JOHN R. HICKS' CONTRIBUTION TO ECONOMICS

William J. Baumol

Princeton and New York Universities, USA

On rereading the works of Professor Hicks one cannot help being struck by the extent to which his ideas have been absorbed by the literature. Almost as soon as his works appear their contents become standard pieces of economic analysis whose source it is almost unnecessary to acknowledge.

The nature of Sir John's work has been such as to give rise to relatively little argument. The exchanges that have occurred, curiously have related to definitional issues. The scarcity of controversial elements in Sir John's work may seem to suggest a propensity to continue along well trodden paths rather than striking out in major new directions. But this is highly misleading—in every field in which he has worked the Hicksian contribution has almost always become a pivotal element in the literature, and has often led to a major redirection of research in the area.

1. Biographical Notes

The bare outlines of Sir John's biography follow the course of the usual academic career. J. R. Hicks was born in 1904 in Leamington Spa. He attended Clifton College and Balliol College, Oxford. His first teaching post was at the London School of Economics where he was a lecturer from 1926–1935. At the end of that time he was appointed fellow at Gonville and Caius in Cambridge, where he remained until 1938. He held his first chair at the University of Manchester from 1938–1946, and accepted a fellowship at Nuffield College, Oxford in 1946. In 1952 he was chosen Drummond Professor of Political Economy at Oxford, and held this chair until his recent retirement. In addition, he served as a member of the Revenue Allocation Commission in Nigeria in 1950, and the Royal Commission on Taxation of Profits and Income in 1951.

However, to understand the character of Hicks' work it is useful to examine in somewhat more detail the early portion of his professional career. It may seem strange in light of his subsequent work that, initially, Hicks did not consider himself a theorist at all. According to his own report, he did not even begin to take economics up seriously until his graduation in 1925. He then intended to become a labor economist. It was only in 1929 when Lionel Robbins joined the faculty of the London School of Economics that Hicks' interest in theory was aroused. But the theoretical materials that Robbins called to Hicks' attention were primarily continental and Scandinavian rather than English. Hicks found himself far better informed on the ideas of Cassel,

Walras, Pareto and Wicksell, than on the writings of Marshall and Pigou. As he wrote *The Theory of Wages* (in about 1930) his grasp of economic theory was, as Hicks recalls, still a bit shaky. He had little inkling of the work of Keynes and his group that was going on at Cambridge. Indeed, it was only through the work of Hayek, particularly through the discussion of *Prices and Production* that took place at the School in 1931 that Hicks was introduced to macroeconomic issues.

This history has clearly colored his subsequent work. It accounts for the limitations of theory of wages that Hicks takes pains to emphasize in the recent reissue of the book. It also shows how Sir John obtained his mastery of the theory of general equilibrium.

After an initial misstep in timing, the date of appearance of his works has been felicitous. The *Theory of Wages*, with its rather preKeynesian treatment of the theory of employment, could not have been published in a more unfortunate year (1932). But, then after the spectacular macroeconomic contributions of the 1920's and 30's in Sweden and the 1930's in Cambridge, the time was clearly ripe for a careful reexamination of these ideas in terms of a rigorous general equilibrium structure. The power of macroeconomics lies in its sweeping and illuminating generalizations rather than its rigor. Fruitful though it is, it unavoidably leaves the theorist a bit uncomfortable about untidy details. Hicks was precisely the man to clear those up and to provide the explicit structure which the course of developments called for.

2. Value Theory and the Theory of Demand

Probably the one area in which Hicks' work has been most influential is the theory of value. Two works, *Value and Capital* (1939) and a *Revision of Demand Theory* (1956) have been devoted largely to this area, and it has entered into many of his other writings. His work runs the gamut of topics in value theory: Equilibrium of the consumer, equilibrium of the firm, theory of interest and wages, and the general equilibrium of the system.

Perhaps the main focus of the Hicksian value analysis is upon the theory of demand. His writing in this area undoubtedly represents the apogee of ordinalist theory. Following through the suggestions of Pareto in the *Manuel* and his famous Encyclopedia article[1] Hicks recognized early that Edgeworth's indifference curve analysis permitted value theory to dispense with cardinal utility notions altogether. Apparently, several subsequent Cambridge contributions[2] along these lines as well as the critical piece by Eugen Slutsky were

[1] Vilfredo Pareto, "Economie Mathématique", *Encyclopédie des Sciences Mathematiques Pures et Appliquees*, Paris 1911, Tome I, Vol. IV, Fasc. 4, pp. 591–640, translation in W. J. Baumol and Stephen Goldfeld (eds.) *Precursors in Mathematical Economics*, London School of Economics, Series of Reprints of Scarce Works on Political Economy, No. 19, London, 1968.

[2] See, e.g., W. E. Johnson, "The Pure Theory of Utility Curves", *Economic Journal* 23: 483–513. Dec. 1913.

unknown to Hicks and R. D. G. Allen when they wrote their famous pair of articles which were to redirect the entire literature. Much of the content of these articles and of the writings that succeeded them in the next few years is too well-known to require detailed recapitulation. The basic discovery was that simply from the assumption that the consumer maximizes *something* which for convenience is referred to as "utility", one can make a *variety of deductions* about that consumer's behavior. The utility function merely has to provide a ranking of the market baskets available to the consumer. This function is taken to have a unique interior maximum and to be twice differentiable. From these assumptions, it is possible to deduce something about the consumer's reactions to price changes, to learn something about his behavior in relation to substitute and complementary goods and to understand more precisely what underlies such concepts as consumers' surplus and index numbers.

But any of these things requires one more step. Slutsky's important discovery, which Hicks and Allen rediscovered independently and applied far more widely, was that changes in prices generate an unruly income effect whose direction cannot be predicted from the weak maximization assumptions. More important, these income effects serve to conceal a remarkable orderliness in the behavior of the remaining price reactions and their relationships. Strip away the income effects and the residual substitution effects can be characterized and studied very effectively.

Obviously in this way one can derive the Slutsky effect—the notion that the substitution effect, $\partial x/\partial p_x$, of a change in the price of x upon the quantity of x purchased, will always be negative. But one can also deduce a number of propositions which Hicks labelled "the secondary substitution theorems", for example, the rather startling proposition $\partial y/\partial p_x = \partial x/\partial p_y$, i.e., that the substitution effect of a change in the price of x on the purchase of y must be precisely equal to the effect of a change in the price of y on the purchase of x.

Hicks goes beyond these secondary substitution theorems in following through this line of analysis. Utilizing what he calls "the compensated demand curve" (the demand curve along which the consumer's income is readjusted so that his utility level remains unchanged as he moves along the curve) Hicks is able to show the limitations of the consumer's surplus concept, to describe the circumstances under which it can be used with some degree of confidence and to indicate the relation to index number concepts. I will not attempt to summarize the index number analysis here, though some of it will appear by implication in the subsequent discussion. However it should be clear that there is a fundamental relation between index number theory and the theory of consumer preferences since the notion of rising or falling real income resulting from a change in composition of the market basket of commodities, the basic issue of index number analysis, is ultimately a matter of preferences.

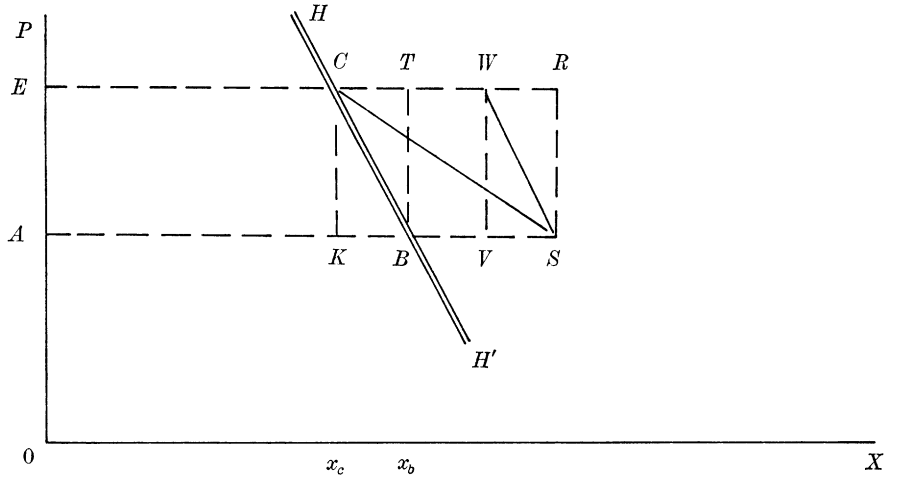

Fig. 1.

