

# Lecture 2

## A Simple Macro Model, Equilibrium & Welfare Theorems

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Macroeconomics EC2B1

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# Today: your first modern macro model

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- As already mentioned: model is very (ridiculously) simple, a baby version
- ... but it will have key features of most modern macro models
  1. micro foundations
  2. general equilibrium
- To make point, will show you what richer modern macro model looks like
  - Heterogeneous Agent New Keynesian (HANK) model from my work  
<https://benjaminmoll.com/HANK/>
  - one of the most complicated models I can think of...
  - ... but will show you: structure is exactly the same

# Plan

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1. Reminder: what we want a macro model to speak to
2. Overview of simple macro model
3. Optimal resource allocation: Robinson Crusoe's problem
4. Decentralized competitive equilibrium
5. First and second welfare theorems: the “invisible hand”
6. What a rich modern macro model looks like: HANK model
7. Putting the simple macro model to use: long-run trends in hours worked

# Useful Readings (most on moodle)

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- Your EC2A1 lecture notes on “The Market System” (Weeks 1 and 2)
  - treatment very complementary though with some differences
- Varian, chapters 32 and 33
  - ch. 32: treatment of first and second welfare theorems
  - ch. 33: application to Robinson Crusoe economy
- Chapter 9 of Kurlat, particularly chapter 9.2
  - but our treatment is simpler: we use a static one-period model (like Varian and Williamson)
  - ... whereas Kurlat uses more complicated two-period model
- Williamson, chapters 4 and 5
- (PhD-level treatment so only read if interested, also not on moodle)  
Chapter 10.B of MasColell-Whinston-Green “Microeconomic Theory”

# Reminder: what we want a macro model to speak to

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Data on macroeconomic aggregates like

1. GDP = aggregate output
2. aggregate hours worked
3. aggregate consumption
4. aggregate investment
5. inflation
6. unemployment
7. ...

Today: build simple model that can speak to 1 to 3. Recall:

- objective is **not** to build one big model we use to address all issues
- instead “custom build” models for particular questions

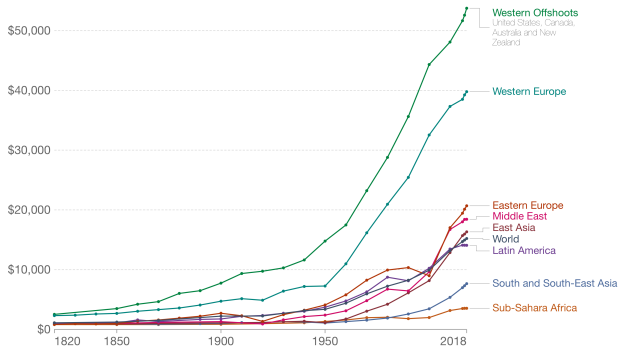
# Reminder from EC1P1: GDP

## Gross domestic product: a statistic

### GDP per capita, 1820 to 2018

GDP per capita adjusted for price changes over time (inflation) and price differences between countries – it is measured in international-\$ in 2011 prices.

Our World  
in Data



Source: Maddison Project Database 2020 (Bolt and van Zanden (2020))

OurWorldInData.org/economic-growth • CC BY

Definition: **GDP** is the market value of goods produced within a country during a period of time (year).

Invented in 1930s-40s by Simon Kuznets at NBER, first published in 1942 for the USA, first UN guide in 1953.

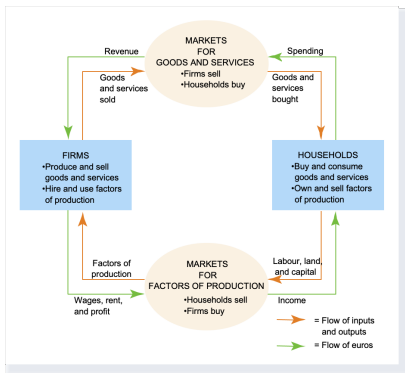
For short, call it output.

# Reminder from EC1P1: GDP

## Measurement of GDP

Figure 1 The Circular Flow

Circular flow in the economy



- What is sold is what is produced
- What is produced is what is paid out as income to those who produced it.
- What is income is what is spent in goods (even in savings)
- *expenditure = income = production*
- GDP is also GDI. (and GDE)

# Reminder from EC1P1: GDP

## Gross domestic product: comparisons across regions

### GDP per capita

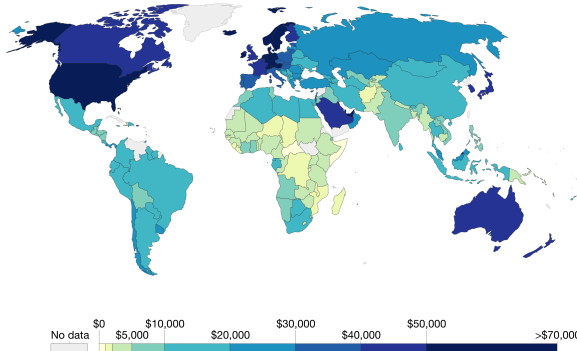
Measured in constant international-\$.



