Appendix for "Asset-Price Redistribution"

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A Appendix on theoretical framework

A.1 Welfare gains in a deterministic environment

We consider the deterministic infinite-horizon economy described in the main text (Section I.B). The following proposition characterizes the welfare effect of a joint deviation in the path of asset prices, labor income, and dividend income. This proposition effectively generalizes Proposition 1, which corresponds to the special case where the deviation in income is zero.

Proposition A1. *The welfare gain of a deviation in asset prices* $\{\mathrm{d}Q_t, \{\mathrm{d}P_{k,t}\}_k\}_{t=0}^{\infty}$ *as well as labor and dividend income* $\{\mathrm{d}Y_{i,t}, \{\mathrm{d}D_{k,t}\}_k\}_{t=0}^{\infty}$ *is*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\left(\sum_{k=1}^{K}(N_{i,k,t-1} - N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t + \sum_{k=1}^{K} N_{i,k,t-1}\,\mathrm{d}D_{k,t} + \mathrm{d}Y_{i,t}\right). \tag{A1}$$

Proof of Proposition A1. To provide some intuition, we first give an heuristic derivation using the Lagrangian corresponding to the individual optimization problem and totally differentiating with respect to the sequence of asset prices $\{Q_t, \{P_{k,t}\}_{k=1}^{K}\}_{t=0}^{\infty}$ and income $\{\mathrm{d}Y_{i,t}, \{\mathrm{d}D_{k,t}\}_{k=1}^{K}\}_{t=0}^{\infty}$. We then provide a more rigorous proof that uses a perturbation of this sequence indexed by a perturbation parameter θ and a version of the envelope theorem due to Oyama and Takenawa (2018).
Heuristic derivation. The Lagrangian associated with the optimization problem is

$$\begin{aligned}\mathcal{L}_i = \sum_{t=0}^{\infty}\beta^t U(C_{i,t}) + \sum_{t=0}^{\infty}\lambda_{i,t}\Bigg(&\sum_{k=1}^{K} N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t} - C_{i,t} \\ &- \sum_{k=1}^{K}(N_{i,k,t} - N_{i,k,t-1})\,P_{k,t} - \sum_{k=1}^{K}\chi_k(N_{i,k,t} - N_{i,k,t-1}) - B_{i,t}Q_t\Bigg).\end{aligned} \tag{A2}$$

Assuming that the value function is differentiable, we can write the infinitesimal change in the value function in terms of the infinitesimal change in the Lagrangian:

$$\begin{aligned}
\mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty}\left(\sum_{k=1}^{K}\frac{\partial \mathcal{L}_i}{\partial P_{k,t}}\,\mathrm{d}P_{k,t} + \frac{\partial \mathcal{L}_i}{\partial Q_t}\,\mathrm{d}Q_t + \sum_{k=1}^{K}\frac{\partial \mathcal{L}_i}{\partial D_{k,t}}\,\mathrm{d}D_{k,t} + \frac{\partial \mathcal{L}}{\partial Y_{i,t}}\,\mathrm{d}Y_{i,t}\right)\\
&= \sum_{t=0}^{\infty}\lambda_{i,t}\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\,\mathrm{d}D_{k,t} + \mathrm{d}Y_{i,t}\right)\\
&= \lambda_{i,0}\sum_{t=0}^{\infty}(Q_0\ldots Q_{t-1})\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\,\mathrm{d}D_{k,t} + \mathrm{d}Y_{i,t}\right)\\
&= U'(C_{i,0})\sum_{t=0}^{\infty}R_{0\to t}^{-1}\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\,\mathrm{d}D_{k,t} + \mathrm{d}Y_{i,t}\right).
\end{aligned}$$

The third equality uses the first-order conditions for $B_{i,t}$, $\lambda_{i,t}Q_t = \lambda_{i,t+1}$, while the last equality uses the first-order condition for $C_{i,0}$, $U'(C_{i,0}) = \lambda_{i,0}$, as well as the definition of the cumulative return $R_{0\to t}^{-1} = Q_0\ldots Q_{t-1}$.

Formal derivation. Consider a deviation in asset prices in the direction $\{\Delta Q_t, \{\Delta P_{k,t}\}_1^K\}_{t=0}^{\infty}$ and of income in the direction $\{\Delta Y_{i,t}, \{\Delta D_{k,t}\}_1^K\}_{t=0}^{\infty}$ Consider a parameter $\theta \in [0,1]$ indexing the size of the perturbation:

$$\begin{aligned}
Q_t(\theta) &\equiv Q_t + \theta\Delta Q_t; \qquad & P_{k,t}(\theta) &\equiv P_{k,t} + \theta\Delta P_{k,t}\\
Y_{i,t}(\theta) &\equiv Y_{i,t} + \theta\Delta Y_{i,t}; \qquad & D_{k,t}(\theta) &\equiv D_{k,t} + \theta\Delta D_{k,t}.
\end{aligned}$$

The optimization problem takes the form $V_{i,0}(\theta) = \max_x f(x,\theta)$ where $x = \{B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}$, and

$$\begin{aligned}
f:(x,\theta) \mapsto \sum_{t=0}^{\infty}\beta^t U\Bigg(&\sum_{k=1}^{K}N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t} - \sum_{k=1}^{K}(N_{i,k,t}-N_{i,k,t-1})P_{k,t}\\
&-B_tQ_{i,t} - \sum_{k=1}^{K}\chi_k(N_{i,k,t}-N_{i,k,t-1})\Bigg).
\end{aligned}$$

Note that f is continuous in x, and that its derivative with respect to θ is

$$\partial_\theta f(x,\theta) = \sum_{t=0}^{\infty}\beta^t U'(C_{i,t})\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right),$$

which is continuous in x and θ. Under this set of assumptions, Proposition 2.1 in Oyama and Takenawa (2018) gives that V is differentiable at 0 and $V'(0) = \partial_\theta f(x^*,0)$, where x^* denote the optimal solution of the maximization problem at $\theta = 0$. Using the expression for $\partial_\theta f$ above gives:

$$\begin{aligned}
V_{i,0}'(0) &= \sum_{t=0}^{\infty}\beta^t U'(C_{i,t})\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right)\\
&= U'(C_{i,0})\sum_{t=0}^{\infty}R_{0\to t}^{-1}\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right),
\end{aligned}$$

where the second line uses the Euler equation. This concludes the proof as $\mathrm{d}V_{i,0} = V'(0)\,\mathrm{d}\theta$, $\mathrm{d}Q_t = \Delta Q_t\,\mathrm{d}\theta$, $\mathrm{d}P_{k,t} = \Delta P_{k,t}\,\mathrm{d}\theta$, $\mathrm{d}Y_{i,t} = \Delta Y_{i,t}\,\mathrm{d}\theta$, and $\mathrm{d}D_{k,t} = \Delta D_{k,t}\,\mathrm{d}\theta$. □

Proof of Proposition 1. The proposition obtains as a special case of Proposition A1 where the deviation in income is taken to be zero, i.e., $\mathrm{d}Y_{i,t} = 0$ and $\mathrm{d}D_{k,t} = 0$ for $1 \le k \le K$. □

A.2 Welfare gains in a stochastic environment

So far, we have focused on the case of a deterministic economy. We now extend our results to the case of a *stochastic* economy. For clarity, we distinguish between two different cases. In Appendix A.2.1, we first consider the case of a baseline stochastic economy that is subject to an unexpected deviation in the path of asset prices and income; that is, the same comparative static exercise as in Proposition 1 but now in a stochastic environment. In Appendix A.2.2, we then consider the more complex case in which the realization of the deviation itself corresponds to a "shock" on which individuals can potentially contract ex-ante; that is, individuals maximize expected values using a probability distribution over different realizations of this shock and can potentially insure against this shock by trading in financial markets.

A.2.1 Extension of Proposition 1 to a stochastic environment (comparative statics exercise)

Setup. We first describe the stochastic economy. At each time period $t \geq 1$, there is a realization of a stochastic event $s_t \in \mathcal{S}$. We denote $s^t = (s_1, \dots, s_t)$ the history of these realizations up to time t. We allow asset prices and income to depend on the entire history of events s^t. More precisely, there is a sequence of one-period bonds with stochastic prices $Q_t(s^t)$, as well as K long-lived assets financial assets with stochastic prices $P_{kt}(s^t)$ and stochastic dividends $D_{kt}(s^t)$ for each asset k. Each individual i receives some stochastic labor income every period $Y_{i,t}(s^t)$.[1] To minimize the notational burden, we omit the explicit dependence of each stochastic process on s^t when there is no risk of ambiguity.

Agents have the same preferences as in the baseline model. The problem of individual i is to choose a history-contingent path for consumption $C_{i,t}$ and asset holdings $B_{i,t}$, $\{N_{i,k,t}\}_k$ to maximize welfare:

$$V_{i,0} \equiv \max_{\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right],$$

subject to initial asset holdings $B_{i,-1}$ and $\{N_{i,k,-1}\}_k$, as well as a sequence of budget constraints for $t \geq 0$

$$C_{i,t} + \sum_{k=1}^{K} (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} + B_{i,t} Q_t + \sum_{k=1}^{K} \chi_k (N_{i,k,t} - N_{i,k,t-1}) = \sum_{k=1}^{K} N_{i,k,t-1} D_{k,t} + B_{i,t-1} + Y_{i,t}. \quad \text{(A3)}$$

As in the baseline model, we assume that there is a unique solution to the agent problem and that it is continuous with respect to asset prices, dividends, and income received in each state of nature.[2]

Welfare gain. We now study the welfare effect of a deviation in the paths of asset prices and income. More precisely, we consider a history-contingent deviation in asset prices, $\{\mathrm{d}Q_t(s^t), \{\mathrm{d}P_{k,t}(s^t)\}_k\}_{t=0}^{\infty}$, as well as in dividend and labor income, $\{\mathrm{d}Y_{i,t}(s^t), \{\mathrm{d}D_{k,t}(s^t)\}_k\}_{t=0}^{\infty}$.[3] As in the main text, this deviation can be interpreted either as a difference between two economies (e.g., comparative static) or as an unexpected MIT shock in the baseline economy.

We define the welfare gain of this deviation as the amount of money received at time $t = 0$ in the baseline economy that would have the same effect on individual welfare. The following proposition expresses this welfare gain in terms of the path of future asset holdings.

[1] While the set of event realizations $\mathcal{S}$ does not depend on i, the mapping from the history of stochastic events s^t to individual labor income $Y_{i,t}$ is individual specific, and so our framework includes models with purely idiosyncratic labor income shocks.

[2] Note that, when the economy is stochastic, this assumption no longer requires the existence of adjustment cost, as different assets can have different risk profiles. Still, this assumption rules out the presence of arbitrage opportunities within the set of assets without adjustment costs.

[3] Note that we allow this deviation to be state-contingent, although it does not play a key role for our results.

Proposition A2. *The welfare gain of a deviation in asset prices* $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^{\infty}$ *as well as labor and dividend income* $\{dY_{i,t}, \{dD_{k,t}\}_k\}_{t=0}^{\infty}$ *is*

$$dV_{i,0}/U'(C_{i,0}) = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}\left(\sum_{k=1}^{K}(N_{i,k,t-1}-N_{i,k,t})\,dP_{k,t} - B_{i,t}\,dQ_t + \sum_{k=1}^{K}N_{i,k,t-1}\,dD_{k,t} + dY_{i,t}\right)\right].$$

Relative to the formula obtained in a deterministic economy (Proposition A1), the baseline formula for the welfare gain changes in two ways. First, in a stochastic world, what matters for (ex-ante) welfare is the expected path of net asset sales, not the realized one. The second is that this expectation is under the individual's risk-neutral measure, which tilts the objective measure by the individual's marginal rate of substitution across states and times $\beta^t U'(C_{i,t})/U'(C_{i,0})$. This adjustment reflects that individuals care about particular states of the world more than others. To be concrete in the context of idiosyncratic labor income risk, consider the ex-ante welfare gain of young individuals who face uncertainty over their future paths of labor income and who plan to buy houses only if they are successful in the labor market. From today's perspective, these individuals "care more" about the states of the world where their income is low, as their marginal utility of consumption will be higher in these states. Hence, their expected housing purchases are lower under the risk-adjusted measure than the objective measure.

Proof of Proposition A2. The Lagrangian associated with the individual problem at time $t = 0$

$$\begin{aligned}\mathcal{L}_i &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(C_{i,t})\right]\\ &\quad + \mathbb{E}_0\left[\sum_{t=0}^{\infty}\lambda_{i,t}\left(Y_{i,t} + \sum_{k=1}^{K}N_{i,k,t-1}D_{k,t} + B_{i,t-1} - C_{i,t} - (N_{i,k,t}-N_{i,k,t-1})P_{k,t} - B_{i,t}Q_t\right)\right],\end{aligned}$$

where $\lambda_{i,t}$ denotes the (stochastic) Lagrange multiplier associated with the budget constraint (A3) at time t. One should understand the expectation $\mathbb{E}_0$ as a probability-weighted sum across different states of nature, and so this Lagrangian can be understood as the one obtained for a static optimization with as many constraints as there are states of nature.

As in the proof of Proposition 1, we apply the Lagrangian method to obtain:

$$\begin{aligned}dV_{i,0} &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\lambda_{i,t}\left(dY_{i,t} + \sum_{k=1}^{K}N_{i,k,t-1}\,dD_{k,t} - (N_{i,k,t}-N_{i,k,t-1})\,dP_{k,t} - B_{i,t}\,dQ_t\right)\right]\\ &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U'(C_{i,t})\left(dY_{i,t} + \sum_{k=1}^{K}N_{i,k,t-1}\,dD_{k,t} - (N_{i,k,t}-N_{i,k,t-1})\,dP_{k,t} - B_{i,t}\,dQ_t\right)\right],\end{aligned}$$

where the second equality uses the first-order condition for $C_{i,t}$, $\beta^t U'(C_{i,t}) = \lambda_{i,t}$. Dividing each side by $U'(C_{i,0})$ gives the result. □

Case of complete markets. We now add the assumption that markets are complete in the baseline economy.[4] In a dynamic environment such as ours, what this assumption means is that there are enough assets with zero adjustment cost to allow individuals to replicate the Arrow-Debreu security associated with any history s^t (i.e., a security that returns one dollar at time t if history s^t is realized and zero otherwise) with some trading strategy started at time $t = 0$ (for any time t and history s^t). In this case, there exists a unique stochastic discount factor, $\Lambda_t(s^t)$ (up to a scaling factor). At each time and in each

[4]Note that we are still considering the case of a comparative statics exercise between two economies (or, equivalently, an unexpected deviation at time $t = 0$; that is, an MIT shock). We will consider the case of markets that are complete regarding the "realization" of the deviation itself in Appendix A.2.2 below.

state, agents equalize their marginal rates of substitution across states and times to the growth of the stochastic discount factor:[5]

$$\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} = \frac{\Lambda_t}{\Lambda_0}. \tag{A4}$$

As a result, the welfare gain formula from Proposition A2 simplifies to:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\left(\sum_{k=1}^{k}(N_{i,k,t-1}-N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t + \sum_{k=1}^{K}N_{i,k,t-1}\,\mathrm{d}D_{k,t} + \mathrm{d}Y_{i,t}\right)\right]. \tag{A5}$$

Because of complete markets, all individuals use the same discount factor to value consumption in any future history relative to today.[6] One implication is that for the case of a pure deviation in asset prices (i.e., no deviation in incomes $\mathrm{d}D_{k,t} = \mathrm{d}Y_{i,t} = 0$), welfare gains aggregate to zero across a set of individuals trading with each other, since transactions aggregate to zero in every state in the world. Note that this critically hinges on the fact that markets are complete, which was de-facto the case in our baseline model (Proposition 1), as the presence of liquid one-period bonds in a deterministic economy is enough to make markets complete.

A.2.2 Welfare effects of potentially insurable shocks

So far, we have considered the case of a stochastic economy with an unexpected deviation in the path of asset prices and income (i.e., a MIT shock). Instead, we now consider an economy with "deviation shocks"; that is, an economy with individuals maximizing expected values using a probability distribution over different realizations of this shock. An important implication of this assumption is that individuals can contract on the realization of the deviation itself. The presence of insurance markets affects the equilibrium trading pattern of individuals and, therefore, their exposure to the "deviation shock". Still, our sufficient statistic formula, which expresses individual welfare gains *given* individual trading patterns, remains unchanged.

Setup. The setup for the baseline economy is very similar to the previous section, except that we now add the realization of a stochastic event s_0 at time $t = 0$. To keep things simple, we assume that there are only two possible realizations for s_0: a baseline scenario $\bar{s}$ and a deviation $\widehat{s}$.[7] We assume that the realization of s_0 does not affect the probability distribution of events in the future, but it does affect the values of asset prices and income associated with each event history. More precisely, labor income, dividend income, and asset prices at time t are stochastic processes that depend on the entire history of shocks from 0 to t, $(s_0, s_1, \ldots, s_t) = (s_0, s^t)$ where $s^t = (s_1, \ldots, s_t)$ denotes the history from $t = 1$ as in the preceding section.

For any stochastic process $X_t(s_0, s_1, \ldots, s_t)$, we denote by $\Delta X_t(s^t) \equiv X_t(\widehat{s}, s^t) - X_t(\bar{s}, s^t)$ the difference in the realized value of X_t if state $s_0 = \widehat{s}$ happens compared to state $s_0 = \bar{s}$. Finally, as in the proof of Proposition A1, we consider a sequence of economies in which the size of the difference in

[5]Denoting $\pi(s^t)$ the probability of history s^t, $\pi(s^t)\Lambda_t(s^t)/\Lambda_0$ corresponds to the price at $t = 0$ of the Arrow-Debreu security associated to history s^t. Hence, this equality can be interpreted as the first-order condition reflecting that the individual i must be indifferent between using one marginal dollar at $t = 0$ to consume or to invest in the Arrow-Debreu security associated to history s^t.

[6]In fact, we only need the markets to be complete starting from time $t = 0$; that is, we do not need agents to be able to insure against the deviation shock.

[7]We could easily allow s_0 to have an arbitrary (finite) number of realizations, $s_0 \in \mathcal{S}$, and focus on the welfare gain of some realization relative to another. However, the notations become much more complex, as one needs to distinguish the ex-post realization of each stochastic process with respect to each possible realization of s_0.

asset prices and income between $\widehat{s}$ and $\bar{s}$ is scaled by θ and we will study the asymptotic limit $\theta \to 0$.[8] To keep notations simple, we will leave implicit the fact that each stochastic process (e.g., asset prices, income, consumption...) is indexed by θ.

Agent preferences are the same as above. Individuals choose a history-contingent path of consumption $C_{i,t}$ and asset holdings $B_{i,t}$, $\{N_{i,k,t}\}_k$ to maximize expected utility

$$V_{i,-1} \equiv \max_{\{C_{i,t},B_{i,t},\{N_{i,k,t}\}_k\}_{t=0}^{\infty}} \mathbb{E}_{-1}\left[\sum_{t=0}^{\infty}\beta^t U(C_{i,t})\right],$$

subject to the following sequence of budget constraints

$$C_{i,t} + \sum_{k=1}^{K}(N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t = \sum_{k=1}^{K} N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t} \quad \text{for } t \geq 0 \tag{A6}$$

$$\sum_{k=1}^{K} N_{i,k,-1}P_{k,-1} + B_{i,-1}Q_{-1} = W_{i,-1} \quad \text{for } t = -1, \tag{A7}$$

with initial financial wealth $W_{i,-1}$. The key difference with the earlier model is that we allow agents to trade before the realization of the deviation shock s_0. Note that, for the sake of simplicity, we also assume away any adjustment costs in the assets, which allows us to get a simple formulation for the individual problem when markets are complete. Finally, we assume that the set of asset prices and dividends is such that there are no arbitrage opportunities, that there is a unique solution to the agent problem, and that it is continuous with respect to θ.

Welfare gain. We now extend our notion of welfare gain (i.e., equivalent variation) to this stochastic economy. Formally, we define the welfare gain of the realization of $s_0 = \widehat{s}$ relative to $s_0 = \bar{s}$ as the quantity of money received at $t = 0$ that would equalize realized welfare across these two states. Denote by $\Delta V_{i,0} \equiv \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(C_{i,t})|s_0 = \widehat{s}\right] - \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(C_{i,t})|s_0 = \bar{s}\right]$ the difference in the welfare of the individual at time $t = 0$ if state $\widehat{s}$ is realized relative to state $\bar{s}$ (given our two-state assumption, $\Delta V_{i,0}$ is just a deterministic scalar). At the first order in θ (i.e., when the difference between the two states is small), the welfare gain corresponds to $\Delta V_{i,0}$ divided by the marginal utility of consumption. Using a similar derivation as in Proposition A2, the welfare gain is:[9]

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}\left(\sum_{k=1}^{K}(N_{i,k,t-1} - N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_{k=1}^{K} N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right)\right] + o(\theta). \tag{A8}$$

The key takeaways are that (i) our sufficient statistic formula can be interpreted as the effect of a realized shock and that (ii) it remains valid even when individuals can ex-ante insure against this shock. Put differently, while the ability to optimally choose portfolios at $t = -1$ impacts the trading patterns of the individual, it does not affect the welfare effect of a shock *given* these trading patterns.

[8] This small shock limit $\theta \to 0$ can be interpreted as the small time horizon limit $\Delta t \to 0$ in an economy in which asset prices and income follow Ito processes.

[9] We use the notation $o(\theta)$ to denote a term that converges to zero faster than θ as $\theta \to 0$ (e.g., a second-order term in θ). Note that while each side of this formula is a random variable contingent on the realization of the state s_0, the difference between the values of these random variables between $s_0 = \widehat{s}$ and $s_0 = \bar{s}$ is second-order in θ. Put differently, as in the deterministic case, welfare gains can be approximated at the first order using our sufficient statistic with either the path of asset transactions in the baseline economy or the path of asset transactions in the perturbed economy.

Case of complete markets We now consider the case where markets are complete starting from time $t = -1$. As discussed above, this means that there is a unique (up to a scaling factor) stochastic discount factor, denoted Λ_t, for $t \geq -1$. Because we abstracted away from adjustment costs, the sequence of asset prices satisfies the following equations:

$$\Lambda_t P_t = \mathbb{E}_t \left[\Lambda_{t+1} \left(D_{t+1} + P_{t+1}\right)\right] \qquad \text{for } 1 \leq k \leq K$$
$$\Lambda_t Q_t = \mathbb{E}_t \left[\Lambda_{t+1}\right].$$

As a result, we can consolidate the sequence of budget constraints (A6) for an individual starting with some initial wealth $W_{i,-1}$ to obtain the following present-value budget constraint:[10]

$$W_{i,-1} = \mathbb{E}_{-1} \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_{-1}} (C_{i,t} - Y_{i,t})\right]. \tag{A9}$$

Hence, because of complete markets, the problem of the agent can simply be expressed as choosing a history-contingent consumption path to maximize utility subject to this present-value budget constraint. Taking the first-order condition of this problem with respect to $C_{i,t}$ pins down consumption in terms of the stochastic discount factor for any time $t \geq 0$:[11]

$$\beta^t U'(C_{i,t}) = \mu_i \frac{\Lambda_t}{\Lambda_{-1}}, \tag{A10}$$

where μ_i is the Lagrange multiplier corresponding to the present-value budget constraint (taking a value such that this constraint is satisfied). Differentiating with respect to the realization of $s_0 = \widehat{s}$ relative to $s_0 = \bar{s}$ gives:

$$\frac{\Delta C_{i,t}}{C_{i,t}} = -\frac{1}{\gamma(C_{i,t})} \frac{\Delta \Lambda_t}{\Lambda_t} + o(\theta), \tag{A11}$$

where $\gamma(C) \equiv -U''(C)C/U'(C)$ denotes the (local) relative risk aversion of the individual. Hence, market completeness pins down the effect of the realization of $s_0 = \widehat{s}$, relative to the baseline $s_0 = \bar{s}$, on consumption: it depends on the relative prices of the Arrow-Debreu securities associated to these events (the sensitivity of the stochastic discount factor to the realization of s_0) divided by the agent relative risk aversion.

We can use this equation to pin down the welfare gain from the realization of $s_0 = \widehat{s}$ relative to the baseline $s_0 = \bar{s}$. We use the fact that the welfare gain of a shock corresponds, at the first order, to the

[10] The derivation is standard: multiply the period t budget constraint (A6) by Λ_t, add the change in asset valuations between $t-1$ and t, $\sum_k N_{i,k,t-1}(\Lambda_t P_{k,t} - \Lambda_{t-1} P_{k,t-1}) + B_{i,t-1}(\Lambda_t - \Lambda_{t-1} Q_{t-1})$, and take the expectation at time $t-1$ to obtain $\Lambda_{t-1} W_{i,t-1} = \mathbb{E}_{t-1}\left[\Lambda_t (C_{i,t} - Y_{i,t}) + \Lambda_t W_{i,t}\right]$ where $W_{i,t} \equiv \sum_k N_{i,k,t} P_{k,t} + B_{i,t} Q_t$ denotes financial wealth at time t. Substituting out $W_{i,t}$ using the next-period budget constraint and solving forward gives the result.

[11] To see why, think of $\mathbb{E}_{-1}[\cdot]$ in the present-value budget constraint as a sum over different states of nature.

present value of the effect of the shock on future consumption, which gives:[12]

$$\begin{aligned}\frac{\Delta V_{i,0}}{U'(C_{i,0})} &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}\Delta C_{i,t}\right] + o(\theta) \\ &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\Delta C_{i,t}\right] + o(\theta) \\ &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}C_{i,t}\left(-\frac{1}{\gamma(C_{i,t})}\frac{\Delta\Lambda_t}{\Lambda_t}\right)\right] + o(\theta), \end{aligned} \tag{A8'}$$

where the second equality uses (A10) and the third equality uses (A11). This equation expresses the welfare effect of the shock $s_0 = \bar{s}$ relative to the baseline $s_0 = \widehat{s}$ in terms of its effect on the stochastic discount factor (i.e., the relative price of insurance for one state relative to the other) as well as individual preferences.

It is important to realize that complete markets do *not* imply that the welfare effect of a shock is equalized across agents. Instead, what complete markets imply is that agents equate the *marginal* benefit of being a bit more insured with its *marginal* cost, given by the price of the Arrow-Debreu security associated with the state (A10). Complete markets do not imply people choose the same level of insurance in equilibrium. In particular, (A8′) implies that more risk-averse agents are typically less exposed to aggregate shocks in equilibrium.[13]

This formula, which pins down the welfare gain in terms of its effect on the stochastic discount factor and individual preferences, is consistent with our earlier "sufficient statistic" formula (A8), which expresses it in terms of endogenous portfolio choices. While the new formula helps to understand the optimal amount of "welfare exposure" chosen by the individual when markets are complete, our "sufficient statistic" formula is still preferable in an empirical context because it is valid whether or not markets are complete.

While our results already imply the equivalence between the two formulas, it is helpful to derive it manually. Note that any portfolio strategy chosen by the agent must satisfy, at each time and in each state of nature:

$$\sum_k (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t = B_{i,t-1} + \sum_k N_{i,k,t-1}D_{k,t} + Y_{i,t} - C_{i,t},$$

where $C_{i,t}$ is given by (A10). Differentiating with respect to the realization of $s_0 = \widehat{s}$ relative to $s_0 = \bar{s}$ gives

$$\begin{aligned}\Delta C_{i,t} = &\sum_k (N_{i,k,t-1} - N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t} \\ &+ \sum_k (\Delta N_{i,k,t-1} - \Delta N_{i,k,t})P_{k,t} + \sum_k (\Delta N_{i,k,t-1})D_{k,t} - (\Delta B_{i,t})Q_t + \Delta B_{i,t-1} + o(\theta).\end{aligned}$$

[12] See Proposition A13 in Appendix E.2.

[13] One analogy is an economy in which entrepreneurs with decreasing return to scale production function can borrow at some interest rate r. Entrepreneurs invest until the marginal return on capital is equal to the interest rate, but that does not necessarily imply that the *average* return to capital is equalized across entrepreneurs.

Taking the present value and rearranging gives

$$\begin{aligned}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\Delta C_{i,t}\right] &= \mathbb{E}_0\Bigg[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\left(\sum_k (N_{i,k,t-1}-N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right) \\ &\quad + \sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\left(\sum_k\left(\frac{\Lambda_{t+1}}{\Lambda_t}(D_{k,t+1}+P_{k,t+1}) - P_{k,t}\right)\Delta N_{i,k,t} + \left(\frac{\Lambda_{t+1}}{\Lambda_t} - Q_t\right)\Delta B_{i,t}\right)\Bigg] + o(\theta) \\ &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\left(\sum_k (N_{i,k,t-1}-N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t}\right)\right] + o(\theta),\end{aligned}$$

where the second equality comes from the properties of the stochastic discount factor. Remember that the left-hand side is pinned down by (A8′). Hence, any trading strategy that implements the agent's optimal consumption plan (A10) will return the same welfare gain when used as an input to our sufficient statistic formula. Notice that if an empirical researcher were to look only at how the realization of s_0 affects capital income $\Delta D_{k,t}$ and labor income $\Delta Y_{i,t}$, they would potentially miss an important component of welfare gains: the one that operates via changes in asset prices $\Delta P_{k,t}$ and interest rates ΔQ_t.