To return to the consumer's surplus issue, in Fig. 1 HH′ represents a Hicksian compensated demand curve. Our first purpose is to determine what Hicks calls "the compensating variation" in income corresponding to some price change, i.e., the change in income which will leave the consumer just as well off as before the price change. Clearly this amount is *one* measure of the net change in consumers' surplus.

Consider a fall in price from E to A. The area AKCE represents the amount of money the consumer would save if he were to continue purchase x_c, the same amount of x as he bought before the price decline. The compensating variation in income must be at least equal to this amount since the consumer always has the option of sticking to the purchase of x_c. Because he normally will buy a different amount after a fall in price, this must be a preferable choice and consequently the compensating variation must be greater than or equal to AKCE. Similarly, we can show that the compensating variation in income must be less than or equal to area ABTE since, if price this time is taken to *rise* from A to E, the consumer could always stick to his purchase quantity x_b by paying the additional amount ABTE. Breaking the price change, EA, into a sequence of smaller changes we see that in the limit the compensating variation for the entire price change, EA, is equal to ABCE, the area to the left of the compensated demand curve between the two price levels.

The significance of this measure becomes clearer when it is compared with other measures of consumer's surplus. In the diagram, segments of two other demand curves are also shown. The first, *CS*, is the ordinary uncompensated (Marshallian) demand curve.

Assuming x is not an inferior good, point S must lie to the right of B because with no income taken away from the consumer he will now purchase more of

x. In fact BS is the income effect of the price fall (and KB the substitution effect). Demand curve SW is a second compensated demand curve. It is the one that is obtained if one starts from point S, the quantity that will actually be bought at the lower price, and price is now increased from A to E. Point W lies to the left of C because it involves no compensation in income to offset the income effect of the price rise.

Now we have three measures of the effect on consumers' surplus of the price change:

1) The compensating variation ABCE which was discussed before;

2) The Marshallian measure, ASCE, to the left of the uncompensated demand curve;

3) The area ASWE to the left of the compensated curve through S. Hicks calls this "the equivalent variation" for it represents that rise in income which would move the consumer to point W and hence put him in a position *equivalent* to (equally desirable with) the lower price positions.

In addition, Hicks brings in two cruder measures which serve as upper and lower bounds to the desired consumers' surplus figure which he points out, correspond to the Laspeyres and Paache index number constructs. These are:

4) The lower bound rectangular area AKCE which gives the price change multiplied (weighted) by the initial purchase quantity EC (The Laspeyres measure).

5) The upper bound, rectangular area ASRE which is the change in price multiplied by the final purchase quantity, AS (the Paache measure).

Two important results emerge from these considerations. First they show us when the Marshallian consumers' surplus measure (the only one we can hope to employ with empirical data) can be used with any degree of equanimity. This will be true when there is good reason to believe that the income effect is likely to be small. For then the three demand curves HH′, CS and WS will all be very close to one another and then measures 1), 2), and 3) will all very nearly be equal. We can then, Hicks tells us, safely use the area under an empirically estimated demand curve to judge the desirability of a public investment project or for whatever other application is at hand. However, if the income effect is substantial, so that the three curves diverge considerably, Hicks suggests that the search for a consumers' surplus measure is rather unprofitable because we have no way, in practice, of getting at the shape of the compensated demand curve, HH′, and an estimate taken from an uncompensated curve is likely to be distorted significantly by the income effect, i.e., in more old fashioned terms, by changes in the marginal utility of money.

The analysis also serves as the link between the utility theoretic concepts and index numbers since, as we have noted, measures 4) and 5) correspond

respectively (in the case of a price fall) to the Laspeyres and Paache index numbers.

By these methods Hicks shows how much he can learn about the behavior of the consumer and its interpretation from an absolute minimum of assumed information. His consumer is restricted by little more than the assumption that he is a maximizer and that his preferences show some minimal degree of consistency. Indeed, Samuelson's revealed preference analysis was later to show how little Hicks' analysis really assumes. We normally expect that weak premises will only lead to weak conclusions and to some extent that is true in this case. After the analysis is all through we know relatively little about the consumer. We can predict that in most circumstances a rise in its price is likely to reduce his demand for a commodity. However we hardly need so sophisticated an analysis for the purpose.

But that misses the essential point. What Hicks has done is to design a set of powerful tools, rather than to use them to derive far reaching generalizations. In enriching the formal procedures of comparative statics in the course of his work on consumer theory Hicks provided the methods which were later to prove essential for the work of Samuelson, Metzler, Patinkin and many others. Much of their comparative statics work is acknowledged by them simply to be extention and redirection of the Hicksian analysis. In application of consumer theory one is also constantly forced to rely on Hicks' work. For example, it is extremely difficult without the Hicksian concepts to provide a rigorous proof of the theorem on optimal pricing policy for a regulated firm in the presence of a budget constraint.[1] This theorem, which has so often been invoked in discussions of nationalized industries in France and of regulated public utilities in the United States, is an excellent example of sophisticated analysis applied to concrete economic problems. Its reliance on the Hicksian analysis and the Hicksian results shows very clearly how fruitful his contribution to value theory has been. Staffan B. Linder's remarkable theorem showing that, because of time limitations, rising rates of pay lead to reduced consumption of services such as the arts and pure idleness which are not complementary with goods consumption is only the *latest application of the Hicksian* methods.

A few words should be said before leaving this subject about the mathematical methods of comparative statics. The basic approach had already been utilized by Cournot in his discussion of the effects of taxation on the price and output of the monopoly firm. However, he dealt only with the case of one decision variable. Moreover, since Cournot steadfastly avoided the use of determinants, it is not easy to see how he could have gone much further. It was essentially Hicks and Allen who developed the method fully, showing how

---

[1] See Marcel Boiteux, "Sur la gestion des Monopoles Publics astreints à l'équilibre budgétaire," *Econometrica* 24, Jan. 1956, pp. 22–40.

total differentiation of the first order conditions produces a set of equations linear in the *differentials* of the variables, once we recognize that the requirement that the decision maker be in equilibrium both before and after the change requires each of the total differentials to be equal to zero. The solution of the system of simultaneous equations can then be compared directly to the second-order conditions to derive qualitative conclusions about the response of the decision variables of the system to changes in the values of the system's parameters. This, in effect, takes us half way to the correspondence principle. For it tells us that the second-order maximum conditions place restrictions on the comparative static behavior of the variables. Samuelson then proceded from this step to show that dynamic stability of the system was also related to the second-order conditions so that a system which was well behaved dynamically was also likely to have appropriate comparative statics properties. However, it must be emphasized that it is the comparative statics side of the analysis which has so far been most fruitful in application, and virtually all subsequent pieces of comparative statics analysis follow the steps laid out by Hicks and Allen.

3. Macro-General Equilibrium Analysis

In a recent summary of Hicks' contribution Professor Samuelson states "'Value and Capital' established for the first time the general properties of the economic market system ... Sir John ... showed what could and what could not be predicted for a general equilibrium system" (*New York Times*, October 26, 1972, p. 71).

