# Asset-Price Redistribution 

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## Rising asset prices



Rising asset prices ... relative to income $=$ rising valuations


## Welfare consequences of rising asset valuations?

- Rising asset valuations had large effects on distribution of wealth

Q: But what are welfare consequences of rising valuations?

- Answer is not obvious. Two polar views regarding effect of $P \uparrow$ :

1. Shift of real resources towards wealthy (Piketty-Zucman, Saez-Yagan-Zucman)
2. Welfare-irrelevant paper gains (Cochrane, Krugman)

## What We Do: Theory

- Formula for money-metric welfare gains/losses (= compensating variation) from asset price changes here for case of one asset and no borrowing
- price deviations holding cashflows constant $\Rightarrow$ pure valuation effects
- envelope theorem $\Rightarrow$ first-order approximation
- +... captures other effects (collateral,GE,..) but 1st term is always there
- Two main lessons: higher valuations ...

1. benefit sellers, not holders
2. are purely redistributive in terms of welfare (for every seller there is a buyer)

- Implication: both polar positions from previous slide are wrong


## What We Do: Empirics

- Implement as sufficient statistic using Norwegian admin data (1994-2019)

$$
\begin{aligned}
\text { Welfare Gain }_{i}= & \sum_{t=0}^{T} \text { Discounting }_{t}\left(\sum_{k=1}^{K}\left(\text { Asset sales }_{i k t} \times \text { Price deviation }_{k t}\right)+\text { Borrowing }_{i t} \times\right. \text { Rate deviation } \\
& + \text { Terms from generalizations }
\end{aligned}
$$

- measure net asset sales, borrowing (housing, stocks, debt, deposits)
- measure price deviations = deviations from constant price-dividend ratios (Gordon growth model) to isolate valuation effect
- Document large redistributive effects of rising asset valuations
- in cross section
- from young towards old
- from poor towards wealthy

Rising asset valuations generate large welfare gains \& losses


## Example: large redistribution from young to old ...



## ... mostly due to house price changes



## Plan

1. Theory: Intuition in two-period model
2. Theory: Sufficient statistic in full dynamic model with multiple assets
3. Empirics: implementation using Norwegian administrative data
4. Empirics: redistribution across households
5. Empirics: generalizations of baseline sufficient statistics approach

## Intuition in two-period model

- Periods $t=0$ and $t=1$, endowments $Y_{0}$ and $Y_{1}$
- Can trade shares $N$ at time $t=0$ that pay dividend $D$ at time $t=1$

$$
\begin{aligned}
& V=\max _{\left\{C_{0}, C_{1}\right\}} U\left(C_{0}\right)+\beta U\left(C_{1}\right) \\
& C_{0}+\left(N_{0}-N_{-1}\right) P_{0}=Y_{0}, \quad C_{1}=Y_{1}+N_{0} D_{1}
\end{aligned}
$$

- Comparative static: effect of $P_{0}$ on welfare $V$ ?

- Rising asset prices benefit sellers ( $N_{-1}-N_{0}>0$ ), not holders ( $N_{-1}>0$ )
- Note: $D_{1}$ held constant, else additional term $+\frac{\beta U^{\prime}\left(C_{1}\right)}{U^{\prime}\left(C_{0}\right)} N_{0} d D_{1}$


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## Intuition in two-period model

- Rising asset prices benefit sellers ( $N_{-1}-N_{0}>0$ ), not holders ( $N_{-1}>0$ )
- How can initial holders not benefit from $P_{0} \uparrow$ ? Two counteracting effects:
$(t=0)$ High initial return $R_{0}=P_{0} / P_{-1} \uparrow$
( $t=1$ ) Low future returns $R_{1}=D_{1} / P_{0} \downarrow$
- For sellers, high initial returns dominate ...
- For buyers, low future returns dominate