Ability to trade financial derivatives. We started this section by specifying an exogenous set of financial assets available to trade and then assuming that this set was big enough to make markets complete. We now ask: what if agents can directly purchase financial derivatives? In this case, we now show that our sufficient statistic formula (A8) still holds, but that one needs to take into account the effect of the shock on the synthetic dividends associated with these new derivatives. Put differently, even if the original shock corresponds to a pure change in the price of fundamental assets (i.e., no difference in dividends $\Delta D_{k,t}$ between the events $s_0 = \hat{s}$ and $s_0 = \bar{s}$), it may generate changes in the synthetic dividends exchanged by agents (as agents may trade derivatives with cash flows dependent on the realization of s_0).

One important example is when the individual trades the entire set of Arrow-Debreu securities at time $t = -1$. That is, the agent decides at time $t = -1$ to buy a financial derivative that gives a flow of synthetic dividends $D_{i,t}(s_0, \ldots, s_t) = C_{i,t}(s_0, \ldots, s_t) - Y_{i,t}(s_0, \ldots, s_t)$ at each time $t \geq 0$, where $C_{i,t}$ is given by (A4). Note that the fair value of this derivative exactly corresponds to the amount of financial wealth that the individual starts with, $W_{i,-1}$, given in (A9). Applying our sufficient statistic formula (A8) to this trading strategy yields:

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\frac{\Lambda_t}{\Lambda_0}\left(\underbrace{(\Delta C_{i,t} - \Delta Y_{i,t})}_{\substack{\text{effect of initial shock on dividends}\\ \text{of derivative bought at } t=-1}} + \underbrace{\Delta Y_{i,t}}_{\substack{\text{effect of initial shock shock on}\\ \text{future labor income}}}\right)\right] + o(\theta).$$

This formula, which captures welfare gains through the present value of changes in (synthetic) dividends of these derivatives, correctly recovers the welfare gain given in Formula (A8′). More generally, any trading strategy used to sustain the optimal path of consumption pinned down by (A4) will end up giving the same welfare gains through our sufficient statistic formula — the only difference is that, when agents trade financial derivatives, it is important to include changes in synthetic dividends to capture the full welfare gains. In practice, when implementing our sufficient statistic approach to the data, we do ignore welfare gains through changes in derivative cash flows as their importance in household portfolios is negligible.[14]

[14] According to data from Statistical Norway, the value of financial derivatives in 2024q1 represents less than 0.0004% of assets in the household sector.

Market clearing. For now, we have focused on the partial equilibrium setup of an individual i facing an exogenous set of financial assets big enough that markets are complete. We now use these results to study the welfare gain of a shock in a general equilibrium economy in which markets are complete. Note that this is a strong assumption in our context, as, in particular, newborns cannot perfectly insure themselves with respect to the state of the economy they are born in.

More precisely, we now consider a general equilibrium endowment economy with I individuals and an exogenous path for individual labor income $Y_{i,t}$ and dividend income $\{D_{k,t}\}_k$. For simplicity, we normalize each asset's total number of shares to one. Equilibrium asset prices are such that total consumption across individuals equals total income:

$$\sum_{i=1}^{I} C_{i,t} = \sum_{i=1}^{I} Y_{i,t} + \sum_{k=1}^{K} D_{k,t}.$$

Differentiating with respect to the realization of $s_0 = \widehat{s}$ relative to $s_0 = \bar{s}$, and combining with Equation (A11) gives

$$\frac{\Delta \Lambda_t}{\Lambda_t} = -\frac{1}{\sum_{i=1}^{I} \frac{C_{i,t}}{\sum_{j=1}^{I} C_{j,t}} \frac{1}{\gamma(C_{i,t})}} \frac{\Delta\left(\sum_{i=1}^{I} Y_{i,t} + \sum_{k=1}^{K} D_{k,t}\right)}{\sum_{j=1}^{I} Y_{j,t} + \sum_{k=1}^{K} D_{k,t}} + o(\theta).$$

This equation expresses the (equilibrium) effect of the realization of $s_0 = \widehat{s}$, relative to $s_0 = \bar{s}$, on the stochastic discount factor in terms of its (exogenous) effect on aggregate labor and dividend income, divided by the consumption-weighted harmonic mean of the relative risk aversion of individuals in the economy. Combining this formula with (A8′) gives the welfare gain from the realization of $s_0 = \widehat{s}$, relative to $s_0 = \bar{s}$, for each individual:

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\frac{\frac{C_{i,t}}{\gamma(C_{i,t})}}{\sum_{j=1}^{I} \frac{C_{j,t}}{\gamma(C_{j,t})}} \right) \Delta \left(\sum_{i=1}^{I} Y_{i,t} + \sum_{k=1}^{K} D_{k,t} \right) \right] + o(\theta). \tag{A8″}$$

This expression recovers the general idea that, with complete markets, agents' welfare is only exposed to shocks that affect aggregate income.

To conclude this section, note that we have derived three distinct expressions for the welfare gain resulting from a small shock: (A8), (A8′), and (A8″). The first expression(A8), which serves as our sufficient statistic, is the most appropriate formula to take to the data, as it holds regardless of whether markets are complete; in contrast, (A8′) requires that the agent can trade a set of financial assets large enough to span all contingencies, while (A8″) requires that *all* agents in the economy can do so.[15]

A.3 Welfare gains in general equilibrium

We now discuss applying our sufficient statistic in a general equilibrium framework. Denote a set of fundamental economic parameters by a vector $\{z_t\}_{t=0}^{\infty}$, which could reflect technological productivity, policy, and so on. The thought experiment that we consider in this context is to perturb the path of z_t by $\theta \Delta z_t$, where θ scales the deviation of the shock, and compute its welfare effect, both through its effect on asset prices and income (i.e., dividend and labor income): $\mathrm{d}P_{k,t} = \partial_\theta P_{k,t}\,\mathrm{d}\theta$, $\mathrm{d}Q_t = \partial_\theta Q_t\,\mathrm{d}\theta$, $\mathrm{d}D_{k,t} = \partial_\theta D_{k,t}\,\mathrm{d}\theta$, $\mathrm{d}Y_{i,t} = \partial_\theta Y_{k,t}\,\mathrm{d}\theta$. An application of Proposition A1 then yields

[15] Of course, the advantage of (A8′) and (A8″) is that they give sharper predictions on welfare gains, as they express them in terms of the (exogenous) shocks in aggregate endowment and household preferences rather than their (endogenous) portfolio decision.

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \Bigg(\underbrace{\sum_{k=1}^{K} (N_{i,k,t-1} - N_{i,k,t})\, dP_{k,t} - B_{i,t}\, dQ_t}_{\text{Effect of } z_t \text{ through asset prices}} + \underbrace{\sum_{k=1}^{K} N_{i,k,t-1} D_{k,t} + dY_{i,t}}_{\text{Effect of } z_t \text{ through income}} \Bigg). \tag{A12}$$

This equation quantify the redistributive effect of a fundamental shock to the economy, which impacts income and asset prices.

We now give a concrete example of asset-price redistribution in general equilibrium. We consider an environment with two assets: an asset in fixed supply (i.e., claim to land) and a reproducible asset (i.e., productive capital). We analyze the redistributive effects associated with a perturbation of cash flows and productivity, disentangling the impacts of asset prices versus income. We start with extending the baseline model with arbitrary household heterogeneity, then specialize to a two-period OLG model with closed-form solutions.

A.3.1 General equilibrium model with production

We now consider a general-equilibrium extension of the baseline model with production motivated by the verbal discussion in Krugman (2021).[16] There are two assets: elastically supplied productive capital and an inelastically supplied long-lived asset (land). Households can convert the final good into physical capital K_{t+1} one-for-one. Capital depreciates at rate $0 \leq \delta \leq 1$. There is a representative firm that operates an AK technology $Y_t = \tilde{A}_t K_t$. Denoting by $A_t \equiv \tilde{A}_t + 1 - \delta$ the total return to capital, the equilibrium rental rate of capital at time t must equal A_t. The other asset (land) can be traded without adjustment costs. As in the baseline model, N denotes the number of shares, D the cash flows per share, and P the price per share. The supply of shares is fixed and normalized to a total of one. The cash flows and productivity $\{D_t, A_t\}_{t=0}^{\infty}$ are exogenous, and land prices $\{P_t\}_{t=0}^{\infty}$ are endogenous.

The household problem is

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t+1}, K_{i,t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$
$$\text{s.t.} \quad C_{i,t} + (N_{i,t} - N_{i,t-1}) P_t + K_{i,t+1} = Y_{i,t} + N_{i,t} D_t + A_t K_{i,t}.$$

The equilibrium price P_t is the solution to a system of two equations:

$$\frac{P_{t+1} + D_{t+1}}{P_t} = A_t, \quad \sum_{i=1}^{I} N_{i,t} = 1.$$

First, the land price P_t must make households indifferent between investing in land or capital. Second, the sum of land holdings must equal its fixed supply $\overline{N}$. Since, in equilibrium, households are indifferent between the two assets, and therefore the equilibrium portfolios are indeterminate, we assume that they all hold the same portfolio shares: $\frac{K_{i,t}}{K_{i,t} + N_{i,t} P_t} = \frac{K_t}{K_t + P_t}$. Solving forward and assuming no bubbles, we obtain a closed-form expression for land prices:

$$P_0 = \sum_{t=0}^{\infty} A_{0\to t}^{-1} D_t,$$

where $A_{0\to t} = A_1 \cdot A_2 \cdots A_t$ is the cumulative gross rental rate of capital. No arbitrage implies that the equilibrium discount rate $R_{0\to t}^{-1} = A_{0\to t}^{-1}$ is simply pinned down only by productivity.

[16]We thank the editor Andy Atkeson for suggesting the material in this section.

Consider an arbitrary perturbation of productivity $\{dA_t\}_{t\geq 0}$. Even though the perturbation does not affect the cash flows generated by land, it does move the value of land due to its effect on equilibrium discount rates:

$$dP_0 = \sum_{t=0}^{\infty} d\left(A_{0\to t}^{-1}\right) D_t.$$

A negative productivity shock *increases* the value of land. A direct application of (A12) yields

$$dV_{i,0}/U'(C_{i,0}) = \underbrace{\sum_{t=0}^{\infty} A_{0\to t}^{-1}(N_{i,t-1} - N_{i,t})\, dP_t}_{\text{Welfare gain through asset prices}} + \underbrace{\sum_{t=0}^{\infty} A_{0\to t}^{-1} K_{i,t}\, dA_t}_{\text{Welfare gain through income}}.$$

The first term, "Welfare gain through asset prices," sums up to zero across households because financial assets are in fixed supply. However, the second term, which accounts for the increased capital income, does not sum up to zero. The idea is that productivity shocks not only lead to *aggregate* changes in consumption, but they also lead to redistribution between buyers and sellers of land due to their effect on discount rates.

The cross-sectional correlation between the two components of welfare change will ultimately depend on the details of the model (i.e., the economic forces that determine household-level financial transactions). We now consider a special case of the model where we can sign the total welfare gains across old and young, both for productivity shocks and cash-flow shocks.

A.3.2 An analytically tractable special case: two-period OLG

(Unless stated otherwise, we use the same notation as in the model above.) Consider an economy where a new cohort is born each year $t \geq -1$. Each cohort lives for two periods and has a subjective discount factor $\beta \in (0,1)$. Households in cohort t are endowed with labor income Y_t when young and zero when old. The cohort born at $t = -1$ is endowed with all the land ($N_{-1} = 1$). As before, denote the (ex-dividend) price of land at time t as P_t, and the one-period holding return on land by $R_{t+1} = (D_{t+1} + P_{t+1})/P_t$, where D_t is the dividend from land at time t. Denote by N_t the share of land owned by cohort t at the end of period t.

Setup and equilibrium. The problem of the young in period $t \geq 0$ is

$$\begin{aligned} V_t = &\max_{C_t, C_t', N_t, K_{t+1}} \log(C_t) + \beta \log(C_t'), \\ &\text{s.t.} \quad C_t + N_t P_t + K_{t+1} = Y_t \\ &\qquad\;\; C_t' = N_t(D_{t+1} + P_{t+1}) + A_{t+1} K_{t+1}, \end{aligned}$$

where C_t and C_t' denote consumption when young and old, respectively.

The equilibrium is characterized by individual optimization and market clearing,[17] with a familiar

[17] The equilibrium at time t is characterized by three equations in three unknowns (C_t, K_{t+1}, P_t):

$$\begin{aligned} \frac{1}{C_t} &= \beta \frac{R_{t+1}}{D_{t+1} + P_{t+1} + A_{t+1} K_{t+1}}, \\ \frac{1}{C_t} &= \beta \frac{A_{t+1}}{D_{t+1} + P_{t+1} + A_{t+1} K_{t+1}}, \\ C_t &= Y_t - P_t - K_{t+1}. \end{aligned}$$

In addition, we impose a restriction on the primitives $\{A_t, D_t\}_{t=0}^{\infty}$ to ensure no bubbles: $\lim_{T\to\infty} \frac{D_t}{P_t} = +\infty$ (see Hirano and Toda, accepted). As before, we assume that all households within a cohort hold the same portfolio.

solution given by

$$C_t = \frac{1}{1+\beta} Y_t, \quad C'_t = D_{t+1} + P_{t+1} + A_{t+1} K_{t+1}$$
$$K_{t+1} = \frac{\beta}{1+\beta} Y_t - P_t., \quad P_t = \sum_{s \geq t+1} A^{-1}_{t \to t+s} D_s.$$

In other words, the young save a fraction $\frac{1}{1+\beta}$ of their income, purchase all of the land from the old, and invest the rest in physical capital. No arbitrage implies that land value is pinned down by future cash flows discounted using the return on capital.

Comparative static. Two things matter for equilibrium land prices: future productivity and cash flows. We now do a comparative static on A_1 and D_1 at time $t = 0$ (i.e., transitory MIT shock). We call the generations born at $t = -1$ and $t = 0$ respectively "the old" and "the young". The old's value function is simply $V_{-1} = \beta \log C'_{-1}$. Applying (A12), which decomposes total welfare gains into the contribution of asset prices and income, we have:

$$\begin{array}{llcrcr}
\text{(Welfare gain of the old)} & \mathrm{d}V'_{-1}/U'(C'_{-1}) & = & \mathrm{d}P_0 & + & 0 \\
\text{(Welfare gain of the young)} & \mathrm{d}V_0/U'(C_0) & = & -\,\mathrm{d}P_0 & + & A_1^{-1}\Big(\mathrm{d}D_1 + K_1\,\mathrm{d}A_1\Big) \\
\hline
\text{(Sum of welfare gains)} & & = & 0 & + & A_1^{-1}\Big(\mathrm{d}D_1 + K_1\,\mathrm{d}A_1\Big).
\end{array}$$

As in Proposition A1, welfare gains are the sum of two terms. The first term is the contribution of changes in asset prices, which has opposite signs for the young (who buy the asset) and the old (who sell the asset). The second term is the contribution of (capital) income.

Table A1: Welfare gain decomposition in general equilibrium

	Welfare gains		
	Through prices	Through income	Total
Productivity shock ($\mathrm{d}A < 0$)			
Old	+	0	+
Young	-	-	-
Aggregate	0	-	-
Cash-flow shock ($\mathrm{d}D > 0$)			
Old	+	0	+
Young	-	+	0
Aggregate	0	+	+

Notes. "Total" is the sum of welfare effect within group; "Aggregate" is the sum of a welfare effect across groups; "+" means positive, "-" means negative.

Using the fact that $\mathrm{d}P_t = 0$ for all $t \geq 1$, we have a closed form solution for the land price deviation: $\mathrm{d}P_0 = (-P_0\,\mathrm{d}A_1 + \mathrm{d}D_1)$. This equation allows us to sign the total welfare gains in the model. Table A1 reports the results. For the productivity shock experiment, we consider $\mathrm{d}A_1 < 0$, and for the cash-flow shock, we consider $\mathrm{d}D_1 > 0$. In both cases, land prices increase (i.e., $\mathrm{d}P_0 > 0$), which implies a redistribution from young to old. However, the contribution of income depends on the shock. In the case of the productivity shock, income declines for the young. In the case of the cash-flow shock, income increases for the young. Summing up, the total welfare gain associated with these shocks is always positive for the old but can be negative or zero for the young.

Stochastic shock. So far, we have considered a comparative static experiment. Following the logic from Appendix A.2.2, we can instead consider the welfare effect of a stochastic shock between two different states.

Suppose that the economy is deterministic as before, except that, at time $t = 0$, the state $s \in \{\bar{s}, \widehat{s}\}$ is drawn with probability $(1 - \pi, \pi)$, which determines the path of exogenous variables $\{A_t(s), D_t(s)\}_{t=0}^{\infty}$. We consider the same experiment as in the deterministic case (i.e., one-time shock). Denoting the size of the shocks to productivity and cash flows by $\Delta A_t \equiv A_t(\widehat{s}) - A_t(\bar{s})$ and $\Delta D_t \equiv D_t(\widehat{s}) - D_t(\bar{s})$, we consider the temporary shock.

$$(\Delta A_t, \Delta D_t) = \begin{cases} (\theta \Delta A, \theta \Delta D), & \text{if } t = 1 \\ 0, & \text{if } t \neq 1, \end{cases}$$

where the parameter θ indexes the size of the shock. Similarly, define the difference in welfare between the two states by $\Delta V_t = V_t(\widehat{s}) - V_t(\bar{s})$.

There are only two possible states of the world and two assets. As a result, markets are complete from the point of view of agents alive at time -1. However, because this cohort cannot trade securities with unborn agents (the young that will arrive at time $t = -1$), they cannot share risk across generations. Hence, the exposure of each type of agent remains unchanged compared to the economy with MIT shocks, and the welfare gains associated with state $s = \widehat{s}$ occurring, relative to $s = \bar{s}$ are given by:

$$\begin{array}{llcrcr} \text{(Welfare gain of the old)} & \Delta V_{-1}/U'(C'_{-1}) & \approx & \Delta P_0 & + & 0 \\ \text{(Welfare gain of the young)} & \Delta V_0/U'(C_0) & \approx & -\Delta P_0 & + & A_1^{-1}\Big(\Delta D_1 + K_1 \Delta A_1\Big) \\ \hline \text{(Sum of welfare gains)} & & \approx & 0 & + & A_1^{-1}\Big(\Delta D_1 + K_1 \Delta A_1\Big). \end{array}$$

The formulas for welfare gains are the same as in the economy with MIT shocks, except that they are now approximate, as the shocks are non-infinitesimal. In particular, the comparative statics from Table A1 remain correct.

Stochastic shocks with ex-ante risk-sharing. Suppose that the environment is exactly as before, except that we now allow young agents, who will only arrive in the economy at time $t = 0$, to trade with other agents at time $t = -1$; that is, before they are "born" — in other words, we are breaking the typical OLG incompleteness. Because there are two possible states and two assets (land and capital), markets are complete, and so, applying the general results in Appendix A.2.2:

$$\frac{\Delta C_0}{C_0} = \frac{\Delta C'_{-1}}{C'_{-1}} = 0.$$

This equation says that the realization of the state $s \in \{\bar{s}, \widehat{s}\}$ does not affect the consumption of either cohort at $t = 0$. This equation is derived as follows: complete markets imply that agents equate the relative growth of their consumption among each other (see Equation A11). Because aggregate income at $t = 0$ is not affected by the realization of the shock, market clearing then implies that the relative growth of consumption is zero for each agent.

How can agents implement this allocation? One way to do so is for the young to buy all the land from the old at $t = -1$ while selling them capital (or issuing riskless bonds) to finance their purchases. In this case, no land sale would be observed at $t = 0$ (i.e., after the realization of the shock), and so applying our sufficient statistic formula would give:

$$\begin{array}{lrcrcr}
\text{(Welfare gain of the old)} & \Delta V_{-1}/U'(C'_{-1}) & \approx & 0 \times \Delta P_0 & + & 0 \\
\text{(Welfare gain of the young)} & \Delta V_0/U'(C_0) & \approx & -0 \times \Delta P_0 & + & A_1^{-1}\Big(\Delta D_1 + K_1 \Delta A_1\Big) \\
\hline
\text{(Sum of welfare gains)} & & \approx & 0 & + & A_1^{-1}\Big(\Delta D_1 + K_1 \Delta A_1\Big).
\end{array}$$

While this ex-ante trading stage affects trading patterns and the welfare gains of an aggregate shock, our sufficient statistic formula, which pins down welfare gains *given* trading patterns, remains valid (see the discussion in Appendix A.2.2). Finally, note that another way to implement this allocation would be for agents to use future contracts instead of buying land early; that is, to use derivatives, allowing them to "lock in" the price of the transaction ex-ante. In this case too, our sufficient statistic formula would remain correct, since there would be no deviation in the transaction price of the asset exchanged by individuals.[18]

One key takeaway from this exercise is that, faced with a shock that affects asset prices at some time $t = 0$ (but that *do not* impact aggregate income at time $t = 0$), optimizing agents would prefer to trade assets *before* the shock takes place to avoid any type of redistributive effect ex-post. Of course, it is not always possible to do so, as, in reality, existing generations cannot contract with future (unborn) generations.

A.4 Extensions

A.4.1 Individual preferences

Assets in the utility function. We now examine the welfare effect of asset-price deviations in the presence of assets in the utility function. Our main finding is that whether or not agents directly derive utility from asset ownership does not matter for our sufficient statistic: it is only when agents directly derive utility from the price of these assets that the welfare-gains formula needs to be adjusted.

For simplicity, we consider a two-asset version of the baseline model:

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + B_{i,t-1}.$$

The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = U(C_{i,t})$, coincides with the baseline model (i.e., assets ownership does not affect flow utility directly). The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = U(C_{i,t}, N_{i,t}, B_{i,t})$ means that individuals value the number of assets that they own directly (e.g., agents value owning a house relative to renting it). The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = N_{i,t}P_t + B_{i,t}Q_t$ means that individuals value the market value of their wealth directly.

Proposition A3. *In the presence of assets in the utility function, the welfare gain implied by a price deviation*

[18] In the context of changes in interest rates, this could also be implemented by interest rate swaps.

$\{\mathrm{d}Q_t, \mathrm{d}P_t\}_{t=0}^{\infty}$ *is:*

$$\frac{\mathrm{d}V_{i,0}}{\partial_C U(C_{i,0}, N_{i,0}, P_0, B_{i,0}, Q_0)} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t}) \, \mathrm{d}P_t - B_{i,t} \, \mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\frac{\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} \, \mathrm{d}P_t + \frac{\partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} \, \mathrm{d}Q_t \right).$$

Proof of Proposition A3. The Lagrangian associated with the individual problem is

$$\mathcal{L}_i = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) + \sum_{t=0}^{\infty} \lambda_{i,t} \left(Y_{i,t} + N_{i,t-1} D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1}) \, P_t - B_{i,t} Q_t - \chi(N_{i,t} - N_{i,t-1}) \right).$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned}
\mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} \, \mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} \, \mathrm{d}Q_t, \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left(- (N_{i,t} - N_{i,t-1}) \, \mathrm{d}P_t - B_{i,t} \, \mathrm{d}Q_t \right) \\
&\quad + \sum_{t=0}^{\infty} \beta^t \left(\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) \, \mathrm{d}P_t + \partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) \, \mathrm{d}Q_t \right) \\
&= \partial_C U(C_{i,0}, N_{i,0}, P_0, B_{i,0}, Q_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t}) \, \mathrm{d}P_t - B_{i,t} \, \mathrm{d}Q_t \right) \\
&\quad + \partial_C U(C_{i,0}, N_{i,0}, P_0, B_{i,0}, Q_0) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\frac{\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} \, \mathrm{d}P_t + \frac{\partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} \, \mathrm{d}Q_t \right),
\end{aligned}$$

where the last line uses the FOC with respect to $B_{i,t}$, $\lambda_{i,t} Q_t = \lambda_{i,t+1}$, as well as the FOC with respect to $C_{i,t}$, $\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = \lambda_{i,t}$. □

Endogenous labor supply. We now examine the welfare effect of price deviations in the case where the agent optimally chooses labor supply. Our main finding is that this extension does not affect the welfare gains of an infinitesimal deviation in asset prices as a result of the envelope theorem.

Formally, we consider the following two-asset model. Agents maximize the utility from consumption and leisure

$$V_{i,0} = \max_{\{C_{i,t}, L_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1 - L_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1}) P_t + B_{i,t} Q_t + \chi(N_{i,t} - N_{i,t-1}) = w_{i,t} L_{i,t} + B_{i,t-1}.$$

Note that, relative to the baseline model, labor income takes the form $w_{i,t} L_{i,t}$ where $w_{i,t}$ denotes the wage and $L_{i,t}$ denotes the quantity of labor, which the agent optimally chooses to maximize utility. We assume that $U(\cdot, \cdot)$ is increasing and concave with respect to each of its arguments.

Proposition A4. *The welfare gain implied by a price deviation* $\{\mathrm{d}Q_t, \mathrm{d}P_t\}_{t=0}^{\infty}$ *remains the same in the presence* *of endogenous labor supply; that is, the welfare gain is:*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t})\, \mathrm{d}P_t - B_{i,t}\, \mathrm{d}Q_t \right).$$

Proof of Proposition A4. The Lagrangian associated with the individual problem is

$$\begin{aligned}\mathcal{L}_i &= \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1 - L_{i,t}) \\ &+ \sum_{t=0}^{\infty} \lambda_{i,t} \left(w_t L_{i,t} + N_{i,t} D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})\, P_t - B_{i,t} Q_t - \chi(N_{i,t} - N_{i,t-1}) \right).\end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned}\mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t}\, \mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t}\, \mathrm{d}Q_t, \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} \left(-\left(N_{i,t} - N_{i,t-1}\right) \mathrm{d}P_t - B_{i,t}\, \mathrm{d}Q_t \right) \\ &= \partial_{C_{i,0}} U(C_{i,0}, 1 - L_{i,0}) \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t})\, \mathrm{d}P_t - B_{i,t}\, \mathrm{d}Q_t \right),\end{aligned}$$

where the last line uses the FOC with respect to $B_{i,t}$, $\lambda_{i,t} Q_t = \lambda_{i,t+1}$ and the FOC with respect to $C_{i,0}$, $\partial_{C_{i,0}} U(C_{i,0}, 1 - L_{i,0}) = \lambda_{i,0}$. □

A.4.2 Finite lives and bequests

We now examine the welfare effect of asset-price deviations when individuals have finite lives. For pedagogical reasons, we first consider the case in which individuals do not care about future generations and hence die with exactly zero assets. We then move on to the more realistic case in which individuals care about future generations, distinguishing between the altruistic and the "warm-glow" bequest motives. Finally, we discuss the extent to which our notion of welfare gains still captures the present value of the deviation of consumption with inter-generational linkages.

Finite lives. We consider an individual who will die with certainty at some horizon T with no offspring and hence no reason to leave bequests. We restrict ourselves to a two-asset version of the baseline model for simplicity. More precisely, the individual solves the following optimization problem:

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{T} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t = 0, 1, ..., T$

$$C_{i,t} + (N_{i,t} - N_{i,t-1}) P_t + B_{i,t} Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + N_{i,t-1} D_t + B_{i,t-1},$$

and with terminal holdings $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$. Note that these constraints will bind at the optimum, implying that the individual will deplete their assets completely by the end of their life.[19] These sales then show up in our welfare-gains formula, as shown in the following proposition.

[19] Alternatively, we could impose the weaker terminal condition that terminal wealth is non-negative; that is, $N_{i,T} P_T + B_{i,T} Q_T \geq 0$. In this alternative formulation, the individual is allowed to die with debt $B_{i,T} Q_T < 0$, in which case an asset sale $N_{i,T} P_T$ would happen right after the individual's death, so as to pay off this debt (e.g.,

Proposition A5. *For an individual with a finite life of length T and no offspring, the welfare gain of an infinitesimal deviation in asset prices is*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{T} R_{0\to t}^{-1}\left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right). \tag{A13}$$

This result is the same as the one obtained in the baseline model (i.e., for the case of an infinitely-lived individual), except for the presence of T in the summation. Intuitively, the individual optimally sells off all of their assets before they die. When asset valuations rise, this generates a welfare gain. While finite lives result in a different time path for optimal asset transactions, the way these asset transactions show up in our welfare-gains formula is the same as with infinitely-lived individuals.

Proof of Proposition A5. The Lagrangian associated with the individual problem is

$$\begin{aligned}\mathcal{L}_i &= \sum_{t=0}^{T} \beta^t U(C_{i,t}) \\ &+ \sum_{t=0}^{T} \lambda_{i,t}\left(Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})\,P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})\right) \\ &+ \mu_{i,N}N_{i,T} + \mu_{i,B}B_{i,T},\end{aligned}$$

where $\mu_{i,N}$ and $\mu_{i,B}$ are the Lagrange multipliers on the terminal conditions $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$. Totally differentiating the welfare function using the envelope theorem and following the same steps as in the proof of Proposition 1

$$\begin{aligned}\mathrm{d}V_{i,0} &= \sum_{t=0}^{T} \frac{\partial \mathcal{L}_i}{\partial P_t}\,\mathrm{d}P_t + \sum_{t=0}^{T} \frac{\partial \mathcal{L}_i}{\partial Q_t}\,\mathrm{d}Q_t, \\ &= \sum_{t=0}^{T} \lambda_{i,t}\left(-\left(N_{i,t} - N_{i,t-1}\right)\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right) \\ &= U'(C_{i,0}) \sum_{t=0}^{T} R_{0\to t}^{-1}\left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right),\end{aligned}$$

where the second equality uses that the terms $\mu_{i,N}N_{i,T}$ and $\mu_{i,B}B_{i,T}$ in the Lagrangian do not depend on asset prices $\{P_t, Q_t\}_{t=0}^{T}$. □

conducted by the bank or executor of the individual's will). Differentiating the corresponding Lagrangian

$$\begin{aligned}\mathcal{L}_i = \sum_{t=0}^{T} \beta^t U(C_{i,t}) + \sum_{t=0}^{T} \lambda_{i,t}\left(Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - \left(N_{i,t} - N_{i,t-1}\right)P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})\right) \\ + \mu_i(N_{i,T}P_T + B_{i,T}Q_T)\end{aligned}$$

and using the first-order conditions for $N_{i,T}$ and $B_{i,T}$, $\lambda_{i,T}P_T = \mu_i P_T$ and $\lambda_{i,T}Q_T = \mu_i Q_T$, would lead to the following formula for welfare gains

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{T} R_{0\to t}^{-1}\left(\left(N_{i,t-1} - N_{i,t}\right)\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right) + R_{0\to T}^{-1}(N_{i,T}\,\mathrm{d}P_T + B_{i,T}\,\mathrm{d}Q_T).$$

This formula is the same as (A13) in Proposition A5, although one needs to take into account the additional asset sales at time T right after the person's death. The intuition is that there is no economic difference in whether the person makes these sales before they die (the formulation in the main text) or the executor of the will doing it (the formulation in this footnote).