Common use: the way we measure how well or badly countries are doing

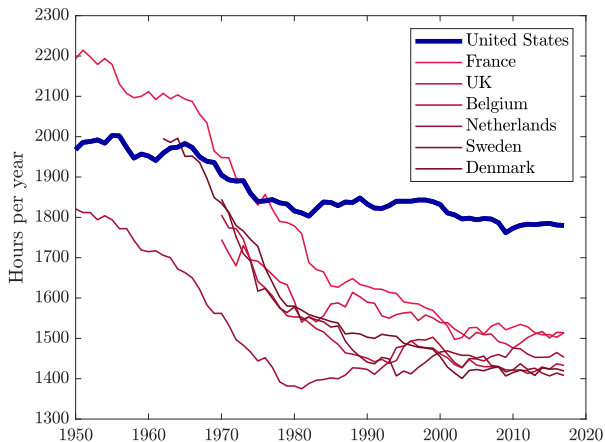
One country versus another: comparison of wellbeing in different places in the globe

Must be in same units “international dollars”



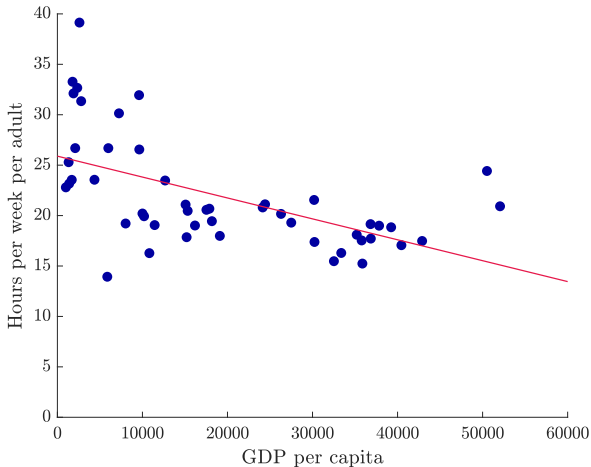


# Aggretate hours worked over time



**Fig. 7.3.3:** Hours worked per year per employed person in the US and selected European countries. Source: OECD.

# Aggregate hours worked and GDP across countries



**Fig. 7.3.2:** Average hours of work per week across countries.  
Source: Bick et al. (2018).

# Overview of simple macro model

# Primitives of the simple macro model

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- **Preferences:**

$$u(c) - v(h)$$

- $c$ : consumption of coconuts,  $u'(c) > 0$ ,  $u''(c) < 0$
- $h$ : hours worked,  $v'(h) > 0$ ,  $v''(h) < 0$

- **Technology:**

$$y = f(n)$$

- $y$ : output, i.e. production of coconuts
- $n$ : hours employed,  $f'(n) > 0$ ,  $f''(n) < 0$

- **Resource constraint:**

$$c = y, \quad n = h$$

# Study two ways of organizing economy with these primitives

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## 1. **“Optimal resource allocation”**

- Robinson has utility function  $u(c) - v(h)$  and production function  $y = f(n)$  and optimally chooses how much to consume and work
- no markets, prices with this organization, only physical quantities of coconuts and labor (the “allocation”)

## 2. **“Decentralized competitive equilibrium”**

- representative household with utility function  $u(c) - v(h)$
- representative firm with production function  $y = f(n)$  (“Crusoe Inc.”)
- (think of large number of identical households, large number of identical firms rather than literally one household and one firm)
- competitive coconut market: households buy, firms sell coconuts
- competitive labor market: households sell, firms buy labor

# Study two ways of organizing economy with these primitives

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Will show:

- even though these sound very different: **close connection**
- there is a deep, much more general reason for this: **welfare theorems**

Logic same as in EC2A1 but

- EC2A1: **exchange economy** (or endowment economy)  $\Rightarrow$  distribution of resources across different individuals, but no production
- here: **production economy** (Robinson Crusoe economy)  $\Rightarrow$  production but representative household so nothing to say about distribution

# Optimal Resource Allocation: Robinson Crusoe's Problem

# Robinson Crusoe's Problem

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- Robinson Crusoe is alone on an island, he lives of coconuts which he harvests, harvesting coconuts takes time
- $\Rightarrow$  he chooses  $c$  and  $n$  to maximize his utility  $u(c) - v(n)$  subject to the constraint that he can only eat the coconuts he harvested  $c = f(n)$

$$\max_{c,n} u(c) - v(n) \quad \text{s.t.} \quad c = f(n)$$

- Note: have used that  $c = y$  and  $n = h$  to drop  $y$  and  $h$
- Optimality condition:  $u'(c)f'(n) = v'(n)$  or

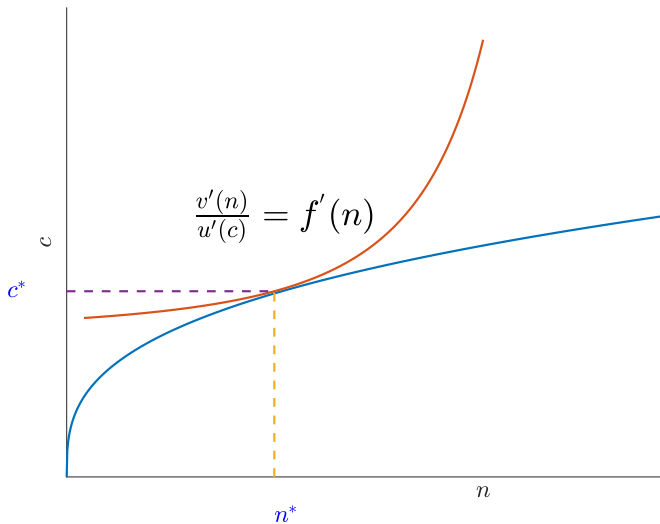
$$\frac{v'(n)}{u'(c)} = f'(n)$$

- Together with  $c = f(n)$  this pins down optimal  $(n^*, c^*)$



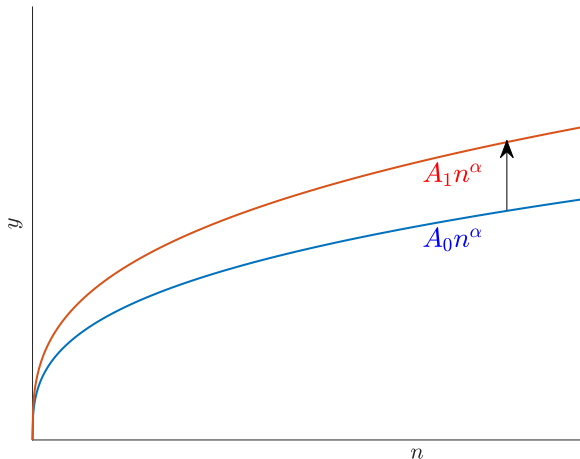
# Graphical representation of optimality condition

