

Asset-Price Redistribution*

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Abstract

Over the last several decades, there has been a large increase in asset valuations across many asset classes. While these rising valuations had important effects on the distribution of wealth, little is known regarding their redistributive effects in terms of welfare. To make progress on this question, we develop a sufficient statistic for the money-metric welfare gain of deviations in asset valuations. This welfare gain depends on the present value of an individual's *net asset sales* rather than asset holdings: higher asset valuations benefit prospective sellers and harm prospective buyers. We estimate this quantity using panel microdata covering the universe of financial transactions in Norway from 1994 to 2019. We also show how to take into account important considerations such as incomplete markets and collateral constraints, and how to quantify the corresponding welfare effects. We find that the rise in asset valuations had large redistributive effects: it redistributed from the young towards the old and from the poor towards the wealthy.

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Introduction

The last few decades have seen large increases in asset valuations across many asset classes.¹ These rising valuations had important effects on the distribution of wealth. This raises the question: what are the *welfare* consequences of such asset price changes? Who wins and who loses from a rise in asset valuations?

One view is that any rise in asset prices represents a welfare-improving shift of resources towards the wealthy, and should be taxed as such (e.g., [Piketty and Zucman, 2014](#); [Saez et al., 2021](#)).² An opposite view is that a rise in asset prices, without a corresponding rise in cash flows, simply generates “paper gains”, with no effect on actual income and therefore welfare (e.g., [Cochrane, 2020](#); [Krugman, 2021](#)).³ Which (if any) of these two opposing views is correct?

To make progress on this question, we develop a sufficient statistic approach that quantifies the individual (money-metric) welfare effect of a deviation in asset prices. We then operationalize this approach using Norwegian administrative panel data on asset transactions from 1994 to 2019 to quantify the redistributive effects of the rise in asset valuations over this time period.

We ask the following question: how much does a given individual value, in monetary terms, a deviation in the trajectory of asset prices, holding everything else (including asset cash flows) constant? The answer to this question is given by the following formula:

$$\text{Welfare Gain}_i = \sum_{t=0}^T R^{-t} \times \text{Sales}_{i,t} \times \text{Price Deviation}_t, \quad (1)$$

where i denotes the individual, T is the length of the sample period, $R > 1$ is a discount rate, $\text{Sales}_{i,t}$ are the net sales of the asset by individual i in year t , and Price Deviation_t is the deviation of the price of the asset relative to a baseline scenario. In words, the welfare gain equals the net present value (NPV) of the trading profits due to the deviation in asset prices. The formula follows from an application of the envelope theorem and thus holds for small price deviations, a point we discuss in more detail below. The welfare gain is in dollar terms and corresponds to the individual willingness to pay for the deviation in asset prices at time $t = 0$ (equivalent variation). The formula is for the case of one asset but the extension to multiple assets is straightforward. Finally, this version of the formula abstracts from a number of important considerations such as incomplete markets and collateral constraints which we take into account below.

¹See, for example, [Farhi and Gourio \(2018\)](#), [Greenwald et al. \(2019\)](#), or [Van Binsbergen \(2020\)](#) for empirical evidence.

²For example, [Piketty and Zucman \(2014\)](#) write “Because wealth is always very concentrated ... [a] high [wealth-to-income ratio] implies that the inequality of wealth, and potentially the inequality of inherited wealth, is likely to play a bigger role for the overall structure of inequality in the twenty-first century than it did in the postwar period. This evolution might reinforce the need for progressive capital taxation.”

³[Cochrane \(2020\)](#) writes “much of the increase in ‘wealth inequality’ ... reflects higher market values of the same income flows, and indicates nothing about increases in consumption inequality”. [Krugman \(2021\)](#) discusses the hypothetical effect of declining interest rates on large fortunes in 19th-century England and writes “So since the ownership of land, in particular, was concentrated in the hands of a narrow elite, would falling interest rates and rising land prices have meant increased inequality? Clearly not. ... The paper value of their estates would have gone up, but so what? The distribution of income wouldn’t have changed at all.”