Finite lives with altruistic preferences. We now consider an individual with altruistic preferences. More precisely, we consider the following value function for individual i

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}, G_{N,i,t}, G_{B,i,t}\}} \sum_{t=0}^{T} \beta^t U(C_{i,t}) + \delta V_{j,0}, \tag{A14}$$

which captures the fact that the individual cares about their consumption and the one of some agent j, which could represent an heir or a parent. We allow individual i to transfer assets to individual j at any point in time and so the sequence of budget constraints becomes

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t, \tag{A15}$$

where $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ denote a path of transfers (through bequests or inter-vivos transfers) from individual i to individual j. As above, we assume that terminal holdings satisfy $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$.

Consider a small deviation in the path of asset prices. Similar to the baseline model, we define the welfare gain of the deviation as the amount of money received at $t = 0$ that would generate an equivalent change in the welfare of individual i (i.e., equivalent variation). Because of altruistic preferences, we need to specify what happens to agent j across these two counterfactuals. Specifically, we assume that the value function of agent j is held constant across these two counterfactuals. This can be interpreted as the equivalent variation for individual i under the assumption that the deviation in asset prices affects only them, or, alternatively, as the equivalent variation for individual *i assuming that agent j has already received their equivalent variation*.

Proposition A6. *With finite lives and altruistic preferences, the welfare gain of an infinitesimal deviation in asset prices is:*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{T} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t} - G_{N,i,t}) \, \mathrm{d}P_t - (B_{i,t} - G_{B,i,t}) \, \mathrm{d}Q_t \right).$$

Proof of Proposition A6. The Lagrangian associated with the optimization problem of individual i is

$$\mathcal{L}_i = \sum_{t=0}^{T} \beta^t U(C_{i,t}) + \delta V_{j,0} + \sum_{t=0}^{T} \lambda_{i,t} \left(N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t \right.$$
$$\left. - C_{i,t} - (N_{i,t} - N_{i,t-1}) \, P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1}) \right).$$

Totally differentiating the welfare function using the envelope theorem gives

$$\begin{aligned} \mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} \, \mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} \, \mathrm{d}Q_t, \\ &= \sum_{t=0}^{T} \lambda_{i,t} \left(- \left(N_{i,t} - N_{i,t-1} + G_{N,i,t} \right) \mathrm{d}P_t - \left(B_{i,t} + G_{B,i,t} \right) \mathrm{d}Q_t \right), \end{aligned}$$

where the second line uses the fact that, in our thought experiment, we set the partial derivative of the value function of j with respect to asset prices to zero. Combining the FOCs for the liquid asset and for consumption at time $t = 0$ gives $\lambda_{i,t} = U'(C_{i,0})R_{0\to t}^{-1}$. Plugging into the previous formula $\mathrm{d}V_{i,0}$ gives the result. □

Finite lives with warm glow preferences. We now discuss the alternative case where individuals have warm glow preferences; that is, individuals care about their own consumption and the value of their bequests. More precisely, we assume that the individual value function is

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}, G_{N,i,t}, G_{B,i,t}\}} \sum_{t=0}^{T} \beta^t \left(U(C_{i,t}) + \mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_t) \right), \tag{A16}$$

where $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ denote the path of asset transfers from individual i to other agents. Finally, we assume that the budget constraints are the same as the one with altruistic preferences (A15). As in the baseline model, we define the welfare gain as the amount of money that would generate an equivalent change in individual welfare.

While this type of preference is often used in the macro-literature (e.g., De Nardi, 2004), note that it is less adapted to welfare assessments than the altruistic model. To take an example, these preferences imply that parents strictly value a deviation that allows them to increase the bequest to their children by one dollar, even if the deviation simultaneously costs their children \$$X$ dollars, where X is an arbitrary number.

Proposition A7. *With finite lives and warm glow preferences, the welfare gain of an infinitesimal deviation in asset prices is:*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{T} R_{0\to t}^{-1} \Bigg((N_{i,t-1} - N_{i,t} - G_{N,i,t})\,\mathrm{d}P_t - (B_{i,t} + G_{B,i,t})\,\mathrm{d}Q_t + \frac{\frac{\partial \mathcal{U}}{\partial P_t}}{\frac{1}{Q_t}\frac{\partial \mathcal{U}}{\partial G_{i,B,t}}}\,\mathrm{d}P_t + \frac{\frac{\partial \mathcal{U}}{\partial Q_t}}{\frac{1}{Q_t}\frac{\partial \mathcal{U}}{\partial G_{i,B,t}}}\,\mathrm{d}Q_t \Bigg).$$

Relative to the sufficient statistic obtained in the baseline model (the one we take to the data), there is an additional term that accounts for the impact of warm glow preferences.[20] The key takeaway of this extension is the same as in Appendix A.4.1 (which examines the case of assets in the utility function): the welfare formula only gains an additional term when agents care about the value of asset prices per se.

More specifically, when individuals only care about the quantity of assets bequeathed to their heirs rather than their price (i.e., $\mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_T) = \mathcal{U}(G_{i,N,t}, G_{i,B,t})$), the additional term attributable to warm glow preferences reduces to zero. In the context of housing, this assumption implies that parents value the physical quantity of real estate (e.g., square meters) passed on to their children, rather than its market value. A contrasting special case occurs when individuals care instead about the total market value of assets transferred to their heirs (i.e., $\mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_T) = \mathcal{U}(G_{i,N,t}P_t + G_{i,B,t}Q_t)$). Under this specification, the additional term associated with warm glow preferences simplifies to $\sum_{t=0}^{T} R_{0\to T}^{-1}(G_{i,N,t}\,\mathrm{d}P_t + G_{i,B,t}\,\mathrm{d}Q_T)$, representing the present value of the effect of price deviations on the market value of transferred assets.

Proof of Proposition A7. The Lagrangian associated with the optimization problem of individual i is

$$\mathcal{L}_i = \sum_{t=0}^{T} \beta^t \left(U(C_{i,t}) + \mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_t) \right) + \sum_{t=0}^{T} \lambda_{i,t} \Big(N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t - C_{i,t} - (N_{i,t} - N_{i,t-1})\,P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1}) \Big).$$

[20]This additional term can be interpreted as the amount the parent could consume to maintain a constant utility value of bequests despite the deviation in asset valuations.

The envelope theorem gives

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \beta^t \left(\frac{\partial \mathcal{U}}{\partial P_t} + \frac{\partial \mathcal{U}}{\partial Q_t} \right) + \sum_{t=0}^{T} \lambda_{i,t} \left(-\left(N_{i,t} - N_{i,t-1} + G_{N,i,t}\right) dP_t - \left(B_{i,t} + G_{i,t}\right) dQ_t \right). \end{aligned}$$

Using the FOC with respect to inter-vivos transfers of the liquid asset at time t gives $\beta^t \frac{\partial \mathcal{U}}{\partial G_{i,B,t}} = \lambda_{i,t} Q_t$. Combining with the previous equation gives

$$dV_{i,0} = \sum_{t=0}^{T} \lambda_{i,t} \left(-\left(N_{i,t} - N_{i,t-1} + G_{N,i,t}\right) dP_t - \left(B_{i,t} + G_{i,t}\right) dQ_t + \frac{\frac{\partial \mathcal{U}}{\partial P_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dP_t + \frac{\frac{\partial \mathcal{U}}{\partial Q_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dQ_t \right).$$

Combining the FOCs for the liquid asset and for consumption at time $t = 0$ gives $\lambda_{i,t} = U'(C_{i,0}) R_{0\to t}^{-1}$. Plugging into the previous formula $dV_{i,0}$ gives the result. □

PV of consumption response with inter-vivos transfers. In the baseline infinite-horizon model, our sufficient statistic formula for welfare gains (i.e., the present value of asset transactions interacted by deviations in asset prices) equals the present value of the actual response in individual consumption to the deviation in asset prices.[21] This equivalence no longer applies in a model with inter-generational links since agents can use their trading profits to either increase consumption or transfer more to other agents. We now show that, with inter-vivos transfers, our sufficient statistic formula corresponds to the present value of the actual response in individual consumption *plus* in individual net transfers.

More precisely, we consider a model where individuals have *either* altruism or warm glow preferences (i.e., with a value function given by Equation A14 or Equation A16). Agents can give or receive assets over time, and we denote $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ the path of net inter-vivos gifts from individual i. We consider a deviation in the path of asset prices $\{dP_t, dQ_t\}_{t=0}^{\infty}$. Differentiating the individual budget constraint (A15) gives

$$\begin{aligned} &dC_{i,t} + P_t(dN_{i,t} - dN_{i,t-1}) + (N_{i,t} - N_{i,t-1})\, dP_t + Q_t\, dB_{i,t} + B_{i,t}\, dQ_t + \chi'(N_{i,t} - N_{i,t-1})(dN_{i,t} - dN_{i,t-1}) \\ &\quad = D_t\, dN_{i,t-1} + dB_{i,t-1} - P_t\, dG_{N,i,t} - G_{N,i,t}\, dP_t - Q_t\, dG_{B,i,t} - G_{B,i,t}\, dQ_t. \end{aligned}$$

Taking the present value, aggregating across time periods, and rearranging gives

$$\begin{aligned} \sum_{0}^{T} R_{0\to t}^{-1} \left(dC_{i,t} + P_t\, dG_{N,i,t} + Q_t\, dG_{B,i,t}\right) &= \sum_{t=0}^{T} R_{0\to t}^{-1} \left(\left(N_{i,t-1} - N_{i,t} - G_{N,i,t}\right) dP_t - \left(B_{i,t} + G_{B,i,t}\right) dQ_t\right) \\ &\quad + \sum_{t=0}^{T} R_{0\to t}^{-1} \left(D_t\, dN_{i,t-1} - P_t(dN_{i,t} - dN_{i,t-1}) - \chi'(N_{i,t} - N_{i,t-1})(dN_{i,t} - dN_{i,t-1})\right) \\ &\quad + \sum_{t=0}^{T} R_{0\to t}^{-1} \left(dB_{i,t-1} - Q_t\, dB_{i,t}\right). \end{aligned}$$

[21] See Proposition A13 for a formal statement.

Rearranging the last two terms:

$$\sum_{0}^{T} R_{0\to t}^{-1} \left(\mathrm{d}C_{i,t} + P_t \,\mathrm{d}G_{N,i,t} + Q_t \,\mathrm{d}G_{B,i,t}\right) = \sum_{t=0}^{T} R_{0\to t}^{-1} \left(\left(N_{i,t-1} - N_{i,t} - G_{N,i,t}\right) \mathrm{d}P_t - \left(B_{i,t} + G_{B,i,t}\right) \mathrm{d}Q_t\right)$$
$$+ \sum_{t=0}^{T} R_{0\to t}^{-1} \left(R_{t+1}^{-1} \left(D_{t+1} + P_{t+1} + \chi'(N_{i,t+1} - N_{i,t})\right) - \chi'(N_{i,t} - N_{i,t-1}) - P_t\right) \mathrm{d}N_{i,t}$$
$$+ \sum_{t=0}^{T} R_{0\to t}^{-1} \left(R_{t+1}^{-1} - Q_t\right) \mathrm{d}B_{i,t}.$$

The FOCs for asset holdings $N_{i,t}$ implies that the second line is zero while the definition of R_{t+1} implies that the third line is zero, and so we get:

$$\underbrace{\sum_{0}^{T} R_{0\to T}^{-1} \left(\mathrm{d}C_t + P_t \,\mathrm{d}G_{N,i,t} + Q_t \,\mathrm{d}G_{B,i,t}\right)}_{\text{PV of response of consumption + net transfers}} = \underbrace{\sum_{t=0}^{T} R_{0\to t}^{-1} \left(\left(N_{i,t-1} - N_{i,t} - G_{N,i,t}\right) \mathrm{d}P_t - \left(B_{i,t} + G_{B,i,t}\right) \mathrm{d}Q_t\right).}_{\text{PV of trading profits (sufficient statistic)}}$$

This equation shows that, in the presence of inter-vivos transfers, our baseline sufficient statistic formula equals the present value of the response of consumption and net transfers to the deviation in asset prices. Intuitively, this equation says that, in a present-value term, any increase in trading profits due to the deviation in asset prices must be consumed or transferred to other agents.

One implication is that, in the presence of intergenerational transfers, our baseline sufficient statistic formula captures the present value of the effect of asset prices on consumption only under the additional assumption that the *quantity* of assets given through inter-vivos transfers does not change in response to the deviation in asset prices. For instance, in the context of housing, this says that changes in asset prices do not affect the physical quantity of real estate (e.g., square meters) parents leave to their children.

In reality, this term could be positive or negative. On the one hand, agents may want to transfer more assets to their children in response to rising asset prices to compensate them for their welfare loss. On the other hand, agents may react to rising asset prices by borrowing more, leaving larger outstanding debt balances to their children (due to a pure substitution effect or a relaxation of borrowing constraints, as in Section IV.B).

A.4.3 Businesses

In the baseline model, we examined the welfare effect of changes in the path of the price of an asset $\{P_t\}_{t=0}^{\infty}$ holding constant its dividends $\{D_t\}_{t=0}^{\infty}$. However, this assumption is not adapted to businesses that themselves buy and sell financial assets, as changes in asset prices will typically affect their dividend payments. This appendix explains how we adapt our methodology to take into account such financial transactions by businesses. For example, it explains the reasoning behind our empirical measure for the equity-valuation ratio used in Section II.B, which is unaffected by share repurchases and is capital-structure neutral.

The case of share repurchase. It is helpful to start with an example in which a business can only make one type of financial transaction: repurchase its own shares. Formally, consider a business that produces an income stream (i.e., earnings minus investment) $\{\Pi_t\}_{t=0}^{\infty}$ from its fundamental (e.g., non-financial) operations. These cash flows are distributed to shareholders through both dividends and share repurchases:

$$\Pi_t = \mathcal{N}_{t-1} D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t) P_t, \tag{A17}$$

where D_t denotes the business dividends per share, P_t denotes the share price, and $\mathcal{N}_t$ denotes the total amount of outstanding shares. When $\mathcal{N}_t < \mathcal{N}_{t-1}$ the business is repurchasing its own shares. From this equation, it is already apparent that share repurchases and dividend payments are equivalent means of distributing cash flows $\{\Pi_t\}_{t=0}^{\infty}$ to shareholders as a whole (more on this shortly). As discussed above, the presence of share repurchases implies that changes in share prices will mechanically affect the path of dividends $\{D_t\}_{t=0}^{\infty}$, as higher share prices will force the firm to either spend more cash to buy the same amount of shares (which reduces dividends per share in the current period) or to buy fewer shares with the same amount of cash (which reduces dividends per share in future periods).

Let us consider the budget constraint of an individual i who, for simplicity, can only invest in the business:

$$(N_{i,t} - N_{i,t-1})P_t = N_{i,t-1}D_t + Y_{i,t} - C_{i,t}. \tag{A18}$$

When the business repurchases its shares (i.e., $\mathcal{N}_t < \mathcal{N}_{t-1}$) this results in an income stream $(N_{i,t-1} - N_{i,t})P_t$ for those individuals selling their shares to the business. Denoting by $s_{i,t} \equiv N_{i,t}/\mathcal{N}_t$ the individual's ownership share of the business, we can combine the individual and business budget constraints, (A18) and (A17), to obtain:

$$(N_{i,t} - N_{i,t-1} + s_{i,t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t))P_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}.$$

Denoting by $M_t \equiv \mathcal{N}_t P_t$ the market value of the business, we obtain:

$$(s_{i,t} - s_{i,t-1})M_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}. \tag{A19}$$

This budget constraint has the same form as (A18), except that (i) the dividend per share D_t is replaced by the income stream from operations Π_t, (ii) the price per share P_t is replaced by the market value of the firm M_t, and (iii) the number of shares held by the individuals $N_{i,t}$ is replaced by the ownership share in the business $s_{i,t}$. An alternative viewpoint on this consolidated budget constraint is to consider the return to investing in the business. As usual, the return implied by the non-consolidated budget constraint is $R_{t+1} \equiv (D_{t+1} + P_{t+1})/P_t$, i.e., the return is the sum of dividend yield and capital gains. Multiplying and dividing by $\mathcal{N}_t$, we have

$$R_{t+1} \equiv \frac{\mathcal{N}_t D_{t+1} + \mathcal{N}_t P_{t+1}}{\mathcal{N}_t P_t} = \frac{\mathcal{N}_t D_{t+1} + (\mathcal{N}_t - \mathcal{N}_{t+1})P_{t+1} + \mathcal{N}_{t+1}P_{t+1}}{\mathcal{N}_t P_t} = \frac{\Pi_{t+1} + M_{t+1}}{M_t},$$

where the last equality uses (A17) and the definition of the market value $M_t \equiv \mathcal{N}_t P_t$. Just like the consolidated budget constraint (A19), writing the return as $R_{t+1} = (\Pi_{t+1} + M_{t+1})/M_t$ again makes clear that what ultimately matters are the business's cash flows $\{\Pi_t\}_{t=0}^{\infty}$ and its market value $\{M_t\}_{t=0}^{\infty}$ and not whether cash flows are distributed to shareholders via dividend payouts or share repurchases.

In our baseline model, we examined the welfare effect of changes in the path of the price of an asset $\{P_t\}_{t=0}^{\infty}$ holding constant its dividends $\{D_t\}_{t=0}^{\infty}$. The consolidated budget constraint (A19) makes clear that, in the presence of share repurchases, the correct analogous experiment is instead to consider deviations in the market value of the business, $\{M_t\}_{t=0}^{\infty}$, holding constant its income stream $\{\Pi_t\}_{t=0}^{\infty}$. In particular, for investors as a whole, it is irrelevant whether the business increases its dividend payments or share repurchases; what matters instead is whether the firm's income stream changes $\{\Pi_t\}_{t=0}^{\infty}$.

Using a similar reasoning as in Proposition 1, we get that the welfare gain of the individual is

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}(s_{i,t-1} - s_{i,t})\,\mathrm{d}M_t. \tag{A20}$$

Hence, in the presence of share repurchases, what matters for welfare is not the number of shares N_t directly traded by the individual, but the overall change in his/her ownership share $s_{i,t}$ in the business. In particular, note that individual welfare gains still aggregate to zero, as ownership shares always aggregate to one in the population. Similarly, and as already noted, what matters is not deviations in the share price P_t holding constant dividends D_t but deviations in the market value $M_t = \mathcal{N}_t P_t$ holding constant the income stream Π_t.

One way to understand expression (A20) is to consider the case of a business that repurchases a given fraction of its shares every period. A rise in valuations benefits individuals who sell shares to the business while hurting the owners of the business as the business needs to spend more cash to purchase the same number of shares. The two effects compensate for individuals who sell a fraction of their holdings equal to the fraction of outstanding shares purchased by the business, i.e, who have $s_{i,t} = s_{i,t-1}$. On the other hand, for individuals who do not sell any of their shares to the business, $N_{i,t} = N_{i,t-1}$ so that $\mathcal{N}_{t-1} - \mathcal{N}_t > 0$ implies $s_{i,t} = N_{i,t}/\mathcal{N}_t < N_{t-1}/\mathcal{N}_{t-1} = s_{i,t-1}$, only the second effect is operational and hence those individuals lose from higher valuations.

The case of arbitrary financial transactions. We now consider the more general case where, every period, the business can (i) repurchase its own shares, (ii) buy and sell one-period bonds, and (iii) buy and sell K financial assets. The business budget constraint is:

$$\Pi_t + \sum_k \mathcal{N}_{k,t-1} D_{k,t} + \mathcal{B}_{t-1} = \mathcal{N}_{t-1} D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t)\, P_t + \sum_k (\mathcal{N}_{k,t} - \mathcal{N}_{k,t-1}) P_{k,t} + \mathcal{B}_t Q_t, \tag{A21}$$

where, as above, Π_t denotes the income stream of a business from its non-financial operations, D_t denotes dividends per share, and $\mathcal{N}_t$ denotes the total amount of outstanding shares. The new part is $\mathcal{N}_{k,t}$, which denotes asset holdings in asset k, and $\mathcal{B}_t$, which denotes bond holdings.

Let us consider an individual investing in K financial assets, one-period bonds, as well as in the business. The individual budget constraint is

$$\begin{aligned}(N_{i,t} - N_{i,t-1}) P_t + \sum_k (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} + B_{i,t} Q_t \\ = N_{i,t-1} D_t + \sum_k N_{i,k,t-1} D_{k,t-1} + B_{i,t-1} + Y_{i,t} - C_{i,t}.\end{aligned} \tag{A22}$$

Combining it with the business budget constraint (A21) gives the following consolidated budget constraint:

$$\begin{aligned}(N_{i,t} - N_{i,t-1} + s_{i,t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t))\, P_t + \sum_k (N_{i,k,t} - N_{i,k,t-1} + s_{i,t-1}(\mathcal{N}_{k,t-1} - \mathcal{N}_{k,t}))\, P_{k,t} + (B_{i,t} + s_{i,t-1}\mathcal{B}_t)\, Q_t \\ = s_{i,t-1}\Pi_t + \sum_k (N_{i,k,t-1} + s_{i,t-1}\mathcal{N}_{k,t-1})\, D_{k,t} + (B_{i,it-1} + s_{i,t-1}\mathcal{B}_{i,t-1}) + Y_{i,t} - C_{i,t},\end{aligned}$$

where, as above, $s_{i,t} \equiv N_{i,t}/\mathcal{N}_t$ denotes the individual ownership share in the business.

We can simplify this expression after denoting $\widetilde{N}_{i,k,t} \equiv N_{i,k,t} + s_{i,t}\mathcal{N}_{k,t}$ the individual's consolidated shares in asset k of the individuals through its ownership of the business, $\widetilde{B}_{i,t} \equiv B_{i,t} + s_{i,t}\mathcal{B}_t$ the individual's consolidated bond holdings, and $\widetilde{M}_{i,t} \equiv \mathcal{N}_{i,t} P_t - \mathcal{B}_{i,t} Q_t - \sum_k \mathcal{N}_{i,k,t} P_{k,t}$ the market value of the firm exclusive of financial assets:

$$(s_{i,t} - s_{i,t-1})\widetilde{M}_t + \sum_k (\widetilde{N}_{i,k,t} - \widetilde{N}_{i,k,t-1}) P_{k,t} + \widetilde{B}_{i,t} Q_t = s_{i,t-1}\Pi_t + \sum_k \widetilde{N}_{i,k,t-1} D_{k,t} + \widetilde{B}_{i,t-1} + Y_{i,t} - C_{i,t}.$$

This has the same form as (A22), except that (i) D_t, the business dividend per share of the business, is replaced by Π_t, the business income stream from its non-financial operations, (ii) P_t, the business price per share, is replaced by $\widetilde{M}_t$, the market value of its fundamental (non-financial) component, (iii) $N_{i,t}$, the number of shares held by the individual, is replaced by $s_{i,t}$, their ownership share in the business, and (iv) individual asset holdings in financial assets and one-period bonds, $\{N_{i,k,t}\}_k$ and $B_{i,t}$, are replaced by their consolidated ones, $\{\widetilde{N}_{i,k,t}\}_k$ and $\widetilde{B}_{i,t}$.

This budget constraint allows us to consider the welfare gain of a deviation in the market value of the fundamental component of a business, $\widetilde{M}_t$, holding constant its income stream Π_t, together with our usual deviations in asset prices $\{P_{k,t}\}_{k=0}^{K}, Q_t$:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\left((s_{i,t-1} - s_{i,t})\,\mathrm{d}\widetilde{M}_t + \sum_k (\widetilde{N}_{i,t-1} - \widetilde{N}_{i,t})\,\mathrm{d}P_{k,t} - \widetilde{B}_{i,t}\,\mathrm{d}Q_t\right).$$

This formula has two main takeaways relative to (A20). First, when measuring individual financial transactions, we should also account for all of the indirect transactions done through the businesses that they own (i.e., $\widetilde{N}_{i,t-1} - \widetilde{N}_{i,t}$ instead of $N_{i,t-1} - N_{i,t}$). Second, when measuring deviations in business valuations, we should only consider deviations in the market value of their non-financial components (i.e., $\mathrm{d}\widetilde{M}_t$ instead of $\mathrm{d}M_t$). Put differently, this formula tells us to split businesses between their financial and non-financial components, and assign their financial components to the individuals who ultimately own them.

A.4.4 Government

We now examine the welfare effect of asset-price deviations in the presence of government transfers. For simplicity, we consider a two-asset version of the baseline model. Suppose the government makes targeted transfers to individuals $i \in \{1, \dots, I\}$, where $T_{i,t}$ denotes the net amount of resources transferred from the government to individual i at time t. The individual problem is now given by

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + T_{i,t} + N_{i,t-1}D_t + B_{i,t-1}.$$

We assume that the government can trade both assets and thus faces, at each period $t \geq 0$, the following budget constraint:

$$(N_{G,t} - N_{G,t-1})P_t + B_{G,t}Q_t = N_{G,t-1}D_t + B_{G,t-1} - \sum_{i=1}^{I} T_{i,t} - \chi(N_{G,t} - N_{G,t-1}), \tag{A23}$$

where, for simplify, χ is assumed to be differentiable. We do not fully specify the government problem, but we assume that the government's portfolio choice satisfies the following cost-minimization condition

$$Q_t^{-1} = \frac{D_{t+1} + P_{t+1} - \chi'(N_{G,t+1} - N_{G,t})}{P_t + \chi'(N_{G,t} - N_{G,t-1})}, \tag{A24}$$

at every $t \geq 0$. The idea is that the government minimizes the cost of borrowing (or alternatively maximizes the return on saving) by adjusting portfolio shares until the marginal return on the long-lived asset (net of adjustment costs) is equalized with the bond return.

The following proposition characterizes the welfare gain in the presence of government transfers.

Proposition A8. *In the presence of government transfers, the welfare gain of individual i is*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t \right) + \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,\mathrm{d}T_{i,t}.$$

Moreover, the aggregate contribution of deviations in government transfers $\mathrm{d}T_{i,t}$ *to individual welfare is*

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1} \sum_{i=1}^{I} \mathrm{d}T_{i,t} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \right).$$

Proof of Proposition A8. The welfare gain formula follows immediately from the envelope theorem, as in the baseline model. This proof focuses on the second equation. Differentiating the government budget constraint (A23), we obtain

$$\begin{aligned}\sum_{i=1}^{I} \mathrm{d}T_{i,t} &= (N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \\ &\quad - \left(\chi'(N_{G,t} - N_{G,t-1}) + P_t\right)\mathrm{d}N_{G,t} + \Big(D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t\Big)\,\mathrm{d}N_{G,t-1} - Q_t\,\mathrm{d}B_{G,t} + \mathrm{d}B_{G,t-1}.\end{aligned}$$

The sum of aggregate net transfer deviations discounted using the liquid asset return is

$$\begin{aligned}\sum_{t=0}^{\infty} R_{0\to t}^{-1} \sum_{i=1}^{I} \mathrm{d}T_{i,t} &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \right) - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\chi'(N_{G,t} - N_{G,t-1}) + P_t \right) \mathrm{d}N_{G,t} \\ &\quad + \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t \right) \mathrm{d}N_{G,t-1} \\ &\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} Q_t\,\mathrm{d}B_{G,t} + \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,\mathrm{d}B_{G,t-1} \\ &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \right) - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(\chi'(N_{G,t} - N_{G,t-1}) + P_t \right) \mathrm{d}N_{G,t} \\ &\quad + \sum_{t'=-1}^{\infty} R_{0\to t'+1}^{-1} \left(D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1} \right) \mathrm{d}N_{G,t'} \\ &\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} Q_t\,\mathrm{d}B_{G,t} + \sum_{t'=-1}^{\infty} R_{0\to t'+1}^{-1}\,\mathrm{d}B_{G,t'} \\ &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \right) \\ &\quad - \sum_{t=0}^{\infty} R_{0\to t}^{-1} \Big(\chi'(N_{G,t} - N_{G,t-1}) + P_t - Q_t \left(D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1} \right) \Big) \mathrm{d}N_{G,t} \\ &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left((N_{G,t-1} - N_{G,t})\,\mathrm{d}P_t - B_{G,t}\,\mathrm{d}Q_t \right).\end{aligned}$$

The second equality uses a change of variables $t' \equiv t - 1$. The third equality uses the fact that $R_{0\to t+1}^{-1} = R_{0\to t}^{-1} Q_t$ as well as $\mathrm{d}N_{G-1} = \mathrm{d}B_{G-1} = 0$. The fourth equality uses the cost-minimization assumption (A24). □

The formula for the welfare gain of individual i differs from that in the baseline model, as it incorporates the present value of deviations in net government transfers. The reason is that the government might respond to asset price changes by adjusting net transfers. Moreover, the second part of Proposition A8 states that the discounted sum of aggregate net transfers to the household sector equals the "welfare gain of the government". Note that we obtain this result without making assumptions about the government's objective. It is merely a consequence of government budget constraints.

