

Lecture 8: Hand-to-Mouth Households (TANK Model)

Macroeconomics EC2B1

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In the New Keynesian model of lecture 7 and lecture 8 thus far, households consume and save according to the permanent income hypothesis: only the present discounted value of their income matters. This has a number of implications, for example that a deficit-financed tax cut (a cut in T_1 financed by an increase in T_2 so that the present value of taxes $P_1 T_1 + \frac{P_2 T_2}{1+i_1}$ stays constant) does not affect consumption at all (“Ricardian equivalence”). However, as discussed in previous lectures, this type of behaviour which is implied by the permanent income hypothesis is at odds with empirical evidence, in particular these models generate marginal propensities to consume (MPCs) that are considerably below empirical estimates. Therefore, we now consider an extension with high-MPC households.

1 Model Economy

The simplest way of introducing high-MPC households is a modeling trick known as the “spender-saver model” after a 1989 paper by Campbell and Mankiw “Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence.” A fraction $0 \leq \lambda \leq 1$ of households are “spenders” that consume their entire income. The remaining households are “savers” that behave as before. Aggregate consumption is

$$C_t = \lambda C_t^{sp} + (1 - \lambda) C_t^{sa} \quad (1)$$

where λ is the share of spenders and where C_t^{sp} is consumption of spenders and C_t^{sa} is consumption of savers.

In the New Keynesian literature, this type of model is sometimes called a Two Agent New Keynesian (TANK) model. We will now spell out this TANK model in more detail. Useful observation: in the special case $\lambda = 0$, i.e. when there are no spenders, the model collapses to the standard New Keynesian model that we have analyzed thus far. When you look at the equations below, you may want to check from time to time that setting $\lambda = 0$ yields the same equations as before, i.e. those in the supplement on the New Keynesian model. If they don’t, then we did something wrong.

Only the savers invest so that aggregate capital and investment are

$$K_2 = I_1 = (1 - \lambda)I_1^{sa}$$

where I_1^{sa} is the per-capita investment of savers.

Because there are now two groups of households rather than a representative household, we have to think about distributional issues, in particular who earns what income and who pays taxes and receives transfers. In this economy, the only source of income are dividends from firm ownership with (real) present value W . We assume each spender receives a fraction or multiple $\gamma^{sp} \geq 0$ of total per-capita income W , i.e. $W^{sp} = \gamma^{sp}W$. Similarly, each saver receives an income $W^{sa} = \gamma^{sa}W$. We assume that γ^{sp} is exogenously given and that γ^{sa} is pinned down residually from the requirement that spender and saver income need to add up to total income

$$\lambda W^{sp} + (1 - \lambda)W^{sa} = W \quad (2)$$

which implies $\gamma^{sa} = (1 - \lambda\gamma^{sp})/(1 - \lambda)$. When $\gamma^{sp} = 1$, then also $\gamma^{sa} = 1$ so that $W^{sp} = W^{sa} = W$ and everyone receives the same income in per-capita terms. In contrast, when $\gamma^{sp} = 0$, then $\gamma^{sa} = 1/(1 - \lambda)$ and $(1 - \lambda)W^{sa} = W$ so that savers receive the entire dividend income. We assume that

$$\lambda\gamma^{sp} < 1$$

so that total income received by spenders $\lambda W^{sp} = \lambda\gamma^{sp}W$ is strictly less than total income W .

We denote the taxes paid by savers by T_t^{sa} and those paid by spenders by T_t^{sp} with negative values representing transfers (as before). The government budget constraint is

$$P_1 G_1 + \frac{P_2 G_2}{1 + i_1} = P_1 (\lambda T_1^{sp} + (1 - \lambda)T_1^{sa}) + \frac{P_2 (\lambda T_2^{sp} + (1 - \lambda)T_2^{sa})}{1 + i_1}$$

For all our fiscal experiments, we treat $(G_1, G_2, T_1^{sp}, T_2^{sp}, T_1^{sa})$ as policy parameters and assume that T_2^{sa} always adjusts in order to satisfy the government budget constraint, in particular any fiscal stimulus via government spending or transfers to households at $t = 1$ is paid via a tax on savers at $t = 2$. Rearranging the government budget constraint we therefore have:

$$\frac{T_2^{sa}}{1 + r_1} = \frac{G_1}{(1 - \lambda)} + \frac{G_2}{(1 + r_1)(1 - \lambda)} - \frac{\lambda}{(1 - \lambda)} T_1^{sp} - T_1^{sa} - \frac{\lambda T_2^{sp}}{(1 + r_1)(1 - \lambda)}$$

As above, we first consider a flexible-price economy and then introduce sticky prices.

