad . \quad . \quad . \quad (4).$$

Eliminating dx, da, db between (2), (3) and (4), we get

$$\frac{\lambda - \sigma}{\eta - \lambda} = \frac{\kappa}{1-\kappa} \cdot \frac{e+\sigma}{e+\eta}$$

or $$\lambda = \frac{\sigma(\eta + e) + \kappa e(\eta - \sigma)}{\eta + e - \kappa(\eta - \sigma)}.$$

This gives us a value for the elasticity of demand for a, in terms of η, e, κ, and σ.[1]

These are in fact the four Marshallian variables. κ, e, η correspond to the rules (II), (III), and (IV) quoted above. σ is a suitable measure for (I); it is the "elasticity of substitution". Its principal component, $\frac{\partial^2 x}{\partial a \partial b}$, gives the rate of change of the marginal product of one factor for a change in the other factor. If $\frac{\partial^2 x}{\partial a \partial b}$ is infinite, $\sigma = o$, and there is no substitution possible at all; the coefficients of production are strictly proportional. If $\frac{\partial^2 x}{\partial a \partial b} = o$, σ is infinite, the factors are perfectly rival or their use is indifferent. If we had a third factor, or more, then $\frac{\partial^2 x}{\partial a \partial b}$ might be negative, and the factors would be rival in the more ordinary sense of the term; an increase in one would diminish the marginal product of the other. But with only two factors, and under the assumption that there can be no "diminishing returns" to all the factors together, this is impossible.

But although $\frac{\partial^2 x}{\partial a \partial b}$ is thus to some extent a test of the amount of substitution possible, it is not a suitable measure of

[1] When $\sigma = 0$, this reduces to Marshall's formula (*Principles*, Mathematical Appendix, Note XV.).

the "elasticity of substitution". For its magnitude depends on the units in which x, a, and b are measured. Just as we have to multiply $\frac{dx}{dp}$ by $\frac{p}{x}$ in order to get the *elasticity* of demand, so we must multiply $\frac{\partial^2 x}{\partial a \partial b}$ by a further factor in order to get the elasticity of substitution. $\frac{p_x{}^2 x}{p_a p_b}$ is a suitable multiplier. But I have taken the reciprocal of this expression, in order to have a measure increasing with the facility of substitution.

Since $\dfrac{p_a p_b}{p_x{}^2 x \dfrac{\partial^2 x}{\partial a \partial b}} = \dfrac{\dfrac{\partial x}{\partial a}\dfrac{\partial x}{\partial b}}{x \dfrac{\partial^2 x}{\partial a \partial b}}$, σ could also have been written in this latter form.

So far we have only shown that the elasticity of derived demand depends upon Marshall's four variables. We have still to examine how it moves with the four variables—*i.e.*, to test the rules.

Taking the formula for λ, and differentiating it partially by each in turn of the four variables on which it depends, we get:

$$(1) \quad \frac{\partial \lambda}{\partial \sigma} = (1 - \kappa) \times \text{a square.}$$

$$(2) \quad \frac{\partial \lambda}{\partial \kappa} = (\eta - \sigma)(\eta + e)(e + \sigma) \times \text{a square.}$$

$$(3) \quad \frac{\partial \lambda}{\partial e} = \kappa (1 - \kappa) \times \text{a square.}$$

$$(4) \quad \frac{\partial \lambda}{\partial \eta} = \kappa \times \text{a square.}$$

The first, third, and fourth of these expressions are always positive. The first, third, and fourth rules are universally true. But the second rule is not universally true. Even if we concern ourselves only with cases where e is positive (η and σ must be positive) the second rule is only true so long as $\eta > \sigma$; so long as the elasticity of demand for the final product is greater than the elasticity of substitution. Of course, in the usual cases taken for illustration of this rule, the condition for its validity is fulfilled. It is supposed that the demand for the product is fairly elastic, while substitution is difficult. But if technical change is easy, while the product has an inelastic demand, the rule works the other way. For example, a factor may find it easier to benefit itself by a restriction in supply if it plays a large part in the process of production than if it plays a small part. *It is "important to be unimportant" only when the consumer can substitute more easily than the entrepreneur.* Further even if $\eta > \sigma$, but if the difference is small, the importance of this second rule will be negligible.

(iv.) The Distribution of the National Dividend

The last part of our enquiry—the application of these results to the wider problem discussed in Chapter VI.—now presents little difficulty. We are now concerned no longer with the money demand for a factor of production engaged in the making of a particular product, but with the real demand for a general group of factors of the traditional kind "labour" or "capital". To this we can still apply our formula, but in a considerably simplified form. Since the total product of a closed community does not need to be sold outside that community, we can write $p_x = 1$, and $\eta =$ infinity. The elasticity of demand for one of these groups of factors is therefore given by the following formula, derived from the formula of the last section:

$$\lambda = \frac{\sigma + \kappa e}{1 - \kappa}.$$

From this formula[1] the second and third of the rules given above in Chapter VI. can be directly derived.

[1] It may be interesting to illustrate the significance of this formula by an arithmetical example. If we suppose $\sigma = 1$, the elasticity of supply of the factors to be zero, and the dividend to be divided between labour and capital in the proportions of 75 per cent. to 25 per cent., then the elasticity of demand for labour (measured in terms of real goods) will be 4; and the elasticity of demand for capital $1\frac{1}{3}$.

For

$$\frac{d}{da}(bp_b) = \frac{p_a(1+e)}{\lambda}$$

$$a\frac{d}{da}\left(\frac{ap_a}{x}\right) = \frac{\kappa(\sigma-1)}{\lambda}.$$

The rules are therefore valid so long as λ is positive; that is to say, in practically every conceivable case. (It was shown above on p. 98, footnote, that e may always be taken to be greater than -1).

It only remains for us now to make a few remarks on the reason which led Dr. Dalton[1] to arrive at a conclusion so different from that which is evidently to be derived from the last of the above formulæ. Dr. Dalton constructed a formula giving a test for the conditions under which an increase in a would increase its relative share. In our notation, his formula is $\lambda > \frac{1}{1-\kappa}$. It is evident that this formula is correct, so long as e can be neglected. He then proceeded to apply to this formula estimates for the elasticities of demand for labour and capital—estimates derived from Marshall's rules, but not from any formula. He thus naturally overlooked the precise way in which λ increases with κ. The larger κ is, the higher is the obstacle that has to be jumped before a factor can increase its relative share; but since the jumper increases in strength at exactly the same rate, the obstacle is irrelevant. The condition for increased relative share depends on σ, and on σ alone.

[1] See above, p. 119.