Taxes on assets In the baseline model, individuals pay no taxes on their asset holdings, transactions or income generated by these assets. We now consider an extension with four types of taxes: wealth taxes, asset transaction taxes, taxes on dividend income, and taxes on interest income.

Formally, we consider: (i) a non-linear wealth tax $\tau_{W,t}$ on the market value of wealth $N_{i,t-1}P_t$, (ii) a non-linear transaction tax $\tau_{\chi,t}$ on the market value of asset sales $(N_{i,t-1} - N_{i,t})P_t$, (iii) a dividend income tax $\tau_{D,t}$ on dividend income $N_{i,t-1}D_t$, and (iv) a linear tax $\tau_{Q,t}$ on interest income or equivalently on the cost of buying bonds B_tQ_t. Individuals maximize

$$V_{i,0} = \max_{\{C_{i,t},N_{i,t},B_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$\begin{aligned}(N_{i,t} - N_{i,t-1})P_t + \tau_{\chi,t}((N_{i,t-1} - N_{i,t})P_t) + \chi(N_{i,t} - N_{i,t-1}) + B_{i,t}Q_t(1 + \tau_{Q,t}) \\ = Y_{i,t} + N_{i,t-1}D_t - \tau_{D,t}(N_{i,t-1}D_t) - \tau_{W,t}(N_{i,t-1}P_t) + B_{i,t-1} - C_{i,t}.\end{aligned}$$

Here the functions $\tau_{\chi,t}(\cdot), \tau_{W,t}(\cdot), \tau_{D,t}(\cdot)$ are non-linear and potentially time-dependent tax functions. Such a specification allows us to capture several features of real-world tax systems. For example, transaction taxes often apply on both sales and purchases (i.e., $\tau_{\chi,t}(\cdot)$ may be positive and increasing when $N_{i,t-1} - N_{i,t} > 0$, positive and decreasing when $N_{i,t-1} - N_{i,t} < 0$ and zero when $N_{i,t-1} - N_{i,t} = 0$). Similarly, there are often large exemption levels, in particular for wealth taxes $\tau_{W,t}(\cdot)$. In contrast, we restrict the tax on interest income to be linear with tax rate $\tau_{Q,t}$ to preserve an Euler equation that is independent of bond holdings $B_{i,t}$. Finally, we assume that the tax functions $\tau_{\chi,t}(\cdot), \tau_{W,t}(\cdot)$ are differentiable.

Proposition A9. *In the presence of taxes on wealth, asset sales, and interest income, $\tau_{W,t}, \tau_{\chi,t}$ and $\tau_{Q,t}$, the welfare gain is*

$$\begin{aligned}\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \widetilde{R}_{0\to t}^{-1} \Big((N_{i,t-1} - N_{i,t}) \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t)\right) \mathrm{d}P_t \\ - \tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1}\,\mathrm{d}P_t - B_{i,t}\left(1 + \tau_{Q,t}\right)\mathrm{d}Q_t\Big).\end{aligned}$$

The presence of taxes changes our baseline formula in Proposition 1 in three noteworthy ways. First, whereas Proposition 1 implied that it is asset transactions and not asset holdings that matter for welfare gains from asset-price changes, holdings do matter whenever there is a wealth tax (i.e., a tax on the market value of asset holdings). In particular, whenever asset prices increase, $\mathrm{d}P_t > 0$, asset holders experience a welfare loss $\tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1}\,\mathrm{d}P_t$.

Second, a transaction tax reduces asset sellers' welfare gains from rising asset prices because the after-tax asset price faced by sellers increases by less than the pre-tax price

$$0 < \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t)\right)\mathrm{d}P_t < \mathrm{d}P_t \quad \text{when} \quad N_{i,t} - N_{i,t-1} > 0 \text{ and } \mathrm{d}P_t > 0.$$

However, it also *increases* asset buyers' welfare losses from rising asset prices because the after-tax asset price faced by buyers increases by *more* than the pre-tax price

$$0 < \mathrm{d}P_t < \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t)\right)\mathrm{d}P_t \quad \text{when} \quad N_{i,t} - N_{i,t-1} < 0 \text{ and } \mathrm{d}P_t > 0.$$

Third and related, both transaction and wealth taxes introduce aggregate welfare losses for the household sector as a whole. Finally, though unsurprisingly, the presence of dividend income taxes leaves welfare gains from asset-price changes unaffected.

Proof of Proposition A9. The Lagrangian is

$$\mathcal{L}_i = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) + \sum_{t=0}^{\infty} \lambda_{i,t} \big(Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - \chi(N_{i,t} - N_{i,t-1})$$
$$-\tau_{\chi,t}((N_{i,t-1} - N_{i,t})P_t) - B_{i,t}Q_t(1+\tau_{Q,t}) - \tau_{W,t}(N_{i,t-1}P_t) - \tau_{D,t}(N_{i,t-1}D_t)\big)\,.$$

The first-order condition for $B_{i,t}$ is

$$\lambda_{i,t+1} = \lambda_{i,t}\widetilde{Q}_t \quad \text{where} \quad \widetilde{Q}_t = Q_t(1+\tau_{Q,t})$$

is the after-tax bond price. The infinitesimal change in the Lagrangian gives the infinitesimal change in the value function:

$$\begin{aligned}
\mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty} \left(\frac{\partial \mathcal{L}_i}{\partial P_t}\,\mathrm{d}P_t + \frac{\partial \mathcal{L}_i}{\partial Q_t}\,\mathrm{d}Q_t \right) \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \Big((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - \tau'_{\chi,t}((N_{i,t-1} - N_{i,t})P_t)(N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t \\
&\qquad -\tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1}\,\mathrm{d}P_t - B_{i,t}(1+\tau_{Q,t})\,\mathrm{d}Q_t \Big) \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \Big((N_{i,t-1} - N_{i,t}) \Big(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t)\Big)\,\mathrm{d}P_t - \tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1}\,\mathrm{d}P_t - B_{i,t}\left(1+\tau_{Q,t}\right)\mathrm{d}Q_t \Big) \\
&= U'(C_{i,0}) \sum_{t=0}^{\infty} \widetilde{R}_{0\to t}^{-1} \Big((N_{i,t-1} - N_{i,t}) \Big(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t)\Big)\,\mathrm{d}P_t \\
&\qquad -\tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1}\,\mathrm{d}P_t - B_{i,t}\left(1+\tau_{Q,t}\right)\mathrm{d}Q_t \Big)\,,
\end{aligned}$$

where the third equality uses the Euler equation for B_t which implies $\lambda_{i,t} = U'(C_{i,0})\widetilde{R}_{0\to t}^{-1}$ with $\widetilde{R}_{0\to t} = (\widetilde{Q}_0 \ldots \widetilde{Q}_{t-1})^{-1}$. □

B Appendix on empirical framework

B.1 Interpreting price deviations away from constant price-dividend ratio

As explained in the main text (Section I.C), we construct the empirical price deviations $\Delta P_{k,t}$ as deviations of asset prices away from a world in which the price-dividend ratio was constant within each asset class (see equation 14 and Figure 2). We now briefly explain why, under the assumption that dividends grow at a constant rate, price deviations around a constant price-dividend ratio can be interpreted as deviations around a constant value for discount rates.

For simplicity, we focus on the case in which dividends are deterministic and grow at a constant rate:

$$D_{t+s} = D_t G^s. \tag{A25}$$

Under this constant-growth assumption, the price of an asset is

$$P_t = \sum_{s=1}^{\infty} R_{t\to t+s}^{-1} D_{t+s} = D_t \sum_{s=1}^{\infty} R_{t\to t+s}^{-1} G^s. \tag{A26}$$

Table A2: Individual wealth at the end of 1993

Asset type	Average	S.D.	p10	p25	p50	p75	p90	p99
Total wealth	781.1	727.6	387.0	529.5	723.4	938.6	1197.7	2075.5
Financial wealth	116.3	616.6	−17.6	5.5	79.9	166.1	280.5	729.6
Housing	131.5	247.8	0.0	0.0	104.6	185.2	289.2	664.7
Debt	−51.0	367.8	−116.5	−67.2	−22.5	−0.2	0.0	21.8
Deposits	23.2	144.8	0.0	1.2	6.6	22.4	55.1	215.1
Public equity	4.2	440.5	0.0	0.0	0.0	0.0	2.1	49.2
Private equity	8.4	268.3	0.0	0.0	0.0	0.0	4.4	111.4
Human wealth	664.8	365.9	299.8	427.8	621.7	824.8	1045.4	1776.1

Notes. The table displays the summary statistics for individual wealth as of December 31st 1993. The total number of observations is 3,268,017. Values are reported in thousands of 2011 U.S. dollars. Each statistic is computed for each variable separately.

When discount rates are constant, $R_t = \overline{R}$ for all t with $\overline{R} > G$, this simplifies to

$$P_t = D_t \times \overline{PD} \quad \text{with} \quad \overline{PD} = \frac{G}{\overline{R} - G}, \tag{A27}$$

i.e., the price-dividend ratio is constant and the price grows at the same rate as dividends. This equation corresponds to the "Gordon growth model" studied in Gordon and Shapiro (1956).

In our exercise, we construct price deviations as deviations of asset prices from a baseline with a constant price-dividend ratio, $\Delta P_t = (PD_t - \overline{PD}) \times D_t$. Combining (A26) and (A27), the difference in prices is:

$$\Delta P_t = \left(\sum_{s=1}^{\infty} \left(R_{t \to t+s}^{-1} - \overline{R}^{-s} \right) G^s \right) D_t.$$

This formula shows that, under the assumption that dividend grows at a constant rate, variations of the price-dividend ratio over time correspond to variations in discount rates. The same ideas hold in a stochastic environment (Campbell and Shiller, 1988).

B.2 Microdata on holdings and transactions

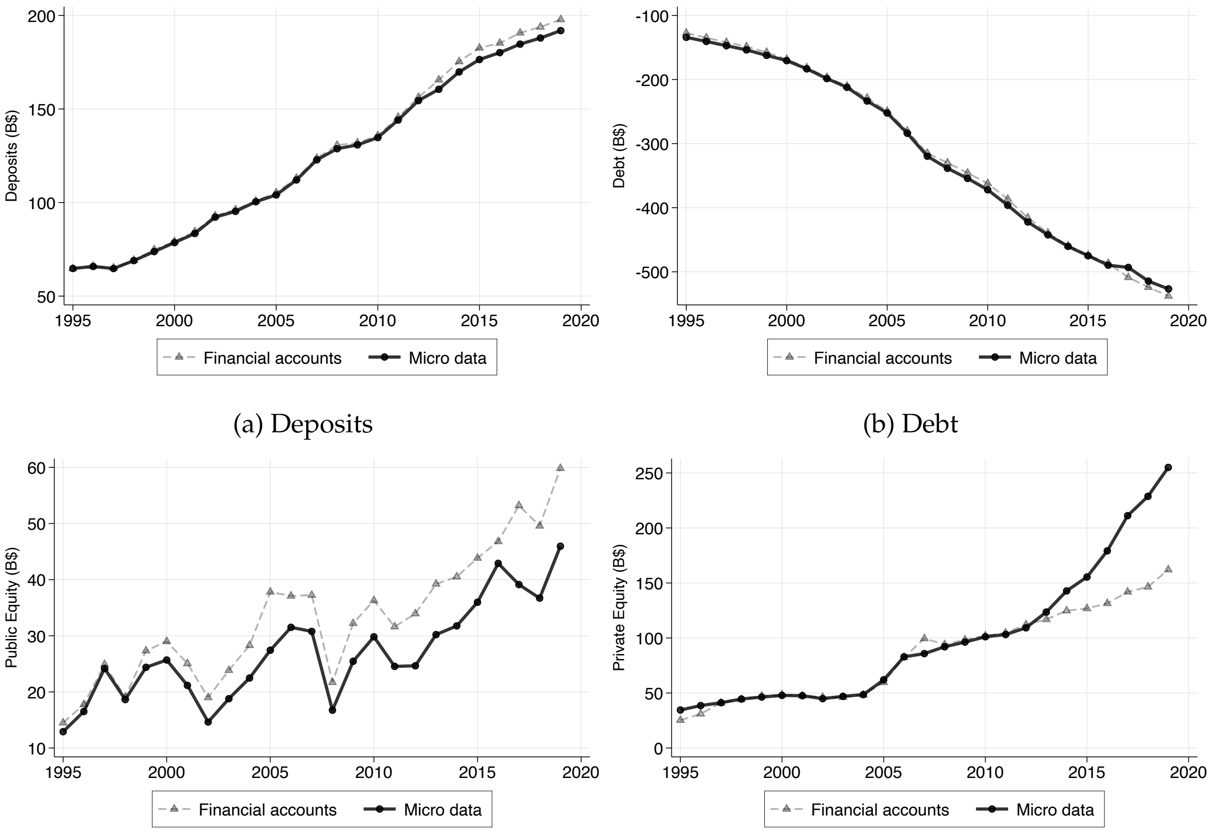
B.2.1 Summary statistics and validation

Table A2 reports summary statistics on the balance sheet of Norwegian individuals at the end of 1993 (the start of our sample).

Figure A1 compares the aggregate value of individuals' net assets for each asset category in the microdata and those reported in the Financial Accounts. Overall, the microdata aligns closely with the Financial Account data. The only notable discrepancies are public equity, which is higher in the microdata than in the National accounts after 2010, and mutual fund equity, which is higher in the Financial Accounts than in our microdata throughout our sample period.

B.2.2 Imputing indirect holdings and transactions

Individuals who own firms are indirectly exposed to asset-price changes through the asset holdings and transactions of the firms they own. We now describe how we impute these indirect holdings and transactions.



(a) Deposits

(b) Debt

(c) Public equity

(d) Private equity

Figure A1: Aggregated administrative microdata versus the Financial Accounts (Holdings)

Private businesses. Beginning in 2005, our dataset includes information on the ownership of limited liability businesses. The data contains information on the number of shares owned by an individual or a firm and the market price if that exists. In addition, we observe the total number of shares issued by a company.

We first compute the direct ownership share of firm j by an individual or firm i. We obtain this number by dividing the number of shares held by owner i by the number of shares outstanding in firm j (i.e., the total number of shares issued by the firm minus the shares held by the firm itself). More precisely, the direct ownership share of an owner i in firm $j \neq i$ is

$$s_{ij} \equiv \frac{N_{ij}}{\sum_{i \neq j} N_{ij}},$$

where N_{ij} denotes the number of shares held by an owner i in firm j.

In our sample, a substantial fraction of businesses are owned by other businesses. For example, a common structure among wealthy individuals is having one umbrella private holding company with several holding companies operating in different sectors. Our goal is to allocate the financial transactions done by all of these businesses to their ultimate owner. Formally, denote s_{ij}^n the ownership share of individual i in firm j through n (and exactly n) intermediate firm layers. When $n = 0$, this corresponds to our direct ownership share $s_{ij}^0 = s_{ij}$. For $n > 0$, we can compute the ownership shares of individual i in firm j at level n recursively:

$$s_{ij}^n \equiv \sum_k s_{ik}^{n-1} s_{kj}.$$

Finally, we obtain the consolidated ownership share of an individual by aggregating the ownership shares at all levels $n \geq 0$:[22]

$$\tilde{s}_{ij} = \sum_{n=0}^{\infty} s_{ij}^{n}.$$

In practice, we only compute indirect ownership shares up to $n = 10$ as indirect ownership shares are close to zero past that point.

Using these ownership shares, we construct an individual-level measure of private business book equity, which we define as the book value of a firm's assets minus net financial assets.[23] We only use book equity to compute the value of private business transactions, which we describe shortly. More generally, we rely on the tax-assessed value of private business equity, which we observe over the full sample (i.e., starting in 1994).

Table A3 reports the average value of indirect holdings and transactions as a fraction of the tax-assessed value of the equity in the firm over the 2005-2019 period. Private firms have, on average, positive net leverage (i.e., debt exceeds deposits). Moreover, private firms hold a significant amount of housing and (publicly-traded) stocks on their balance sheet, with a small amount of yearly transactions. Before 2005, we do not observe the balance sheet of private firms. Hence we do not have data on indirect holdings and transactions. From 1994 to 2004, we, therefore, attribute indirect holdings and transactions by using the values in Table A3 multiplied by the tax-assessed value of equity.

Table A3: Indirect holdings through private businesses (share of tax-assessed value, 2005–2019 average)

Asset class	Holdings	Transactions
Deposits	0.40	–
Debt	1.11	–
Housing	0.65	−0.03
Stocks	0.16	−0.00

To measure the net transactions in private business equity for individual i in firm j in year t, we use the formula

$$\text{private equity transaction}_{ijt} = (\tilde{s}_{ij,t+1} - \tilde{s}_{ij,t}) \times \text{book equity}_{jt} \times Q,$$

where, as above, $\tilde{s}_{ij,t}$ denotes the ownership share of individual i in firm j at time t. If the firm does not exist at time t and enters at time $t+1$, we set the net transactions in private business equity to zero. Note that this formula automatically accounts for equity issuance. For instance, when a firm issues equity to finance its growth, the existing owners get diluted (i.e., their ownership share declines). Regarding exposure to asset-price changes, this is equivalent to the owners selling equity shares.

The term Q represents the ratio between the market value of private business equity and its book value. While we do not observe Q directly, we set it to a value of 0.80, which corresponds to the aggregate share of the tax-assessed value of private business equity to book value of private business equity, averaged over the 2005-2019 period. Before 2005, we do not observe ownership shares and, therefore, we set private equity transactions to zero.

[22] Formally, denote Ω the matrix of ownership within firms, that is, $\Omega_{ij} = s_{ij}$ for $i \neq j$ and $\Omega_{ij} = 0$ for $i = j$. Then, the vector of consolidated ownership of an individual i with direct ownership shares $\boldsymbol{s}_i = (s_{ij})_j$ is given by $(I - \Omega')^{-1}\boldsymbol{s}_i = \sum_{n=0}^{\infty}(\Omega')^n \boldsymbol{s}_i$.

[23] For instance, suppose that a firm has \$2 of assets, which includes \$1 of stocks, and \$0.25 of debt outstanding. The net financial assets of the firm is then \$1 − \$0.25 = \$0.75. Book equity is then \$2 − \$0.75 = \$1.25.

Public businesses. Finally, we attribute indirect holdings and transactions due to the ownership of publicly-traded stocks. We start individuals' indirect holdings and transactions through their ownership of the aggregate corporate sector, as reported in the Financial Accounts (see Appendix C for more details). We then subtract the aggregate indirect holdings and transactions due to their ownership of private businesses, as computed above. We, therefore, obtain residually the indirect aggregate holdings and transactions of public businesses that must be allocated to individuals. We then allocate these indirect holdings and transactions to individuals for every year in our sample, in proportion to their equity holdings of public firms.

B.3 Data on asset prices

Figure A2 plots the relative price deviations (defined in equation 14) for our four asset classes: housing, equity, debt, and deposits.

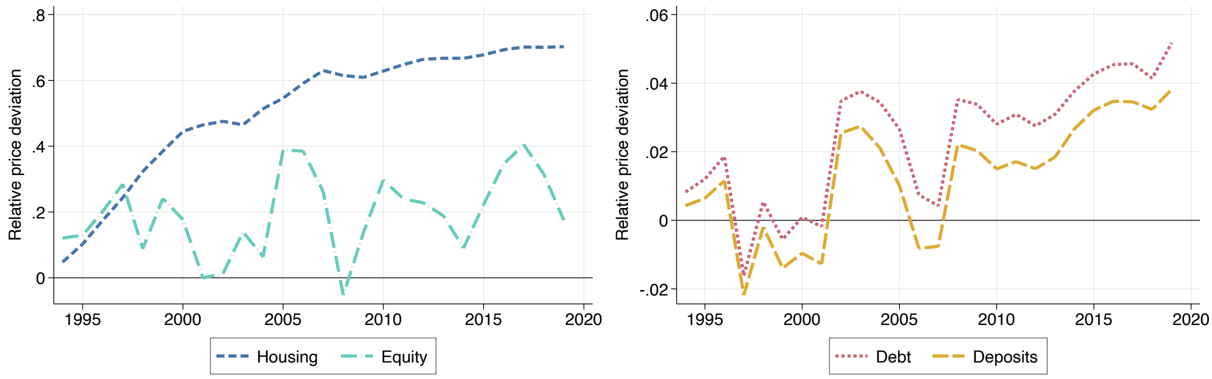


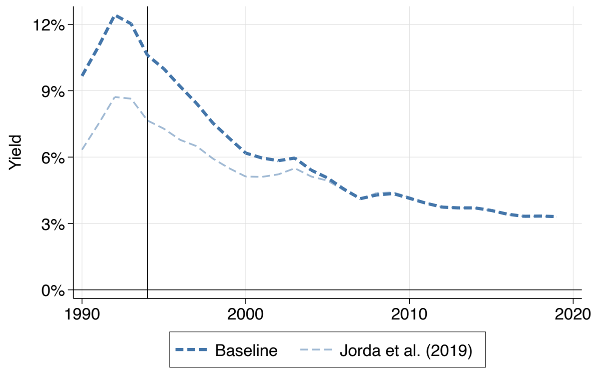
Figure A2: Relative price deviations for four asset classes in Norway

Notes. Note that the scale of the price deviation for debt (or deposits) is much smaller than for housing. Still, these two asset classes will have comparable effects on welfare, as the size of debt holdings (i.e., issuance of one-period bonds) is much larger than the size of housing transactions (Figure 7).

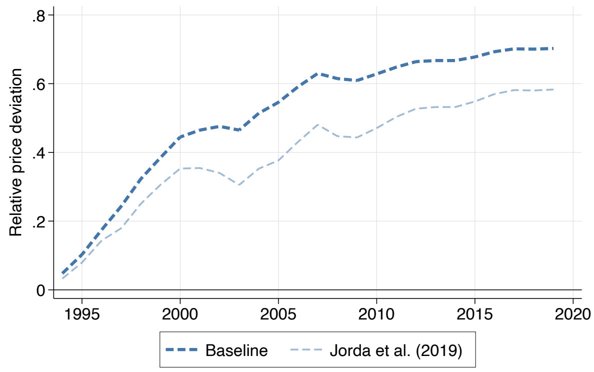
B.4 Alternative measure of the housing price-to-rent ratio

We now compare our construction of the price-to-rent ratio measure with that of Jordà et al. (2019). Although the latter paper employs a similar methodology, it inflates the growth in the rental index reported by SSB. Specifically, the authors write in the Data Appendix: "we stipulate that the rental index during late 1990s and early 2000s—a period when house prices increased substantially—understated the growth of rents relative to prices, leading the rent-price approach to overstate the historical rental yields. To correct for this presumed bias, we adjust the growth in rents up by a factor of 1.5 for the years 1990 to 2005."

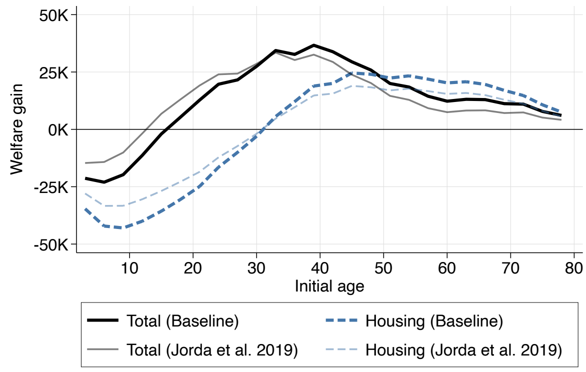
Table A3a plots both series. By construction, both ratios are equal in 2013. However, due to the gradual rescaling done by Jordà et al. (2019), their series gradually diverge from our baseline series before 2005. Using these alternative series leads to a lower increase in the price-dividend ratio and, therefore, a lower price deviation relative to our 1992-1996 baseline. To visualize the difference, Figure A3b reports the implied deviation in housing price relative to 1992-1996 (as defined in equation 14) for our baseline and the alternative series. On average, the housing price deviation implied by Jordà et al. (2019) represents 0.91 of our baseline. Hence, as a rule of thumb, one can expect the welfare gains due to housing to be scaled by 0.91 if one were to reproduce all our figures with this alternative ratio. For example, we recompute the welfare gains across generations using this alternative series in Figure A3c.



(a) Housing yield (ratio of rents to house prices)



(b) Price deviation



(c) Welfare gains across cohorts

Figure A3: Comparing our series for house-to-rent ratio with Jordà et al. (2019)

C Redistribution across sectors

As discussed in the main text (Section III), welfare gains within the household sector do not aggregate to zero. The reason is that individuals trade with other non-household entities, such as the government and foreigners. We now conduct a systematic investigation of welfare gains across sectors. This analysis is particularly significant in the Norwegian context, given the scale of the sovereign wealth fund, which invests in domestic and foreign assets on behalf of Norwegian households.

Specifically, we group all entities in the economy into three sectors: households (H), the government (G), and foreigners (F). The central accounting identity underpinning our analysis states that every asset purchased by one sector must be sold by another. With this in mind, it is immediate that in a multisector economy, our measurement of welfare gains implies:

$$\text{Welfare Gain}_{\text{H}} + \text{Welfare Gain}_{\text{G}} + \text{Welfare Gain}_{\text{F}} = 0. \tag{A28}$$

In other words, a positive welfare gain for the household sector must be exactly offset by a welfare loss in another sector. We first present the data in Section C.0.1, and discuss the results in Section C.0.2.

C.0.1 Data sources

We use publicly available data from the *Financial Accounts*, which covers all holdings and transactions of financial assets in the Norwegian economy since 1995. For our analysis, we combine the government sector with the central bank and the non-profit sector. Importantly, our government sector includes the *Government Pension Fund of Norway*, financed by income taxes on the energy (oil and gas) sector. It comprises the Government Pension Fund Global — which invests in foreign assets — and the Government Pension Fund Norway — which is smaller and invests in domestic and Scandinavian assets.[24]

We consider the following sectors of the economy:

1. Households (14);
2. Government (121, 13, 15);
3. Foreigners (2).
4. Corporations
    4.1 Nonfinancial corporations (11)
    4.2 Monetary financial institutions (122-123)
    4.3 Non-MM investment funds (124)
    4.4 Other financial institutions (125-127)
    4.5 Insurance corporations and pension funds (128-129)

The numbers in parentheses denote the sector codes from the Financial Accounts we aggregate. Note that our definition of "Government" includes the central bank and the non-profit sector (i.e., institutions that serve the domestic household sector). We consolidate the different sectors constituting the corporate sector to their ultimate owner (i.e., either households, the government, or foreigners) by using the exact formula provided in Footnote 22. The consolidation process, therefore, adjusts the measures of holdings and transactions by households, the government, and foreigners by accounting for their indirect holdings and transactions through their corporate sector ownership. Note that this consolidation maintains the Financial Accounts' identities, and, in particular, financial transactions remain in zero-sum.

We consider the following asset categories:

1. Deposits (22);
2. Loans and debt securities (30, 40);
3. Public equity shares (511);
4. Private equity shares (512);
5. Fund equity shares (520);
6. Other (10, 21, 519, 610–800).

The numbers in parentheses denote the line items from the Financial Accounts that we aggregate. The category "other" contains either quantitatively unimportant or illiquid assets. We can further decompose each asset category using the identity of the sector issuing the security (e.g., public equity shares issued by the corporate sector versus the foreign sector). Real estate is a real rather than a financial asset, meaning housing holdings and transactions are not recorded in the Financial Accounts. Therefore, we augment the Financial Accounts with between-sector housing holdings and transactions, which we construct by aggregating the housing transaction registry data described in the main text (Section II.C).

[24] Over our sample period, the Government Pension Fund Global fund's value grew from approximately zero in 1997 to approximately 1B$ in 2019. Its portfolio mandate first prescribed 40 percent equities and 60 percent fixed-income assets. In 2007, this was changed to 60 percent equities. In 2010, the fund's portfolio was extended to real estate with a 5 percent weight, and the fixed income share was cut to 35. A fiscal policy rule states that the expected real rate of return, first 4% and since 2017 3%, of the current fund value can be spent over the national budget each year. As the fund grew over our sample period, so did government spending. Details regarding the fund's mandate and investment strategy are provided at `https://www.nbim.no/en/the-fund/how-we-invest`.

The resulting dataset covers the total asset holdings and transactions for three sectors (households, government, and foreigners) and four asset classes (housing, deposits, debt, equity) over the 1995–2019 period.

C.0.2 Results

Sectoral transactions. Before quantifying the welfare gains by sector, we briefly discuss the main pattern of housing and equity transactions as well as debt and deposit holdings across sectors, as reported in Table A4.

The annual net housing purchases across sectors are very low (less than $1,000 per capita in absolute value). The reason is that most housing transactions are within the household sector, with minimal transactions between sectors. Regarding equity purchases, households have a positive but small level of net equity purchase on average. In contrast, the government is a net buyer of foreign equities via the sovereign wealth fund described above. Those transactions are pretty large, and amount to more than $7,000 per capita per year. The fact that the government has been a net buyer of equity in our time sample reflects that inflows into the sovereign wealth fund (which are proportional to oil and gas revenues) have exceeded outflows (which are proportional to the fund's market value). This dynamic may shift in the future as the sovereign wealth fund continues to grow relative to the Norwegian economy.