1.1 Flexible Prices

Savers solve

$$\max_{C_1^{sa}, C_2^{sa}} U(C_1^{sa}) + \beta U(C_2^{sa}) \quad \text{s.t.} \quad C_1^{sa} + \frac{C_2^{sa}}{1+r_1} = W^{sa} - \left(T_1^{sa} + \frac{T_2^{sa}}{1+r_1} \right)$$

Spenders simply consume their income

$$C_1^{sp} = W^{sp} - T_1^{sp}, \quad C_2^{sp} = -T_2^{sp}$$

Note that we assume that the spenders obtain all their income from firm ownership in the first period and have no income source in the second period except government transfers. So we need to assume that $T_2^{sp} \leq 0$.

Solving for this economy's equilibrium allocation with flexible prices is left as an exercise.

1.2 Sticky Prices

We now turn to the main application of interest, namely the economy with high-MPC households (i.e. $\lambda > 0$) and sticky prices. This is the TANK model.

Claim: the equilibrium with sticky prices and high-MPC households satisfies

$$\begin{aligned} C_1 &= \frac{1}{1-\lambda\gamma^{sp}} \left[-\lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)P_1} + \lambda T_2^{sp} \right) \right] + \frac{\lambda\gamma^{sp}}{1-\lambda\gamma^{sp}} \left[G_1 + \frac{M_2}{(1+i_1)P_1} + \frac{G_2}{A_2} \right] \\ I_1 &= \frac{M_2}{(1+i_1)P_1} + \frac{G_2}{A_2} \\ Y_1 &= \frac{1}{1-\lambda\gamma^{sp}} \left[G_1 - \lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1+i_1)P_1} + \frac{G_2}{A_2} \right] \\ C_2 &= \frac{A_2 M_2}{(1+i_1)P_1} \\ Y_2 &= \frac{A_2 M_2}{(1+i_1)P_1} + G_2. \end{aligned} \tag{3}$$

Similarly consumption of savers and spenders in the two time periods are

$$\begin{aligned} C_1^{sa} &= \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda} \right) \\ C_2^{sa} &= \frac{A_2 M_2}{(1+i_1)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda} \\ C_1^{sp} &= \gamma^{sp} Y_1 - T_1^{sp} \\ C_2^{sp} &= -T_2^{sp} \end{aligned} \tag{4}$$

Considering the aggregate variables in (3), the important observations are

1. The government spending multiplier is

$$\frac{\partial Y_1}{\partial G_1} = \frac{1}{1 - \lambda \gamma^{sp}}$$

which is strictly greater than one whenever $\lambda \gamma^{sp} > 0$, i.e. when there are some spenders $\lambda > 0$ and when they receive some income $\gamma^{sp} > 0$.

2. The transfer multiplier for transfers to spenders is

$$\frac{\partial Y_1}{\partial (-T_1^{sp})} = \frac{\lambda}{1 - \lambda \gamma^{sp}}$$

which is strictly positive whenever $\lambda > 0$.

3. While fiscal stimulus payments now affect aggregate consumption (in contrast to the standard New Keynesian model without spenders, i.e. the case $\lambda = 0$), this policy still cannot restore the first-best allocation. When there is a demand-driven recession, investment with sticky prices will generally differ from investment with flexible prices (while we haven't worked out the case with flexible prices and $\lambda > 0$ here, the logic is the same as in the case $\lambda = 0$ – see the previous supplement on the New Keynesian model). The same is true for second-period consumption C_2 . But we can see in the expressions above that fiscal stimulus in the form of transfers to spenders $-T_1^{sp}$ does not affect aggregate investment and consumption in the second period (see the expression for I_1 and C_2 above). As a result, the two are not equal and hence fiscal stimulus alone cannot restore the first best (flexible price) allocation. In contrast, monetary policy can achieve this job. However, we will show below that a combination of fiscal stimulus with appropriate investment policies can restore the first-best allocation just like monetary policy (Wolf, 2022).