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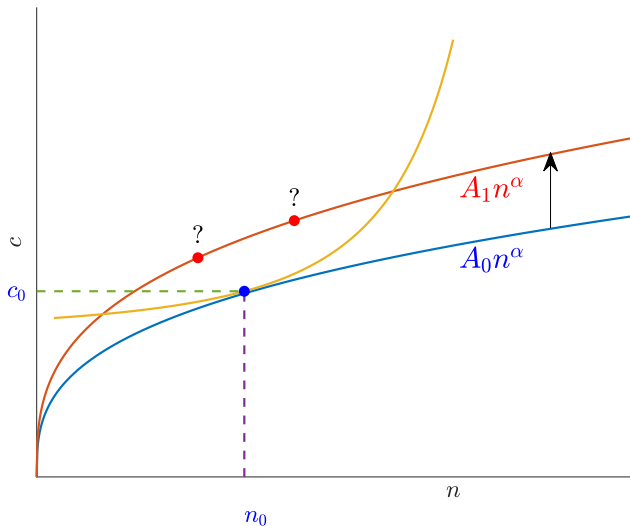


# An increase in productivity from $A_0$ to $A_1 > A_0$

Example: Robinson figures out that ladder  $\Rightarrow$  more coconuts per hour worked



# What will happen to consumption and hours worked?



# Intuition: income and substitution effects

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## Substitution effect:

- $A \uparrow \Rightarrow$  Robinson's marginal product of labor  $\uparrow$
- $\Rightarrow$  Robinson works more (he substitutes leisure for consumption)

## Income effect:

- $A \uparrow \Rightarrow$  Robinson gets more output per hour worked (he is richer)
- $\Rightarrow$  Robinson works less (he consumes more leisure because leisure is a normal good, i.e. a good whose consumption increases with income)

# Parametric example: functional forms from last lecture

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- Production function

$$f(n) = An^\alpha, \quad 0 < \alpha < 1, A > 0$$

- Utility function

$$u(c) = \log c, \quad v(n) = \theta \frac{n^{1+1/\varepsilon}}{1+1/\varepsilon}, \quad \theta, \varepsilon > 0 \quad \left(\text{note: } v'(n) = \theta n^{1/\varepsilon}\right)$$

- Optimality condition

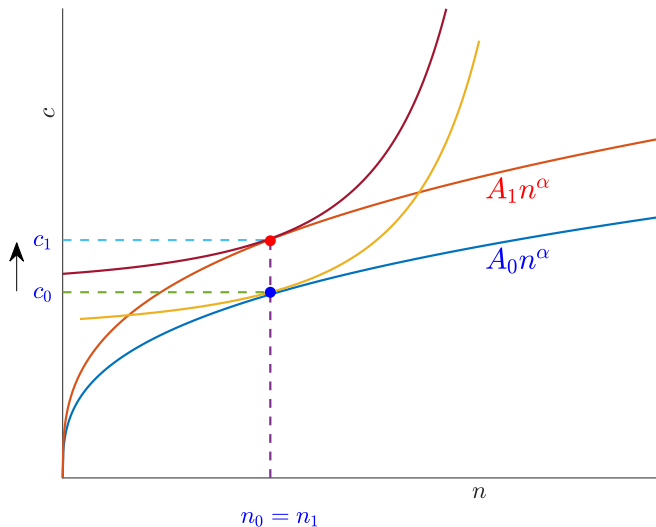
$$\frac{v'(n)}{u'(c)} = f'(n) \quad \Rightarrow \quad \frac{\theta n^{1/\varepsilon}}{1/c} = \alpha An^{\alpha-1}$$

- Using that  $c = An^\alpha$ , solution is

$$n^* = \left(\frac{\alpha}{\theta}\right)^{\frac{\varepsilon}{1+\varepsilon}}, \quad c^* = A \left(\frac{\alpha}{\theta}\right)^{\frac{\alpha\varepsilon}{1+\varepsilon}}$$

- How do these vary with  $A, \theta, \dots$  and why?

# An increase in productivity in the parametric example



# Competitive Equilibrium

# Competitive equilibrium: Plan

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1. Define “competitive equilibrium” for a general, abstract economy
  - only sketch this in lecture notes, see **supplement** for details
  - important thing is not the precise maths but to understand general structure
2. Apply analysis to Robinson Crusoe economy



# Preview of general definition of competitive equilibrium

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The following will be useful only in retrospect after you've seen next slides / supplement

A **competitive equilibrium (CE)** are quantities and prices that satisfies **three types of conditions**

1. Households maximize taking prices as given
2. Firms maximize taking prices as given
3. All markets clear

Next: flesh this out in more detail

## Definition of CE for general, abstract economy (supplement)

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- $I$  consumers (households) indexed by  $i = 1, \dots, I$
- $J$  producers (firms) indexed by  $j = 1, \dots, J$
- $K$  factors of production (inputs) indexed by  $k = 1, \dots, K$
- $L$  final goods indexed by  $\ell = 1, \dots, L$
- **Preferences:** household  $i$ 's utility over  $L$  goods and  $K$  factors
- **Technology:** firm  $j$ 's production function for producing good  $\ell$
- **Resource constraints (feasibility):**

total demand of good  $\ell$  = total supply of good  $\ell$ ,    all  $\ell = 1, \dots, L$   
total demand of factor  $k$  = total supply of factor  $k$ ,    all  $k = 1, \dots, K$

# Prices and price-taking (from EC1A1 notes)

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Final bit of notation: prices