Our formula for the welfare gains of asset-price changes (1) highlights that these welfare effects depend on asset *transactions*, not asset holdings. Intuitively, higher asset valuations are good news for prospective sellers (those with $\text{Sales}_{i,t} > 0$) and bad news for prospective buyers (those with $\text{Sales}_{i,t} < 0$). A particularly interesting case is an individual who owns assets but does not plan to buy or sell (i.e., $\text{Sales}_{i,t} = 0$). For such an individual, rising asset valuations are merely “paper gains”, with no effect on welfare.

It is useful to contrast these results with the two polar views described earlier. The first view posited that higher asset valuations redistribute toward existing asset holders. Our formula shows that, it is *sellers* that benefit, not *holders*: if asset holders never sell, they do not benefit from the unrealized capital gains generated by the price deviation. The second view held that all (or at least most) of rising asset valuations are irrelevant for welfare. As our formula shows, this is only true if assets are not traded (e.g., in an economy with a representative agent). But when heterogeneous individuals buy and sell assets like they do in the real world, fluctuations in asset prices do generate welfare gains and losses. In short, both views are incomplete.

As we show in the paper, the formula easily extends to multiple assets including bonds and long-lived assets subject to transaction costs (e.g., housing). Our key contribution is an empirical implementation of this welfare formula for the Norwegian economy. We compute welfare gains and losses due to the observed asset price path for the time period from 1994 to 2019 relative to a baseline where asset prices grew in tandem with dividends (i.e., relative to a balanced growth path as the baseline). Formally, we compute the relative price deviation in (1) as the relative difference between the actual price-dividend ratio PD_t and a baseline price-dividend ratio \overline{PD} :

$$\text{Price Deviation}_t = \frac{PD_t - \overline{PD}}{PD_t}. \quad (2)$$

For our application, we use the 1992–1996 average price-dividend ratio as the baseline (i.e., a 5-year window around the beginning of the sample). Importantly, all of the variables in (1) and (2) are readily observable in our data. Price deviations in Norway have been particularly large for real estate (i.e., house prices have grown much faster than rents) and debt (i.e., real interest rates have declined sharply).

Our main findings are as follows. First, rising asset valuations have had large redistributive effects. While the average individual-level money metric welfare gain is around \$10,000, it is $-\$184,000$ at the 1st percentile and $\$270,000$ at the 99th percentile (in 2011 US dollars). As a fraction of total wealth (i.e., financial wealth plus human wealth), the average welfare gain is 0.1%, while it is -28% at the 1st percentile and 27% at the 99th percentile. Importantly, the distribution of welfare gains differs substantially from the distribution of revaluation gains (defined as the discounted sum of asset holdings times the changes in asset valuations), which are positive for almost everyone (and, in magnitude, equal to 15.9% on average).

Second, we quantify the amount of redistribution *across cohorts*. Overall, we find a large amount of redistribution from young to old. For instance, the average welfare gain is approximately $-\$13,000$ for the cohorts aged 15 or younger at the end of 1993 (Millennials), and

around \$23,000 for the cohorts aged 30 and older at the end of 1993 (Baby boomers). This is primarily due to the fact that the young are net buyers of housing. Declining interest rates of mortgage debt offset the welfare losses of the young due to rising house prices, but do so only partially.

Third, we quantify the amount of redistribution *across the wealth distribution*. We rank adults according to their total initial wealth (measured at the end of 1993) within cohort and find that welfare gains have been concentrated at the top of the wealth distribution. The wealthiest 1% experienced on average a \$53,000 welfare gain while the corresponding number is nearly zero at the 10th percentile, reflecting the fact that (perhaps surprisingly) the wealthy tend to be net sellers of equity and borrowers. However, average welfare gains track total wealth almost one-for-one along most of the wealth distribution: the average welfare gain as a fraction of total wealth remains approximately constant from the 20th through the 80th percentile, at around 1.7%. This reflects the fact that transactions are roughly proportional to wealth in that part of the wealth distribution.

