

Supplement to Lecture 2: Definition of Competitive Equilibrium

Macroeconomics, EC2B1

Benjamin Moll

In the lecture notes we sketched the definition of a competitive equilibrium for a general, abstract economy. This supplement fills in the gaps and provides the equations.

While the definition of a competitive equilibrium presented here is fairly general, it simplifies the notation somewhat and the maths is less precise than in a fully rigorous treatment. For such full generality and precision, see chapter 10.B of MasColell-Whinston-Green “Microeconomic Theory” (MWG) referenced in the lecture notes. Either way, as stated in the lecture notes: the important thing is not the precise maths but to understand general structure.

1 Some Notation

- I consumers (households) indexed by $i = 1, \dots, I$
- J producers (firms) indexed by $j = 1, \dots, J$
- K factors of production (inputs) indexed by $k = 1, \dots, K$
- L final goods indexed by $\ell = 1, \dots, L$
- Quantities
 - $x_{\ell i}$: household i 's consumption of good ℓ
 - $y_{\ell j}$: firm j 's production of good ℓ
 - $e_{\ell i}$: household i 's endowment of good ℓ
 - \tilde{x}_{ki} : household i 's supply of factor k
 - \tilde{y}_{kj} : firm j 's use of factor k
 - \tilde{e}_{ki} : household i 's endowment of factor k

Note: many books (MasColell-Whinston-Green,...) use more general notation: they denote both goods and inputs by y_{1j}, \dots, y_{Lj} and use negative numbers $y_{\ell j} \leq 0$ for inputs. I personally find this confusing which is why I chose to use the somewhat less general but simpler notation presented here.

2 Primitives of the general economy

- Preferences: household i 's utility

$$u_i(x_{1i}, \dots, x_{Li}, \tilde{x}_{1i}, \dots, \tilde{x}_{Ki})$$

- Technology: firm j 's production function for producing good ℓ

$$y_{\ell j} = f_j(\tilde{y}_{1j}, \dots, \tilde{y}_{Kj})$$

- Resource constraints (feasibility):

$$\begin{array}{ll} \text{Goods:} & \underbrace{\sum_{i=1}^I x_{\ell i}}_{\text{total demand of good } \ell} = \underbrace{\sum_{j=1}^J y_{\ell j} + \sum_{i=1}^I e_{\ell i}}_{\text{total supply of good } \ell}, \quad \text{all } \ell = 1, \dots, L \\ \text{Factors:} & \underbrace{\sum_{j=1}^J \tilde{y}_{kj}}_{\text{total demand of factor } k} = \underbrace{\sum_{i=1}^I \tilde{x}_{ki} + \sum_{i=1}^I \tilde{e}_{ki}}_{\text{total supply of factor } k}, \quad \text{all } k = 1, \dots, K \end{array}$$

Note: as usual $\sum_{i=1}^I x_{i\ell} = x_{1\ell} + x_{2\ell} + \dots + x_{I\ell}$ and similarly for the other summations

3 Definition of competitive equilibrium (CE)

Definition: a competitive equilibrium are quantities $\{x_{\ell i}, y_{\ell j}, \tilde{x}_{ki}, \tilde{y}_{kj}\}$ and prices $\{p_\ell, \tilde{p}_k\}$ for $\ell = 1, \dots, L$, $k = 1, \dots, K$, $i = 1, \dots, I$ and $j = 1, \dots, J$ such that:

1. Utility maximization: taking as given prices $\{p_\ell, \tilde{p}_k\}$, households maximize utility subject to their budget constraints
2. Profit maximization: taking as given prices $\{p_\ell, \tilde{p}_k\}$, firms maximize profits

3. Market clearing: demand = supply for each good and each factor

$$\begin{array}{lcl}
 \text{Goods:} & \underbrace{\sum_{i=1}^I x_{\ell i}}_{\text{total demand of good } \ell} & = \underbrace{\sum_{j=1}^J y_{\ell j} + \sum_{i=1}^I e_{\ell i}}_{\text{total supply of good } \ell}, \quad \text{all } \ell = 1, \dots, L \\
 \text{Factors:} & \underbrace{\sum_{j=1}^J \tilde{y}_{kj}}_{\text{total demand of factor } k} & = \underbrace{\sum_{i=1}^I \tilde{x}_{ki} + \sum_{i=1}^I \tilde{e}_{ki}}_{\text{total supply of factor } k}, \quad \text{all } k = 1, \dots, K
 \end{array}$$

As emphasized in the lecture notes, the important thing is the general structure of a competitive equilibrium:

1. Households maximize taking prices as given
2. Firms maximize taking prices as given
3. All markets clear

This structure will come up over and over again.