An air of aridity hangs over much of the general equilibrium theory that preceded Hicks' work. It was no mean accomplishment to point out that, ultimately, prices can be explained only in terms of a system of simultaneous equations or inequalities. But having specified the nature of these relationships, what could be done with them tended to remain somewhat obscure.

Sir John has given us a variety of powerful methods capable of deriving the implications of such a system. We have just considered one such technique —the comparative statics method as brought to maturity in Hicks' writing. Another such contribution of at least comparable importance is his careful use of aggregation. In *Value and Capital* we are given "... the very important principle—that if the prices of a group of goods change in the same proportion, that group of goods behaves just as if it were a single commodity" (mathematical appendix, section 10). This enables the theorist to have it both ways at once. It permits the construction of theoretical models with almost as few variables as characterize a partial analysis, models in which we can easily follow the economic implications of each of its components. On the other hand it does not leave unaccounted for the effects of developments in one part of the economy upon the remaining sectors, effects which may in turn rebound

upon the initiating sector and undermine conclusions about it drawn from partial analysis. In Hicksian aggregative analysis the rest of the economy is always in the picture, but it is put into manageable form.

In "Mr. Keynes and the 'Classics'" this process of aggregation had been carried out a somewhat different way. There the economy was subdivided into a goods and a money market through the mechanism of the IS and LM curves. Here again the economy is all there, but with the aid of Hicks' mechanism we can understand what happens to the system under a variety of important changes in circumstances—the model tells us about the things that counts for the workings of the economy and not just about the mathematical relationships.

This approach, we may call it macro general equilibrium analysis, has predecessors in Quesney and Marx, but in modern analysis it is a Hicksian contribution. There is reason to believe its potential has only begun to be plumbed. In applied welfare economics, for example, it promises to pave the way for significant results as it has already done for the theory of employment and inflation.

4. Monetary Theory

Though *The Theory of Wages* depicts what is essentially a world without money, the influence of Keynes *Treatise on Money* soon made its impact on Hicks' work. Soon after the appearance of the *The Theory of Wages* Hicks wrote his important paper "A Suggestion for Simplifying the Theory of Money". In that essay Hicks laid out the line of approach that he follows in most of his subsequent monetary analyses. One basic idea was that money is not an asset completely distinct in character from all other assets. Rather it constitutes one form of a continuum of possible types of asset differing in return and risk, or in degree of liquidity. Money is, at least in some circumstances, an extreme case in this continuum—it represents (in a world of stable prices) the one asset which is riskless, perfectly liquid and which bears a zero rate of return. But there are various interest bearing assets that have liquidity in varying degrees. This means that the appropriate way to analyze the demand for money is not to consider it simply as a choice between holding money and "other things" but to think in terms of the design of a portfolio which consists of a combination of assets that best meets the requirements of the holder.

This way of looking at the matter gives rise to two basic issues; first it requires an explanation of the reason anyone will want to hold money at all. Second it requires an analysis of the appropriate combination of assets in a portfolio. Both of these subjects have in recent decades been investigated in detail and the sophisticated body of portfolio analysis which this work has produced follows directly in the lines of the Hicksian analysis. The holding of cash is shown to depend upon three phenomena: the presence of risk, the cost

of transactions and the imperfect coordination of receipts and payments. The basic idea is clear: money is held despite its zero rate of return because it is the only asset that can be used without delay or loss in value (in a non-inflationary world) to make unanticipated purchases or purchases whose timing cannot be foreseen with any degree of confidence. But even if risk were completely absent it would pay to hold some cash. If the transactions cost is high enough and cash is now being held for a payment to be made in a month, then it will not pay to purchase an interest-bearing security now and then sell it when the payment is due. This observation was later used by other economists as the basis for the inventory theoretic model of the transactions demand for cash in which the relevant relationships are deduced from an optimization assumption.

The other side of the Hicksian monetary analysis, the portfolio theoretic approach, has also since undergone a considerable elaboration at the hands of Tobin, Markowitz and Hicks himself, among others. The bulk of the theory bases itself on a characterization of any asset in terms of two of its parameters: its expected earnings and the variance of its earnings, taken as a measure of its risk. The convenience of this formulation is that given these figures for any set of assets plus data on the covariances of every pair of assets in the set one can utilize standard formulas to determine the expected earnings and variance of the earnings of a portfolio of these assets combined in any desired proportions. Let x_i be the proportion of the cost of a portfolio invested in asset i. It is then a straightforward matter to calculate the values of the x_i that minimize the variance of the portfolio's assets for any given level of the portfolio's expected earnings (taken as the constraint). Repeating this calculation for all values of expected earnings in the relevant range, we generate what may be described as Pareto efficient locus of portfolios. If we arbitrarily take money to be the nth in the set of assets considered for inclusion in the portfolio, then x_n in an efficient portfolio indicates the proposition of total portfolio value that is efficient to hold in cash, given the expected return that is desired. This is how the portfolio analysis explains the individual's demand for money. Note that it makes no explicit use of such standard concepts as transactions and precautionary demand, and that it does not treat money as a unique item. The logic of the analysis permits us to go much further than this. For example, it enables us to specify the shape of the efficient frontier. It permits us to deal with the effects on asset choice of risk aversion and risk preference. In sum, it is a rather rich body of theoretical analysis.

Hicks did not only lay the foundations for this theory. He has continued to make significant contributions to it. For example, in a recent paper (in *Critical Essays on Monetary Theory*) he has joined the group of economists who have pointed out the limitations of the analysis based on variance and expected return alone, showing that it certainly cannot characterize an assymetric probability distribution, and that it can lead to the inclusion of

some anomalous portfolios in the calculated efficiency locus. Hicks then goes on to carry out a pathbreaking calculation showing just how the inclusion of the third moment of the probability distribution as a measure of asymmetry affects the nature of the efficient locus. This and a reexamination of the manner in which the Keynesian analysis must be reinterpreted in the light of portfolio theory represent the main contributions of his latest incursion into monetary theory.

5. The Term Structure of Interest Rates

In discussing Hicks' work on the subject of money it is appropriate also to refer to his contributions to the theory of interest. These have occurred at many points in his writing; implicitly in all of his portfolio analyses, in his graphic translation of the Keynesian and classical systems ("Mr. Keynes and the 'Classics': a Suggested Interpretation,") and in his recent volume on growth theory. There is, however, at least one contribution that must be singled out because of the important line of further work for which it provided the foundation. This is the Hicks-Lutz model of the structure of interest rates. Basing itself on expectations regarding the short term rate of interest it asserts simply that the n period (long) rate must in equilibrium equal the geometric mean of the expected short rates for the next n periods. The reason is, of course, that if, say, the long rate were less than that average, potential holders of long securities would be better off lending short in a sequence of loans renewed at the end of every period for the desired n periods. This would drive down the price of long term bonds and drive up the price of short term bonds to a point where effective yields had adjusted themselves to the Hicksian relationship.

This analysis has given rise to a substantial literature on the term structure of interest rates. For example, J. M. Culbertson ("The Term Structure of Interest Rates," *Quarterly Journal of Economics* 71, Nov. 1957) has argued that the term structure cannot possibly exhibit the tight interrelationships postulated by Hicks because the markets for lonas of various duration are segmented, some institutions being required by law, tradition or the nature of their activities to deal exclusively in short loans, and some in other markets. As a result he maintains, a rise in the supply of short lending, for example, will drive down the short term rate of interest relative to other rates, contrary to what the Hicks–Lutz analysis asserts. On the other hand, Meiselman (*The Term Structure of Interest Rates*, Englewood Cliffs, N. J., 1966) has constructed an "error-learning model" in which expectations are not taken to be formed instantaneously but are assumed to change in light of how accurate previous expectations have proved to be. Meiselman has carried out careful econometric studies which seem to show that a Hicksian model modified to include a learning procedure can account rather closely for the facts. This line of analysis

reached its most sophisticated expression in the work of B. G. Malkiel (*The Term Structure of Interest Rates*, Princeton, 1966). The Malkiel analysis takes account of the fact that transactions costs will prevent the perfect adjustment required in the Hicksian analysis, and that the expectations of different investors will vary from one another. He shows empirically that there is some degree of overlap provided by borrowers and lenders who can and do switch from one maturity period to another *nearby* maturity when it is profitable. Malkiel shows that this analysis is consistent with some responsiveness of term structure to changes of relative supplies in the various maturities, but that, ultimately, expectations do provide the framework about which rates of interest for loans of various durations tend to array themselves.