Norwegian households trade not just with each other but also with the rest of the world and the government. We show that the net welfare gain of the household sector came at the expense of the Norwegian government, which, through the sovereign wealth fund, is a net saver. The government intertemporal budget constraint implies that Norwegian households will eventually have to bear the cost of this “government welfare loss” through lower future net transfers.

Our baseline welfare gain formula is derived in the context of a deterministic model without borrowing constraints. Taking advantage of the flexibility of the envelope theorem, we consider a number of model extensions and explain how they affect our formula. Building on these theoretical results, we then empirically implement a version of our sufficient statistic to address what we view as the most important omissions of our baseline empirical exercise: borrowing constraints with collateral effects, incomplete markets, second-order effects from the large observed asset price changes, and valuation changes beyond the end of our sample period. These generalizations affect our estimated welfare gains and losses quantitatively but not qualitatively. We also discuss the interpretation of our sufficient statistic in more general environments, in particular when asset prices are determined in general equilibrium.

Literature. Our paper contributes to several strands of literature. In recent decades, there has been a sustained rise in valuations across many asset classes (e.g., [Piketty and Zucman, 2014](#), [Farhi and Gourio, 2018](#), [Greenwald et al., 2019](#)). As a response to this trend, a growing literature focuses on understanding the effect of rising asset prices (and declining interest rates) on wealth inequality (e.g., [Kuhn et al., 2020](#); [Gomez, 2016](#); [Wolff, 2022](#); [Gomez and Gouin-Bonenfant, 2024](#); [Cioffi, 2021](#); [Catherine et al., 2020](#); [Greenwald et al., 2021](#)). Relative to this literature, our contribution is to study the heterogeneous effect of rising asset prices on welfare.⁴ More broadly, we contribute to a large literature that uses microdata to study the het-

⁴Our theoretical results build on [Moll \(2020\)](#) who studied a two-period model similar to that in Section 1.1. Our result that the welfare of an individual who never buys or sells an asset is unaffected when the asset’s price changes is related to (but different from) a result by [Sinai and Souleles \(2005\)](#) that an individual with an infinite

erogeneity in saving and portfolio choices over the life cycle (e.g., Berger et al., 2018; Feiveson and Sabelhaus, 2019; Calvet et al., 2021; Black et al., 2022) and along the wealth distribution (e.g., Bach et al., 2017; Fagereng et al., 2019; Mian et al., 2020; Bach et al., 2020).

Dávila and Korinek (2018) study the externalities associated with asset price fluctuations in economies with financial frictions. In this context, they obtain a similar formula as our sufficient statistic for the welfare effect of small asset-price changes (see their Lemma 1). We generalize this expression along empirically relevant dimensions (e.g., more than three time periods, inter-generational linkages, the government sector, and financial transactions done via businesses), we develop a methodological framework to measure these welfare effects at the individual level, and we implement it using household-level transaction data. Dávila and Korinek show that deviations in asset prices generate two types of externalities: distributive externalities (when agents do not equate their marginal rates of substitutions across states or times) and collateral externalities (when asset prices matter for financial constraints). Building on these two insights, we stress that, while our baseline measures of welfare gains aggregate to zero in the population, they no longer do when we modify the formula to take into account incomplete markets (Section 4.1) or collateral constraints (Section 4.2).

Our formula for welfare gains is also related to Auclert (2019) who derives the welfare and consumption effects of deviations in interest rates. Relatedly, Greenwald et al. (2021) stress that the welfare effect of a permanent decline in interest rates can be measured as the duration mismatch between consumption and income which they estimate using U.S. data. While there is a profound connection between the two approaches, our sufficient statistic has two main advantages for our application.⁵ First, it allows us to consider the welfare effect of arbitrary valuation changes across asset classes, rather than the ones induced by a uniform change of discount rates in all asset classes. Second, it allows us to measure welfare gains using financial transactions, which we observe directly, rather than in terms of the path of consumption and income, which is typically harder to observe. Finally, our focus on the heterogeneous welfare effect of asset price fluctuations connects this paper to Doepke and Schneider (2006), who study the redistributive effect of inflation episodes using data from the Survey of Consumer Finances, and Glover et al. (2020), who stress the role of asset prices for inter-generational redistribution during the Great Recession in a calibrated model.