Derivation of (3) and (4) : We proceed like in the representative-agent case. As before, the price level P_2 is pinned down from the firm's first-order condition which implies that the real rate equals $1 + r_1 = A_2$, the Fisher equation, and the assumption that P_1 is fixed:

$$P_2 = \frac{1 + i_1}{A_2} P_1$$

Then second-period consumption is pinned down from the cash-in-advance constraint

$$M_2 = P_2 C_2 \quad \Rightarrow \quad C_2 = M_2 / P_2 = \frac{A_2 M_2}{(1 + i_1) P_1}$$

which is the expression in (3). The savers' consumption is then pinned down from the definition of aggregate consumption (1) at $t = 2$ and that $C_2^{sp} = -T_2^{sp}$:

$$C_2^{sa} = \frac{C_2 + \lambda T_2^{sp}}{1 - \lambda} = \frac{A_2 M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{\lambda T_2^{sp}}{1 - \lambda}$$

which is the expression for C_2^{sa} in (4). Using the Euler equation in real terms:

$$C_1^{sa} = \left(\frac{1}{\beta A_2} \right)^\sigma C_2^{sa} = \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{\lambda T_2^{sp}}{1 - \lambda} \right)$$

which is the expression for C_1^{sa} in (4). Now we can use the second period resource constraint $C_2 + G_2 = Y_2$ to solve for I_1 and I_1^{sa} :

$$-\lambda T_2^{sp} + (1 - \lambda) \left(\frac{A_2 M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{\lambda T_2^{sp}}{1 - \lambda} \right) + G_2 = A_2 (1 - \lambda) I_1^{sa}$$

so that

$$\begin{aligned} I_1^{sa} &= \frac{G_2}{A_2 (1 - \lambda)} - \frac{\lambda T_2^{sp}}{A_2 (1 - \lambda)} + \left(\frac{M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{\lambda T_2^{sp}}{(1 - \lambda) A_2} \right) \\ &= \frac{M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{G_2}{A_2 (1 - \lambda)} \end{aligned}$$

Using that $I_1 = (1 - \lambda) I_1^{sa}$ we obtain the expression for I_1 in (3).

Output in the first period is:

$$\begin{aligned} Y_1 &= G_1 + \lambda (W^{sp} - T_1^{sp}) + (1 - \lambda) \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{\lambda T_2^{sp}}{1 - \lambda} \right) \\ &\quad + (1 - \lambda) \left(\frac{M_2}{(1 + i_1) P_1 (1 - \lambda)} + \frac{G_2}{A_2 (1 - \lambda)} \right) \\ &= G_1 + \lambda (W^{sp} - T_1^{sp}) + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1 + i_1) P_1} + \frac{G_2}{A_2}. \end{aligned}$$

Using that $W^{sp} = \gamma^{sa} Y_1$ we have

$$Y_1 = G_1 + \lambda \gamma^{sa} Y_1 - \lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1 + i_1) P_1} + \frac{G_2}{A_2}.$$

Here we can see a Keynesian Cross type loop in terms of how increasing government expenditure G_1 can increase output Y_1 . Increasing G_1 increases Y_1 , which in turn increases the income of spenders ($\gamma^{sp}Y_1$). As a result, spenders will demand more consumption goods, which further increases output and the loop goes on.

Solving for Y_1 we have:

$$Y_1 = \frac{1}{1 - \lambda\gamma^{sp}} \left[G_1 - \lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1 + i_1) P_1} + \frac{G_2}{A_2} \right]$$

which is the expression for Y_1 in (3). Finally, we can solve for

$$\begin{aligned} C_1 &= Y_1 - I_1 - G_1 \\ &= \frac{1}{1 - \lambda\gamma^{sp}} \left[-\lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1 + i_1) P_1} + \frac{G_2}{A_2} + G_1 \right] - \frac{M_2}{(1 + i_1) P_1} - \frac{G_2}{A_2} - G_1 \\ &= \frac{1}{1 - \lambda\gamma^{sp}} \left[-\lambda T_1^{sp} + \left(\frac{1}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1 + i_1) P_1} + \lambda T_2^{sp} \right) \right] + \frac{\lambda\gamma^{sp}}{1 - \lambda\gamma^{sp}} \left[G_1 + \frac{M_2}{(1 + i_1) P_1} + \frac{G_2}{A_2} \right]. \end{aligned}$$

2 Restoring the First Best: Wolf's Equivalence Result

As we discussed above, while fiscal stimulus affects aggregate consumption in this TANK model, it cannot restore the entire (first best) flexible-price allocation. This means that monetary policy is preferable to fiscal policy. A natural question is whether one can combine fiscal stimulus with other policy tools to achieve this goal, i.e. whether fiscal stimulus plus other policy tools can replicate what can be done with monetary policy?