In his most recent writings in monetary theory Hicks also returns briefly to the term structure and emerges with considerations very similar to those emphasized by Malkiel. But the essential point of the discussion is that here is yet another example of a rich and extensive literature (most of it not even mentioned here) that acknowledges its origins in the work of Hicks. The work is of interest both as a matter of pure theory and for the most direct sort of application. It stands as a most effective tribute to the fruitfulness of the Hicksian interest analysis.

6. Price Flexibility and Employment

One of the many untidy elements in the *General Theory* is the role assigned to changes in money wages. At one point Keynes makes the startling assertion that real wages and, hence, employment will be totally unaffected by a general change in money wages. But he never really explains the mechanism by which he believes all prices will be led to change precisely in proportion with money wages. Later on, however, Keynes offers quite a different conclusion, pointing out that with the quantity of money fixed a fall in money wages will lead to some fall in prices which in turn will increase the real money supply, reduce interest rates (unless the economy is caught in the liquidity trap) and thereby it will tend to increase employment. Keynes does argue that this is normally not a very efficient way to go about increasing the supply of jobs but, analytically, the damage is done; the system becomes one in which falling wages can restore equilibrium between supply and demand in the money market.

Nevertheless, the mechanism of the equilibrating process, and the relationship between the two contradictory assertions did not become clear before Hicks and Pigou, followed by Lange, Patinkin and others, provided an explicit analysis of the matter. The Hicksian analysis points out that each of the Keynesian assertions is a logically possibility though, of course, they cannot simultaneously be true. What happens to the supply of jobs when all money wages fall depends, in a general equilibrium system, on what happens to two elements: the nominal money supply and expectations. If the nominal money

supply falls in proportion with wages then fall a in prices will indeed lead to an increase in its purchasing power and hence it will generate both a real balance effect on the demand for commodities and real investment, and a Keynesian effect depressing the interest rates, and in both ways employment will be stimulated. However, this need not happen in a Wicksellian pure credit economy. For there every monetary asset involves both a lender and a borrower. The fall in prices will increase the wealth of the lender but it will decrease that of the borrower commensurately. In that case the fall in wages should lead to a disequilibrium in which other prices must also fall so long as relative prices are not restored. We are indeed in the world in which no fall in money wages can produce a decrease in real wages.

However, it is also possible for expectations to prevent real wages from remaining unchanged. For the fall in money wages may either lead people to postpone purchase and investment plans or they may cause them to be advanced. In the former case, in the pure credit economy, the fall in money wages will actually lead to a decrease in employment today.

The issue then is how expectations have to be affected to produce these results. The obvious (but incorrect) answer is that an expectation of falling prices will lead to postponement of demands and that rising prices will bring demands forward. However, in a comparative statics context that is not the relevant point. Hicks gives us a measure of the relevant relationship: the elasticity of expectations, defined as the percentage *change* in anticipated future prices (over what they had been expected to be previously) divided by the percentage change in wages that causes the modification in anticipations. That is, what is relevant is not the difference between prices expected in 1973 and prices expected in 1974, but the difference between the prices expected in 1974 after the wage change and the prices that had been expected *for that same year*, 1974 before the wage change. For suppose prices had been expected to fall initially. A 10 percent wage fall today will increase demand if and only if it causes the anticipated future price to fall by less than 10 percent from the level previously expected. Otherwise the ratio between current and expected future price will either remain unchanged or rise. Thus a wage fall will increase employment if expectations are inelastic: if they are elastic, the wage decrease will actually tend to reduce the current demand for employment.

When Lange produced an exhaustive statement of these relationships (*Price Flexibility and Employment*, The Principia Press, Bloomington, Indiana 1944) it was criticized rather severely by Milton Friedman (*American Economic Review* 36, Sept. 1946, pp. 613–31) as "implicit theorizing" which consists of pure classification described in such a way as to imply which of the alternatives is most likely in fact. But this criticism surely is largely beside the point. Of course the mere invention of categories by itself does not settle the issues which ultimately can only be resolved by empirical investigation. We do not know from the Hicksian analysis whether a wage decrease will in fact decrease

real wages and stimulate employment. But we do understand how it can happen—when there is only credit money and expectations are unit elastic—that a fall in money wages will leave real wages unchanged, and how, in other circumstances, which we can now specify, it can stimulate employment. The mystery has thus been removed from the original Keynesian assertions. Moreover, we now know what to look for when conducting an empirical investigation of the matter.

7. Dynamic Theory

Hicks' work has dealt with various types of dynamic analysis. Of course, the connotation of the term itself is not clear on the face of it. But virtually every form of dynamics has appeared somewhere in the Hicksian writings. In *Value and Capital* Hicks may be considered to stretch the meaning of the term more than a little because the bulk of the material to which he assigns this label deals with intertemporal equilibrium, but does so in precisely the same manner as in the purely static analysis. Hicks implies then that the mere recognition of the existence of a future takes us into the realm of dynamics; but that surely does not leave much over for static analysis since it would imply that static theory can only explain the behavior of individuals who believe that every moment is their last moment on earth, hardly a very interesting class of persons even for theoretical analysis. Even in his latest writing in the area some issue can be taken with his choice of definition. He now suggests that statics should deal primarily with stationary processes while dynamics should refer to economic situations which are undergoing change. One cannot quarrel about the validity of definitions; a word can be defined in any way its user wishes. But one can suggest that some definitions correspond to entes that are analytically interesting and some do not. It is on precisely these grounds that one feels uncomfortable with the Hicksian views on the term "dynamics." If statics must deal with a world without change it surely must deal with materials whose relevance to reality is rather tenuous. It would seem that the point of view adopted by Frisch, Samuelson and others is more useful. They distinguish between statics and dynamics not in terms of the stationariness or nonstationariness of the economy studied, but upon the degree of emphasis of the analysis upon the *process* of change. A dynamic analysis is one that focuses upon some aspect of the process of change: its mechanism, its equilibrium, or its optimality, on whatever standard one chooses to apply. Statics is a method of analysis that deals with a world in which change occurs but the analysis concerns itself with *other* characteristics of the system.

If we adopt this criterion, the Hicksian trade cycle model is dynamic because it deals with the process of change; the analysis in *Capital and Growth* is also dynamic because it is concerned with growth equilibrium and optimality of the growth trajectory. But the intertemporal analysis in *Value and Capital*

is, on this criterion, purely static. Indeed, therein lies the beauty of the device Hicks invents for the purpose, for he shows how, by the use of discounted present values of future prices, the intertemporal equilibrium analysis can be made absolutely equivalent to the analysis of equilibrium at a moment of time. One merely needs to treat a commodity at time t as though it were a different item from the same good at some other time, s. Thus, a model involving h periods and n different goods in effect becomes a static model of equilibrium with the hn quantities x_{it} and the hn discounted prices p_{it}. This is a very convenient way of dealing with the matter whether or not any change occurs during the n periods. But the method is essentially static—that is just the point of the device.