Table A4: Transactions across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	0.9	−0.4	−0.5	0.0
Debt	−74.4	65.1	9.3	0.0
Household debt	−65.2	24.8	40.4	0.0
Corporate debt	−26.8	−9.9	36.7	0.0
Government debt	6.4	−38.6	32.2	0.0
Foreign debt	11.3	88.8	−100.1	0.0
Deposits	20.4	−7.9	−12.4	0.0
Corporate deposits	15.6	−11.2	−4.3	0.0
Government deposits	0.7	−1.9	1.3	0.0
Foreign deposits	4.1	5.2	−9.4	0.0
Equity	0.6	7.0	−7.6	0.0
Corporate equity	−0.6	−0.7	1.3	0.0
Foreign equity	1.2	7.6	−8.9	0.0

Notes. All transactions (net purchases) are in thousands of 2011 U.S. dollars and divided by Norway's population. Averages over 1995-2019. "Household debt" is debt taken by households (mostly mortgages); " Corporate debt" is debt issued by the corporate sector (i.e., bonds and bank loans); "Foreign debt" contains all debt issued by foreigners (e.g., foreign corporate entities, foreign households, and foreign governments); "Corporate deposits" is deposits issued by private banks; "Government deposits" is central bank reserves; "Corporate equity" is equity issued by corporations; "Foreign equity" is equity issued by foreign corporations.

Table A4 reveals that the household sector has a large amount of debt. Most of it is household debt (mainly mortgages), but some is corporate debt, which individuals indirectly hold through their ownership of businesses. While households, on net, hold debt securities as liabilities (i.e., they are indebted), the government, on net, holds debt securities as assets (i.e., they are lenders). The debt level of households is approximately equal to the government's net holding of debt securities (roughly $70,000 per capita). The foreign sector only holds a small amount of debt on net. While households do not borrow directly from the government, the effect is the same in terms of welfare redistribution: a decline in interest rates redistribute from the government towards households.[25]

A similar pattern holds for deposits, although the magnitudes are much smaller. The household sector is a net holder of deposits, while the government and foreign sector hold these deposits as liabilities. The reason is that deposits are a liability for the financial sector: since the government includes the central bank, and since foreigners are important holders of financial business equity, they are ultimately liable for interest payments on these deposits.

Sectoral welfare gains. Table A5 reports the detailed welfare gains asset class by asset class, including a breakdown within an asset class (i.e., equity is the sum of domestic corporate equity and foreign corporate equity). All welfare gains are scaled by the number of individuals in Norway in 1994. Note

Table A5: Welfare gains across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	−4.4	1.8	2.6	0.0
Debt	16.4	−15.6	−0.8	0.0
Household debt	14.6	−5.5	−9.2	0.0
Corporate debt	5.7	2.8	−8.5	0.0
Government debt	−1.3	8.4	−7.1	0.0
Foreign debt	−2.7	−21.3	24.0	0.0
Deposits	−2.5	1.0	1.4	0.0
Corporate deposits	−1.8	1.4	0.4	0.0
Government deposits	−0.1	0.2	−0.2	0.0
Foreign deposits	−0.6	−0.7	1.2	0.0
Equity	−1.5	−9.0	10.5	0.0
Corporate equity	1.1	1.5	−2.6	0.0
Foreign equity	−2.6	−10.5	13.1	0.0
Total	8.0	−21.7	13.7	0.0

that welfare gains sum up to zero within each asset class by construction and that the welfare gain per capita in the household sector is very similar to the one estimated in our microdata (see Table 2). The slight difference is because our microdata does not aggregate exactly to the Norwegian Financial Accounts (see Appendix B.2.1), as well as the fact that our microdata starts in 1994 while the Norwegian Financial Accounts only start in 1995.

The household sector has a positive welfare gain of roughly \$8,000 per capita. Breaking down the welfare gain by asset class, we find a large positive contribution of debt (\$16,400) and a small contribution of deposits (−\$2,500). Equity transactions make a negligible contribution (−\$1,500), and housing transactions are a more important one (−\$4,400). Overall, the household sector's positive welfare gain is mostly due to declining interest rates, which have benefited households since they are net debtors (i.e., their debt exceeds their bank deposits).

If the household sector has experienced a positive welfare gain, who is the counterparty that experienced a welfare loss? For the most part, it was the government. As discussed earlier, the government

[25] Most of household debt is mortgages, which are then securitized into mortgage bonds by private banks. Then, these bonds are, for the most part, sold to domestic pension funds as well as foreigners. However, foreigners also issue a large amount of debt that ends up being held by the sovereign wealth fund. This explains why the net foreign debt position is close to zero in Table A4. The sovereign wealth fund's holding of foreign bonds then accounts for most of the government's net holding of debt securities, while a small fraction is held by other public pension funds that invest domestically. The main domestic public pension funds are *Folketrygdfondet* and *Kommunenes Landspensjonskasse* (see Bank, 2021 for an overview of Norway's financial system).

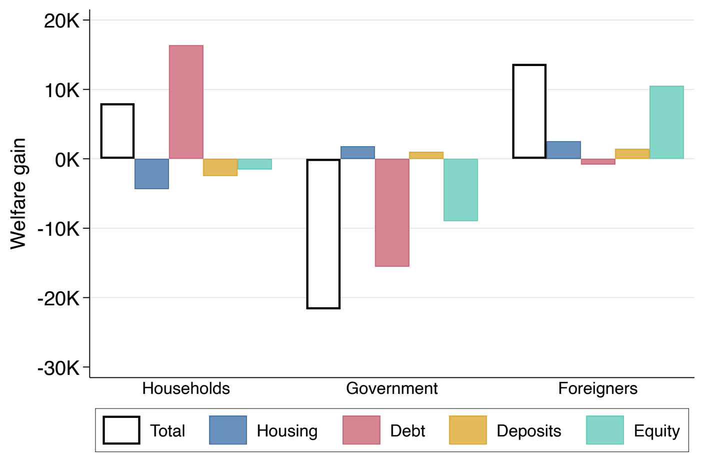


Figure A4: Welfare gains across sectors

Notes. The figure plots the welfare gain for each sector of the economy, disaggregated by asset class. To make it comparable to the other figures in our paper, the aggregate welfare gain of each sector is divided by the number of individuals in Norway. Units are 2011 U.S. dollars.

is a net saver and is thus hurt by declining interest rates. Overall, the welfare gain of the government is negative, with a large contribution of debt and equity. As reported in Table A4, this comes from the fact that the government is a net holder of debt and a net purchaser of equity. In contrast, the contributions of deposits and housing for welfare gains are negligible (<$2,000 in absolute value).

The fact that the Norwegian government is hurt by rising asset prices and declining interest rates can seem surprising from a U.S. perspective. In the U.S., the government is a net debt issuer, so it tends to benefit from a rise in asset prices at the expense of households and foreigners who hold its debt. The same effect holds true in Norway: as shown in Table A5, if we restrict ourselves to the debt issued by the government (i.e., the row "Government debt"), the rise in asset prices does benefit the government at the expense of households and foreigners. However, this effect is swamped by the fact that the Norwegian government holds a large amount of debt issued by households and foreigners: rising asset prices and declining interest rates have ultimately hurt the Norwegian government.

As discussed in Section I.C, the loss of the government represents a loss of real resources available for net transfers to the household sector. While it is beyond our paper's scope to quantify how the Norwegian government has adjusted (and will adjust) net transfers in response to persistently lower interest rates and higher asset prices, it is entirely possible that the very individuals who experienced welfare losses (i.e., the young) will also be the ones to bear the brunt of future reductions in government transfers such as pension benefits.

C.0.3 Domestic versus foreign price indices

In all of these exercises, we use the same price deviation for foreign and domestic assets (Section II). This assumption was innocuous when computing the average welfare gain within the household sector, as most of the financial transactions between Norwegians are transactions of domestic assets. However, this assumption becomes more restrictive when discussing welfare gains across sectors, as the Norwegian government buys a large amount of foreign assets. Price deviations for foreign equity and debt may differ from the ones for Norwegian (domestic) assets.

For robustness, we now re-estimate sectoral welfare gains using different price indices for domestic versus foreign assets. For holdings of foreign debt, we use as a measure of yield the OECD average 3-year government bond yield (series from Global Financial Data). For transactions of foreign equity, we use the ratio of total firm payout to total enterprise value in the universe of firms from Worldscope as a measure of yield (after removing financial and public utility firms).[26] Figure A6 plots the evolution of these two quantities relative to the ones used for domestic equity and debt. One can see that, while yields follow similar dynamics in Norway and in the rest of the world, the equity yield decreased less in Norway relative to the rest of the world. Figure A6 plots the resulting relative price deviation (Equation 14 in the main text) used in the welfare-gains formula.

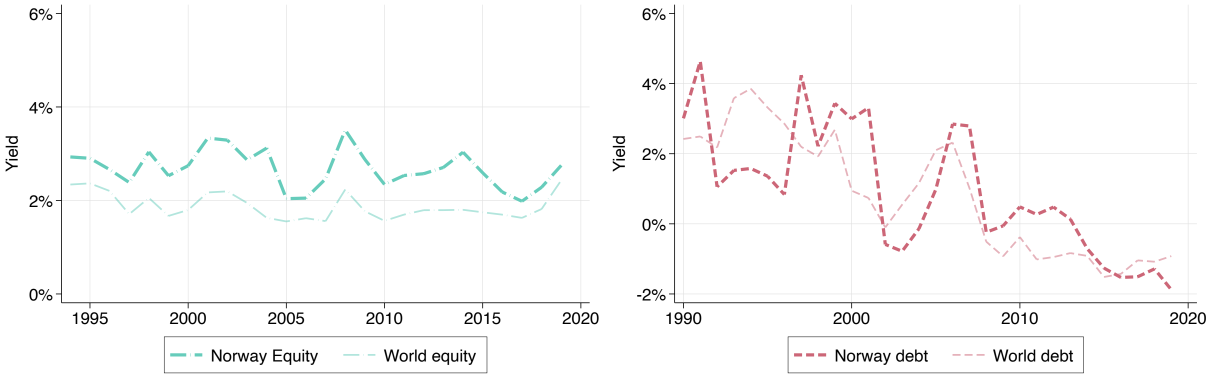


Figure A5: Yields for domestic and foreign assets

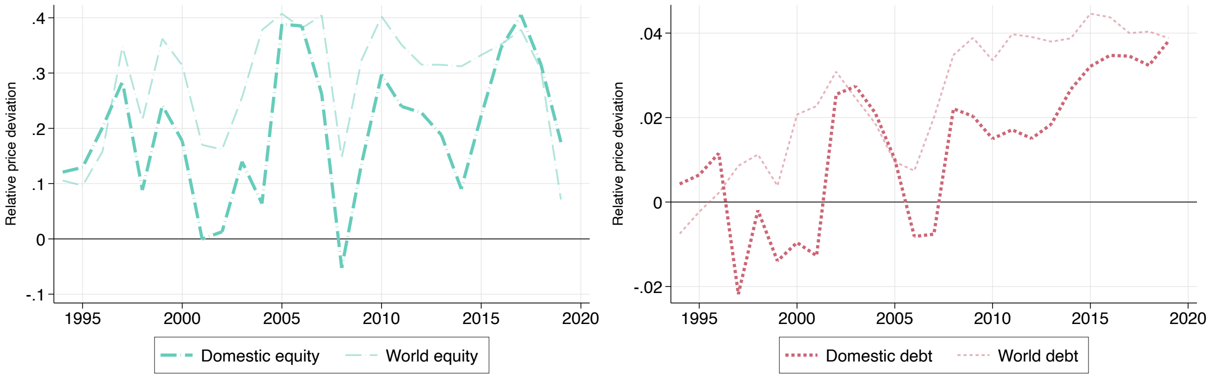


Figure A6: Price deviations for domestic and foreign assets

Table A6 reports the welfare gains using these foreign price indices for debt, deposit, and equity. The dagger † sign indicates the rows that change relative to Table A5. Overall, one can see that we obtain very similar results. The main difference is that the welfare gains of the Norwegian government are more negative in magnitude, which reflects the fact that they disproportionately purchased foreign equity, whose valuation increased more than the valuation of domestic equity.

D Appendix on generalizing the baseline sufficient statistic

D.1 Uninsurable income risk

Proof of Proposition 2. The proposition obtains as a special case of Proposition A2 in the case without deviation in labor or dividend income; that is, $\mathrm{d}Y_{i,t} = 0$ and $\mathrm{d}D_{k,t} = 0$ for $1 \leq k \leq K$. □

[26]We also exclude firms from Venezuala and Brazil due to issues in Worldscope datas on the units used in reporting.

Table A6: Welfare gains across sectors using heterogeneous price indices

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	−4.4	1.8	2.6	0.0
Debt	16.6	−14.8	−1.8	0.0
Household debt	14.6	−5.5	−9.2	0.0
Corporate debt	5.7	2.8	−8.5	0.0
Government debt	−1.3	8.4	−7.1	0.0
Foreign debt†	−2.5	−20.6	23.0	0.0
Deposits	−2.9	0.4	2.5	0.0
Corporate deposits	−1.8	1.4	0.4	0.0
Government deposits	−0.1	0.2	−0.2	0.0
Foreign deposits†	−1.0	−1.3	2.3	0.0
Equity	−2.7	−17.2	19.9	0.0
Corporate equity	1.1	1.5	−2.6	0.0
Foreign equity†	−3.8	−18.7	22.4	0.0
Total	6.6	−29.8	23.2	0.0

D.1.1 Empirical implementation of Equation 20

We now detail how we estimate the incomplete market adjustment term (i.e., the covariance term given in Equation 20). We defined income as net non-financial income (i.e., labor income plus net government transfers, exactly as we do when computing human wealth). We measure consumption as the residual of income minus net asset purchases, consistently with the budget constraint (9). We ensure that our measure of income and consumption is always higher than twice the basic amount (grunnbeløp) used in the Norwegian social security system, and we winsorize the top 1% of observations every year. Finally, we construct asset savings for each individual and asset class: for housing and equity, it is transactions-based — that is, $S_{i,k,t} = (N_{i,k,t} - N_{i,k,t-1})P_{k,t}$, while, for debt and deposits, it is holdings-based — that is, $S_{i,k,t} = B_{i,t}Q_t$.

Remember that our covariance term (Equation 20 in the main text) should capture the uncertainty at the individual level. Hence, to account for this, we residualize our variables (asset savings, log income, and log consumption) on individual characteristics known at $t = 0$ for each cohort c and horizon t. The set of controls is (i) highest lifetime education achievement (i.e.,"less than high school", "high school", "college" dummies), (ii) deciles of within-cohort financial wealth at the beginning of our sample (i.e., ten dummies), and (iii) the average of income in the first three years of our sample.

After residualizing all of these quantities, we then regress our measure of consumption (or spending) on labor income for each cohort c and horizon t:

$$\log C_{i,t} = \alpha_{c,t} + \beta_{c,t} \log Y_{i,t} + u_{i,c,t}.$$

We then compute, for each cohort c and horizon t and asset class k, the covariance between asset sales $S_{i,k,t}$ and our predicted measure of consumption $\widehat{\log C_{i,t}} = \beta_{c,t} \log Y_{i,t}$. Alternatively, this methodology can be interpreted as a regression of asset sales on consumption instrumented by labor income for each cohort and horizon, using initial individual characteristics as controls.

D.1.2 Quantifying welfare gains in models of incomplete markets

We now study the welfare effect of deviation in asset prices in heterogeneous-agent incomplete market models. The goal is to complement our sufficient statistic approach with a more standard approach based on a calibrated model. We focus on two classes of incomplete market models. First, we consider the welfare effect of asset-price deviations in a Bewley-type model, in which individual labor income is subject to permanent and transitory shocks. We then study the welfare effect of asset-price deviations in a random growth model of wealth accumulation, for instance where entrepreneurs face idiosyncratic return risk.

Quantifying the effect of stochastic labor income We consider a model in which agents live for 65 years. They spend 40 years as workers and 25 as retired. The permanent labor income of workers Y_t evolves as a random walk (in log) with innovations drawn from a normal distribution with mean μ and volatility σ. Moreover, workers switch from being employed to unemployed, with transition probabilities given by $\lambda_{U\to E}$ and $\lambda_{E\to U}$; when unemployed, workers only earn a fraction χ_U of their permanent income. Finally, during retirement, agents earn a fraction χ_R of their permanent income. Agents can save in an asset that delivers a constant flow of dividends. The asset's price is such that investing in the asset returns an interest rate R. We denote $s_t \in \{E, U, R\}$ the current state of a household with age t, corresponding to employment, unemployment, and retirement.

Households have homothetic utilities with relative risk aversion γ, impatience parameter β. Moreover, agents have a preference over the size of bequest they leave $bW^{1-\gamma}/(1-\gamma)$. Formally, the problem of an agent with age t is to maximize

$$V(Y_{i,t}, W_{i,t}, x_{i,t}) = \mathbb{E}_t \left[\sum_{s=t}^{65} \beta^{s-t} \frac{C_{i,s}^{1-\gamma}}{1-\gamma} + \beta^{65-t} b \frac{W_{i,T}^{1-\gamma}}{1-\gamma} \right],$$

subject to

$$\begin{aligned} W_{i,t+1} &= R_{i,t+1} W_{i,t} + \chi_{s_{i,t}} Y_{i,t} - C_{i,t} \\ \log Y_{i,t+1} &= \log Y_{i,t} + \epsilon_{t+1} 1_{x_{it} \in \{E,U\}}, \quad \text{where } \epsilon_{i,t+1} \sim N(\mu, \sigma^2). \end{aligned}$$

and $x_{i,t}$ evolves as a Markov chain on $\{E, U\}$ with transition probabilities $\lambda_{U\to E}$ and $\lambda_{E\to U}$ for $t \leq 40$, and then switches to R for retirement at $t = 40$. Initial wealth is log normally distributed and, on average, equals the terminal wealth of deceased households. Initial income is set to one as a normalization. Because the lower bound on labor income is zero, the natural borrowing constraint is $W_{i,t} \geq 0$.

We calibrate the growth and volatility of the innovations of log labor income $\mu = 0.01$ and $\sigma = 0.10$ following Wang et al. (2016). We set the annual probability of switching from employment to unemployment to 5%, the annual probability of switching from employment to unemployment to 80%, and the income multiplier when unemployed to $\chi_U = 0.6$ following Krueger et al. (2016). Finally, we pick the impatience parameter β and the bequest motive parameter b to match a ratio of average wealth to average labor income of 6 and a ratio of terminal wealth (at 85) to average wealth of 0.7, which gives $\beta = 0.95$ and $b = 0.02$.

As in Wang et al. (2016), the problem is homogeneous in the financial wealth $W_{i,t}$ and permanent labor income $Y_{i,w}$ and so one only needs to solve the value function as a function of $w_{i,t} = W_{i,t}/Y_{i,t}$. We plot the solution of the model in Figure A7. Figure A7a reports the annual purchases of households as a function of their financial wealth and states $x \in \{E, U, R\}$ (averaged across ages). One can see that unemployed agents are net asset sellers, while employed agents tend to be net asset buyers — they only start selling if their financial wealth is 15 times their labor income. At retirement, agents sell until their

Table A7: Parameters

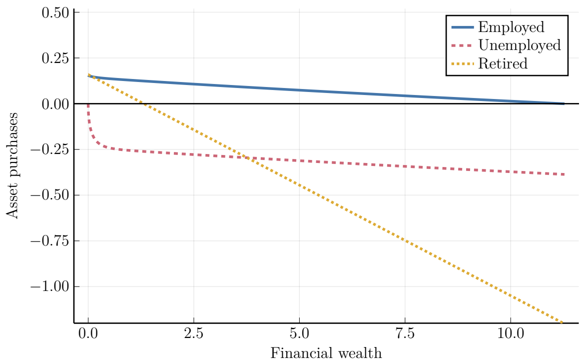
Description	Symbol	Value
Income process		
Labor income growth	μ	0.01
Labor income volatility	σ	0.10
Income multiplier when retired	χ_R	0.70
Transition probability to unemployment	λ_{EU}	0.05
Transition probability to employment	λ_{UE}	0.80
Income multiplier when unemployed	χ_U	0.60
Asset prices		
Interest rate	$\log R$	0.05
Household preferences		
Relative risk aversion	γ	1.50
Impatience parameter	β	0.95
Bequest preference	b	0.02

financial wealth equals approximately 3 times their labor income pre-retirement. Figure A7b reports the overall density of assets scaled by permanent labor income within employed, unemployed, and retired. Note that the density of assets is even more dispersed as the permanent labor income is log-normally distributed across agents, with a variance increasing linearly with age for workers. Finally, figure A7c plots the ratio of marginal utility between employed and unemployed. Naturally, this ratio tends towards $\chi_R^{-\gamma}$ (the ratio of labor income) as assets tend to zero and 1 (perfect insurance) as assets tend to infinity.

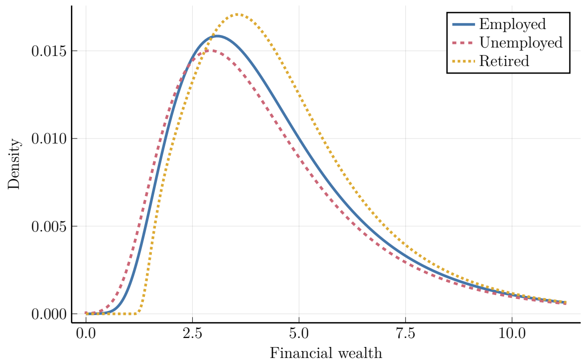
We now examine the difference between our baseline sufficient statistic formula and the actual welfare gains of a deviation in asset prices in the calibrated model. More precisely, we consider a mean-reverting deviation in the interest rate $\mathrm{d}R_t/R_t = \phi^t \,\mathrm{d}R_0/R_0$ with $\mathrm{d}R_0/R_0 = -5\%$ and $\phi = 0.95$, as plotted in Figure A8a. Since the asset's price is given by $P_t = \sum_{t=1}^{\infty} R_{0\to t}^{-1} D$, this implies a deviation in the asset's price given by $\mathrm{d}P_t/P_t = -\left(1-\phi/R\right)^{-1} \mathrm{d}R_t/R_t$. We compute two quantities associated with this deviation in asset prices: the actual welfare gain, which discounts future trading profits by the individual-specific marginal rate of substitution $\beta^t U'(C_{i,t})/U'(C_{i,0})$ across states and times (Equation 18 in the main text), and the one returned by our baseline sufficient statistic, which simply discounts them by the interest rate (Equation 10 in the main text).

Figure A8b compares these two quantities for workers as a function of their financial wealth. One can see that the "true" welfare gains (discounted by the individual-specific marginal rate of substitution or MRS) are always above the one discounted by the interest rate. Two distinct forces drive this positive covariance between individual-specific marginal rates of substitutions and asset sales. The first force is transitory income shocks (the transition between employment and unemployment). As shown in (A7), workers tend to sell assets when they are unemployed, i.e., when their marginal utility of consumption is high. This effect is particularly high for agents in the left tail of the wealth distribution, as the ratio of their marginal utility between being employed and unemployed is particularly high (Figure A7c). The second force is permanent income shocks: young workers who experience a positive permanent income shock scale up their asset purchases, as they now target a higher level of savings. Hence, like transitory income shocks, permanent income shocks generate a positive covariance between the growth of marginal utility and asset sales.

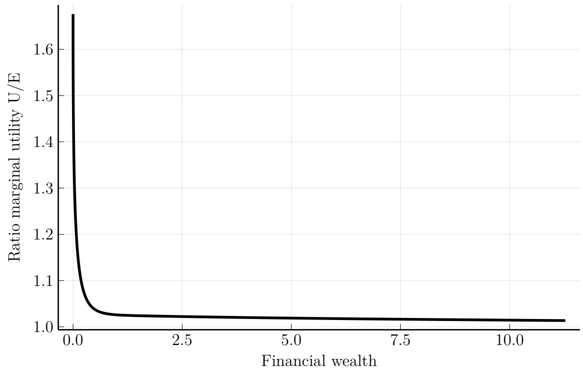
Finally, Figure A8c plots the average welfare gains across cohorts. Similarly to the empirical results plotted in Figure 10, the wedge between the two quantities is particularly significant for younger generations: this reflects the fact that, in the model as in the data, younger households face more id-



(a) Annual asset purchases



(b) Density of assets within groups



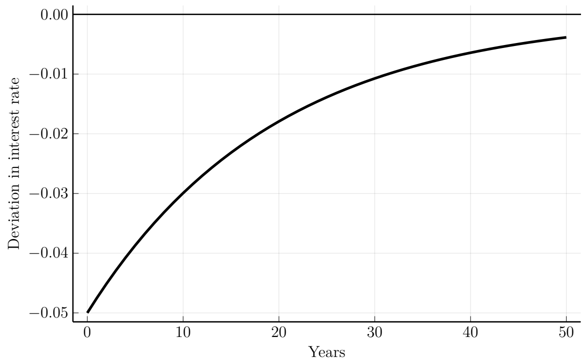
(c) Marginal utility unemployed v.s. employed

Figure A7: Model solution

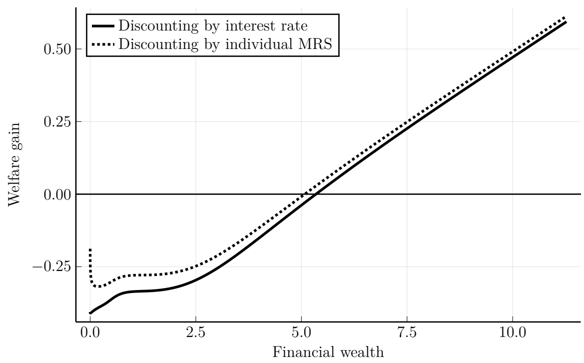
Notes. These panels plot important economic quantities in the model calibrated using the parameter values in Table A7. Asset purchases and financial wealth are scaled by permanent labor income.

iosyncratic risk (through labor income risk) than older households. Finally, note that the model yields substantially higher welfare gains for older households relative to the data — this reflects the fact that, in the data, asset prices only gradually increase over time, whereas they jump to their new values in our experiment.

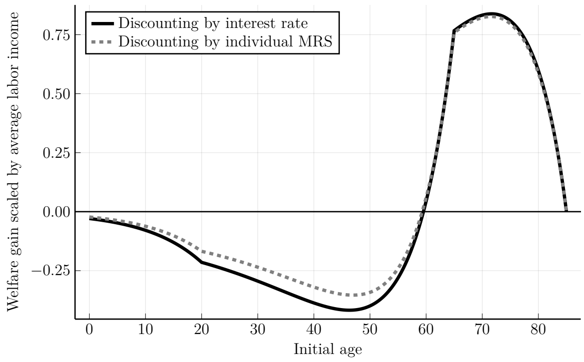
The key advantage of a model, relative to our reduced-form approach, is that we can examine how different calibrations would affect the wedge between our baseline statistic and actual welfare gains. Hence, the model is useful for understanding whether we should expect the wedge between the two quantities to matter in settings outside Norway (external validity). To examine this question, we re-solve the models after successively increasing the degree of income risk (through permanent innovations σ or transitory ones χ_U), the degree of risk aversion for household (relative risk aversion γ), and the persistence of the price deviation (ϕ). For each alternative calibration, we report the average absolute value of the welfare gain across households, the average absolute value of our sufficient statistic, the root mean squared error between the two quantities, as well as the correlation between the two (across all ages and household states). We report the result in Table A8. Intuitively, increasing the degree of income risk or the risk aversion of households increases the wedge between our sufficient statistic formula and the actual welfare gains. Still, note that the difference between the two notions of welfare gains (the theoretical one and our empirical proxy) remains relatively small across the calibrations – in particular, the correlation between the two measures across individuals is very close to one. Overall, these results suggest that the effect of uninsurable labor income risk is likely to be small not only for Norway but also for other countries.



(a) Deviation in interest rate



(b) Welfare gains across states



(c) Welfare gains across cohorts

Figure A8: Welfare effect of a deviation in interest rates in the model

Notes. This figure plots the welfare effect of a mean-reverting 5% drop in interest rates in the model calibrated using the parameter values in Table A7. Unless specified otherwise, financial wealth and welfare gains are scaled by permanent labor income. Welfare gains discounting by interest rate correspond to the first term in Equation 19 in the main text (i.e., the output of our baseline sufficient statistic approach). The difference between discounting by the individual-specific marginal rate of substitution (MRS) versus the interest rate corresponds to the covariance term in Equation 19.

Table A8: Welfare gains with alternative parameters

	Welfare gains discounted by ...		RMSE	Correlation
	Interest rate (abs. value)	Individual MRS (abs. value)		
Baseline	0.22	0.20	0.05	0.99
σ: 0.10 → 0.20	0.56	0.61	0.24	0.97
χ_U: 0.60 → 0.30	0.22	0.19	0.05	0.99
γ: 1.50 → 3.00	0.29	0.28	0.09	0.99
ϕ: 0.95 → 0.99	0.27	0.25	0.10	0.97

Quantifying the effect of idiosyncratic returns The previous model covered the case of idiosyncratic shocks in labor income, an essential source of market incompleteness in the left tail of the distribution. We now discuss the case of idiosyncratic shocks in wealth shocks, which is the relevant source of market incompleteness in the wealth distribution's right tail. We treat this case separately from the previous model for the sake of clarity and because, in this case, we can obtain closed-form formulas for the effect of idiosyncratic shocks in the limit of high wealth (i.e., in the limit where labor income is zero).