- $p_\ell$ : price of good  $\ell = 1, \dots, L$
- $\tilde{p}_k$ : price of factor  $k = 1, \dots, K$

**Assumption:** both households and firms are **price takers**. From EC1A1 notes:

## Price taking

- Price taking implies the firm has no market power
- Nothing it can do affects the prices it gets for its output (or pays for its inputs)
- ...but price-taking does not mean prices do not change

### When is price taking plausible?

- Price taking is plausible if the firm has a small market share
- Many small producers produce a homogeneous good
- Everyone can observe prices

# General definition of competitive equilibrium (CE)

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**Definition:** a competitive equilibrium are quantities and prices  $\{p_\ell, \tilde{p}_k\}$  for  $\ell = 1, \dots, L$ ,  $k = 1, \dots, K$ ,  $i = 1, \dots, I$  and  $j = 1, \dots, J$  such that:

1. Utility maximization: taking as given prices  $\{p_\ell, \tilde{p}_k\}$ , households maximize utility subject to their budget constraints
2. Profit maximization: taking as given prices  $\{p_\ell, \tilde{p}_k\}$ , firms maximize profits
3. Market clearing: demand = supply for each good and each factor

total demand of good  $\ell$  = total supply of good  $\ell$ ,    all  $\ell = 1, \dots, M$   
total demand of factor  $k$  = total supply of factor  $k$ ,    all  $k = 1, \dots, K$

## Useful facts about CEs: numeraire good and Walras' law

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- **Nnumeraire good**: only relative prices  $p_\ell/p_{\ell'}$  matter  $\Rightarrow$  can always “normalize” price of one good or factor to 1, e.g. price of good 1,  $p_1 = 1$ 
  - only  $N - 1$  variable where  $N = K + L =$  number of goods and factors
- **Walras' law** in words: if we have found prices  $\{p_\ell, \tilde{p}_k\}$  such that all markets clear except one (i.e. demand = supply in all markets except one), then the remaining market must also clear (i.e. demand must also equal supply in that remaining market)
  - only  $N - 1$  equations which makes sense given  $N - 1$  variables
- Idea of proof (see Varian and MWG textbooks for formal proof):
  - add up all budget constraints and all firm profits
  - ... use all market clearing conditions except one
  - then the algebra implies the one remaining market clearing condition
- Useful implication: when computing a competitive equilibrium in practice we can drop one market clearing condition (and we will frequently do so)

# Now: apply analysis to Robinson Crusoe economy

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Special case with:

- $I = 1$  consumer: representative household
- $J = 1$  producer: representative firm
- $K = 1$  factor of production: labor
- $L = 1$  final good: coconuts

Prices:

- make coconuts the numeraire and normalize its price to 1
- only price is the wage (= price of labor)

Ownership: the representative household owns the representative firm and therefore receives its profits as dividends

# Competitive equilibrium in Robinson Crusoe economy

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**Definition:** a competitive equilibrium in the Robinson Crusoe economy are quantities  $(c, y, n, h)$  and a wage  $w$  such that

1. Utility maximization: taking as given  $w$  (and also  $\Pi$ ), the representative household chooses  $(c, h)$  to solve

$$\max_{c, h} u(c) - v(h) \quad \text{s.t.} \quad c = wh + \Pi$$

2. Profit maximization: taking as given  $w$ , the representative firm chooses  $n$  and  $y = f(n)$  to solve

$$\Pi = \max_n f(n) - wn$$

3. Market clearing: demand = supply for both coconuts and labor

$$\text{coconut market:} \quad c = y$$

$$\text{labor market:} \quad n = h$$

# Comments on CE in Robinson Crusoe economy

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Taken literally, the CE defined on previous slide has following features

1. one household, one firm
2. household = firm's only employee
3. household = firm's only customer
4. firm is fully owned by household

**Question:** isn't this completely schizophrenic?

- if it's literally just one household and one firm, why would they trade with each other in these competitive markets and even act as price takers?
- surely the household would just decide how much to work, tell the firm how much to produce, then eat it?
- why would Robinson artificially split himself in two and trade with himself?

**Answer:** yes, this is 100% correct, taken literally this definition of a competitive equilibrium is completely schizophrenic



## Alternative interpretation: large number of households, firms

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- **BUT** there is an alternative interpretation that is considerably less crazy:
  - there is a **large number of households**  $i = 1, \dots, I$
  - there is a **large number of firms**  $j = 1, \dots, J$
  - all households are **identical**: have same utility function  $u(c_i, h_i)$
  - all firms are **identical**: have same production function  $f(n_j)$
  - resource constraints / market clearing conditions:

$$\sum_{i=1}^I c_i = \sum_{j=1}^J y_j, \quad \sum_{j=1}^J n_j = \sum_{i=1}^I h_i,$$

- In this interpretation, trade and price-taking are much more sensible
- But equations same: clearly all households choose same  $(c_i, h_i)$ , all firms choose same  $(y_j, n_j)$
- $\Rightarrow$  think of  $(c, h, y, n)$  as variables in per capita terms

# Equations characterizing CE in Robinson Crusoe economy

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**Definition:** a competitive equilibrium in the Robinson Crusoe economy are quantities  $(c, y, n, h)$  and a wage  $w$  such that

1. Utility maximization: taking as given  $w$ , the representative household chooses  $(c, h)$  to solve

$$\max_{c, h} u(c) - v(h) \quad \text{s.t.} \quad c = wh + \Pi$$

2. Profit maximization: taking as given  $w$ , the representative firm chooses  $n$  and  $y = f(n)$  to solve

$$\Pi = \max_n f(n) - wn$$

3. Market clearing: demand = supply for both coconuts and labor

$$\text{coconut market:} \quad c = y$$

$$\text{labor market:} \quad n = h$$

# Equations characterizing CE in Robinson Crusoe economy

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**Definition:** a competitive equilibrium in the Robinson Crusoe economy are quantities  $(c, y, n, h)$  and a wage  $w$  such that