In Hicks' little book on the trade cycle a very interesting methodological advance is offered on the mathematics of cycle models. Up to the time of its appearance, virtually all formal mathematical models of economic fluctuations utilized *linear* difference or differential equations either separately or in combination. Apparently, the only exception was Goodwin's work ("The Nonlinear Accelerator and the Persistence of Business Cycles," *Econometrica* 19, Jan. 1951) which employed a nonlinear differential equation construct. Hicks set out to utilize a nonlinear difference equation model for the same purpose. The objective in leaving the linearity assumption is to avoid the two unpleasant properties of the linearity assumption. These are first, the fact that the nature of the time path in a linear model is totally unaffected by initial conditions, i.e., in a linear world there can in effect be no decisions that change the course of history; and second, with a trivial exception, the cycles generated by a linear model must necessarily either explode into cateclysmic proportions or they must fade away and tend to disappear. Since in the absence of effective countercyclical policy, there is no evidence that either of these types of time path characterizes reality one is driven to seek models that behave differently from the time path generated by a linear construct.

Earlier, Frisch ("Propagation Problems and Impulse Problems in Dynamic Economics," *Economic Essays in Honor of Gustav Cassel*, London, 1938) had dealt with the same issue by assuming that the cycle was in fact damped but that the system was reactivated from time to time by the effects of random shocks. Hicks shows that one can also construct plausible models that generate cycles of more or less constant amplitude without the intervention of stochastic components.

Essentially, there are two ways in which a difference equation model can be modified to produce stable cycles. These are illustrated in Figs. 2 and 3. Fig. 2 shows an $t+1=f(y_t)$ curve with basically two negatively curved segments—a steep segment crossing the 45° line and a much flatter segment off the 45° line. This means that if our time path begins somewhat far from the 45° line it will lead to a convergent cobweb (the dotted curve in Fig. 2) while if we start near the 45° line the cobweb will be explosive (the broken curve).

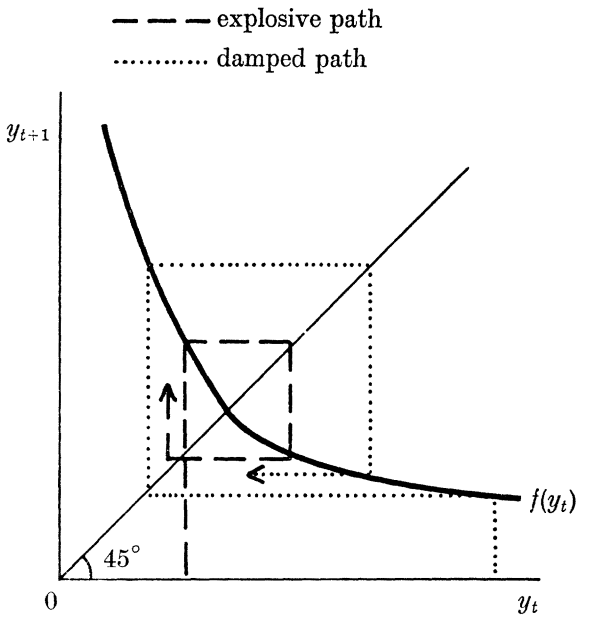

Fig. 2

Where the damped and explosive cobwebs meet there will be a stable *limit cycle* of constant amplitude towards which all paths lead.[1]

A second way in which a nonlinear model can produce a cycle of constant amplitude is via an appropriately closed $y_{t+1}=f(y_t)$ curve. In Fig. 3 we see how such a curve can lead to an indefinitely repeated time path ABCDEA-ABCDEAB ...[2] This second approach to stability of the generated cycles is the one taken by Hicks in his cycle model. His closed $f(y_t)$ curve (Fig. 4) consists, essentially of four linear segments: One, corresponds to the upswing of the business cycle during which there is a demand for more capital and so the demand for investment is governed by the acceleration principle (segment AB); a second segment corresponding to Harrod's full employment natural rate of growth, which slows down the expansion of real income and hence cuts the accelerator investment demand sharply (segment BC); a third segment corresponding to the downswing, when there is disinvestment proceeding at the rate of deterioration of capital (segment CD); and, finally, a fourth segment (DA) corresponding to the end of the downswing, when autonomous invest-

[1] If the curvature of the $f(y_t)$ locus changes several times then there may be a number of different limit cycles. To which one the system will converge depends on the initial position. In that case, the choice of initial state can change the history of the entire system, which, as we have seen, can never happen in the linear case.

[2] Such a path may not repeat itself perfectly each time round the closed curve. After the system goes to point E it may, with a slightly different shape of $f(y_t)$ curve, move back to some point in the vicinity of A rather than to A itself. In that case the next time round it will follow a path a bit different from the initial one, but the amplitude of the overall cycles will be similar. Certainly, they will neither die out nor explode.

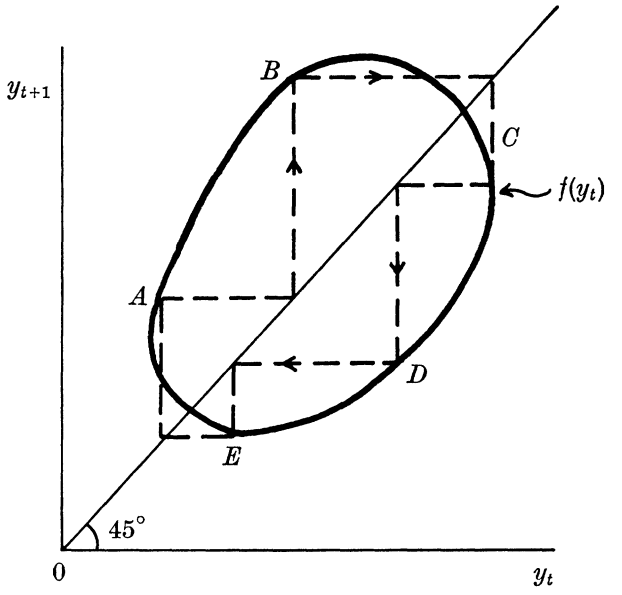

Fig. 3

ment demand takes over and prevents a further decrease in the stock of capital. This leads to precisely the sort of time path we found generated by the closed curve in Figure 3.

The trade cycle model is a clever piece of work. It served to take dynamic analysis beyond the linear process to which it had largely been confined

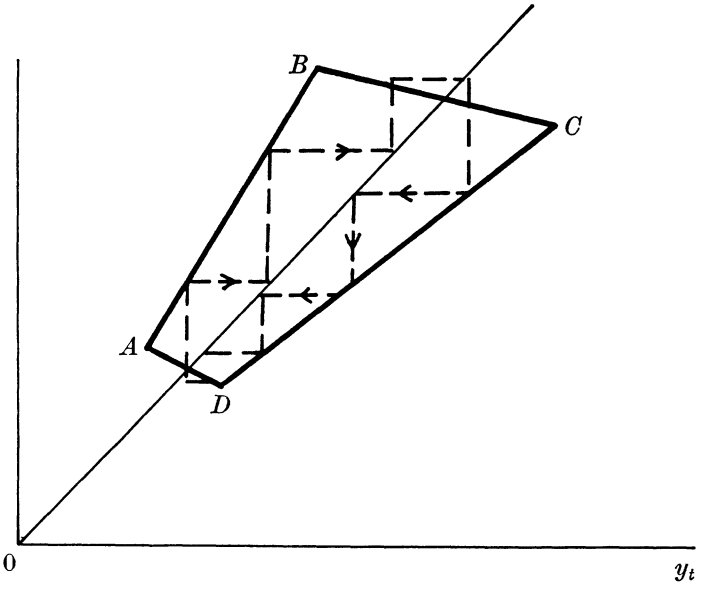

Fig. 4

hitherto. In that sense it is, indeed, a contribution. However, at least so far, unlike most of the other novel ideas produced by Hicks it does not seem to have led to any significant amount of follow-up by other economists.