This is the question taken up in a nice recent paper by Christian Wolf (2022) “Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result”. His answer is “yes”, in particular that fiscal stimulus combined with an appropriately chosen investment policy can replicate the allocation implemented via monetary policy. Citing from Wolf’s abstract “ I derive a general condition on consumer behavior ensuring that, in a simple textbook model of demand-determined output, any path of aggregate inflation and output that is implementable via interest rate policy is also implementable through time-varying uniform lump-sum transfers (“stimulus checks”) alone. [...] My results extend to environments with investment if transfers are supplemented by a second standard fiscal tool – bonus depreciation [Note: this is the “appropriately chosen investment policy” mentioned above; below it will just be an investment subsidy.]”

This result is important because it contrasts with the standard New Keynesian model with $\lambda = 0$ in which this is not possible. We now derive Wolf’s result in the context of the simple TANK model analyzed in this supplement.

2.1 TANK Model with investment subsidy

We now introduce an additional policy tool into the TANK model of this supplement, namely an investment subsidy which we denote by s . Relative to the model above, there are two changes. First, the firm problem is now

$$\Omega = \max_{K_2} P_1(Y_1 - (1 - s)K_2) + \frac{P_2}{1 + i_1} A_2 K_2. \quad (5)$$

where s is the investment subsidy. Second, the government budget constraint is now given by

$$P_1 G_1 + s P_1 I_1 + \frac{P_2 G_2}{1 + i_1} = P_1(\lambda T_1^{sp} + (1 - \lambda)T_1^{sa}) + \frac{P_2(\lambda T_2^{sp} + (1 - \lambda)T_2^{sa})}{1 + i_1},$$

i.e. spending on the investment subsidy $s P_1 I_1$ is an additional expenditure for the government.

Claim: In the TANK model with an investment subsidy at rate s , output, consumption

and investment are given by

$$\begin{aligned}
C_1 &= \frac{1}{1 - \lambda\gamma^{sp}} \left[-\lambda T_1^{sp} + \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1} + \lambda T_2^{sp} \right) \right] + \frac{\lambda\gamma^{sp}}{1 - \lambda\gamma^{sp}} \left[G_1 + \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2} \right] \\
I_1 &= \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2} \\
Y_1 &= \frac{1}{1 - \lambda\gamma^{sp}} \left[G_1 - \lambda T_1^{sp} + \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2} \right] \\
C_2 &= \frac{A_2 M_2}{(1+i_1)(1-s)P_1} \\
Y_2 &= \frac{A_2 M_2}{(1+i_1)(1-s)P_1} + G_2.
\end{aligned} \tag{6}$$

Derivation: The first-order condition for the firm's problem (5) is

$$-P_1(1-s) + \frac{P_2}{1+i_1} A_2 = 0 \quad \Rightarrow \quad 1+i_1 = \frac{P_2 A_2}{P_1(1-s)} \quad \Rightarrow \quad 1+i_1 = \frac{(1+\pi)A_2}{1-s}$$

and hence the real interest rate is

$$1+r_1 = \frac{1+i_1}{1+\pi} = \frac{A_2}{1-s},$$

which is higher than in the case without investment subsidy.