## Graphical intuition: welfare effect of $P_{0} \uparrow$

A seller's investment decision


A buyer's investment decision


## Graphical intuition: welfare effect of $P_{0} \uparrow$

Effect of $P_{0} \uparrow$ on seller


Effect of $P_{0} \uparrow$ on buyer


## Full dynamic model with multiple assets

- Infinite horizon, no risk
- One-period bond $\left\{B_{i, t}\right\}_{t=0}^{\infty}$ with prices $\left\{Q_{t}\right\}_{t=0}^{\infty}$ (deposits)
- one-period return: $R_{t+1}=1 / Q_{t}$
- return from 0 to $t: R_{0 \rightarrow t} \equiv R_{1} \cdot R_{2} \cdots R_{t}$
- $K$ long-duration assets $\left\{N_{i k, t}\right\}_{t=0}^{\infty}$, prices $\left\{P_{k, t}\right\}_{t=0}^{\infty}$ \& dividends $\left\{D_{k, t}\right\}_{t=0}^{\infty}$
- adjustment cost $\chi_{i k}\left(N_{i k, t}-N_{i k, t-1}\right)$, potentially kinked (inaction)
- asset returns: $R_{k, t+1} \equiv \frac{D_{k, t+1}+P_{k, t+1}}{P_{k, t}}$


## Extensions: see paper, show you two of these today

1. Borrowing and collateral constraints
2. Incomplete markets
3. Bequests
4. Consolidating businesses with their owners.
5. Government sector
6. Taxes on assets
7. Housing and wealth in the utility function
8. General equilibrium

## Welfare gains/losses in full dynamic model

- Households solve

$$
\begin{aligned}
& V_{i}=\max _{\left\{C_{i, t}, B_{i, t},\left\{N_{i, t},\right\}_{k=1}^{\}}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} U\left(C_{i, t}\right) \quad \text { s.t. } \\
& C_{i, t}+\sum_{k=1}^{K}\left(N_{i k, t}-N_{i k, t-1}\right) P_{k, t}+B_{i, t} Q_{t}+\sum_{k=1}^{K} \chi_{i k}=\sum_{k=1}^{K} N_{i k, t-1} D_{k, t}+B_{i, t-1}+Y_{i, t}
\end{aligned}
$$

- Proposition: welfare effect of perturbation $\left\{\mathrm{d} P_{1, t}, \ldots, \mathrm{~d} P_{K, t}, \mathrm{~d} Q_{t}\right\}_{t=0}^{\infty}$ is

$$
\mathrm{d} V_{i}=U^{\prime}\left(C_{i, 0}\right) \times \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}\left(\sum_{k=1}^{K}\left(N_{i k, t-1}-N_{i k, t}\right) \mathrm{d} P_{k, t}-B_{i, t} \mathrm{~d} Q_{t}\right)}_{\text {Welfare Gain }_{i}}
$$

- As in two-period model, rising asset prices benefit net sellers
... but portfolio choice + timing of purchases also matters


## Aggregation

- Corollary: Suppose that initial prices clear all asset markets, i.e. asset sales and purchases add up to zero for each asset class. Then

$$
\sum_{i=1}^{1}{\text { Welfare } \text { Gain }_{i}=0}
$$

so that asset price deviations are purely redistributive

## Extension: general equilibrium

- Claim: in GE, our formula does not capture full welfare effect but rather welfare effect working through equilibrium asset price changes
- Fundamental drivers of asset prices: vector $z_{t}=\bar{z}_{t}+\theta \Delta z_{t}$
- Equilibrium prices: $\Gamma_{t}(\theta)=\left\{\left\{P_{k, t}(\theta), D_{k, t}(\theta)\right\}_{k}, Q_{t}(\theta), Y_{t}(\theta)\right\}$
- Welfare $V\left(\left\{\Gamma_{t}(\theta)\right\}_{t=0}^{\infty}, \theta\right)$. Hence

$$
\begin{aligned}
\mathrm{d} V= & \underbrace{\sum_{t=0}^{\infty}\left(\sum_{k=1}^{K} \frac{\partial V}{\partial P_{k, t}} \frac{\partial P_{k, t}}{\partial \theta}+\frac{\partial V}{\partial Q_{t}} \frac{\partial Q_{t}}{\partial \theta}\right) \mathrm{d} \theta}_{\text {Welfare gain through asset prices = our main formula }} \\
& +\underbrace{\sum_{t=0}^{\infty}\left(\sum_{k=1}^{K} \frac{\partial V}{\partial D_{k, t}} \frac{\partial D_{k, t}}{\partial \theta}+\frac{\partial V}{\partial Y_{t}} \frac{\partial Y_{t}}{\partial \theta}\right) \mathrm{d} \theta}_{\text {Welfare gain through dividends, labor income, ... }}+\underbrace{\frac{\partial V}{\partial \theta} \mathrm{~d} \theta}_{\text {Direct effect }}
\end{aligned}
$$