In his more recent book, *Capital and Growth* Hicks turns to another type of dynamic analysis, the study of optimal growth paths, turnpike theory and other related matters. This time the objective is quite different from the last. His subject, as Hicks himself describes it, is comparative dynamics. That is, he follows the host of others writing in this area in the past two decades in studying an economy following a stable growth equilibrium path. The problem is to characterize the effect of such parameters as the rate of saving upon the speed of growth of the associated equilibrium path. The problem is a very difficult one and as in most of the analyses in the area Hicks is forced to utilize a number of highly artificial assumptions. He must deal almost exclusively with the case in which the production function is perfectly linear and homogeneous. In part of the discussion one must also, following Von Neumann, assume that all resources are absolutely unlimited in supply, and that there is no autonomous final demand, human consumption being simply the input needed to produce more labor. In the turnpike theory he must follow the standard premise that it is only the terminal stock of capital that matters. That is, if one selects some arbitrary horizon date an optimal path is defined to be one which maximizes outputs in some prespecified proportions on this final day of reckoning. This assumption is, of course, the explanation of the working of the noted turnpike theorem, originally offered as a conjecture by Dorfman, Samuelson and Solow, the theorem that under the simplifying premises specified, the optimal growth path will tend toward the Von Neumann path of *proportionate* expansion, even if initial output proportions are not the same as those desired on the horizon date, so that proportionate expansion of all outputs will not get the economy directly where it wishes to go. With a linear homogeneous production function, whatever input (output) proportions make for fastest growth at any one scale of activity will also make for fastest growth at any other scale. Hence the "turnpike—the route with the greatest possible speed—will indeed involve proportionate growth. Moreover, the turnpike analogy really does explain the remainder of the proposition—for we do know that turnpikes are necessarily the best routes only if we are concerned with nothing but speed and if we care nothing about what we see "along the way." If the economy's objective is to maximize some sort of trajectory of consumption taking into account what becomes available to the consumer "along the way" i.e., at intermediate points before the horizon, the turnpike theorem no longer holds.

The assumptions are emphasized here because one of Hicks' main contributions in the book is to stress the restrictiveness of the premises of the theory, and to show the routes that would have to be followed to widen the applicability of the analysis. He deals with important issues such as the transition

between equilibrium paths, the role of prices and interest rates in the process, the place of consumer demands in periods before the horizon date, etc. Hicks is acutely aware of the difficulty of progress in these areas and his claims for his own work are severely restricted. What he offers us primarily is a sense of proportion about the work on growth equilibria, which is certainly appropriate in this area. Moreover he gives the reader a relatively accessible discussion of most of the materials and a program for further research. The illumination he provides is of a sort that could only be offered by an economist who is a master of an older tradition in which a price or an interest rate really represents a money payment, not just the value of a variable in the dual program. For, ultimately, if work in the area is to represent more than formal exercises it is just such a translation that will have to be carried out. It is too early to say yet, but it is hard to believe that there will not be a great deal of fruitful work by others following out Hicks' promising leads in the field of growth equilibrium analysis.

8. Distribution: The Adding-Up Problem

There is at least one item in *The Theory of Wages* that deserves more attention than it has received. That is the appendix on the role of linear homogeneity in the adding-up problem of the theory of distribution. The problem, it will be recalled, is a matter of the consistency of marginal productivity theory—if under pure competition every input is paid the value of its marginal product, will the sum of the payments actually add up to the value of the total product, or will it, for example, call for payments exceeding the amount of product to be distributed? There is no need to summarize in detail the tortured history of the discussion, though since Hicks' contribution here may perhaps be considered a piece of doctrinal history, some elements of the story do have to be recalled. The initial solution offered was that, by Euler's Theorem, the sum of the marginal product would indeed equal the total product if the production function were linear and homogeneous. This solution, usually erroneously attributed to Wicksteed, but in fact first proposed by Flux in his review of Wicksteed, at once drew Edgeworth's scathing sarcasm. In the third edition to his *Éléments* Walras came up with an alternative solution, one which we accept today, but which Walras himself apparently did not appreciate fully. We know that Walras withdrew this discussion in the next edition and it was never restored in subsequent printings.

Hicks' accomplishment here was to rediscover the Walrasian solution and to restate it in more readable mathematical terms so that it can be followed without difficulty by the reader today. In a way, it may almost be attributed to Hicks in the sense that he may have been the first to understand its logic and its relevance fully, Walras himself evidently not having done so, and his tortured mathematics hidden away in a defunct edition of his main work having made it very difficult for others.

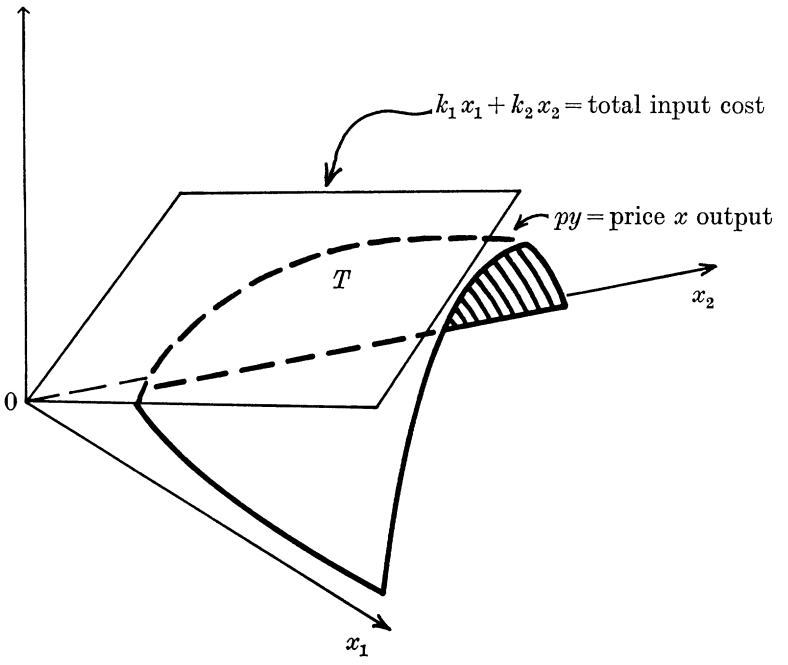

Fig. 5

The basic proposition asserts simply that whether or not the production function is linear an homogeneous, at a *point* of competitive equilibrium it must be *locally* homogeneous of the first degree. That is, at the point of equilibrium all of the partial derivatives of the firm's revenue production function must be the same as those of a hyperplane through the origin and through that equilibrium point. Since by Euler's theorem $\Sigma p_i \partial y/\partial x_i = y$ on the hyperplane (where p is product price, y is product output and $\partial y/\partial x_i$ is the marginal physical product of input i) the same must be true of the firm's production surface to which the hyperplane is tangent. In Figure 5, T is the point of tangency between the firm's total revenue surface, py, and its total input cost plane $k_1 x_1 + k_2 x_2$. Since the latter is a plane through the origin the production surface is locally linear and homogeneous at T. Specifically, if each input is paid the value of its marginal product so that $k_1 = p\partial y/\partial x_1$ and $k_2 = p\partial y/\partial x_2$ then, obviously, at T we have $py = k_1 x_1 + k_2 x_2 = p\partial y/\partial x_1 + p\partial y/\partial x_2$ so that payments to the inputs always add up to the value of the total product. This is not a matter of the nature of the production function, i.e., linear homogeneity is not required to hold any place but the vicinity of T. Rather, it results from the nature of the competitive process which, by forcing profits to zero, produces an equilibrium point in which factor payments just eat up total revenue. Thus, it has taken the Walras–Hicks mathematics to return us to a common sense solution to the adding-up problem.