Formally, consider an individual with CRRA utility that can invest in a risk-free asset with constant interest rate $R = Q^{-1}$ or in a linear technology with i.i.d. stochastic return $A_{i,t}$. Because of the linearity of investment opportunities and the homotheticity of preferences, an agent maximizing their expected utility chooses to invest a fixed fraction of their wealth in the risky technology, α and to consume a fixed fraction of their wealth, ρ. As a result, the wealth of the agent evolves as

$$W_{i,t+1} = (R + \alpha (A_{i,t+1} - R) - \rho) W_{i,t}.$$

Such models where wealth evolves with random innovations ("random-growth" models) are particularly important to generate a realistic concentration of wealth at the very top. It turns out that, in these models, one can also characterize the effect of incomplete markets in closed form.

Proposition A10. *Under the assumptions discussed above, the welfare gain of a deviation in the risk-free rate* $\mathrm{d}Q_t$ *is*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \left(Re^{c(1)+c(-\gamma)-c(1-\gamma)}\right)^{-t} (-\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_{i,t}), \tag{A29}$$

where $c(\xi) \equiv \log \mathbb{E}\left[(R + \alpha (A_{i,t} - R) - \rho)^{\xi}\right]$ *denotes the Cumulating Generating Function (CGF) of log wealth growth and where* $B_{i,t} = (1-\lambda)W_{i,t}/Q_t$ *denotes agent i's holdings of one-period bonds at time t.*

This formula for welfare gains is the same as in the baseline model except that, to adjust for idiosyncratic wealth returns, one needs to adjust the baseline discount rate, R, by a quantity dependent on the dispersion of wealth growth, $e^{c(1)+c(-\gamma)-c(1-\gamma)}$. In the presence of stochastic wealth growth, this adjustment is higher than one as the function $c(\cdot)$ is convex.[27]

This proposition implies that idiosyncratic wealth shocks systematically decrease the magnitude of welfare gains. The intuition is that, due to the linearity of policy functions, individuals jointly increase their consumption and financial transactions after a positive wealth shock. As a result, the growth of marginal utility covaries negatively with financial transactions, which dampens welfare gains (in magnitude) relative to the baseline formula. Such a formula would still hold in a model with multiple types of assets: the key requirement is that the agent policy decisions (including financial transactions) scale linearly with the level of wealth. Finally, note that there is a key difference between the effect of transitory labor income shocks and the effect of wealth shocks. As discussed above, the presence of labor income shocks tends to *increase* the welfare gains of rising asset prices (relative to the baseline formula), as agents tend to purchase assets in good times and sell assets in bad times. In contrast, the presence of idiosyncratic return shocks tends to *dampen* welfare gains relative to the baseline formula, as agents tend to scale up their more significant financial transactions (sales or purchases) in good times and scale them down in bad times.

We now discuss how big this adjustment is quantitatively. For instance, consider the case in which R_t^K is log-normally distributed with mean μ and variance σ^2. In this case, using a Taylor approximation, the growth in wealth is also log-normally distributed with mean $\log R + \alpha(\mu - \log R) + \frac{1}{2}\alpha(1-\alpha)\sigma^2$ and variance $\alpha^2\sigma^2$.[28] For a log normal variable X, we have $\log \mathbb{E}\left[X^{\xi}\right] = \xi\mu + \frac{1}{2}\xi^2\mu^2$. As a result, the formula for welfare gains (A29) simplifies to:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \left(Re^{\gamma\alpha^2\sigma^2}\right)^{-t} (-\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_t).$$

[27]See Martin (2013) for similar results on discount rates in the context of aggregate shocks as well as a geometric interpretation for the discount rate adjustment.

[28]As shown in Campbell and Viceira (2002), we have

$$\log (R + \alpha (A_{i,t} - R) - c) \approx \log R + \alpha (\log A_{i,t} - \log R) - \rho + \frac{1}{2}\alpha(1-\alpha)\sigma^2.$$

This formula tells us that, to adjust for idiosyncratic wealth shocks, one simply needs to adjust discount rates by $\gamma\alpha^2\sigma^2$. To take a realistic example, consider the case $\alpha = 1.5$, $\sigma = 10\%$, and $\gamma = 1$, in which case the adjustment is $\gamma\alpha^2\sigma^2 \approx 2$ pp. This result justifies using a relatively high discount rate in our empirical exercise — remember that we use $R = 1.05$, which is higher than the average midpoint rate between deposit and debt rates in our time sample.

Proof of Proposition A10. We start from the formula for welfare gains in a stochastic environment given in Proposition A2:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = -\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{U'(C_{i,t})}{U'(C_{i,0})} B_{i,t}\right]\mathrm{d}Q_t.$$

For an agent with CRRA preferences, we have $U'(C_{i,t})/U'(C_{i,0}) = (C_{i,t}/C_{i,0})^{-\gamma} = (W_{i,t}/W_{i,0})^{-\gamma}$. Now, we have $B_{i,t} = ((1-\alpha)/Q_t)\, W_{i,t}$ and so

$$\begin{aligned}
\mathrm{d}V_{i,0}/U'(C_{i,0}) &= -\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{W_{i,t}^{-\gamma}}{W_{i,0}^{-\gamma}}\left((1-\alpha)/Q_t\right) W_{i,t}\right]\mathrm{d}Q_t \\
&= -\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\left[\frac{W_{i,t}^{1-\gamma}}{W_{i,0}^{1-\gamma}}\right]\mathbb{E}_0\left[\frac{W_{i,t}}{W_{i,0}}\right]^{-1}\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_t \\
&= -\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\left[\frac{W_{i,1}^{1-\gamma}}{W_{i,0}^{1-\gamma}}\right]^t\mathbb{E}_0\left[\frac{W_{i,1}}{W_{i,0}}\right]^{-t}\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_t,
\end{aligned}$$

where the last equation results from the fact that returns (and therefore wealth growth) are i.i.d. Now, Euler equation implies $R^{-1} = \beta\mathbb{E}_0\left[\frac{W_{i,1}^{-\gamma}}{W_{i,0}^{-\gamma}}\right]$. Plugging this into the previous equation gives

$$\begin{aligned}
\mathrm{d}V_{i,0}/U'(C_{i,0}) &= -\sum_{t=0}^{\infty}\left(R\frac{\mathbb{E}_0\left[\frac{W_{i,1}^{1-\gamma}}{W_{i,0}^{1-\gamma}}\right]}{\mathbb{E}_0\left[\frac{W_{i,1}^{-\gamma}}{W_{i,0}^{-\gamma}}\right]\mathbb{E}_0\left[\frac{W_{i,1}}{W_{i,0}}\right]}\right)^{-t}\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_t \\
&= -\sum_{t=0}^{\infty}\left(Re^{c(1-\gamma)-c(1)-c(-\gamma)}\right)^{-t}\mathbb{E}_0[B_{i,t}]\,\mathrm{d}Q_t,
\end{aligned}$$

which concludes the proof. □

D.2 Borrowing constraints and collateral effects

D.2.1 Proof of Proposition 3

Proof of Proposition 3. The Lagrangian associated with the individual problem is

$$\begin{aligned}
\mathcal{L}_i &= \sum_{t=0}^{\infty}\beta^t U(C_{i,t}) \\
&\quad + \sum_{t=0}^{\infty}\lambda_{i,t}\left(Y_{i,t} + N_{i,t-1}D_{i,t} + B_{i,t-1} - C_{i,t} - (N_{i,t}-N_{i,t-1})\,P_t - B_{i,t}Q_{i,t} - \chi(N_{i,t}-N_{i,t-1})\right).
\end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned}
\mathrm{d}V_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t}\,\mathrm{d}P_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t}\,\mathrm{d}Q_t, \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t \right) \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t}\,\mathrm{d}Q_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\,\mathrm{d}P_t \right) \right),
\end{aligned}$$

where the last equality uses the definition of the interest schedule (Equation 23 in the main text). Combining this equation with the first-order condition with respect to consumption $C_{i,t}$, $\beta^t U'(C_{i,t}) = \lambda_{i,t}$, gives:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t}\,\mathrm{d}Q_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\,\mathrm{d}P_t \right) \right). \tag{A30}$$

We can go further by using the first-order condition with respect to bond holdings $B_{i,t}$ for $t \geq 0$:

$$\lambda_{i,t} \left(Q_{i,t} + B_{i,t} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right) = \lambda_{i,t+1}.$$

Combined with the first-order condition for consumption, we obtain a modified Euler equation for $t \geq 0$

$$\frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} = Q_{i,t} \left(1 + \frac{B_{i,t}}{Q_{i,t}} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right). \tag{A31}$$

This equation says that agents equalize their marginal rate of substitutions between t and $t+1$ to the effective *marginal* interest rate they face given the schedule (Equation 23). Plugging this equation into (A30) gives

$$\begin{aligned}
\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} & \left(\prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right) \right. \\
& \left. \times \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t}\,\mathrm{d}Q_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\,\mathrm{d}P_t \right) \right) \right),
\end{aligned}$$

which concludes the proof after defining $\widetilde{R}^{-1}_{i,0\to t} \equiv \prod_{s=0}^{t-1} Q_{i,s} \left(\left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right)$. □

D.2.2 Empirical implementation

Proof of Corollary 4. The parametric form $Q_{i,t} = Q_t e^{-\xi \mathrm{LTV}_{i,t}}$ with $\mathrm{LTV}_{i,t} \equiv -B_{i,t}/(N_{i,t}P_t)$ implies the following expressions for the derivatives of the individual-specific bond price $Q_{i,t}$ with respect to asset prices and asset holdings:

$$\frac{\partial Q_{i,t}}{\partial Q_t} = \frac{Q_{i,t}}{Q_t} \quad ; \quad \frac{\partial Q_{i,t}}{\partial B_{i,t}} = \xi \frac{Q_{i,t}}{N_{i,t}P_t} \quad ; \quad \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} = \xi \frac{Q_{i,t}\mathrm{LTV}_{i,t}}{N_{i,t}P_t}.$$

Plugging these derivatives into the expression for welfare gains obtained in Proposition 3 gives

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \widetilde{R}^{-1}_{i,0\to t} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}Q_{i,t} \left(\frac{\mathrm{d}Q_t}{Q_t} + \xi \times \mathrm{LTV}_{i,t} \times \frac{\mathrm{d}P_t}{P_t} \right) \right),$$

with $\widetilde{R}^{-1}_{i,0\to t} = \prod_{s=0}^{t-1} Q_{i,s} \left(1 - \xi \times \mathrm{LTV}_{i,t} \right)$, which concludes the proof. □

Reduced-form evidence on ξ. We now describe how we estimate the parameter ξ (Equation 26 in the main text). We start with the full sample of individuals over the 1994–2019 period. To compute the implied mortgage interest rate, we first compute the interest costs by outstanding debt, which are readily available in our data. We compute the interest rate and loan-to-value of individual i at time t as

$$\text{Interest rate}_{i,t} = \frac{\text{Interest costs}_{i,t}}{\frac{1}{2}\text{Debt}_{i,t-1} + \frac{1}{2}\text{Debt}_{i,t}}, \qquad \text{LTV}_{i,t} = \frac{\text{Debt}_{i,t}}{\text{Housing value}_{i,t}},$$

where $\text{Debt}_{i,t} \equiv -B_{i,M,t}Q_{i,M,t}$. Both the numerator and denominator pertain to directly held debt and housing.

To estimate ξ, we use the approximation $\text{Interest rate}_{i,t} \approx -\log Q_{i,t}$ and estimate a regression of the form

$$\text{Interest rate}_{i,t} = \alpha_t + \xi \times \text{LTV}_{i,t-1} + u_{i,t},$$

where α_t is a year fixed-effect, and $u_{i,t}$ is an error term. We remove observations with a loan-to-value lower than 0.2 as (i) the interest rate estimate is imprecise for low debt levels and (ii) these low values are more likely to be driven by consumer debt rather than mortgage debt (empirically, we find that interest rates decrease between 0 and 0.2). Specification (1) in Table A9 reports the results. The implied value of ξ is approximately 0.0025: a 10 pp. increase in the loan-to-value ratio implies a 0.025 pp. (2.5 basis point) increase in the interest rate.

One potential concern with the regression evidence is the presence of omitted variable bias. For instance, if some households are deemed safer by banks, they may face an interest-rate schedule that is shifted downward and they may endogenously choose a higher loan-to-value ratio. This bias would dampen the relationship between interest rates and loan-to-value ratios. Therefore, we also estimate a specification with age dummies and education groups as controls. Specification (2) in Table A9 reports the results. The implied value of ξ is approximately 0.004: a 10 pp. increase in the loan-to-value ratio implies a 0.04 percentage point (i.e., a 4 basis points) increase in the interest rate.

Table A9: Regression of mortgage interest rate on loan-to-value

Mortgage interest rate	(1)	(2)
Loan-to-value	0.0024	0.0039
	(0.00001)	(0.00002)
Year fixed effects	✓	✓
Age and education controls		✓
Sample size	28,298,160	27,501,724
R^2	(0.519)	(0.533)

Notes. Standard errors in parentheses. The education groups are: "less than high school", "high school", "college". To estimate this regression, we restrict attention to observations with an interest rate in the 5%-95% range within each year and with a ratio of debt to housing value between 0.2 and 0.99.

Another potential concern with the regression evidence is that the presence of measurement error in the loan-to-value variable will generate an attenuation bias (i.e., bias the estimate of ξ towards zero). Therefore, we provide external evidence on the relationship between loan-to-value and mortgage interest rate using posted interest-rate schedules published by banks. Although data for the largest banks is unavailable, some smaller banks do report mortgage interest rates as a function of the loan-to-value ratio. Table A10 presents an example of such a schedule from *Bulder Bank* in 2022.

Using the last four rows of Table A10 and using the midpoints of the loan-to-value range, we obtain a linear slope of $\xi = 0.011$; that is, a 10 pp. increase in the loan-to-value ratio implies an 11 basis point

Table A10: Example of interest-rate schedule

Loan-to-value	Interest rate
$< 50\%$	3.33%
$50-55\%$	3.41%
$55-60\%$	3.51%
$60-65\%$	3.56%
$65-70\%$	3.60%
$70-75\%$	3.64%

Notes. Extracted on October 26, 2022, from Bulder Bank's website (https://www.bulderbank.no/priser).

rise in the interest rate. This slope is roughly four times as large as the regression evidence without controls and twice as large as the regression evidence with controls. As a result, for robustness, we estimate the effect of borrowing constraints using two potential estimates $\xi \in \{0.005, 0.01\}$.

Implementation. Finally, to implement our formula for welfare gains taking into account borrowing constraints (Equation 27 in the main text), we use the following modified version of our sufficient statistic formula:

$$
\begin{aligned}
\text{Welfare Gain}_i^{BC} &= \sum_{k\in\{\text{housing,debt,deposit,equity}\}} \text{Welfare Gain}_{i,k}^{BC}, \\
\text{Welfare Gain}_{\text{i,housing}}^{BC} &= \sum_{t=0}^{25} \widetilde{R}_{i,0\to t}^{-1}\Big((N_{i,H,t-1}-N_{i,H,t})P_{H,t}+\xi\times \text{LTV}_{i,t}^2\times N_{i,H,t}P_{H,t}\Big)\frac{PD_{H,t}-\overline{PD}_H}{PD_{H,t}}, \\
\text{Welfare Gain}_{\text{i,debt}}^{BC} &= \sum_{t=0}^{25} \widetilde{R}_{i,0\to t}^{-1}(-B_{i,M,t}Q_{M,t})\times\frac{Q_{M,t}-\overline{Q}_M}{Q_{M,t}} \\
\text{Welfare Gain}_{\text{i,deposit}}^{BC} &= \sum_{t=0}^{25} \widetilde{R}_{i,0\to t}^{-1}(-B_{i,D,t}Q_{D,t})\times\frac{Q_{D,t}-\overline{Q}_D}{Q_{D,t}}, \\
\text{Welfare Gain}_{\text{i,equity}}^{BC} &= \sum_{t=0}^{25} \widetilde{R}_{i,0\to t}^{-1}(N_{i,E,t-1}-N_{i,E,t})P_{E,t}\times\frac{PD_{E,t}-\overline{PD}_E}{PD_{E,t}},
\end{aligned}
\tag{A32}
$$

where the individual-specific discount factor $\widetilde{R}_{i,0\to t}^{-1}$ and loan-to-value ratios $\text{LTV}_{i,t}$ are given by:

$$
\begin{aligned}
\widetilde{R}_{i,0\to t}^{-1} &\equiv R^{-t}e^{-\xi\sum_{s=0}^{t-1}\text{LTV}_{i,s}}\prod_{s=0}^{t}(1-\xi\times\text{LTV}_{i,s}) \approx R^{-t}e^{-2\xi\sum_{s=0}^{t-1}\text{LTV}_{i,s}}, \\
\text{LTV}_{i,t} &\equiv \max\left(0,\min\left(-\frac{B_{i,M,t}Q_{M,t}}{N_{i,H,t}P_{H,t}},1\right)\right).
\end{aligned}
$$

As in the baseline implementation of our sufficient statistic (Equation 16 in the main text), our measure of debt holdings $B_{i,M,t}Q_{M,t}$ include both directly-held debt and indirectly-held debt (i.e., the debt on the balance sheet of businesses owned by individuals). Similarly, our measure of housing wealth $N_{i,H,t}P_{H,t}$ $B_{i,M,t}Q_{M,t}$ include both directly-held and indirectly-held housing. Hence, this modification of the sufficient statistic formula affects not only individuals who buy a home via a mortgage but also entrepreneurs who use real estate assets as collateral (as in Chaney et al., 2012).

Figure A9 decomposes the average wealth gain in each cohort into three terms: the baseline welfare gains (the one obtained in the baseline case without borrowing constraints), a term due to the discount-rate channel, and a term due to the collateral channel (this decomposition follows Equation 25 in the main text). Note that the term due to the discount-rate channel tends to be negative, while the term due to the collateral channel is always positive. Hence, the two effects tend to cancel out, which explains why the overall impact of borrowing constraints is negligible in our setting.

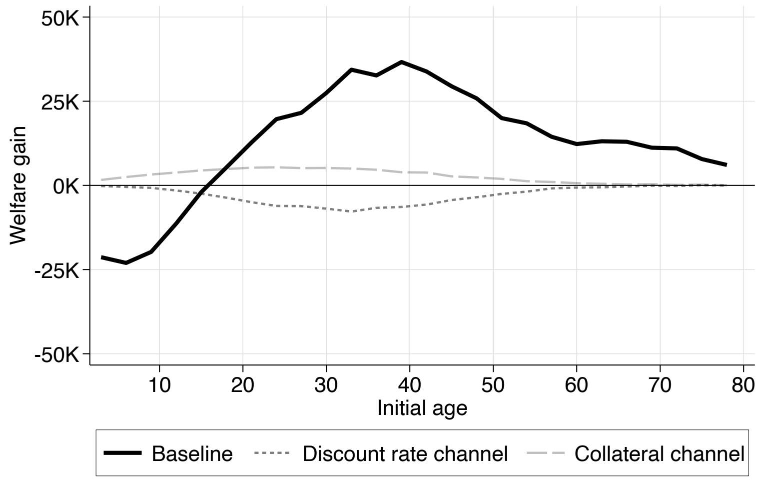


Figure A9: Decomposing the effect of borrowing constraints on welfare gains

Notes. This figure decomposes welfare gains in the presence of borrowing constraints as a sum of three terms: our baseline formula, a discount-rate channel, and a collateral channel (see Equation 25 in the main text). The discount-rate channel corresponds to the effect of increasing individual discount rates by $\xi \frac{1}{t} \sum_{s=0}^{t-1} \text{LTV}_{i,s}$. The collateral channel corresponds to the effect of increasing the annual home sales of individuals by $\xi \times \text{LTV}_{i,t}^2 \times N_{i,H,t} P_{H,t} (PD_{H,t} - \overline{PD}_H) / PD_{H,t}$. See (A32) for more detail.

Histograms. In the main text, we focus on the *average* effect of borrowing constraints on welfare gains within cohorts. However, there is also an important heterogeneity in the effect of this correction within cohorts. In particular, we can expect individuals with more debt to disproportionately benefit from the collateral channel.

Table A11 reports the average effect of borrowing constraints on welfare gains in the population for each of our $\xi \in \{0.005, 0.01\}$ (as the sum of the discount rate and collateral channels). We also report it in six percentile bins. Looking at the row with $\xi = 0.01$, the average correction due to borrowing constraints is close to zero.

Similarly to debt holdings, the effect of borrowing constraints on welfare gains is right-skewed. As shown in Table A11, the welfare gains reach \$98,000 for the top 1% of the most affected individuals. Note that this remains much smaller than the baseline welfare gains for the top 1% individuals, which is \$662,000 (Table 2).

Table A11: Distribution of the effect of borrowing constraints on welfare gains

Value for ξ	Average	Average by percentile groups					
		p0-1	p1-10	p10-50	p50-90	p90-99	p99-100
$\xi = 0.005$	−0.0	−79.5	−3.9	−0.1	0.6	4.6	52.4
$\xi = 0.01$	−0.1	−151.2	−7.6	−0.3	1.1	8.8	97.5

Notes. We compute for each individual the effect of borrowing constraints on welfare gains, computed as the difference between welfare gains taking into account borrowing constraints (A32) and the baseline formula (Equation 16 in the main text). As shown in Equation 25 in the main text, this difference can be interpreted as the sum of the terms corresponding to the discount-rate and collateral channels. The table reports the average effect of borrowing constraints by percentile group. All numbers are in thousands of 2011 U.S. dollars.

Figure A10 plots the distribution of the effect of welfare gains across individuals in Norway. To avoid scaling issues, we do not plot the density at zero (i.e., observations with zero debt), as they account for roughly 50% of our observations (see Table A11).

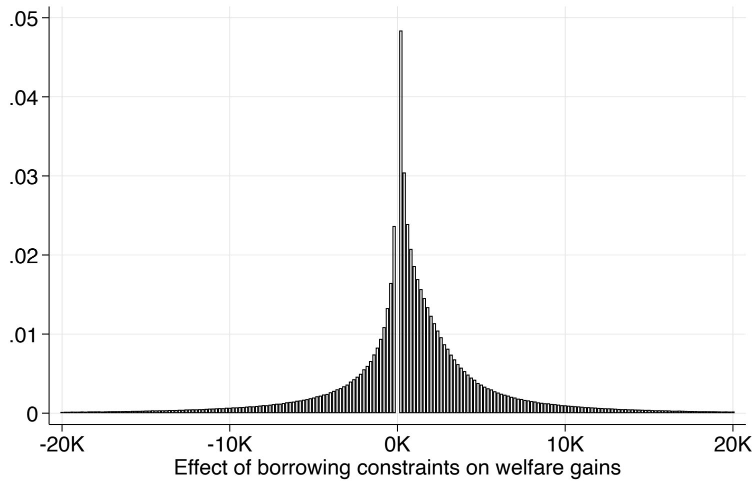


Figure A10: Distribution of the effect of borrowing constraints on welfare gains

Notes. We compute for each individual the effect of borrowing constraints on welfare gains, computed as the difference between welfare gains taking into account borrowing constraints (A32) and the baseline formula (Equation 16 in the main text). As shown in Equation 25 in the main tex, this difference can be interpreted as the sum of the terms corresponding to the discount-rate channel and to the collateral channel. The figure plots the density of this object across individuals in Norway for $\xi = 0.01$. More precisely, the figure plots the relative mass of individuals within equally-spaced bins of welfare gains (width of $100). For the sake of legibility, we do not report the relative mass of individuals with a welfare gain lower than 100 in absolute value, as approximately half of the population has no debt. All numbers are in thousands of 2011 U.S. dollars.

D.2.3 Case of hard borrowing constraints

In the main text, we have focused on quantifying theoretically and empirically the effect of a smooth borrowing constraint (i.e., a smooth interest-rate schedule). This methodology choice is guided by the data, as the gradient of mortgage rate to loan-to-value appears to be fairly smooth in Norway. For external validity, we now discuss how to quantify theoretically and empirically the effect of hard borrowing constraints. We obtain simple formulas that are analogous, in spirit and in magnitude, to those derived for a smooth interest-rate schedule.

Theory. We consider an economy in which individual i faces a hard borrowing constraint; that is, when there is a strict upper bound on the amount of borrowing:

$$-B_{i,t} \leq \Phi(N_{i,t}P_t), \tag{A33}$$

where $\Phi(\cdot)$ is a smooth function. This general specification includes both ad-hoc borrowing limits of the form $-B_{i,t} \leq \phi$, where $\phi > 0$ is a borrowing limit, as well as collateral constraints of the form $-B_{i,t} \leq \lambda N_{i,t}P_t$ where $\lambda > 0$ is a maximum ratio of loan to asset value.

Collateral constraints may impact even wealthy individuals. One particular reason is asset illiquidity; that is, the case where adjustment costs $\chi(N_{i,t} - N_{i,t-1})$ are large (the "wealthy hand-to-mouth" of Kaplan and Violante, 2014). Another reason is tax avoidance: in particular, individuals may borrow against their assets to consume as part of a "buy, borrow, die" tax avoidance strategy.[29] Just like in the

[29] One main reason individuals use a "buy, borrow, die" strategy is step-up in basis at death. This feature of the U.S. tax system (and some other countries) means that dying without ever having sold an asset and passing it on to an heir greatly reduces the heir's capital gains tax bill if they sell the inherited asset.

case of asset illiquidity, individuals may want to borrow as much as possible and may run into collateral constraints precisely *because* of the differential tax treatment of borrowing and asset sales. Another reason is that rich individuals may get direct utility from asset ownership (as in Appendix A.4.1). In this case, individuals may not want to sell their assets precisely *because* of the utility benefit from ownership and may be pushed into collateral constraint instead.[30]

Note that the hard borrowing constraint (A33) can be seen as a limiting case of our general interest schedule framework. To see this formally, consider a particular interest-rate schedule of the form:[31]

$$Q_{i,t} = Q_t \left(1 + e^{-\theta\left(B_{i,t}+\Phi(N_{i,t}P_t)+1/\sqrt{\theta}\right)}\right)^{-1}. \tag{A34}$$

As $\theta \to \infty$, this interest-rate schedule converges to

$$\lim_{\theta\to\infty} Q_{i,t} = \begin{cases} Q_t & \text{if } -B_{i,t} \leq \Phi(N_{i,t}P_t) \\ 0 & \text{otherwise.} \end{cases}$$

In other words, individuals can borrow at the reference interest rate Q_t^{-1} whenever the borrowing limit $-B_{i,t} \leq \Phi(N_{i,t}P_t)$ is satisfied but they face an interest rate of infinity otherwise; hence, this limiting economy corresponds to the hard borrowing constraint (A33). See Figure A11 for a graphical representation.

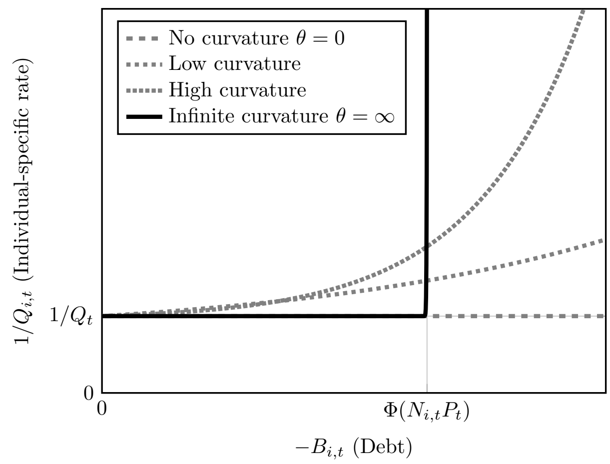


Figure A11: Interest-rate schedule (A34) for different values of θ

The following proposition expresses the welfare gains of a deviation in asset prices in the presence of a hard borrowing constraint.

Proposition A11. *When agent i faces a hard borrowing constraint* (A33)*, the welfare gain of a deviation in asset prices is*

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t + \left(Q_t - \beta \frac{U'(C_{i,t+1})}{U'(C_{i,t})}\right) \Phi'(N_{i,t}P_t) N_{i,t}\,\mathrm{d}P_t \right).$$

This proposition says that hard borrowing constraint affect our welfare gain formula through two

[30] Note that, in the case of tax avoidance, rising asset prices will also affect the *relative* welfare of asset sellers who use the strategy relative to those who do not. When asset prices rise, asset sellers who do not use the "buy, borrow, die" strategy pay higher capital gains taxes, which attenuates their welfare gain (as in Appendix A.4.4). In contrast, this attenuation effect is smaller (or non-existent) for individuals who use the strategy because they pay less (or no) capital gains taxes in the first place.

[31] The term in $1/\sqrt{\theta}$ is to ensure that $Q_{i,t} \to Q_t$ as $\theta \to \infty$ at the upper bound $-B_{i,t} = \Phi(N_{i,t}P_t)$.

channels: a discount-rate channel and a collateral channel. First, when the borrowing constraint binds, the Euler equation is not satisfied, and $\beta U'(C_{i,t+1})/U'(C_{i,t}) < 1/Q_t$, which means that agents discount more future dollars compared to the interest rate on the liquid asset ("discount rate" channel). Second, when $\Phi'(N_{i,t}P_t) > 0$, a rise in asset prices relaxes the borrowing constraint, which allows the individual to consume more today at the expense of tomorrow. This is welfare improving because, in this case, the benefit of increasing consumption today at time t, $U'(C_{i,t})$, is strictly higher than the cost of decreasing consumption tomorrow $\beta U'(C_{i,t+1})/Q_t$ ("collateral" channel).

One implication of this proposition is that, in this setting, the welfare gains of a deviation in asset prices no longer aggregate to zero. First, there is an effect due to the discount-rate channel. With $\Phi(\cdot) > 0$, agents at the borrowing constraint tend to be net borrowers, so they disproportionately benefit from the welfare effect of lower interest rates. Because they are also the ones with higher discount rates, this tends to reduce the sum of welfare gains due to lower rates across individuals. Second, there is an effect due to the collateral channel: when $\Phi'(N_{i,t}P_t) > 0$, the formula for welfare gains has an additional term that is positive for every asset holder. While the first effect generally has an ambiguous sign, the second effect is always positive.