## Extension: collateral constraints $-B_{i, t} \leq \theta N_{i, t} P_{t}$

- With collateral constraint, money-metric welfare gains are

where $\mu_{i, t}$ is Lagrange multiplier on collateral constraint
- Alternatively, household-specific rate schedule $Q_{i, t}=Q_{t} F\left(N_{i, t} P_{t}, B_{i, t}\right)$ :


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Welfare Gain $_{i}=\sum_{t=0}^{\infty} \frac{\beta^{t} U^{\prime}\left(C_{i, t}\right)}{U^{\prime}\left(C_{i, 0}\right)}\left(\left(N_{i, t-1}-N_{i, t}\right) \mathrm{d} P_{t}-B_{i, t} \mathrm{~d} Q_{t}\right)+\sum_{t=0}^{\infty} \frac{\beta^{t} \mu_{i, t}}{U^{\prime}\left(C_{i, 0}\right)} \theta N_{i, t} \mathrm{~d} P_{t}$
where $\mu_{i, t}$ is Lagrange multiplier on collateral constraint

- Alternatively, household-specific rate schedule $Q_{i, t}=Q_{t} F\left(N_{i, t} P_{t}, B_{i, t}\right)$ :

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&{\text { Welfare } \text { Gain }_{i}=} \sum_{t=0}^{\infty} \frac{\beta^{t} U^{\prime}\left(C_{i, t}\right)}{U^{\prime}\left(C_{i, 0}\right)}\left(\left(N_{i, t-1}-N_{i, t}\right) \mathrm{d} P_{t}-B_{i, t} Q_{i, t} \frac{\mathrm{~d} Q_{t}}{Q_{t}}\right) \\
&+\sum_{t=0}^{\infty} \frac{\beta^{t} U^{\prime}\left(C_{i, t}\right)}{U^{\prime}\left(C_{i, 0}\right)}\left(-B_{i, t} \frac{\partial Q_{i, t}}{\partial\left(N_{i, t} P_{t}\right)} N_{i, t} \mathrm{~d} P_{t}\right)
\end{aligned}
$$

- Later: implement by taking interest rate schedule from data


## Empirics

## Implementation

1. Replace infinitesimal changes $\mathrm{d} P_{k, t}$ by discrete changes $\Delta P_{k, t}$

- robustness: second-order effects

2. Consider deviations in prices away from constant price-dividend ratios

$$
\Delta P_{k, t}=P_{k t}-\overline{P D}_{k} \times D_{k, t} \quad \Leftrightarrow \quad \frac{\Delta P_{k, t}}{P_{k, t}}=\frac{P D_{k, t}-\overline{P D}_{k}}{P D_{k, t}}
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3. Truncate formula at finite date $T$ (end of sample)

- robustness: extrapolate trading and price deviations beyond $T$ (later)


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3. Truncate formula at finite date $T$ (end of sample)

- robustness: extrapolate trading and price deviations beyond $T$ (later)
- $\Rightarrow$ Formula we implement empirically

$$
\text { Welfare gain }_{i} \approx \sum_{t=0}^{T} R_{0 \rightarrow t}^{-1}\left(\sum_{k=1}^{K}\left(N_{i k, t-1}-N_{i k, t}\right) P_{k, t} \times \frac{P D_{k, t}-\overline{P D}_{k}}{P D_{k, t}}-B_{i, t} Q_{t} \times \frac{Q_{t}-\bar{Q}}{Q_{t}}\right)
$$

## Data on Holdings and Transactions

- Administrative data covering the universe of Norwegians over 1993-2019
- Focus on 4 broad asset categories that cover most of household wealth

1. deposits ( $15 \%$ )
2. debt (mortgage, student loan, ..., $-35 \%$ )
3. equity (individual stocks, mutual funds, private businesses, ..., 10\%)
4. housing (110\%)

- For deposits/debt, we only need to measure holdings
- For equities/housing, we use data on individual transactions
- Take into account indirect transactions/holdings through equity ownership

Rising valuations, declining yields in all asset classes



Gross real interest rate (debt/deposits); Rents/Price (housing); Cashflows/EV (equity)

## Data on housing and equity transactions



## Data on debt and deposits



## Rising asset valuations generate large welfare gains \& losses



| Average |  | Average by percentile groups of welfare gains |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{p} 0-1$ | $\mathrm{p} 1-10$ | $\mathrm{p} 10-50$ | $\mathrm{p} 50-90$ | $\mathrm{p} 90-99$ | $\mathrm{p} 99-100$ |
| 10.1 |  | -546.2 | -94.5 | -13.7 | 16.7 | 100.2 | 701.5 |