More generally, our paper is related to a large asset pricing literature on the role of discount rate shocks. One key finding in the literature is that discount rate shocks account for most of asset price fluctuations (Shiller, 1981; Campbell and Shiller, 1988). The distinction between cash flow and discount rate shocks has important implications for portfolio allocation (e.g., Merton, 1973, Campbell and Viceira, 2002, Campbell and Vuolteenaho, 2004, Catherine et al., 2022). Relative to these papers, we examine the effect of discount rate shocks on *welfare*, both theoretically and empirically.

Finally, our emphasis that rising asset valuations benefit sellers and not asset holders has some historical precedent in the works of Paish (1940), Kaldor (1955) and Whalley (1979) which

expected residence spell is insulated from house price risk.

⁵We discuss the precise mapping between the two approaches in Appendix E.4.

were, in turn, part of a debate in the public finance literature whether unrealized capital gains are a form of income and should therefore be taxed (Haig, 1921; Simons, 1938).⁶

Roadmap. This paper is organized as follows. In Section 1, we present our theoretical framework to quantify the welfare effect of a deviation in asset prices. In Section 2, we discuss the implementation of our sufficient statistic approach using administrative data from Norway. In Section 3, we report our estimates for the redistributive effects of asset price changes. Finally, we discuss generalizations of our sufficient statistic approach in Section 4.

1 Theoretical framework

This section presents our sufficient statistic approach. To focus on the intuition, we first examine the welfare effect of asset price deviations in a two-period model with only one asset in Section 1.1. We then generalize the result to an infinite horizon model with multiple assets and adjustment costs in Section 1.2. We then discuss some extensions of our results in Section 1.3 to more general models.

1.1 Intuition in a two-period model

Time is discrete with two time periods $t = 0, 1$. Individual i receives labor income $Y_{i,0}$ at time 0 and $Y_{i,1}$ at time 1. There is one asset available for trading at time $t = 0$ with price $P_0 > 0$, which pays a dividend $D_1 > 0$ at time 1. Individuals have time separable preferences with a differentiable utility function $U(\cdot)$ that is increasing and strictly concave and with a subjective discount factor $\beta < 1$.

Individual problem. Denote by $C_{i,t}$ the consumption of individual i at time t , and $N_{i,t}$ the number of shares owned at the end of period t . Given initial asset holdings $N_{i,-1}$, the problem of the individual is to choose consumption and asset holdings to maximize utility

$$V_{i,0} \equiv \max_{\{C_{i,0}, C_{i,1}, N_{i,0}\}} U(C_{i,0}) + \beta U(C_{i,1}), \quad (3)$$

subject to the following budget constraints:

$$\begin{aligned} C_{i,0} + (N_{i,0} - N_{i,-1})P_0 &= Y_{i,0}, \\ C_{i,1} &= N_{i,0}D_1 + Y_{i,1}. \end{aligned} \quad (4)$$

These budget constraints say that, in each period t , consumption plus net asset purchases (the left-hand side) must equal income (the right-hand side).⁷

⁶For example, Kaldor (1955) writes: "We may now turn to the other type of capital appreciation which [comes] without a corresponding increase in the flow of real income accruing from that wealth ... [insofar] as a capital gain is realized and spent ... the benefit derived from the gain is equivalent to that of any other casual profit. If however it is not so realized, there is clearly only a smaller benefit."

