

# Supplement to Lecture 2: Definition of Competitive Equilibrium

Macroeconomics, EC2B1

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In the lecture notes we sketched the definition of a competitive equilibrium for a general, abstract economy. This supplement fills in the gaps and provides the equations.

While the definition of a competitive equilibrium presented here is fairly general, it simplifies the notation somewhat and the maths is less precise than in a fully rigorous treatment. For such full generality and precision, see chapter 10.B of MasColell-Whinston-Green “Microeconomic Theory” (MWG) referenced in the lecture notes. Either way, as stated in the lecture notes: the important thing is not the precise maths but to understand general structure.

## 1 Some Notation

- $I$  consumers (households) indexed by  $i = 1, \dots, I$
- $J$  producers (firms) indexed by  $j = 1, \dots, J$
- $K$  factors of production (inputs) indexed by  $k = 1, \dots, K$
- $L$  final goods indexed by  $\ell = 1, \dots, L$
- Quantities
  - $x_{\ell i}$ : household  $i$ 's consumption of good  $\ell$
  - $y_{\ell j}$ : firm  $j$ 's production of good  $\ell$
  - $e_{\ell i}$ : household  $i$ 's endowment of good  $\ell$
  - $\tilde{x}_{ki}$ : household  $i$ 's supply of factor  $k$
  - $\tilde{y}_{kj}$ : firm  $j$ 's use of factor  $k$
  - $\tilde{e}_{ki}$ : household  $i$ 's endowment of factor  $k$

Note: many books (MasColell-Whinston-Green,...) use more general notation: they denote both goods and inputs by  $y_{1j}, \dots, y_{Lj}$  and use negative numbers  $y_{\ell j} \leq 0$  for inputs. I personally find this confusing which is why I chose to use the somewhat less general but simpler notation presented here.

## 2 Primitives of the general economy

- Preferences: household  $i$ 's utility

$$u_i(x_{1i}, \dots, x_{Li}, \tilde{x}_{1i}, \dots, \tilde{x}_{Ki})$$

- Technology: firm  $j$ 's production function for producing good  $\ell$

$$y_{\ell j} = f_j(\tilde{y}_{1j}, \dots, \tilde{y}_{Kj})$$

- Resource constraints (feasibility):

$$\begin{array}{l} \text{Goods:} \\ \text{Factors:} \end{array} \quad \begin{array}{l} \underbrace{\sum_{i=1}^I x_{\ell i}}_{\text{total demand of good } \ell} \\ \underbrace{\sum_{j=1}^J \tilde{y}_{kj}}_{\text{total demand of factor } k} \end{array} = \begin{array}{l} \underbrace{\sum_{j=1}^J y_{\ell j} + \sum_{i=1}^I e_{\ell i}}_{\text{total supply of good } \ell} \\ \underbrace{\sum_{i=1}^I \tilde{x}_{ki} + \sum_{i=1}^I \tilde{e}_{ki}}_{\text{total supply of factor } k} \end{array}, \quad \begin{array}{l} \text{all } \ell = 1, \dots, M \\ \text{all } k = 1, \dots, K \end{array}$$

Note: as usual  $\sum_{i=1}^I x_{i\ell} = x_{1\ell} + x_{2\ell} + \dots + x_{I\ell}$  and similarly for the other summations

## 3 Definition of competitive equilibrium (CE)

**Definition:** a competitive equilibrium are quantities  $\{x_{\ell i}, y_{\ell j}, \tilde{x}_{ki}, \tilde{y}_{kj}\}$  and prices  $\{p_\ell, \tilde{p}_k\}$  for  $\ell = 1, \dots, L$ ,  $k = 1, \dots, K$ ,  $i = 1, \dots, I$  and  $j = 1, \dots, J$  such that:

1. Utility maximization: taking as given prices  $\{p_\ell, \tilde{p}_k\}$ , households maximize utility subject to their budget constraints
2. Profit maximization: taking as given prices  $\{p_\ell, \tilde{p}_k\}$ , firms maximize profits

3. Market clearing: demand = supply for each good and each factor

$$\begin{array}{l}
 \text{Goods:} \quad \underbrace{\sum_{i=1}^I x_{\ell i}}_{\text{total demand of good } \ell} = \underbrace{\sum_{j=1}^J y_{\ell j} + \sum_{i=1}^I e_{\ell i}}_{\text{total supply of good } \ell}, \quad \text{all } \ell = 1, \dots, M \\
 \\
 \text{Factors:} \quad \underbrace{\sum_{j=1}^J \tilde{y}_{kj}}_{\text{total demand of factor } k} = \underbrace{\sum_{i=1}^I \tilde{x}_{ki} + \sum_{i=1}^I \tilde{e}_{ki}}_{\text{total supply of factor } k}, \quad \text{all } k = 1, \dots, K
 \end{array}$$

As emphasized in the lecture notes, the important thing is the general structure of a competitive equilibrium:

1. Households maximize taking prices as given
2. Firms maximize taking prices as given
3. All markets clear

This structure will come up over and over again.