We now briefly compare Proposition A11, which focuses on the effect of a hard borrowing constraint, to Proposition 3 (Section IV.B), which focuses on the impact of a smooth interest-rate schedule. Both propositions say that borrowing constraints affect welfare gains through a discount-rate channel and a collateral channel. However, the intuition for the collateral channel is slightly different across the two propositions. Proposition 3 says that, in the presence of an interest-rate schedule, collateral constraints affect welfare gains because higher asset prices allow agents to borrow at a lower interest rate. In comparison, Proposition A11 says that, in the case of a hard borrowing constraint, collateral constraints affect welfare gains because higher asset prices allow constrained individuals to better smooth their consumption over time. To understand the connection between the two results, note that $U'(C_{i,t})/(\beta U'(C_{i,t+1}))$ can be seen as the shadow interest rate of individual i; that is, to the interest rate at which the individual is indifferent to lend or to borrow a dollar. Seen this way, as in the case of the interest-rate schedule of Proposition 3, Proposition A11 says that a rise in asset prices increases welfare because it allows individuals at the borrowing constraint to decrease their borrowing costs, as they can borrow an infinitesimal amount $\Phi'(N_{i,t}P_t)N_{i,t}\,\mathrm{d}P_t$ at rate $1/Q_t$ rather than at the shadow rate $U'(C_{i,t})/(\beta U'(C_{i,t+1}))$.

Finally, as in the case of the interest-rate schedule (Proposition 3), we can formally write the expression for welfare gains in the presence of a hard borrowing constraint as a sum of three terms, which capture, respectively, welfare gains in the baseline (frictionless) model, a term capturing the discount-rate channel, and a term capturing the collateral channel:

$$
\begin{aligned}
\mathrm{d}V_{i,0}/U'(C_{i,0}) = & \underbrace{\sum_{t=0}^{\infty} R_{0,t}^{-1}\left((N_{i,t-1}-N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right)}_{\text{Baseline}} \\
& + \underbrace{\sum_{t=0}^{\infty}\left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} - Q_0\dots Q_{t-1}\right)\left((N_{i,t-1}-N_{i,t})\,\mathrm{d}P_t - B_{i,t}\,\mathrm{d}Q_t\right)}_{\text{Discount-rate channel}} \\
& + \underbrace{\sum_{t=0}^{\infty}\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}\left(Q_t - \beta\frac{U'(C_{i,t+1})}{U'(C_{i,t})}\right)\Phi'(N_{i,t}P_t)N_{i,t}\,\mathrm{d}P_t.}_{\text{Collateral channel}}
\end{aligned}
\tag{A35}
$$

Proof of Proposition A11. One way to derive this expression is to apply the usual Lagrangian method augmented with the hard borrowing constraint (A33). Instead, for the sake of intuition, we use a derivation that relies on the expression for welfare gains obtained in the case of an interest-rate schedule and passes to the limit of a "hard" borrowing constraint. As stated in Proposition 3, the welfare gain of a deviation in asset prices when the agent faces an interest-rate schedule is:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right)$$
$$\times \left((N_{i,t-1} - N_{i,t})\, dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\, dP_t \right) \right).$$

Remember that the modified Euler equation (A31) in the case of an interest-rate schedule is

$$Q_{i,t} \left(1 + \frac{B_{i,t}}{Q_{i,t}} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right) = \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})}. \tag{A36}$$

Equation A36 implies that the discount rate in the expression for welfare gains can be rewritten as:

$$\prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right) = \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}. \tag{A37}$$

Equation A36 also implies that the term capturing the effect of collateral constraints can be rewritten as:

$$-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\, dP_t = \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \frac{\partial Q_{i,t} / \partial (N_{i,t}P_t)}{\partial Q_{i,t} / \partial B_{i,t}} N_{i,t}\, dP_t,$$

where the quantity $\frac{\partial Q_{i,t}/\partial (N_{i,t}P_t)}{\partial Q_{i,t}/\partial B_{i,t}} N_{i,t}\, dP_t$ can be interpreted as the additional amount of cash that agent i can borrow as a result of the change in asset prices while keeping the interest rate constant. Hence, this expression says that the collateral channel in an interest schedule model can be seen in two equivalent ways: as the monetary value of lower interest costs in the left-hand-side (corresponding to the case where the agent holds the amount of debt fixed), or as the welfare value of better consumption smoothing in the right-hand-side (obtained when the agent borrows more in response to the rise in asset prices, keeping the individual-specific interest rate fixed).

In the particular case of the interest-rate schedule (A34), the last equation simplifies to

$$-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t}\, dP_t = \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t}P_t) N_{i,t}\, dP_t. \tag{A38}$$

Plugging (A37) and (A38) into the expression for welfare gains given above yields

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}$$
$$\times \left((N_{i,t-1} - N_{i,t})\, dP_t - B_{i,t} Q_{i,t} \frac{dQ_t}{Q_t} + \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t}P_t) N_{i,t}\, dP_t \right).$$

Since this expression holds for any θ, it also holds in the limit $\theta \to \infty$ (i.e., in the limit of a hard borrowing constraint), which concludes the proof. □

Calibration approach. We now quantify the effect of a hard borrowing constraint on our welfare-gains formula. As seen in (A35), the effects of the discount rate and collateral channels depend on two critical statistics: (i) the difference between the shadow rate at which the individual is willing to borrow and the market rate, as represented by $Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t})$, and (ii) the extent to which higher asset prices relax the borrowing limit, $\Phi'(N_{i,t}P_t)$.

We first discuss the quantification of $Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t})$. This term is zero for individuals outside the borrowing constraint but positive for individuals at the borrowing constraint. As discussed above, this term can be interpreted as the difference between the highest rate the individual would be willing to borrow and the cost of debt. As a specific example, it is useful to think of an entrepreneur (or an investor) who has access to a decreasing-return-to-scale technology (or an investment opportunity) and who cannot equalize the marginal return of the technology with the marginal cost of funds because of collateral constraint.

We now turn to the quantification of $\Phi'(N_{i,t}P_t)$. Many models assume that borrowing constraints take the form $\Phi(N_{i,t}P_t) = \lambda_i N_{i,t}P_t$, where λ_i corresponds to the limiting debt to asset ratio, which implies $\Phi'(N_{i,t}P_t) = \lambda_i$. Alternatively, other models assume that borrowing limits depend on earnings rather than collateral values (e.g., Lian and Ma, 2021, Drechsel, 2023), in which case $\Phi'(N_{i,t}P_t) = 0$.[32]

Given an estimate for these two objects, we can use (A35) to quantify the effect of borrowing constraints on welfare gains. For the sake of calibration, we now assume that the wedge between the shadow interest rate and the cost of funding averages to $Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t}) = 5\%$ for agents at the constraint, that $\pi = 10\%$ of agents are at the borrowing constraint, and that the maximum loan-to-value ratio averages to $\lambda_i = 0.5$.[33] In this case, (A35) tells us that, on average, borrowing constraints increase the discount rate of households by $\pi\,(Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t})) \approx 0.5$pp (discount-rate channel) and that collateral constraints increase the welfare exposure of homeowners to a rise in asset prices by an amount equivalent to an increase in their annual rate of home sales by $\pi\,(Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t}))\,\lambda_i \approx 0.25$pp. (collateral channel).

Note that this quantification exercise ends up being very similar, in spirit and in magnitude, to the quantification exercise done for interest-rate schedules in the main text (Section IV.C). Indeed, we estimated that for the interest-rate schedule $Q_{i,t} = Q_t e^{-\xi \times \text{LTV}_{i,t}}$ with $\xi = 0.01$ and an agent loan-to-value ratio $\text{LTV}_{i,t} = 0.5$, borrowing constraints increased the agent discount rate of an agent by $2\xi \times \text{LTV}_{i,t} \approx 1$pp. (discount-rate channel) and that collateral constraints increased the welfare exposure of homeowners to a rise in asset prices by an amount equivalent to an increase in their annual rate of home sales by $\xi \times \text{LTV}_{i,t}^2 \approx 0.25$pp. (collateral channel). Hence, the two models end up giving similar results. The main difference between the two models, however, is that, with an interest-rate schedule, borrowing constraints affect all individuals in the economy (as long as they have some outstanding debt), while, with a hard borrowing constraint, borrowing constraints purely affect the subset of individuals for which the borrowing constraint is binding.[34]

D.3 Second-order

We now detail how we implement our second-order approximation of welfare gains. Multiplying and dividing each term in the second-order approximation of welfare gains (Equation 29 in the main text)

[32] We would also obtain a derivative of zero in a model where the borrowing limit only depends on the "book-value" of asset holdings $N_{i,t}$ rather their market value $N_{i,t}P_t$.

[33] These last two numbers are roughly consistent with Chaney et al. (2012), who estimates that firm investment increases by \$0.03 in response to a \$1 rise in house prices. Direct evidence on the number of firms that are borrowing constrained is more scarce, Champagne and Gouin-Bonenfant (2023) estimate that 4% of Canadian SMEs are constrained in a given year.

[34] This distinction would not be as stark in a stochastic environment, as most individuals would have some probability of hitting the constraint in the future.

by the price of the corresponding asset gives:

$$\mathrm{EV}_i(\theta) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,(\theta/2)\left\{\sum_{k=1}^{K}\left(\frac{(N_{i,k,t-1}(0)-N_{i,k,t}(0))P_{k,t}}{2}+\frac{(N_{i,k,t-1}(\theta)-N_{i,k,t}(\theta))P_{k,t}}{2}\right)\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{B_{i,t}(0)Q_t+B_{i,t}(\theta)Q_t}{2}\frac{\Delta Q_t}{Q_t}\right\}+o(\theta^2).$$

Averaging across cohorts gives

$$\overline{\mathrm{EV}}_a(\theta) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,(\theta/2)\left\{\sum_{k=1}^{K}\left(\frac{(\overline{N}_{i,k,t-1}(0)-\overline{N}_{i,k,t}(0))P_{k,t}}{2}+\frac{(\overline{N}_{i,k,t-1}(\theta)-\overline{N}_{i,k,t}(\theta))P_{k,t}}{2}\right)\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\overline{B}_{a,t}(0)Q_t+\overline{B}_{a,t}(\theta)Q_t}{2}\frac{\Delta Q_t}{Q_t}\right\}+o(\theta^2).$$

Our assumption on counterfactual asset transactions (Equation 30 in the main text) implies

$$\begin{aligned}\left(\overline{N}_{a,k,t}(\theta)-\overline{N}_{a,k,t-1}(\theta)\right)P_{k,t} &= \frac{G^t PD_{k,t}}{PD_{k,0}}\left(\overline{N}_{a,k,0}(0)-\overline{N}_{a,k,-1}(0)\right)P_{k,0}\\ \overline{B}_{a,t}(\theta)Q_t &= G^t\frac{Q_t}{Q_0}\overline{B}_{a,0}(0)Q_0.\end{aligned} \tag{A39}$$

Plugging into the formula for welfare gains above gives

$$\overline{\mathrm{EV}}_a(\theta) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,(\theta/2)\left\{\sum_{k=1}^{K}\left(\frac{(\overline{N}_{i,k,t-1}(0)-\overline{N}_{i,k,t}(0))P_{k,t}}{2}+\frac{G^t PD_{k,t}}{PD_{k,0}}\frac{\left(\overline{N}_{a,k,0}(0)-\overline{N}_{a,k,-1}(0)\right)P_{k,0}}{2}\right)\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\overline{B}_{a,t}(0)Q_t+G^t\frac{Q_t}{Q_0}\overline{B}_{a,0}(0)Q_0}{2}\frac{\Delta Q_t}{Q_t}\right\}+o(\theta^2).$$

Note that all terms are now empirically observable, so we can implement this formula in the data. Finally, to ensure robustness, we proxy the initial transactions $\overline{N}_{a,k,0}(0)-\overline{N}_{a,k,-1}(0)$ and $\overline{B}_{a,0}(0)$ with the average transactions between 1994 and 1999 ($0 \le t \le 4$) instead of the transactions in 1994 alone ($t = 0$).

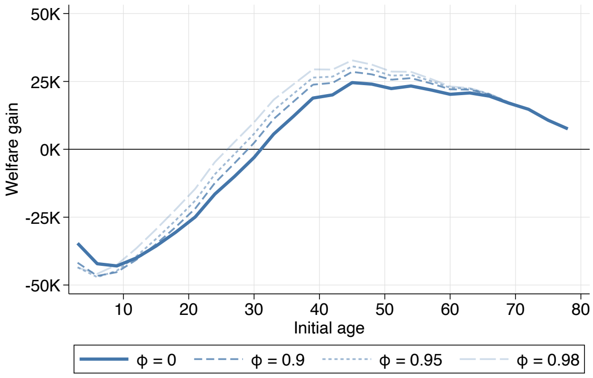
D.4 Extrapolation

We now explain how we construct individuals' transactions in the future. As discussed in the main text, we assume that the average quantity of assets sold by a given cohort in a given year will equal the average amount of assets sold by individuals of the same age in our sample after adjusting for economic growth $G = 1.01$ (which corresponds to the per-capita growth rate of Norway's GDP over our time sample). Formally, we assume that the average transactions of individuals of age a at time $t > T$ is given by

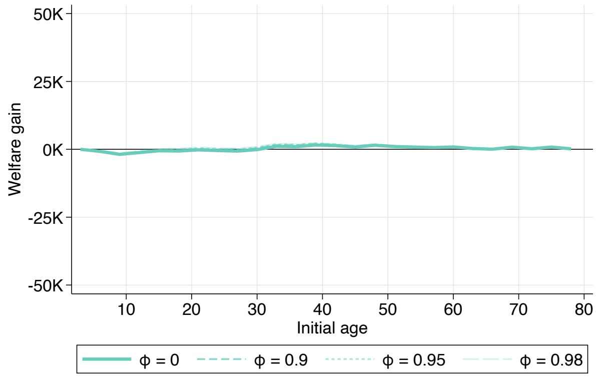
$$\begin{aligned}\left(\overline{N}_{a,k,t}-\overline{N}_{a,k,t-1}\right)PD_{k,t} &= \frac{1}{T+1}\sum_{s=0}^{T} G^{t-s}\frac{PD_{k,t}}{PD_{k,s}}\left(\overline{N}_{a,k,s}-\overline{N}_{a,k,s-1}\right)PD_{k,s},\\ \overline{B}_{a,t}Q_t &= \frac{1}{T+1}\sum_{s=0}^{T} G^{t-s}\frac{Q_t}{Q_s}\overline{B}_{a,s}Q_s,\end{aligned} \tag{A40}$$

where the right-hand side is observable. Finally, we predict the population size in each cohort using the average death rate by age group in our sample (and that the initial size of future cohorts is the same as the average size of the last cohort in our sample).

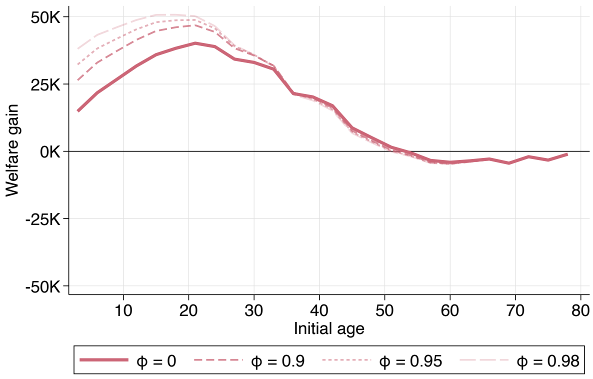
Figure A12 decomposes the welfare gains asset class by asset class. Interestingly, the majority of the impact of extrapolating price deviations is attributable to the welfare effect of lower interest rates through future debt holdings.



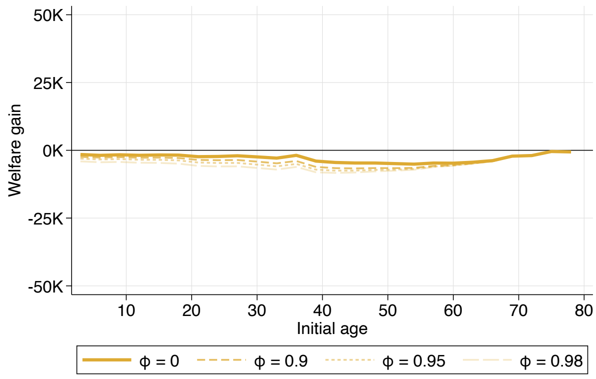
(a) Housing



(b) Equity



(c) Debt



(d) Deposits

Figure A12: Welfare gain depending on the behavior of asset prices in the future, asset class by asset class

D.5 Combining all extensions

For the sake of transparency, we have analyzed the effect of each extension separately. We conclude this section by assessing the combined impact of all these extensions. We start with a straightforward exercise: adding the effects of each extension linearly. Figure A13 plots the result. Overall, the cumulative impact of the extensions increases welfare gains across the distribution, especially for middle-aged cohorts.

Additionally, we do a second exercise where we combine all extensions simultaneously, taking into account the interaction terms between the different extensions (e.g., using the individual-specific discount rate from the borrowing constraints to discount the covariance terms between consumption and asset transactions). One significant challenge is that we must extrapolate the covariance terms between asset transactions and consumption (computed in Section IV.A) beyond our time sample. To do so, for cohorts born before 1974 (i.e., older than 20 years old at the start of our sample), we use a log-linear extrapolation of the covariance terms estimated in the last five years of the sample. For cohorts born after 1974, we use the life-cycle pattern of covariance terms estimated for the 1970-1974 cohorts (including the log-linear extrapolation mentioned above). Figure A13 plots the result of combining all extensions.

We find that the result is relatively close to the effect of adding extensions linearly, which validates our decision to consider each extension separately in the main text (for the sake of simplicity).

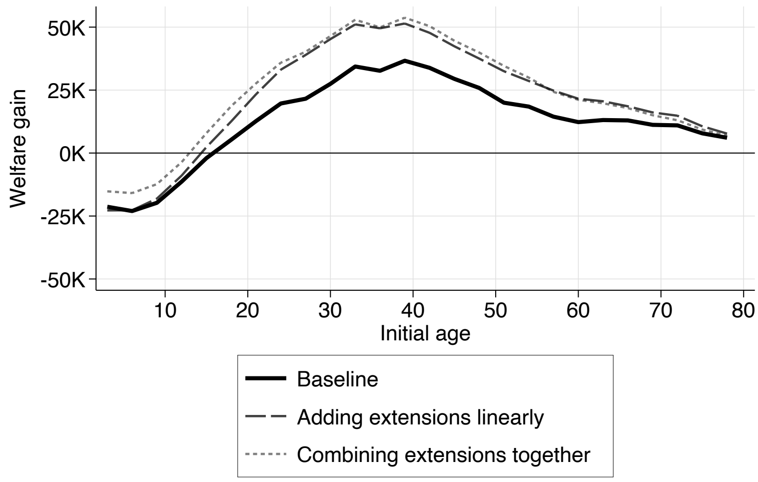


Figure A13: Welfare gains across cohorts after combining all extensions

E Additional theoretical results

E.1 Two-periods model

Deriving Equation 5. We first provide a derivation of Equation 5 in the main text. The Lagrangian associated with the agent optimization problem is

$$\mathcal{L}_i = U(C_{i,0}) + \beta U(C_{i,1}) + \lambda_{i,0}(Y_{i,0} + (N_{i,-1} - N_{i,0})P_0 - C_{i,0}) + \lambda_{i,1}(N_{i,0}D_1 + Y_{i,1} - C_{i,1}).$$

The envelope theorem gives

$$\frac{\partial V_{i,0}}{\partial P_0} = \frac{\partial \mathcal{L}_i}{\partial P_0} = \lambda_{i,0}(N_{i,-1} - N_{i,0})P_0.$$

The first-order condition with respect to consumption at time $t = 0$ gives $\lambda_{i,0} = U'(C_{i,0})$, which concludes the proof.

Solving for the response of consumption As discussed in Proposition A13, our notion of welfare gains can be interpreted as the present value of the deviation in consumption in response to the deviation in asset prices; indeed, by definition, we have

$$\mathrm{d}V_{i,0} = \mathrm{d}U(C_{i,0}) + \beta\,\mathrm{d}U(C_{i,1}) = U'(C_{i,0})\,\mathrm{d}C_{i,0} + \beta U'(C_{i,1})\,\mathrm{d}C_{i,1}.$$

Dividing by $U'(C_{i,0})$ and using the Euler equation implies

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \mathrm{d}C_{i,0} + R_1^{-1}\,\mathrm{d}C_{i,1},$$

where $R_1 \equiv D_1/P_0$. Hence, this equation says that the present value of the deviation in consumption in response to asset prices is pinned down by our sufficient statistic formula (Equation 5). However, our results are silent on how much of the consumption response happens at time $t = 0$ relative to time $t = 1$. We now derive the "term structure" of the consumption response in the context of the two-period model and emphasize that, in contrast with our expression for welfare gains, it depends critically on individual preferences and, in particular, on the individual elasticity of intertemporal substitutions.

Lemma A12. *In the two-period model, the consumption response to an asset-price deviation is:*

$$
\begin{aligned}
\mathrm{d}C_{i,0} &= \underbrace{\mathrm{MPC}_{i,0} \times (N_{i,-1} - N_{i,0})\,\mathrm{d}P_0}_{\mathrm{d}C_{i,0}^{income}} && \underbrace{-\psi(C_{i,1}) \times \mathrm{MPC}_{i,0} \times C_{i,1} R_1^{-1}\,\mathrm{d}\log R_1}_{\mathrm{d}C_{i,0}^{substitution}} \\
\mathrm{d}C_{i,1} &= \underbrace{(1 - \mathrm{MPC}_{i,0}) \times R_1 \times (N_{i,-1} - N_{i,0})\,\mathrm{d}P_0}_{\mathrm{d}C_{i,1}^{income}} && \underbrace{+\psi(C_{i,1}) \times \mathrm{MPC}_{i,0} \times C_{i,1}\,\mathrm{d}\log R_1}_{\mathrm{d}C_{i,1}^{substitution}},
\end{aligned}
\tag{A41}
$$

where $\mathrm{MPC}_{i,0} \equiv \left(1 + R_1^{-1}\frac{C_{i,1}\psi(C_{i,1})}{C_{i,0}\psi(C_{i,0})}\right)^{-1}$ *is the marginal propensity to consume out of income at time* $t = 0$ *and* $\psi(C) \equiv -U'(C)/(U''(C)C)$ *denotes the local elasticity of intertemporal substitution.*

The expressions in obtained for $\mathrm{d}C_{i,0}$ and $\mathrm{d}C_{i,1}$ are similar to those in Auclert (2019). Classically, the consumption response to a change in asset prices can be decomposed into an income effect and a substitution effect. The income effect, denoted $\mathrm{d}C_{i,t}^{\mathrm{income}}$, equals the consumption response to a change in initial income that is welfare-equivalent to the change in prices, $(N_{i,-1} - N_{i,0})\,\mathrm{d}P_0$. In comparison, the substitution effect, denoted $\mathrm{d}C_{i,t}^{\mathrm{substitution}}$, equals the consumption response to the change in prices holding welfare constant. Note that the substitution effect of a higher asset price (i.e., a lower asset return) is positive at $t = 0$ and negative at $t = 1$. Consistently with these definitions, we have the present value of the income response equates the welfare gain while the present value of the substitution effect equates zero.

$$
\begin{aligned}
\mathrm{d}V_{i,0}/U'(C_{i,0}) &= \mathrm{d}C_{i,0}^{\mathrm{income}} + R_1^{-1}\,\mathrm{d}C_{i,1}^{\mathrm{income}}, \\
0 &= \mathrm{d}C_{i,0}^{\mathrm{substitution}} + R_1^{-1}\,\mathrm{d}C_{i,1}^{\mathrm{substitution}}.
\end{aligned}
\tag{A42}
$$

Proof of Lemma A12. Differentiating the budget constraints at $t = 0$ and $t = 1$ gives

$$
\begin{aligned}
\mathrm{d}C_{i,0} + (N_{i,0} - N_{i,-1})\,\mathrm{d}P_0 + \mathrm{d}N_{i,0}P_0 &= 0 \\
\mathrm{d}C_{i,1} &= D_1\,\mathrm{d}N_{i,0}.
\end{aligned}
$$

Substituting out $\mathrm{d}N_{i,0}$ gives the consolidated (differentiated) budget constraint:

$$
\mathrm{d}C_{i,0} + \frac{P_0}{D_1}\,\mathrm{d}C_{i,1} = (N_{i,-1} - N_0)\,\mathrm{d}P_0. \tag{A43}
$$

Now, differentiating the Euler equation $\beta U'(C_{i,1})/U'(C_{i,0}) = P_0/D_1$ gives

$$
\frac{U''(C_{i,1})}{U'(C_{i,1})}\,\mathrm{d}C_{i,1} - \frac{U''(C_{i,0})}{U'(C_{i,0})}\,\mathrm{d}C_{i,0} = \frac{\mathrm{d}P_0}{P_0}. \tag{A44}
$$

Hence, we obtain a system of two equations, (A43) and (A44), and two unknowns, $\mathrm{d}C_{i,0}$ and $\mathrm{d}C_{i,1}$. This system can be represented as a matrix equation:

$$
\begin{pmatrix} 1 & \frac{P_0}{D_1} \\ -\frac{U''(C_{i,0})}{U'(C_{i,0})}\frac{U'(C_{i,1})}{U''(C_{i,1})} & 1 \end{pmatrix}
\begin{pmatrix} \mathrm{d}C_{i,0} \\ \mathrm{d}C_{i,1} \end{pmatrix}
=
\begin{pmatrix} (N_{i,-1} - N_{i,0})\,\mathrm{d}P_0 \\ \frac{U'(C_{i,1})}{U''(C_{i,1})}\frac{\mathrm{d}P_0}{P_0} \end{pmatrix}.
$$

Solving this system gives

$$\begin{pmatrix} \mathrm{d}C_{i,0} \\ \mathrm{d}C_{i,1} \end{pmatrix} = \frac{1}{1+\frac{U''(C_{i,0})}{U'(C_{i,0})}\frac{U'(C_{i,1})}{U''(C_{i,1})}\frac{P_0}{D_1}} \begin{pmatrix} 1 & -\frac{P_0}{D_1} \\ \frac{U''(C_{i,0})}{U'(C_{i,0})}\frac{U'(C_{i,1})}{U''(C_{i,1})} & 1 \end{pmatrix} \begin{pmatrix} (N_{i,-1}-N_{i,0})\,\mathrm{d}P_0 \\ \frac{U'(C_{i,1})}{U''(C_{i,1})}\frac{\mathrm{d}P_0}{P_0} \end{pmatrix}.$$

Using the definition $R_1 = D_1/P_0$ and $\psi(C) = -U'(C)/(U''(C)C)$, this simplifies to

$$\begin{pmatrix} \mathrm{d}C_{i,0} \\ \mathrm{d}C_{i,1} \end{pmatrix} = \frac{1}{1+R_1^{-1}\frac{C_{i,1}\psi(C_{i,1})}{C_{i,0}\psi(C_{i,0})}} \begin{pmatrix} 1 & -R_1^{-1} \\ \frac{C_{i,1}\psi(C_{i,1})}{C_{i,0}\psi(C_{i,0})} & 1 \end{pmatrix} \begin{pmatrix} (N_{i,-1}-N_{i,0})\,\mathrm{d}P_0 \\ \psi(C_{i,1})C_{i,1}\,\mathrm{d}\log R_1 \end{pmatrix}. \tag{A45}$$

To conclude the proof, we now split the total consumption response into an income and substitution effect. By definition, the income effect is the response of consumption to a change in income at $t = 0$ that is welfare-equivalent to the change in prices; that is, $\mathrm{d}Y_{i,0} = (N_{i,-1} - N_{i,0})\,\mathrm{d}P_0$. It can be obtained by setting $\mathrm{d}\log R_1$ to zero in (A45). Conversely, the substitution effect is defined as the consumption response to a change in price $\mathrm{d}P_0$ holding welfare constant. It can be obtained by setting $\mathrm{d}P_0$ to zero in (A45), which concludes the proof. □

E.2 Equivalent interpretations of welfare gains

In our main text, we defined the welfare gain of a deviation in asset prices as the amount of money received in the baseline economy that would have the same impact on welfare as the deviation in asset prices (equivalent variation). As long as the deviation in asset prices is infinitesimal, it is well known that this quantity is the same as (minus) the amount of money received in the perturbated economy that would return the agent to their original welfare level (compensating variation). We now list five other equivalent ways of thinking about our notion of welfare gains in our baseline model.