⁷Recall that the environment has only two periods, with no market for transactions at time $t = 1$ (alternatively, the price of the asset is zero at $t = 1$). We consider the multi-period case below, in which case we add appropriate

Welfare effect. Consider a small change in the price of the asset at time $t = 0$, holding everything else constant. We are interested in the welfare gain associated with this change in asset prices, which we define as the amount of money that would have an equivalent effect on individual welfare (“equivalent variation”). For brevity we will simply refer to this quantity as “welfare gain” in the rest of the paper but it is important to keep in mind that it is a money metric.⁸ For an infinitesimal price change dP_0 , the welfare gain simply corresponds to the change in welfare $dV_{i,0}$ scaled by the marginal utility of consumption $U'(C_{i,0})$. An application of the envelope theorem yields the following expression for the welfare gain:⁹

$$dV_{i,0}/U'(C_{i,0}) = (N_{i,-1} - N_{i,0}) dP_0. \quad (5)$$

The effect of a rise in P_0 is given by the extent to which it relaxes the budget constraint at $t = 0$, namely asset sales $N_{i,-1} - N_{i,0}$. More precisely, a rise in the price of the asset benefits individuals who plan to sell the asset (i.e., $N_{i,0} < N_{i,-1}$) and hurts individuals who plan to buy the asset (i.e., $N_{i,0} > N_{i,-1}$). Importantly, a rise in the price of the asset does not affect individuals who do not plan to trade (i.e., $N_{i,0} = N_{i,-1}$): for those individuals, the rise in the price of the asset is merely a “paper gain” with no corresponding effect on welfare. Similar expressions for the welfare effect of asset prices were previously obtained by [Dávila and Korinek \(2018\)](#) and [Moll \(2020\)](#) in similar two- and three-period environments.

Welfare versus revaluation gains. The result in equation (5) may be surprising at first. How can an asset holder not benefit from a rise in prices given that the market value of their initial wealth unambiguously increases? The reason is that we consider a rise in P_0 holding everything else (in particular the dividend of the asset D_1) constant.¹⁰ While a rise in P_0 increases the return of holding the asset at time $t = 0$, it simultaneously *decreases* the return of holding the asset at $t = 1$. On net, only individuals whose holdings decline over time (i.e., sellers) end up benefiting from the rise in asset price.

To see this formally, denote R_t the return of the asset at time t ; that is $R_0 = P_0/P_{-1}$ and $R_1 = D_1/P_0$. Note that a rise in P_0 increases R_0 , via a higher capital gain, but *decreases* R_1 , via a lower dividend yield:

$$\frac{dR_0}{dP_0} = 1/P_{-1} > 0, \quad \frac{dR_1}{dP_0} = -R_1/P_0 < 0. \quad (6)$$

terminal conditions.

⁸Consistently with standard consumer theory, we focus on a money-metric measure of welfare to respect the notion that preferences are ordinal, rather than cardinal, in nature ([Mas-Colell et al., 1995](#), [Baqaee and Burstein, 2023](#),...). Our measure is similar to the welfare-equivalent increase in consumption defined in [Lucas \(2000\)](#) (see Proposition 17 in Appendix E.2 for more detail).

⁹See Appendix E.1 for the explicit derivation.

¹⁰To put this more precisely, it is useful to adopt the asset pricing perspective that the asset price at $t = 0$ is the present discounted value of future cash flows: $P_0 = D_1/R_1$ where R_1 is the asset required rate of return, which we take as exogenous. An increase in the price P_0 without a change in the dividend D_1 is then equivalent to a fall in the required rate of return R_1 . We develop this general point in Appendix E.4.

The welfare gain due to this change in asset returns can then be written as:¹¹

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= \underbrace{N_{i,-1}P_{-1} \times dR_0}_{\text{contribution of return at } t=0} + \underbrace{R_1^{-1}N_{i,0}P_0 \times dR_1}_{\text{contribution of return at } t=1} \\ &= N_{i,-1} dP_0 - N_{i,0} dP_0. \end{aligned} \quad (7)$$

where the second line is obtained via (6). This alternative derivation highlights that the welfare effect (5) can be seen as the sum of two terms: the first term $N_{i,-1} dP_0$ accounts for the positive effect of a rise in P_0 on today's return (via a higher capital gain) while the second term, $-N_{i,0} dP_0$, accounts for the negative effect of a rise in P_0 on tomorrow's return (via a lower dividend yield). For an individual who does not trade, the two terms offset each other: as a result, a change in asset prices has no welfare effect. We will stress the difference between the welfare effect of a deviation in asset prices—the left-hand side of equation (7)—and its *revaluation* effect—the first term on the right-hand side of equation (7) in our empirical application.