1. Utility maximization: taking as given  $w$ , the representative household chooses  $(c, h)$  such that

$$\frac{v'(h)}{u'(c)} = w$$

2. Profit maximization: taking as given  $w$ , the representative firm chooses  $n$  and  $y = f(n)$  such that

$$f'(n) = w$$

3. Market clearing: demand = supply for both coconuts and labor

$$\text{coconut market:} \quad c = y$$

$$\text{labor market:} \quad n = h$$

# Equations characterizing CE in Robinson Crusoe economy

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Summary: a CE are are quantities  $(c, y, n, h)$  and a wage  $w$  such that

$$\frac{v'(h)}{u'(c)} = w \quad (1)$$

$$f'(n) = w \quad (2)$$

$$y = f(n) \quad (3)$$

$$c = y \quad (4)$$

$$n = h \quad (5)$$

Always want to check: same number of equations and unknowns?

Here: 5 equations (1)-(5) in 5 unknowns  $(c, y, n, h, w)$  so this looks promising

# Equations characterizing CE in Robinson Crusoe economy

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- Let's solve these equations as far as possible:

- equating (1) and (2) and using (4)

$$\frac{v'(n)}{u'(c)} = f'(n) \quad (*)$$

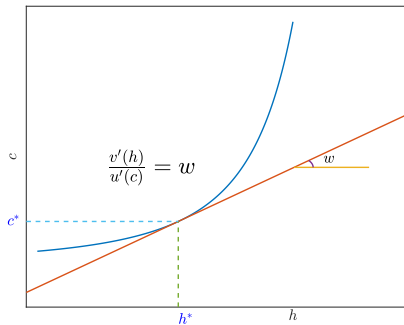
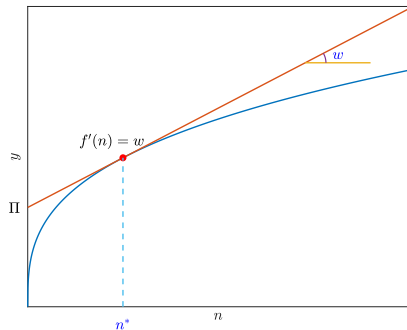
- using (3) and (4)

$$c = f(n) \quad (**)$$

- So equilibrium allocation  $(c, n)$  satisfies  $(*)$  and  $(**)$
- But these are same equations as in optimal resource allocation problem!
- So equilibrium allocation  $(c, n)$  satisfies same two equations as optimal allocation  $(c^*, n^*) \Rightarrow$  CE allocation = optimal allocation!
- What's going on?

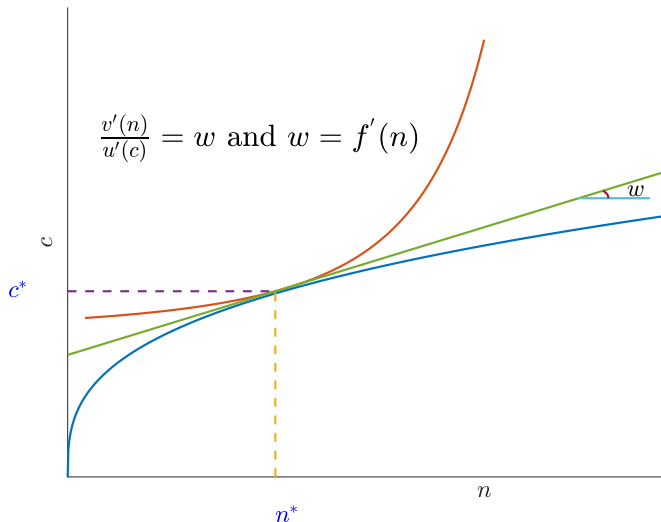
# What's going on? Recall utility and profit maximization

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# Competitive equilibrium in Robinson Crusoe economy

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# A remarkable fact about the competitive equilibrium

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- Have studied equilibrium in which households and firms maximize taking as given prices and trade in competitive markets
- Somehow magically the resulting competitive equilibrium allocation = optimal allocation!
- So even though both households and firms act purely in their own self-interest and only do what's individually optimal for them (maximize utility and profits), this generates an allocation that is jointly optimal!
- Also has a convenient practical implication for us: instead of solving for CE can just solve for optimal allocation (which is much easier).
- Natural questions:
  - is this true more generally?
  - if so, when does this hold? Always? Sometimes? Under reasonable conditions? Under crazy conditions?



# Welfare Theorems: the “Invisible Hand”

# Pareto efficiency (or Pareto optimality)

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- Consider general economy from earlier with many individuals  $i = 1, \dots, I$
- **Definition:** an allocation is Pareto efficient (PE) if it is impossible to find another feasible allocation that improves everyone's welfare
  - more precisely: there is no feasible allocation that makes one individual better off without making another worse off
  - feasible allocation = allocation that respects resource constraints
- If PE, government intervention cannot make everyone better off
- But note: while Pareto efficiency is desirable it is also a very weak requirement (a single person consuming everything may be PE)
  - “A society can be Pareto optimal and still perfectly disgusting” (Sen)
- To find Pareto efficient allocations: solve problem of **fictitious “social planner”** who maximizes weighted sum of individual utilities s.t. feasibility

# Pareto efficiency in Robinson Crusoe economy

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- One individual or everyone identical (and gets same): no dist'nal conflict
- PE = allocation that attains highest utility of rep household s.t. feasibility
- **Definition:** a Pareto efficient allocation in the Robinson Crusoe economy are quantities  $(c, y, n, h)$  that maximize household utility

$$u(c) - v(h)$$

subject to being technologically feasible

$$y = f(n), \quad c = y, \quad n = h.$$

- Can interpret as problem of fictitious **benevolent social planner**
- But exact same problem we solved earlier!  $\max u(c) - v(n)$  s.t.  $c = f(n)$
- So when we studied “optimal resource allocation” or “Robinson Crusoe’s problem” we actually studied the Pareto efficient allocation in this economy
- **In this economy, CE allocation = PE allocation.** True more generally?