This does not mean that the issue is entirely trivial. On the contrary, recent work in duality theory in nonlinear programming shows how essential these considerations are. One of the basic properties of the primal and dual programs is the theorem that in an optimal solution the maximum value of the primal objective function is equal to the minimum value of the dual objective function. In economic terms this means that all profits are imputed to the firm's inputs by their shadow prices, where the shadow prices are, roughly speaking, the marginal revenue products of the inputs. Now this requirement causes no problem in the linear programming case since there the relevant functions are indeed linear and homogeneous. But in the nonlinear case the functions may show increasing or decreasing returns to scale throughout, so there may exist no point on the relevant surface that is linear and homogeneous. For that reason mathematicians have had to adopt a form for the objective function of the nonlinear dual that is more complicated than the obvious nonlinear generalization of that of the linear dual. The objective function for the nonlinear dual contains a number of additional terms which turn out on closer examination to be, simply, economic rents (positive or negative) that eat up the difference between the value of the total output, and the values imputed to the inputs by their marginal yields. Thus, the Walras–Hicks analysis has explained what lies behind the adding-up problem and helped us to understand a good deal of more recent work. But the adding-up problem simply has refused to go away completely.

9. Hicks and Economic History

In his most recent book, Sir John has turned to economic history—an area which has obviously always been close to his heart but which appears in his previous works only in a sprinkling of footnotes. However, even here he approaches matters from a theoretical viewpoint. Clearly, we economists cannot venture to judge the book from the viewpoint of the professional historian. Nor can we evaluate it as pure theory since it makes no pretense at a formal structure or a rigorous chain of analysis of the sort to which we are habituated. Yet anyone who has done any amount of reading in the historical literature of the relevant periods cannot help being struck by the insights the little work provides. As an illustration, consider the fascinating chapter on the finances of the sovereign. Hicks begins with a discussion of what he calls "the middle period", apparently in an attempt to generalize, but quite obviously referring to the later middle ages and the early renaissance. He points out that the most striking characteristic of the kings of this period is that they were perpetually short of funds. That helps to explain the origins of parliaments on whose mercy the monarch was often dependant for financing.

Hicks points out that the poverty of the sovereign can be attributed in

large part to the unavailability of economic activities that could easily be taxed. Record keeping was minimal and so trade could be taxed effectively only if it could be intercepted physically; but with trading activities spread out widely over the countryside this too was very difficult. Hicks points out that in this respect the English kings were relatively fortunate in ruling an island since so much of their trade passed through a few geographic points, the "cinque ports." One may suspect that Slussen may have played a somewhat similar role in the history of Sweden. The critical change in this situation, according to Hicks, occurred with the invention of the Joint Stock Company since, by its very nature, it must keep records for its stockholders. That means that the government was for the first time able to share in the growing wealth of the community.

The earlier monarchs did not even find it easy to borrow money, paradoxically, the difficulty of obtaining it being ascribable to the fairly great power of the King. For as Schelling and Strotz have indicated, the ability to undertake an *enforcable* commitment is itself a very valuable asset. But the King had no way in which to commit himself. If he were to default there would be no higher court in which the debtor could sue for nonpayment of debt, and so the fact that there was no higher authority made the King a very bad lending risk. Hicks suggests that it is only with the founding of the central banks and the establishment of republics that borrowing became easier for governments. The central banks imparted credibility to the borrowing commitment because it was clear that defaulting on a debt might undermine the institution itself and this was too high a price to make it worth-while to the government. The Republican form of government was helpful because it provided continuity and eliminated the danger that with his accession a new monarch might repudiate the debts of his predecessor.

From all this Hicks attempts to explain why modern inflations have been so much sharper than those of the middle period where it is not easy to find examples of prices increasing as rapidly as 2 per cent per year over any protracted period. Hicks points out that inflation would in fact hurt the finances of the middle period sovereign in at least two ways. First, it would reduce the attractiveness of his coinage and hence might cut sharply into the flow of precious metals sent to him for coinage by private persons—a minting operation that seems to have been extremely profitable. Second, since it is very difficult to revalue taxable *property* as prices rise (both as a political matter, and because of the sheer difficulty of judging "fair market value"), and because, at best, such revaluation occurs only after a considerable lag, rising prices could cut sharply into the real value of the resources flowing into the treasury.

With the institutions of public finance today, on the contrary, inflation has become an instrument which increases the real income of the treasury. Progressive income tax rates are defined in terms of money income, not real

income, and so, via a well-known process, the public coffers benefit from the rising effective tax rates caused by increasing prices.

The difference in the profitability of inflation to older and more recent governments, suggests Hicks, may go a long way toward explaining the difference in the rate of increase in prices characteristic of the two periods. It is, of course, easy to think of other explanations; surely, employment policy in modern states plays a significant role in rising prices. However, the relationships Hicks calls to our attention may also account for a good deal, and certainly, even as a conjecture, it is highly illuminating.

This brief summary is intended to offer some notion of the Hicksian analysis of economic history. Hicks is careful to point out that any generalizations about so enormous a set of events must run into many exceptions. Some degree of superficiality is also an unavoidable characteristic of generalization on this scale. However, as a set of insights and a new way of looking at a body of facts I think most readers will agree that the Hicksian view of history is a most substantial piece of work.

10. Other Areas and Concluding Comment

Sir John has contributed in his writings to a host of theoretical areas. This paper has, however, already grown too long and in any event, one cannot hope within the compass of a reasonable space to cover all his rich and extensive work. We have said nothing about his writings on international trade, on the pure theory of capital, on the theory and practice of public finance (much of it written jointly with Lady Hicks), on the theory of monopoly and a number of other areas. In each of these, Sir John has had interesting things to bring to our attention. But aside from noting these gaps in the present paper no more will be said here about this work. A glance at the bibliography will indicate how many pieces have been written in these areas that have not been discussed and how many of the titles will strike one as very familiar.

Having covered so much territory it is appropriate to attempt an overall evaluation of Hicks' work. But first an apology is called for, with respect to the approach that has been adopted. More than seems to be customary in such a piece, this has gone into detail on the structure of analyses, and in many cases, on the nature of subsequent writings in the areas covered. The analyses have been described so extensively because so much of the Hicksian contribution resides not in the theorems that are derived but in the methods by which the results are reached, the logic and power of these methods and their transferability to other investigations. The work that has followed Hicks' is discussed in such detail because much of his contribution lies in the paths he has opened up to others. There is no better way of documenting this than by showing how others have in fact made use of his ideas.

To summarize, the main strength of Sir John's theoretical work resides in

his mastery of general equilibrium methods. He is one of the few theorists working in the area who have actually been able to make general equilibrium models produce interesting results about the workings of the economy. He is one of the few who have been able to take the approach beyond the counting of variables and equations or the more sophisticated approaches toward the testing of the existence and uniqueness of a solution. Most economists who have been able to produce illuminating results about economics have done so with the aid of the simplifications of macroeconomics or partial analysis. That current general equilibrium analysis is no longer a purely formal exercise is in no small part a tribute to the success of the Hicksian analysis.

Even though on occasion Hicks' work bears the stamp of pure abstraction, he is usually acutely aware of the relevance of a particular line of analysis, and as already indicated, this may well be one of the most significant contributions of his recent work in growth theory.

But, most of all, one should emphasize the work of others that has been inspired by Sir John's writings. On rereading his works and even in reading those of his papers which one has not seen before, one is immediately impressed by a feeling of *déja vu*. Even if the reader has not come across the materials in their original form in Sir John's writings themselves he is likely to have absorbed them from the many pieces of literature that have adopted them and built on them further. These writings constitute the most eloquent of possible tributes to his accomplishments.

Partial Bibliography, Sir John Hicks

I. *Books and monographs*

The Theory of Wages, London 1935, 247 pp., 2nd ed. London 1963, 388 pp.

Théorie Mathématique de la valeur en régime de libre concurrence, Paris 1937, 55 pp.

Value and Capital, Oxford 1939, 331 pp., 2nd ed. Oxford 1946, 340 pp.

(with U. K. Hicks and L. Rostas), *The Taxation of War Wealth*, Oxford 1941, 304 pp.