Proposition A13. *Consider the baseline economy described in Section I.A with an infinitesimal deviation in asset prices. The following objects are equal:*

(i) Willingness to pay for the deviation in asset prices (equivalent variation)

(ii) Deviation in welfare divided by the marginal utility of consumption

(iii) Present value of the deviation in consumption

(iv) Present value of the deviation of consumption if the individual were to maintain their original path of asset holdings

(v) Present value of consumption times the relative deviation in consumption in every period that would generate the same increase in welfare (Lucas's notion of welfare gain)

(vi) Minus the deviation in initial wealth necessary for individuals to maintain their original paths of consumption and asset holdings (Slutsky's wealth compensation)

Proof of Proposition A13. We show each equivalence in succession. We first prove the equality between (i) and (ii) Denote $V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t, \{P_{k,t}\}_k\}_{t=0}^{\infty})$ the value function (or indirect utility) of the individual problem maximizing (8) subject to (9). Define the equivalent variation of a non-infinitesimal deviation in asset prices $\{\theta\Delta Q_t, \{\theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty}$, denoted $EV_i(\theta)$, as the amount of money, at time $t = 0$, which would have the equivalent change in welfare as the deviation in asset prices; that is,

$$V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1} + EV_i(\theta), \{Q_t, \{P_{k,t}\}_k\}_{t=0}^{\infty}) = V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta\Delta Q_t, \{P_{k,t} + \theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty}).$$

The implicit function theorem gives

$$\partial_{\theta=0} EV_i(\theta) = \frac{\partial_{\theta=0} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta\Delta Q_t, \{P_{k,t} + \theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty})}{\partial_{B_{i,-1}} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t, \{P_{k,t}\}_k\}_{t=0}^{\infty})}$$
$$= \frac{\partial_{\theta=0} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta\Delta Q_t, \{P_{k,t} + \theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty})}{U'(C_{i,0})},$$

where the second line uses the first-order condition for consumption. This implies

$$\mathrm{d}EV_i = \mathrm{d}V_{i,0}/U'(C_{i,0}),$$

using the notations $\mathrm{d}EV_i \equiv (\partial_{\theta=0} EV_i)\,\mathrm{d}\theta$ and $\mathrm{d}V_{i,0} \equiv (\partial_{\theta=0} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta\Delta Q_t, \{P_{k,t} + \theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty}))\,\mathrm{d}\theta$.

We now prove the equality between (*ii*) and (*iii*). Totally differentiating the definition of welfare (8) gives

$$\mathrm{d}V_{i,0} = \sum_{t=0}^{\infty} \beta^t \,\mathrm{d}U(C_{i,t}) = \sum_{t=0}^{\infty} \beta^t U'(C_{i,t})\,\mathrm{d}C_{i,t}.$$

Combining with the Euler equation $U'(C_{i,0}) = \beta^t R_{0\to t} U'(C_{i,t})$ gives

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,\mathrm{d}C_{i,t},$$

which proves the statement.

We now prove the equality between (*ii*) and (*iv*). Consider a hypothetical individual that would not change asset holdings in response to the deviation in asset prices and denote $\mathrm{d}C_{i,t}^{\text{fixed holdings}}$ the resulting deviation in consumption. Differentiating the budget constraint (9) gives:

$$\mathrm{d}C_{i,t}^{\text{fixed holdings}} \equiv \sum_{k=1}^{K} (N_{i,k,t-1} - N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t.$$

Taking the present value of both terms over multiple periods and using Proposition 1 gives:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\,\mathrm{d}C_{i,t}^{\text{fixed holdings}},$$

which proves the statement. Note that this statement results from the envelope theorem: re-optimizing asset holdings in response to a change in asset prices does not have a (first-order) effect on welfare. This equivalence, however, breaks in more complex cases, such as when the individual faces a binding borrowing constraint (see Proposition A11).

We now prove the equality between (*ii*) and (*v*). Consider a non-infinitesimal deviation in asset prices $\{\theta\Delta Q_t, \{\theta\Delta P_{k,t}\}_k\}_{t=0}^{\infty}$ indexed by a scale θ. Denote $c_i(\theta)$ the relative increase in consumption in each period that would generate the same increase in welfare as the deviation for individual i. By definition, this quantity satisfies

$$\Delta V_{i,0}(\theta) = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}(1 + c_i(\theta))).$$

Differentiating around $\theta = 0$ gives

$$\mathrm{d}V_{i,0} = \sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) C_{i,t}\,\mathrm{d}c_i,$$

where $\mathrm{d}V_{i,0} \equiv (\partial_{\theta=0}\Delta V_{i,0})\,\mathrm{d}\theta$ and $\mathrm{d}c_i \equiv c_i'(0)\,\mathrm{d}\theta$. Combining with Euler equation $U'(C_{i,0}) = \beta^t U'(C_{i,t})R_{0\to t}$ gives

$$\mathrm{d}V_{i,0} = U'(C_{i,0}) \sum_{t=0}^{\infty} R_{0\to t}^{-1} C_{i,t}\,\mathrm{d}c_i$$
$$\implies \mathrm{d}V_{i,0}/U'(C_{i,0}) = \left(\sum_{t=0}^{\infty} R_{0\to t}^{-1} C_{i,t}\right) \mathrm{d}c_i,$$

which proves the statement. Note that this derivation allows us to interpret our notion of welfare gain dividend by the present value of consumption (or "total wealth") as the percentage increase in consumption in every period that would have the same effect on welfare as the deviation, which corresponds to the notion of welfare gain in Lucas (2000).

We now prove the equality between (ii) and (vi). Denote $\mathrm{d}B_{i,t}^{\text{fixed consumption and holdings}}$ the deviation in bond holdings that would allow a hypothetical individual to maintain their original path of consumption and asset holdings (beyond bonds) despite the deviation in asset prices. Differentiating the budget constraint (9) gives:

$$Q_t\,\mathrm{d}B_{i,t}^{\text{fixed consumption and holdings}} - \mathrm{d}B_{i,t-1}^{\text{fixed consumption and holdings}} = \sum_{k=1}^{K}(N_{i,k,t-1} - N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t,$$

Solving this forward and using the no-Ponzi condition gives

$$\mathrm{d}B_{i,-1}^{\text{fixed consumption and holdings}} = -\sum_{t=0}^{\infty}(Q_0 \dots Q_{t-1})\left(\sum_{k=1}^{K}(N_{i,k,t-1} - N_{i,k,t})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t\right)$$
$$= -\sum_{t=0}^{\infty} R_{0\to T}^{-1}\left(\sum_{k=1}^{K}(N_{i,k,t-1} - N_{i,k,t-1})\,\mathrm{d}P_{k,t} - B_{i,t}\,\mathrm{d}Q_t\right).$$

Combining with Proposition 1 gives

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = -\,\mathrm{d}B_{i,-1}^{\text{fixed consumption and holdings}},$$

which proves the statement. □

Generalization to a stochastic environment. Proposition A13 shows the equivalence between our notion of welfare gains and five other concepts in the context of the baseline (deterministic) economy described in Section I.A. We now discuss how this result would generalize to stochastic economies. First, the proposition would still hold as long as markets are complete: all the derivations in the proof of Proposition A13 would remain the same after replacing the return of the risk-free rate asset $R_{0,t}^{-1}$ by the growth of the stochastic discount factor Λ_t/Λ_0 (see Appendix A.2).

However, the equivalents in Proposition A13 would no longer hold in the more realistic case where markets are incomplete. More precisely, the equivalence between (i) and (ii) still holds. The equivalence between (i)/(ii) and (iii)/(iv)/(v) only holds if one understands the term "present value" as meaning "discounted by the marginal rate of substitution of the individual" (which may differ from the economy-wide stochastic discount factor when markets are incomplete).

More importantly, the equivalence between (i)/(ii) and (vi) no longer holds: when markets are incomplete, the agent cannot invest some initial cash in a way that ensures that the cash flow from the trading strategy exactly covers the deviation in the budget constraint due to the deviation in asset prices in each state of the world. As discussed in the main text, however, our baseline implementation of the sufficient statistic formula (Equation 16), which simply discounts realized transactions using a homogeneous discount rate $R_{0\to t}$, is still equivalent to (vi) since it corresponds to the amount of wealth at $t = 0$ that, invested in the liquid asset, would have been enough, *ex-post*, to cover the realized deviations in the budget constraint due to the deviation in asset prices.

E.3 Present-value budget constraint

Present-value budget constraint. We now derive the present-value budget constraint implied by the period budget constraints (9) and the no-Ponzi condition stated in the main text. At the optimum, the no-Ponzi condition holds with equality and is therefore given by

$$\lim_{T\to\infty} R_{0\to T}^{-1}\left(B_T Q_T + \sum_{k=1}^{K} N_{i,k,T} P_{i,k,T}\right) = 0. \tag{A46}$$

To derive the present-value budget constraint, it is useful to express a period budget constraint in terms of the individual's financial wealth

$$W_{i,t} \equiv B_{i,t} Q_t + \sum_{k=1}^{K} N_{i,k,t} P_{k,t}.$$

Using this definition, (9) can be rewritten as

$$C_{i,t} + W_{i,t} = Y_{i,t} + R_t W_{i,t-1} + \sum_{k=1}^{K} \left(R_{k,t} - R_t\right) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^{K} \chi(N_{i,k,t} - N_{i,k,t-1}), \tag{A47}$$

where the reader should recall that $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ is the return of asset k and $R_{t+1} = 1/Q_t$ is the return on the one-period bond. The right-hand side of this constraint says that the individual's capital income equals total wealth times the bond return $R_t W_{i,t-1}$ plus the excess return from holding long-lived assets $\sum_{k=1}^{K} \left(R_{k,t} - R_t\right) N_{k,t-1} P_{k,t-1}$ minus any adjustment costs from transacting these. The no-Ponzi condition (A46) becomes

$$\lim_{T\to\infty} R_{0\to T}^{-1} W_{i,T} = 0. \tag{A48}$$

Recursive forward substitution of the sequence of budget constraints (A47) (eliminating $W_{i,0}$, $W_{i,1}$ and so on) yields

$$R_{0\to T}^{-1} W_{i,T} + \sum_{t=0}^{T} R_{0\to t}^{-1} C_{i,t} = \sum_{t=0}^{T} R_{0\to t}^{-1}\left[Y_{i,t} + \sum_{k=1}^{K} \left(R_{k,t} - R_t\right) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^{K} \chi(N_{i,k,t} - N_{i,k,t-1})\right] + R_0 W_{i,-1},$$

where we have used the definition of the cumulative return $R_{0\to t} = R_1 \cdot R_2 \cdots R_t$. Taking the limit as $T \to \infty$ and imposing the no-Ponzi condition (A48) yields the present-value budget constraint:

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1} C_{i,t} = \sum_{t=0}^{\infty} R_{0\to t}^{-1}\left[Y_{i,t} + \sum_{k=1}^{K} \left(R_{k,t} - R_t\right) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^{K} \chi(N_{i,k,t} - N_{i,k,t-1})\right] + R_0 W_{i,-1}, \tag{A49}$$

where $W_{i,-1} \equiv \sum_{k=1}^{K} N_{i,k,-1} P_{k,-1} + B_{i,-1} Q_{-1}$ denotes initial financial wealth.

Alternative expression for welfare gains. While the proof of Proposition 1 uses the sequence of period budget constraints, our welfare-gains formula can also be derived from the corresponding present-value constraint. Here, we explain the connection between the two proofs. To illustrate most transparently and avoid cluttered notation, we here consider a version of the baseline model without bonds and only one long-lived asset N_t without adjustment costs. Everything generalizes to the case covered in our baseline model; that is, to the case of a bond and K long-lived assets that are potentially subject to adjustment costs.

In this one-asset version, the problem of the agent can be reformulated as maximizing the discounted value of utility $\sum_{t=0}^{\infty} \beta^t U(C_{i,t})$ subject to the following present-value budget constraint.

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1} C_{i,t} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} Y_{i,t} + N_{i,-1}(P_0 + D_0).$$

where the asset's return is $R_{t+1} = (D_{t+1} + P_{t+1})/P_t$, and the no-Ponzi condition is $\lim_{T\to\infty} R_{0\to T} N_{i,T} P_T = 0$. Proposition 1 implies that the welfare gain is:

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(N_{i,t-1} - N_{i,t}\right) \mathrm{d}P_t. \tag{A50}$$

We now show how to derive our expression for the welfare effect of asset prices from this alternative maximization problem. Along the way, we derive two alternative expressions for the welfare-gains formula (equations (A51) and (A51) below). Both are instructive and carry important economic intuition. The Lagrangian corresponding to the maximization problem is

$$\mathcal{L}_i = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) + \lambda_i \left(N_{i,-1}(P_0 + D_0) + \sum_{t=0}^{\infty} R_{0\to t}^{-1} (Y_{i,t} - C_{i,t}) \right).$$

where λ_i is the Lagrange multiplier on the present-value budget constraint. Using the envelope condition to compute $\mathrm{d}V_{i,0}$ and that $\lambda_i = U'(C_{i,0})$, the welfare gain from a deviation in asset prices $\{\mathrm{d}P_t\}_{t=0}^{\infty}$ and the associated deviation in asset returns $\{\mathrm{d}R_{t+1}\}_{t=0}^{\infty}$ equals

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = N_{i,-1}\,\mathrm{d}P_0 + \sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t})\,\mathrm{d}R_{0\to t}^{-1}. \tag{A51}$$

This equation says that the welfare gain from an asset-price change is the change in initial wealth, $N_{i,-1}\,\mathrm{d}P_0$, plus an extra term $\sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t})\,\mathrm{d}R_{0\to t}^{-1}$. This formulation of the welfare-gains formula clarifies that when asset discount rates change, price deviations result in a revaluation of initial wealth $N_{i,-1}\,\mathrm{d}P_0$ and affect the discounting in the present-value budget constraint. This is the same intuition as the one discussed in the two-period model (Figure 1).

To derive the second alternative formulation of our welfare-gains formula, we now write this extra term as:

$$\begin{aligned}
\sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t})\,\mathrm{d}R_{0\to t}^{-1} &= \sum_{t=0}^{\infty} R_{0\to t}^{-1} (Y_{i,t} - C_{i,t}) \left(-\sum_{s=1}^{t} \mathrm{d}\log R_s \right) \\
&= -\sum_{t=1}^{\infty} \left(\sum_{s=t}^{\infty} R_{0\to s}^{-1} (Y_{i,s} - C_{i,s}) \right) \mathrm{d}\log R_t \\
&= \sum_{t=1}^{\infty} R_{0\to t-1}^{-1} N_{i,t-1} P_{t-1}\,\mathrm{d}\log R_t \\
&= \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,t} P_t\,\mathrm{d}\log R_{t+1},
\end{aligned}$$

where the third equality uses the consolidated budget constraint from the point of view of time t. Plugging this expression into (A51) gives

$$dV_{i,0}/U'(C_{i,0}) = N_{i,-1}\, dP_0 + \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,t} P_t \, d\log R_{t+1}. \tag{A52}$$

The intuition is that welfare gains depend not only on prices today but also on returns going forward. This is the same intuition as the one discussed in the two-period model (Section I.A).

Finally, to get back our main welfare formula (A50), we need to express the deviation in interest rates in terms of the deviation in asset prices. Differentiating the definition of the asset return gives

$$d\log R_{t+1} = R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t}.$$

Plugging into (A52) gives

$$dV_{i,0}/U'(C_{i,0}) = N_{i,-1}\, dP_0 + \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,t} P_t \left(R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t} \right) = \sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(N_{i,t-1} - N_{i,t} \right) dP_t,$$

where the second equality uses the assumption that $R_{0\to t+1}^{-1}\, dP_{t+1} \to 0$ as $t \to \infty$, and we have therefore recovered our main welfare-gains formula (A50).

E.4 Equivalence between deviations in asset prices and discount rates

We mentioned in the main text that deviation in asset prices, holding dividends constant, are equivalent to deviations in discount rates. We now express this idea more formally. Consider a sequence of dividends $\{D_{k,t}\}_{t=0}^{\infty}$ and a sequence of discount rates $\{R_{k,t}\}_{t=0}^{\infty}$. Assuming the no-bubble condition, the price of the asset is then given by

$$P_{k,t} = \sum_{s=t+1}^{\infty} \frac{D_{k,s}}{R_{k,t+1} \dots R_{k,s}}.$$

This formula makes clear that, in the absence of dividend changes, price changes must come from changes in discount rates. The following proposition gives a formal mapping between the two.[35]

Proposition A14. *Holding dividends constant, a given deviation in the path of asset discount rates* $\{dR_{k,t}\}_{t=0}^{\infty}$ *generates the following deviation in asset prices* $\{dP_{k,t}\}_{t=-1}^{\infty}$*:*

$$\frac{dP_{k,t}}{P_{k,t}} = -\sum_{s=t}^{\infty} \left(\prod_{u=t}^{s} \left(1 + \frac{D_{k,u}}{P_{k,u}} \right)^{-1} \right) \frac{dR_{k,s+1}}{R_{k,s+1}}. \tag{A53}$$

This equation gives the deviation in asset prices generated by a given deviation path in discount rates. In spirit, this equation is very similar to Campbell and Shiller (1988)'s log linearized present-value identity: indeed, in the particular case where the price-dividend ratio is constant on the baseline path, Equation (A53) simplifies to

$$d\log P_{k,t} = -\sum_{s=t}^{\infty} \rho^{s-t}\, d\log R_{k,s+1},$$

where $\rho \equiv 1/(1 + D_{k,t}/P_{k,t})$.

Proof of Proposition A14. Differentiating the definition of returns $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ gives

$$dR_{k,t+1} = \frac{dP_{k,t+1} - R_{k,t+1}\, dP_{k,t}}{P_{k,t}}.$$

[35] See also Knox and Vissing-Jorgensen (2022) for a similar result.

Rearranging this equation gives the deviation in prices today as a function of the deviation in returns and prices tomorrow:

$$\mathrm{d}P_{k,t} = R_{k,t+1}^{-1} P_{k,t}\, \mathrm{d}R_{k,t+1} + R_{k,t+1}^{-1}\, \mathrm{d}P_{k,t+1}.$$

Iterating forward,

$$\mathrm{d}P_{k,t} = \sum_{s=t}^{\infty} (R_{k,t+1} \dots R_{k,s})^{-1} P_{k,s} \frac{\mathrm{d}R_{k,s+1}}{R_{k,s+1}}.$$

Dividing by $P_{k,t}$,

$$\frac{\mathrm{d}P_{k,t}}{P_{k,t}} = \sum_{s=t}^{\infty} (R_{k,t+1} \dots R_{k,s})^{-1} \frac{P_{k,s}}{P_{k,t}} \frac{\mathrm{d}R_{k,s+1}}{R_{k,s+1}}. \tag{A54}$$

Now, we have

$$R_{k,s+1} = \frac{D_{k,s+1} + P_{k,s+1}}{P_{k,s}} = \left(1 + \frac{D_{k,s}}{P_{k,s}}\right) \frac{P_{k,s+1}}{P_{k,s}},$$

which implies

$$(R_{k,t+1} \dots R_{k,s})^{-1} \frac{P_{k,s}}{P_{k,t}} = \prod_{u=t}^{s} \left(1 + \frac{D_{k,u}}{P_{k,u}}\right).$$

Plugging this equality into (A54) gives the result. □

E.5 Connection with Auclert (2019)

Auclert (2019) examines the effect of a one-time perturbation in the path of interest rates on consumption and welfare. We now discuss how this result relates to our Proposition 1. Consider an economy where, at time $t = 0$, individuals can trade bonds of all maturities. Denote Q_h the price of the bond with maturity $h \geq 1$. The long-term interest rate between 0 and h is $R_{0\to h} = 1/Q_h$.

As in the baseline model, the individual receives labor income $Y_{i,t}$ at time t, and they initially own $N_{i,-1}$ shares of a long-lived asset that pays a sequence of dividends $\{D_t\}_{t=0}^{\infty}$. The individual i chooses consumption and holdings to maximize utility

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}_0^\infty} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

with the following sequence of budget constraints

$$\begin{aligned} C_{i,0} + \sum_{h=1}^{\infty} B_{i,h} Q_{h,0} &= N_{i,-1} D_0 + Y_{i,0} && \text{for } t = 0, \\ C_{i,t} &= N_{i,-1} D_t + B_{i,h} + Y_{i,t} && \text{for } t \geq 1, \end{aligned}$$

where $B_{i,h}$ denotes the number of bonds with maturity h bought at time $t = 0$. Proposition 1 states that the welfare gain of a perturbation in the price of bonds with different maturities depends on transactions:

$$\begin{aligned} \mathrm{d}V_{i,0}/U'(C_{i,0}) &= -\sum_{h=1}^{\infty} B_{i,h}\, \mathrm{d}Q_{h0} \\ &= \sum_{h=1}^{\infty} (N_{i,-1} D_h + Y_{i,h} - C_{i,h})\, \mathrm{d}Q_{h0} \\ &= \sum_{h=1}^{\infty} R_{0\to h}^{-1} (C_{i,h} - Y_{i,h} - N_{i,-1} D_h)\, \mathrm{d}\log R_{0\to h}. \end{aligned}$$

This equation corresponds to Appendix formula (A.37) in Auclert (2019).

In the special case in which the perturbation is a level shift in the yield curve (i.e., $\mathrm{d}\log R_{0\to h} = h\,\mathrm{d}\log R$ for $h > 1$), the formula simplifies to

$$\mathrm{d}V_{i,0}/U'(C_{i,0}) = \left(\sum_{h=1}^{\infty} R_{0\to h}^{-1}(C_{i,h} - Y_{i,h} - N_{i,-1}D_h)h\right)\mathrm{d}\log R.$$

This formula expresses the welfare gain of a permanent rise in interest rate on welfare, as a share of total wealth, as the difference between the duration of consumption and the duration of income, where "duration" is defined as the value-weighted time to maturity of a sequence of cash flows (see Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021).[36]

E.6 Comparing welfare and revaluation gains

We now explain the revaluation gains defined in Equation 17 in the main text and how these differ from our baseline welfare-gains formula. We first consider infinitesimal price deviations $\{\mathrm{d}P_{k,t}\}_{t\geq 0}$ and then discuss non-infinitesimal deviations $\{\Delta P_{k,t}\}_{t\geq 0}$.

Infinitesimal Deviations. We now discuss the relationship between welfare and revaluation gains due to infinitesimal price deviations.

Proposition A15. *Consider an asset $1 \leq k \leq K$ and a sequence of price deviations $(\mathrm{d}P_{k,t})_{t\geq 0}$. We have:*

$$\begin{aligned} \underbrace{\sum_{t=0}^{\infty} R_{0\to t}^{-1}\left(N_{i,k,t-1} - N_{i,k,t}\right)\mathrm{d}P_{k,t}}_{\textit{Welfare gain}} &= \underbrace{\sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1}P_{k,t-1}\,\mathrm{d}\left(\frac{P_{k,t}}{P_{k,t-1}}\right)}_{\textit{Revaluation gain}} \\ &\quad + \underbrace{\sum_{t=0}^{\infty} R_{0\to t-1}^{-1} N_{i,k,t-1}P_{k,t-1}\frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}}\,\mathrm{d}\left(\frac{D_{k,t}}{P_{k,t-1}}\right)}_{\textit{Effect of price deviations on dividend yields}}. \end{aligned} \tag{A55}$$

The proposition decomposes the welfare effect of the deviation in asset prices (the left-hand side in Equation A55) into two terms. The first term ("revaluation gains") corresponds to the positive effect of a rise in asset prices on returns through higher capital gains. The second term corresponds to the negative effect of higher prices on returns through lower dividend yields.

This proposition generalizes the intuition of the two-period model in a model with infinite horizon and multiple assets. The key message is that, following a rise in asset prices, revaluation gains overestimate welfare gains because they only consider the positive effect of rising prices on capital gains without taking into account their negative effects in the future through lower yields.

Finally, note that the capital gains deviation that enters the revaluation gain in (A55) can also be written as

$$\mathrm{d}\left(\frac{P_{k,t}}{P_{k,t-1}}\right) = \frac{P_{k,t}}{P_{k,t-1}}\left(\frac{\mathrm{d}P_{k,t}}{P_{k,t}} - \frac{\mathrm{d}P_{k,t-1}}{P_{k,t-1}}\right). \tag{A56}$$

Proof of Proposition A15. Using summation by parts on the sequence $(R_{0\to t}^{-1}\,\mathrm{d}P_{k,t})_{t\geq 0}$ and $(N_{i,k,t})_{t\geq 0}$, the

[36] More specifically, the duration of consumption is $\sum_{h=1}^{\infty} R_{0\to h}^{-1}\frac{C_h}{\sum_{h=1}^{\infty} R_{0\to h}^{-1}C_h}h$ while the duration of income is $\sum_{h=1}^{\infty} R_{0\to h}^{-1}\frac{Y_{i,h}+N_{i,-1}D_h}{\sum_{h=1}^{\infty} R_{0\to h}^{-1}(Y_{i,h}+N_{i,-1}D_h)}h$.

welfare gain for asset k can be rewritten as:

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(N_{i,k,t-1} - N_{i,k,t}\right) \mathrm{d}P_{k,t} = \sum_{t=0}^{\infty} N_{i,k,t-1} \left(R_{0\to t}^{-1} \mathrm{d}P_{k,t} - R_{0\to t-1}^{-1} \mathrm{d}P_{k,t-1}\right)$$
$$= \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1} P_{k,t-1} \frac{\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1}}{P_{k,t-1}}. \tag{A57}$$

This equation highlights a duality between measuring welfare gains as the present value of sales interacted with price deviations (the left-hand side) and the present value of asset holdings interacted with return deviations (the right-hand side).

To see why $(\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1})/P_{k,t-1}$ can be interpreted as the deviation in returns, note that we have:

$$\frac{\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1}}{P_{k,t-1}} = \frac{\mathrm{d}P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} \mathrm{d}P_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}} \mathrm{d}P_{k,t-1} - R_t \, \mathrm{d}P_{k,t-1}}{P_{k,t-1}}$$
$$= \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{\mathrm{d}P_{k,t}}{P_{k,t}} - \frac{\mathrm{d}P_{k,t-1}}{P_{k,t-1}}\right) + \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t\right) \frac{\mathrm{d}P_{k,t-1}}{P_{k,t-1}}$$
$$= \mathrm{d}\left(\frac{P_{k,t}}{P_{k,t-1}}\right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} \left(-\frac{D_{k,t}}{P_{k,t-1}}\right) \frac{\mathrm{d}P_{k,t-1}}{P_{k,t-1}}$$
$$= \mathrm{d}\left(\frac{P_{k,t}}{P_{k,t-1}}\right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} \mathrm{d}\left(\frac{D_{k,t}}{P_{k,t-1}}\right), \tag{A58}$$

where the third line uses the definition of the return of asset k at time t $R_{k,t} \equiv (D_{k,t} + P_{k,t})/P_{k,t-1}$. This equation decomposes $(\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1})/P_{k,t-1}$ into a part due to the deviation in capital gains (the first term in the RHS) and a part due to the deviation in dividend yields (the second part in the RHS). In the particular case where $R_{k,t} = R_t$ (no adjustment costs), we have $(\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1})/P_{k,t-1} = \mathrm{d}\left(P_{k,t}/P_{k,t-1}\right) + \mathrm{d}\left(D_{k,t}/P_{k,t-1}\right)$; that is, $(\mathrm{d}P_{k,t} - R_t \, \mathrm{d}P_{k,t-1})/P_{k,t-1}$ corresponds exactly to the deviation in the return of asset k. Combining (A57) with (A58) concludes the proof. □

Non-infinitesimal deviations. We measure welfare and revaluation gains using non-infinitesimal price changes in our empirical application. We now derive a counterpart of Proposition A15 above for non-infinitesimal price deviations.

Corollary A16. *Consider an asset $1 \leq k \leq K$ and a sequence of non-infinitesimal changes in prices $(\Delta P_{k,t})_{t\geq 0}$. We have:*

$$\underbrace{\sum_{t=0}^{\infty} R_{0\to t}^{-1} \left(N_{i,k,t-1} - N_{i,k,t}\right) \Delta P_{k,t}}_{\textit{Welfare gain}} = \underbrace{\sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1} P_{k,t-1} \, \Delta\left(\frac{P_{k,t}}{P_{k,t-1}}\right)}_{\textit{Revaluation gain}}$$
$$+ \underbrace{\sum_{t=0}^{\infty} R_{0\to t-1}^{-1} N_{i,k,t-1} P_{k,t-1} \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t\right) \frac{\Delta P_{k,t-1}}{P_{k,t-1}}}_{\textit{Effect of price deviations on dividend yields}}. \tag{A59}$$

where we define

$$\Delta\left(\frac{P_{k,t}}{P_{k,t-1}}\right) \equiv \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\Delta P_{k,t-1}}{P_{k,t-1}}\right) \tag{A60}$$

as the deviation in the capital gains component $P_{k,t}/P_{k,t-1}$ of asset returns caused by the price deviation $\{\Delta P_{k,t}\}_{t\geq 0}$.

Note that $\Delta\left(P_{k,t}/P_{k,t-1}\right)$ defined in (A60) is the natural discrete counterpart to $\mathrm{d}\left(P_{k,t}/P_{k,t-1}\right)$ in (A56).

Proof of Corollary A16. The proof follows the same steps as the proof of Proposition A15, except with non-infinitesimal price deviations. Using summation by parts:

$$\sum_{t=0}^{\infty} R_{0\to t}^{-1}\left(N_{i,k,t-1} - N_{i,k,t}\right)\Delta P_{k,t} = \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1} P_{k,t-1} \frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

In turn, we can write

$$\frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}} = \frac{\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}\Delta P_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}}\Delta P_{k,t-1} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

Plugging into the previous equation gives

$$\begin{aligned}\sum_{t=0}^{\infty} R_{0\to t}^{-1}\left(N_{i,k,t-1} - N_{i,k,t}\right)\Delta P_{k,t} =& \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1}\left(\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}\Delta P_{k,t-1}\right) \\ &+ \sum_{t=0}^{\infty} R_{0\to t}^{-1} N_{i,k,t-1}\left(\frac{P_{k,t}}{P_{k,t-1}}\Delta P_{k,t-1} - R_t \Delta P_{k,t-1}\right).\end{aligned}$$

Rearranging gives the result. Finally, note that as price deviations become infinitesimal, each term in the formula converges to the respective term in Proposition A15. □

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