# First and second welfare theorems

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Very roughly:

- **1st welfare theorem:** (under some strong assumptions) CE allocations are PE, i.e.  $CE \Rightarrow PE$
- **2nd welfare theorem:** (under even stronger assumptions\*) PE allocations are CE, i.e.  $CE \Leftarrow PE$
- Together:  $CE \Leftrightarrow PE$

\*even stronger assumptions = same assumptions as 1st welfare theorem + “convexity assumptions”, e.g. utility and production functions are concave

For more precise treatment, see references in beginning of slides in particular Varian and MWG

# First welfare theorem: the “invisible hand”

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Focus on first welfare theorem and unpack assumptions a bit

- **1st welfare theorem:** if

1. perfect competition (individuals and firms are price takers)
2. individuals and firms are rational
3. no externalities
4. perfect information

then CE allocations are Pareto efficient

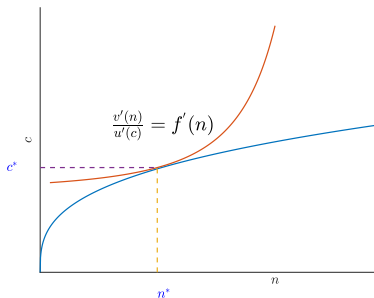
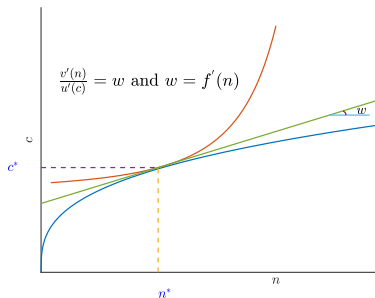
- How to think about this result and the “invisible hand”?
- Does it perhaps mean that free markets always deliver socially optimal outcomes and government intervention is never desirable?

# Limitations and uses of the first welfare theorem

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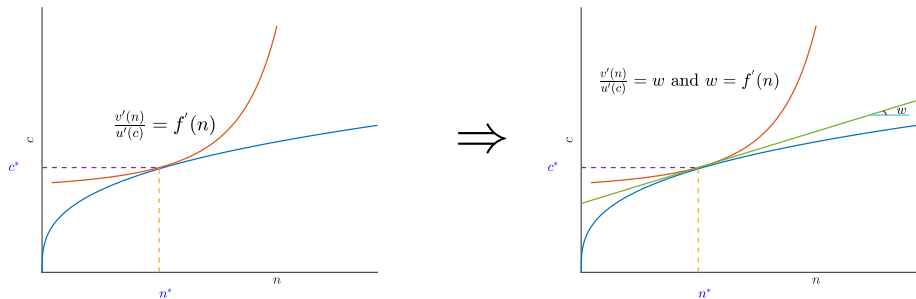
- Does 1st welfare theorem mean that free markets always deliver socially optimal outcomes and government intervention is never desirable?
- Definitely not. Instead **1st welfare theorem is all about its assumptions.**
  - violations of these are called “market failures” or “frictions”
  - in real world and also in modern macro, market failures and frictions are everywhere
  - “The reason that the invisible hand often seems invisible is that it is often not there” (Stiglitz)
- ... but 1st welfare theorem is nevertheless an important benchmark
- In my view, perhaps the most important use of 1st welfare theorem: force you to think about rationale for policy intervention
  - what market failures, frictions, externalities are there?
  - once identified, what policies can tackle these? (targeting principle)

# Idea of proof of 1st welfare theorem: $CE \Rightarrow PE$



Idea: if indifference curves and production possibilities are tangent to price line, then they are also tangent to each other and hence allocation is optimal

## Idea of proof of 2nd welfare theorem: $PE \Rightarrow CE$

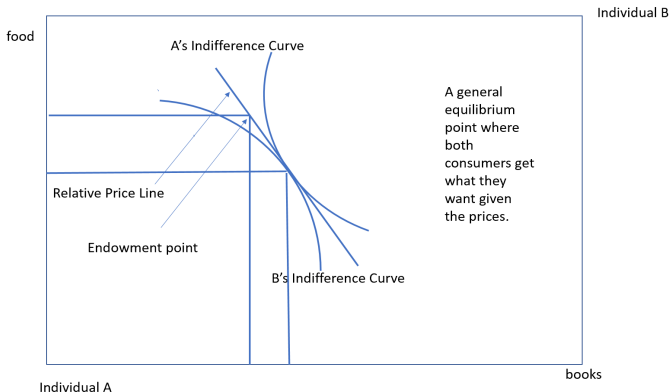


Idea: if indifference curves and production possibilities are tangent to each other, then can find a price line that separates them in a tangent fashion

- in general (see MWG): a “separating hyperplane”



# Link to EC2A1: logic exactly same as in exchange economy



- EC2A1: **exchange economies** (or endowment economies)
- Here: **production economies**, e.g. Robinson Crusoe economy
- Different uses but same logic, welfare theorems are very general

## Another use of welfare theorems: a shortcut for finding CE

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- In many applications, planning problem is much easier to solve than CE
  - think of Robinson Crusoe economy
- In such cases: if welfare theorems hold, can use planning problem as a shortcut to find CE allocation
  - will often see this in macro, and will sometimes do this in this course
- Obviously only works if assumptions of welfare theorems hold, i.e. **if there are no market failures / frictions**
  - technically need PE  $\Rightarrow$  CE so need 2nd welfare theorem assumptions (recall = same as 1st welfare theorem + “convexity assumptions”)
  - in practice assumptions of 1st welfare theorem will fail so check those

# Equilibrium vs Planning Problem: Key Differences

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- Even though the competitive equilibrium and planning problem may lead to the same allocation, they are very different animals
- Some key differences to remember when you solve these in practice
  1. equilibrium features **prices**, the planning problem does not
  2. equation for resource constraint plays mathematically different roles:
    - in competitive equilibrium: market clearing condition
    - in planning problem: constraint on planner's maximization problem
- A typical mistake that students make: write planning problem as planner maximizing utility subject to budget constraint (which features prices)
  - Please don't do this. If in exam: zero points on that subquestion.