The Social Framework. An Introduction to Economics, Oxford 1942, 212 pp., 2nd ed. 1952, 283 pp., 4th ed. 1971, 317 pp.

(with U. K. Hicks), *Standards of Local Expenditure*, Cambridge 1943, 61[1]p.

(with U. K. Hicks and C. E. V. Leser), *The Problem of Valuation for Rating*, Cambridge 1944, 90 [6]pp.

(with U. K. Hicks), *The Incidence of Local Rates in Great Britain*, Cambridge, 1945 64 pp.

The Problem of Budgetary Reform, Oxford 1948, 95 pp.

(with U. K. Hicks), *Report on Finance and Taxation in Jamaica*, Kingston, Jamaica, 171 pp.

A Contribution to the Theory of the Trade Cycle, Oxford 1950, 201 pp.

A Revision of Demand Theory, Oxford 1956, 196 pp; 1959, 198pp.

Essays in World Economics, Oxford 1959, 274 pp.

International Trade: The Long View, Central Bank of Egypt Lectures, Cairo 1963, 29 pp.

Capital and Growth, Oxford 1965, 339 pp.

Critical Essays in Monetary Theory, Oxford 1967, 219 pp.

A Theory of Economic History, Oxford 1969, 181 pp.

II. *Articles*

Wage fixing in the building industry, *Economica* 8: 159–67, June 1928.

The early history of industrial conciliation in England, *Economica* 10: 25–39. March 1930.

A reply, *Economic Journal* 41: 145–46, March 1931.

The theory of uncertainty and profit, *Economica 11, 170–89*, May 1931.

Marginal productivity and the principle of variation, *Economica* 12: 79–88, Feb. 1932.

A rejoinder [Marginal productivity and the Lausanne School], *Economica* 12: 297–300, Aug. 1932.

A note on Mr Kahn's paper, [On elasticity of substitution], *Rev. of Econ. Stud.* 1, 78–80, Oct. 1933.

(with Allen, R. D. G.), A reconsideration of the theory of value, Parts I–II, *Economica* N. S. 1: 52–76, 196–219, Feb., May 1934.

Léon Walras, *Econometrica* 2: 338–48, Oct. 1934.

A note on the elasticity of supply, *Rev. of Econ. Studies* 2: 31–37, Oct. 1934.

Annual survey of economic theory: The theory of monopoly *Econometrica* 3: 1–20, Jan. 1935.

A suggestion for simplifying the theory of money, *Economica* N. S. 2: 1–19, Feb. 1935.

Wages and interest: The dynamic problem, *Econ. Journal* 45: 456–68, Sept. 1935.

Mr Keynes theory of employment, Review, *Econ. Journal*, 46: 238–53, June 1936.

Distribution and economic progress: A revised version, *Rev. of Econ. Stud.* 4: 1–12, Oct. 1936.

Mr Keynes and the 'Classics'; A suggested interpretation, *Econometrica* 5: 147–59, April 1937.

(with Hicks, U. K.), Public finance in the national income, *Rev. of Econ. Stud.* 6: 147–55, Feb. 1939.

The foundations of welfare economics, *Econ. Journal* 49: 696–712, Dec. 1939.

Mr Hawtrey on bank rate and the long-term rate of interest, Review, *Manch. School* 10: 21–37, No. 1, 1939.

A reply, *Manch. School* 10: 152–55, No. 2, 1939.

The valuation of the social income, *Economica* N. S. 7: 105–24, May 1940.

A comment [on O. Lange, Complementarity and interrelations of shifts in demand], *Rev. of Econ. Stud.* 8, 64–65, Oct. 1940.

The rehabilitation of consumers' surplus, *Rev. of Econ. Stud.* 8: 108–16, Feb. 1941.

Saving and the rate of interest in wartime, *Manch. School.* 12: 21–27, April 1941.

The monetary theory of D. H. Robertson, Review, *Economica* N. S. 9: 53–57, Feb. 1942.

Maintaining capital intact: a further suggestion, *Economica* N. S. 9: 174–79, May 1942.

Consumers' surplus and index numbers, *Rev. of Econ. Stud.* 9: 126–39, No. 2, 1942.

(with Hicks, U. K.), "The Beveridge Plan and local government finance, *Rev. of Econ. Stud.* 11: 1–19, No. 1, 1943.

The four consumers' surpluses, *Rev. of Econ. Stud.* 11: 31–41, No. 1, 1943.

History of economic doctrine, Review, *Econ. Hist. Rev. 13:* 111–15, No. 1–2, 1943.

Comment [The inter-relations of shifts in demand], *Rev. of Econ. Stud. 12*: 72–75, No. 1, 1944.

Recent contributions to general equilibrium economics, Review, *Economica* N. S. 12: 235–42, Nov. 1945.

The generalized theory of consumers' surplus, *Rev. of Econ. Stud*, 13: 68–74, No. 2, 1945.

World recovery after war: A theoretical analysis, *Econ. Journal* 57: 151–64, June 1947.

The empty economy, *Lloyds Bank Rev.* 5: 1–13, July 1947.

The valuation of the social income: A comment on Professor Kuznets' Reflections, *Economica* N. S. 15: 163–72, Aug. 1948.

Mr Harrod's dynamic theory, Review, *Economica* N. S. 16: 106–21, May 1949.

Devaluation and world trade, *Three Banks Review* 4: 3–23, Dec. 1949.

A comment on Mr Ichimura's definition [of related goods], *Rev. of Econ. Stud.* 18: 184–87, No. 3, 1951.

Comments [Monetary policy and the crisis], *Oxford Institute of Statist.* 14: 157–59, No. 4 & 5, 1952.

An-inaugural lecture, *Oxford Econ. Papers* N. S. 5: 117–35, June 1953.

The process of imperfect competition, *Oxford Econ. Papers* N. S. 6: 41–54, Feb. 1954.

Robbins on Robertson on utility, *Economica* N. S. 21: 154–57, May 1954.

A Reply [to a comment on value and capital], *Rev. of Econ. Stud.* 21: 218–21, No. 3, 1954.

Economic foundations of wage policy, *Econ. Journal* 65: 389–404, Sept. 1955.

The instability of wages, *Three Banks Review* 31: 3–19, Sept. 1956.

A rehabilitation of 'classical' economics? Review, *Econ Journ.* 67: 278–89, June 1957.

The measurement of real income, *Oxford Econ. Papers* N. S. 10: 125–62, June 1958.

A 'Value and capital' growth model, *Rev. of Econ. Stud.* 26: 159–73, June 1959.

Thoughts on the theory of capital: the Corfu Conference, *Oxford Econ. Papers* N. S. 12: 123–32, June 1960.

Linear theory, *Econ. Journal* 70: 671–709, Dec. 1960.

Prices and the turnpike. The story of a mare's nest, *Rev. of Econ. Stud.* 28: 77–88, Feb. 1961.

Pareto revealed, Review, *Economica* N. S. 28: 318–22, Aug. 1961.

Marshall's third rule: A further comment, *Oxford Econ. Papers* N. S. 13: 262–65, Oct. 1961.

Economic theory and the evaluation of consumers' wants, *Journal of Business* 35: 256–63, July 1962.

Liquidity, *Econ. Journal* 72: 787–802, Dec. 1962.

(with Hicks, U. K.), The reform of budget accounts, *Oxford Inst. of Stat.* 25: 119–26, May 1963.

Dennis Robertson, British Academy, 1965.

After the boom, Institute of Economic Affairs, 1966.

Growth and anti growth, *Oxford Econ. Papers* N. S. 18: 257–69, Nov. 1966.

Saving, Investment and taxation: An international comparison *Three Banks Review* 78: June 1968.

Elasticity of substitution again: Substitute and complements, *Oxford Econ. Papers* N. S. 22: 289–96, Nov. 1970.