## Large gains and losses as \% of initial wealth



In theory, we have $\frac{\text { Welfare gain }}{\text { Total weath }}=\frac{\sum_{t=0}^{\infty} R_{0}^{-1} \mathrm{~d} C_{t}}{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_{t}}=$ welfare gain as a share of lifetime consumption

## Welfare vs wealth gains (revaluation gains)



Revaluation gain ${ }_{i}=\sum_{t=0}^{T} R_{0 \rightarrow t}^{-1} \sum_{k=1}^{K} N_{i k, t-1} P_{i k, t-1} \mathrm{~d}\left(P_{k, t} / P_{k, t-1}\right) \quad$ (recall asset returns $\left.\frac{D_{k, t}+P_{k, t}}{P_{k, t-1}}\right)$

## Joint distribution of welfare and revaluation gains



## Redistribution from the young to the old



## Redistribution from the young to the old



## Redistribution from the poor to the rich



## Welfare vs revaluation gains across wealth distribution



## Redistribution across sectors

- Households welfare gains aggregate to $\approx \$ 10 \mathrm{~K}$ per capita
- Who is the losing counterparty?

Welfare Gain Households $=-$ Welfare Gain $_{\text {Government }}-$ Welfare Gain $_{\text {Foreigners }}$

## Redistribution across sectors



Generalizations of baseline sufficient statistics approach

## Collateral effects

- Recall extension: interest rate schedule $Q_{i, t}=Q_{t} F\left(B_{i, t}, N_{i, t} P_{t}\right)$

$$
\begin{aligned}
\text { Welfare Gain }_{i}= & \sum_{t=0}^{\infty} \frac{\beta^{t} U^{\prime}\left(C_{i, t}\right)}{U^{\prime}\left(C_{i, 0}\right)}\left(\left(N_{i, t-1}-N_{i, t}\right) \mathrm{d} P_{t}-B_{i, t} Q_{i, t} \frac{\mathrm{~d} Q_{t}}{Q_{t}}\right) \\
& +\sum_{t=0}^{\infty} \frac{\beta^{t} U^{\prime}\left(C_{i, t}\right)}{U^{\prime}\left(C_{i, 0}\right)}\left(-B_{i, t} \frac{\partial Q_{i, t}}{\partial\left(N_{i, t} P_{t}\right)} N_{i, t} \mathrm{~d} P_{t}\right)
\end{aligned}
$$

- Estimate second term by measuring effect of LTV on mortgage rates


## Mortgage interest rates increase with loan-to-value ratio



## Collateral effects




## Valuations changes beyond end of our sample period

- We extend our baseline formula to account for future valuation changes:

Welfare Gain ${ }_{i} \approx \sum_{t=0}^{T} R_{0 \rightarrow t}^{-1}\left(N_{i, t-1}-N_{i, t}\right) \Delta P_{t}+\underbrace{\sum_{t=T+1}^{\infty} R_{0 \rightarrow t}^{-1}\left(N_{i, t-1}-N_{i, t}\right) \Delta P_{t}}_{\text {future valuation changes }}$

- Estimate second term assuming: for $t \geq T$

$$
\begin{aligned}
& \frac{\Delta P_{t}}{P_{t}}=\frac{P D_{t}-\overline{P D}_{t}}{P D_{t}} \text { with } \log \left(\frac{P D_{t}}{\overline{P D}}\right)=\phi^{t-T} \log \left(\frac{P D_{T}}{\overline{P D}}\right), \phi<1 \\
& N_{a, t-1}-N_{a, t}=N_{a, T-1}-N_{a, T} \text { where } a=\text { age }
\end{aligned}
$$

## Valuations changes beyond end of our sample period




## Conclusion

- Simple framework to quantify welfare effects of asset price deviations
- Framework can be extended to take into account collateral effects, incomplete markets, ...
- Application to Norway (1994-2019)

1. large heterogeneity in welfare gains across households
2. welfare gains $\neq$ revaluation gains
3. redistribution from young to old and from poor to rich
4. negative "welfare gain" for government $\Rightarrow$ future net transfers $\downarrow$

- Could apply in other contexts, e.g. collapsing asset prices in recessions
- Optimal taxation? (Aguiar-Moll-Scheuer)