A much richer modern macro model:  
HANK

# A richer modern macro model

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- I picked one that I know well: a Heterogeneous Agent New Keynesian (HANK) model from my own work <https://benjaminmoll.com/HANK/>

## Monetary Policy According to HANK<sup>†</sup>

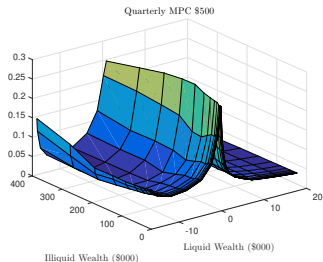
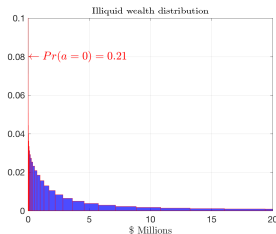
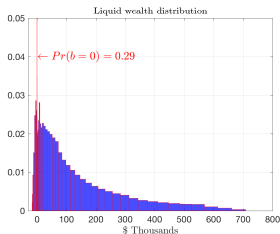
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*We revisit the transmission mechanism from monetary policy to household consumption in a Heterogeneous Agent New Keynesian (HANK) model. The model yields empirically realistic distributions of wealth and marginal propensities to consume because of two features: uninsurable income shocks and multiple assets with different degrees of liquidity and different returns. In this environment, the indirect effects of an unexpected cut in interest rates, which operate through a general equilibrium increase in labor demand, far outweigh direct effects such as intertemporal substitution. This finding is in stark contrast to small- and medium-scale Representative Agent New Keynesian (RANK) economies, where the substitution channel drives virtually all of the transmission from interest rates to consumption. Failure of Ricardian equivalence implies that, in HANK models, the fiscal reaction to the monetary expansion is a key determinant of the overall size of the macroeconomic response. (JEL D31, E12, E21, E24, E43, E52, E62)*

- ... but could have also picked lots of other papers / models
- See Carlin-Soskice chapter on moodle for good accessible discussion of the heterogeneous agent and HANK literatures

# A richer modern macro model

- A very complicated model with lots of bells & whistles
  - lots of heterogeneity and inequality
  - lots of assumptions that invalidate welfare theorems (borrowing constraints, incomplete insurance markets, monopolistic comp., ...)



- Still the model's structure is exactly the same as our baby model

# Primitives of the HANK economy

- Preferences:**

Households receive a utility flow  $u$  from consuming  $c_t \geq 0$  and a disutility flow from supplying labor  $\ell_t$ , where  $\ell_t \in [0, 1]$  are hours worked as a fraction of the time endowment, normalized to 1. The function  $u$  is strictly increasing and strictly concave in consumption, and strictly decreasing and strictly convex in hours worked. Preferences are time-separable and, conditional on surviving, the future is discounted at rate  $\rho \geq 0$ :

$$(10) \quad E_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t, \ell_t) dt,$$

- Technology:**

*Intermediate Goods Producers.*—Each intermediate good  $j$  is produced by a monopolistically competitive producer using effective units of capital  $k_{j,t}$  and effective units of labor  $n_{j,t}$  according to the production function

$$(15) \quad y_{j,t} = k_{j,t}^\alpha n_{j,t}^{1-\alpha}.$$

- Resource constraints (feasibility):**

The liquid asset market clears when

$$(25) \quad B_t^h + B_t^g = 0,$$

where  $B_t^g$  is the stock of outstanding government debt and  $B_t^h = \int b d\mu_t$  are total household holdings of liquid bonds. The illiquid asset market clears when physical capital  $K_t$  plus the equity value of monopolistic producers  $q_t$  (with the total number of shares normalized to 1) equals households' holdings of illiquid assets  $A_t = \int a d\mu_t$ ,

$$(26) \quad K_t + q_t = A_t.$$

The labor market clears when

$$(27) \quad N_t = \int z \ell_t(a, b, z) d\mu_t.$$

Finally, the goods market clearing condition is

$$(28) \quad Y_t = C_t + I_t + G_t + \Theta_t + \chi_t + \kappa \int \max\{-b, 0\} d\mu_t.$$

# Equilibrium in the HANK model

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## B. *Equilibrium*

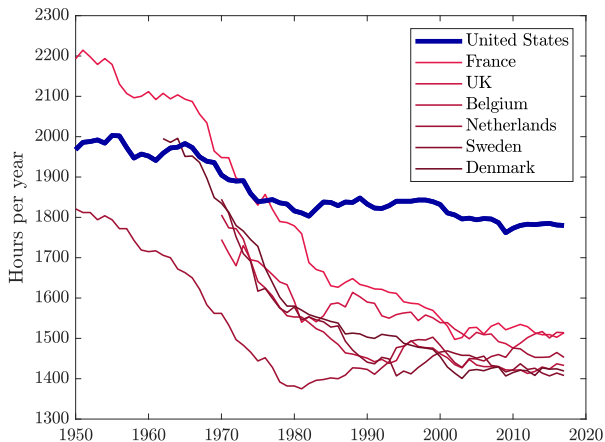
An equilibrium in this economy is defined as paths for individual household and firm decisions  $\{a_t, b_t, c_t, d_t, \ell_t, n_t, k_t\}_{t \geq 0}$ , input prices  $\{w_t, r_t^k\}_{t \geq 0}$ , returns on liquid and illiquid assets  $\{r_t^b, r_t^a\}_{t \geq 0}$ , the share price  $\{q_t\}_{t \geq 0}$ , the inflation rate  $\{\pi_t\}_{t \geq 0}$ , fiscal variables  $\{\tau_t, T_t, G_t, B_t\}_{t \geq 0}$ , measures  $\{\mu_t\}_{t \geq 0}$ , and aggregate quantities such that, at every  $t$ : (i) households and firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) all markets clear. There are five markets in our economy: the liquid asset (bond)