As before, the price level P_2 is pinned by the Fisher equation, and the assumption that P_1 is fixed:

$$P_2 = \frac{(1+i_1)(1-s)}{A_2} P_1$$

Then second-period consumption is pinned down from the cash-in-advance constraint

$$M_2 = P_2 C_2 \quad \Rightarrow \quad C_2 = M_2/P_2 = \frac{A_2 M_2}{(1+i_1)(1-s)P_1}$$

which is the expression in (6). The savers' consumption is then pinned down from the definition of aggregate consumption (1) at $t=2$ and that $C_2^{sp} = -T_2^{sp}$:

$$C_2^{sa} = \frac{C_2 + \lambda T_2^{sp}}{1-\lambda} = \frac{A_2 M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda}$$

To find savers' consumption in the first period, C_1^{sa} , we use the Euler equation in real terms. Here we must take into account the fact that the real interest rate has changed from $1+r_1 = A_2$

to $1 + r_1 = \frac{A_2}{1-s}$, so the Euler equation is

$$C_1^{sa} = \left(\frac{1}{\beta(1+r_1)} \right)^\sigma C_2^{sa} = \left(\frac{1-s}{\beta A_2} \right)^\sigma C_2^{sa} = \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda} \right)$$

The remainder of the derivation follows the same steps as before. Specifically, we use the second period resource constraint $C_2 + G_2 = Y_2$ to solve for I_1 and I_1^{sa} :

$$-\lambda T_2^{sp} + (1-\lambda) \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda} \right) + G_2 = A_2(1-\lambda)I_1^{sa}$$

so that

$$\begin{aligned} I_1^{sa} &= \frac{G_2}{A_2(1-\lambda)} - \frac{\lambda T_2^{sp}}{A_2(1-\lambda)} + \left(\frac{M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{(1-\lambda)A_2} \right) \\ &= \frac{M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{G_2}{A_2(1-\lambda)} \end{aligned}$$

Using that $I_1 = (1-\lambda)I_1^{sa}$ we obtain the expression for I_1 .

Output in the first period is:

$$\begin{aligned} Y_1 &= G_1 + \lambda(W^{sp} - T_1^{sp}) + (1-\lambda) \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{\lambda T_2^{sp}}{1-\lambda} \right) \\ &\quad + (1-\lambda) \left(\frac{M_2}{(1+i_1)(1-s)P_1(1-\lambda)} + \frac{G_2}{A_2(1-\lambda)} \right) \\ &= G_1 + \lambda(W^{sp} - T_1^{sp}) + \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2}. \end{aligned}$$

Using that $W^{sp} = \gamma^{sa} Y_1$ we have

$$Y_1 = G_1 + \lambda \gamma^{sp} Y_1 - \lambda T_1^{sp} + \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2}.$$

Solving for Y_1 we have:

$$Y_1 = \frac{1}{1-\lambda \gamma^{sp}} \left[G_1 - \lambda T_1^{sp} + \left(\frac{1-s}{\beta A_2} \right)^\sigma \left(\frac{A_2 M_2}{(1+i_1)(1-s)P_1} + \lambda T_2^{sp} \right) + \frac{M_2}{(1+i_1)(1-s)P_1} + \frac{G_2}{A_2} \right]$$

which is the expression for Y_1 in (6). Finally, as above, we can solve for C_1 from $C_1 = Y_1 - I_1 - G_1$ which yields the expression in (6).

2.2 Wolf's equivalence result: restoring the first best

Having derived the expressions for the equilibrium variables in (6), we now show that an appropriately chosen combination of fiscal stimulus (T_1^{sp}, T_2^{sp}) and the investment subsidy s can restore the first best allocation.

From an entirely mathematical perspective, we can see that transfers (T_1^{sp}, T_2^{sp}) only appear in the expressions for C_1 and Y_1 , but not in C_2 , Y_2 , and I_1 . So, transfers alone do not affect any variable in the second period and also cannot restore them to their first-best levels. On the other hand, the investment subsidy s *does* appear in the expressions for C_2 , Y_2 , and I_1 , so the government can affect the values of these variables by changing the investment subsidy. In particular, to restore the first-best allocation, the government can first choose the investment subsidy s so that $(1 + i_1)(1 - s)$ makes the expressions for C_2 , Y_2 , and I_1 equal to their first best. Then, it sets T_1^{sp} or T_2^{sp} so that we achieve the first best for C_1 and Y_1 as well.

The economic intuition for the above is that investment in the first period determines the output and consumption in the second period. In the second period, prices are flexible, so output is no longer demand-determined. Hence, taking income from savers and giving it to spenders will not affect output (unlike in the first period). The only way to increase the output is to arrive in the second period with more capital. The government can achieve this by subsidizing investment.