As we discuss in more detail in our multi-period model, when dividend income D_1 rises as well, it still remains true that a rising asset price P_0 benefits sellers and not holders. However, there is now an additional effect: a rise in dividend income directly benefits all asset holders. Equivalently, it offsets the decline in the dividend yield and hence the return $R_1 = D_1/P_0$.

Graphical intuition. Building on Whalley (1979), Figure 1 presents a graphical intuition for the welfare consequences of asset price changes based on the Fisher diagram, the standard graphical apparatus for intertemporal consumption choice problems. The red line represents the present-value budget constraint of the agent's problem, with slope $-D_1/P_0$, while the black curve represents the agent's indifference curve.¹²

Consider the welfare consequences of a rise in the asset price P_0 for a hypothetical seller (panel a) and buyer (panel b). When the asset price P_0 rises, the budget constraint rotates through the endowment point and becomes flatter (the slope is $-D_1/P_0$). The figure shows clearly that the seller ends up on a higher indifference curve (increase in welfare) whereas the buyer ends up on a lower indifference curve (decrease in welfare).¹³

1.2 Baseline model

We now extend this simple intuition to an infinite horizon deterministic economy with multiple assets and adjustment costs (henceforth the “baseline model”), which is key to bringing

¹¹This follows from rewriting the budget constraints (4) as $C_{i,0} + A_{i,0} = R_0 A_{i,-1} + Y_{i,0}$ and $C_{i,1} = R_1 A_{i,0} + Y_{i,1}$, where $A_{i,t} \equiv N_{i,t} P_t$, and using the envelop theorem to compute the effect of deviations in R_0 and R_1 on the value function.

¹²More precisely, the present value budget constraint for the agent problem is given by

$$C_{i,0} + \frac{P_0}{D_1} C_{i,1} = Y_0 + \frac{P_0}{D_1} Y_{i,1} + N_{-1} P_0. \quad (4')$$

¹³In fact, our notion of money-metric welfare gain corresponds, at the first-order, to the horizontal distance between the initial $C_{i,0}$ and the new budget line (as indicated by the solid arrows), as this distance measures the extent to which $C_{i,0}$ would need to adjust if $C_{i,1}$ was held constant: $\Delta C_{i,0} = (N_{i,-1} - N_{i,0}) \Delta P_0$.