Households maximize (10) subject to (11)–(14). They take as given equilibrium paths for the real wage  $\{w_t\}_{t \geq 0}$ , the real return to liquid assets  $\{r_t^b\}_{t \geq 0}$ , the real return to illiquid assets  $\{r_t^a\}_{t \geq 0}$ , and taxes and transfers  $\{\tau_t, T_t\}_{t \geq 0}$ . As we explain below,



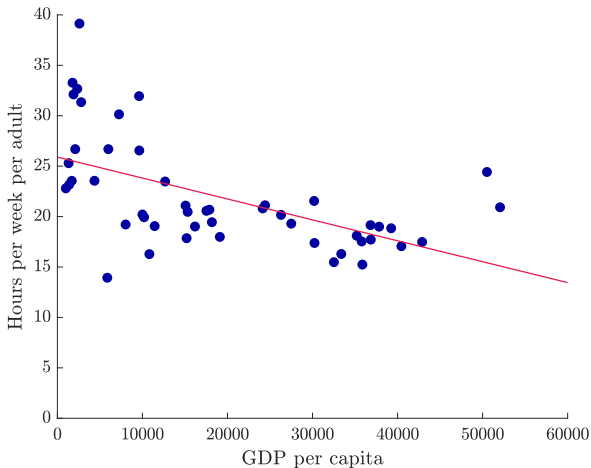
# Long-run trends in hours worked

# Aggretate hours worked over time



***Fig. 7.3.3:** Hours worked per year per employed person in the US and selected European countries. Source: OECD.*

# Aggregate hours worked and GDP across countries

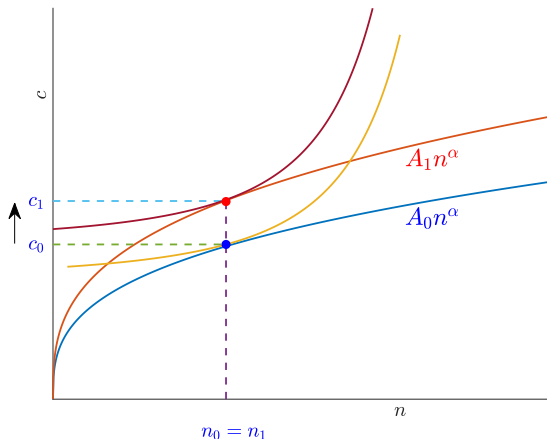


**Fig. 7.3.2:** Average hours of work per week across countries.  
Source: Bick et al. (2018).

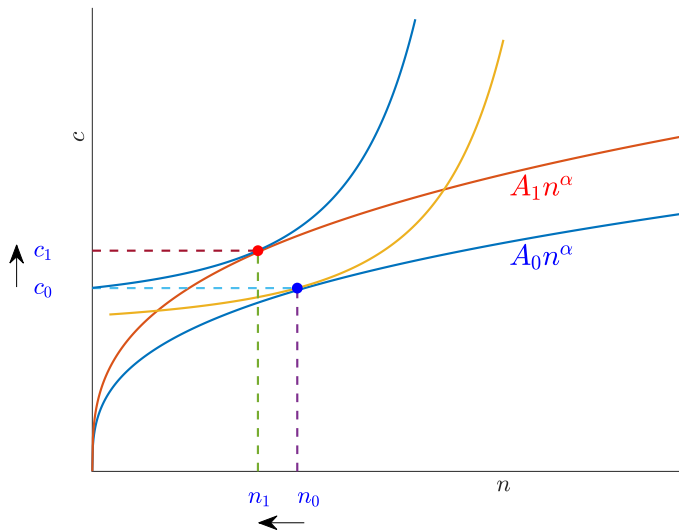
# Can our parametric example reproduce these trends?

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$$n^* = \left(\frac{\alpha}{\theta}\right)^{\frac{\varepsilon}{1+\varepsilon}}, \quad c^* = A \left(\frac{\alpha}{\theta}\right)^{\frac{\alpha\varepsilon}{1+\varepsilon}}$$



# What alternative model specification can?



# Interesting recent book: “Friday is the new Saturday”

‘Fingers crossed that this book will shake up the five-day working week.’  
Sir Christopher Pissarides, 2010 Nobel Laureate in Economics

## FRIDAY IS THE NEW SATURDAY

HOW A FOUR-DAY  
WORKING WEEK  
WILL SAVE THE  
ECONOMY

PEDRO GOMES

### *Friday is the New Saturday*

“Pedro Gomes presents a compelling approach to the topic, rooting his arguments in a range of economic theories, history and data — focused on the improvement of society.” **Financial Times Business Books of the Month: August Edition (2021)**

“I often just read a chapter or two of books that are sent to me but in this case I kept wanting to read more— both because of the importance of the idea, the nice manner in which it was presented, and the way in which the author’s genial and enthusiastic persona radiated through so clearly.” **Jason Furman**

“Written in an easy style, with humour, the book tells stories from economics, history and even the author’s own experience with childcare, that directly or indirectly support the four-day week. Social institutions change slowly, unless a big shock, like a war or a pandemic, comes and shakes them up. Fingers crossed that this book will shake up the five-day working week without the need for another war or a new covid virus.”  
**Christopher Pissarides, 2010 Nobel Laureate in Economics**