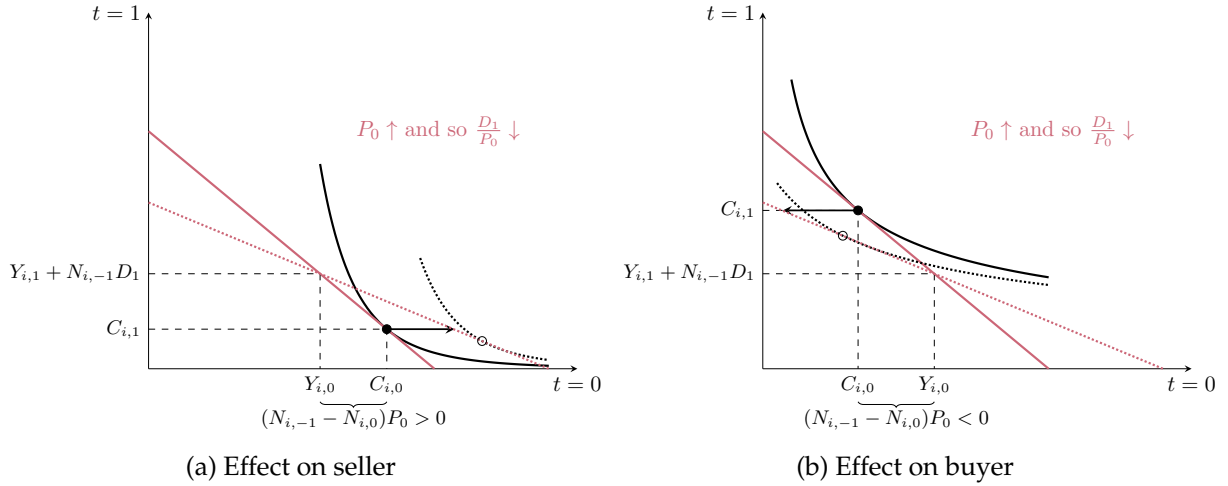


Figure 1: Welfare effect of a rise in the asset price P_0 (two-period model)

Notes. Figure 1 represents the effect of an increase in the asset price P_0 on the welfare of a seller (panel a) and that of a buyer (panel b). The red line represents the agent present-value budget constraint ($4''$), which goes through the endowment point $C_{i,0} = Y_{i,0}$ and $C_{i,1} = Y_{i,1} + N_{i,-1}D_1$ and has slope $-D_1/P_0$. In both panels, the solid budget constraint and indifference curve correspond to the allocation at the initial asset price and the dotted lines are those at the new, higher price. When the asset price P_0 increases, the budget constraint slope $-D_1/P_0$ flattens, rotating through the endowment point. The seller's welfare increases (panel a) and the buyer's welfare decreases (panel b).

the theory to the data.

Financial markets. There is a sequence of liquid one-period bonds with a face value of one and price $Q_t > 0$ available for trading. Purchasing a one-period bond is equivalent to investing in a deposit account with an interest rate $R_{t+1} = 1/Q_t$ between time t and $t+1$. Denote by $R_{0 \rightarrow t} = R_1 \cdot R_2 \cdots R_t$ the cumulative return of these one-period bonds between time 0 and t . There are also K long-lived assets available for trading (e.g., housing, stocks, private businesses, or long-term bonds). A share of asset $1 \leq k \leq K$ is a claim to a stream of dividends $\{D_{k,t}\}_{t=0}^{\infty}$, with price $P_{k,t}$ at the end of period t . The return of asset k between t and $t+1$ is thus $R_{k,t+1} \equiv (D_{k,t+1} + P_{k,t+1})/P_{k,t}$.

We assume that trading these long-lived assets is subject to adjustment costs which may be large or small depending on the asset. These adjustment costs capture the fact that some assets, such as houses and privately-traded equity, are illiquid. For other assets, such as publicly traded equity, the adjustment costs—which may be arbitrarily small—are instead a technical assumption required to have well defined asset demand functions in a deterministic economy.¹⁴ We assume that the adjustment costs, denoted $\chi_k(\cdot)$, are continuous functions of the number of assets purchased each period but they can be kinked (non-differentiable) to capture infrequent adjustment and inaction regions (as in Bertola and Caballero, 1990 or Kaplan et al., 2018).

Individual problem. Individuals have time-separable preferences with a differentiable utility function $U(\cdot)$ that is increasing and strictly concave and a subjective discount factor $\beta \in$

¹⁴These adjustment costs are no longer necessary when the economy is stochastic (if assets have heterogeneous risk profiles, see Appendix A.2) or when agents have non-monetary benefits of owning certain assets (e.g., owning a house versus renting it, see Appendix A.4.1).

$(0, 1)$. They receive labor income $Y_t > 0$ at time t and they can trade financial assets: we denote by B_t the holdings of the one-period bond and by $N_{k,t}$ those of asset k at the end of period t . Individuals take asset prices as given and choose path of consumption and asset holdings to maximize utility

$$V_{i,0} \equiv \max_{\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \quad (8)$$

subject to initial asset holdings $B_{i,-1}$ and $\{N_{i,k,-1}\}_k$, as well as a sequence of budget constraints

$$C_{i,t} + \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t + \sum_{k=1}^K \chi_k (N_{i,k,t} - N_{i,k,t-1}) = \sum_{k=1}^K N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t}. \quad (9)$$

The budget constraint says that consumption plus net purchases of financial assets (the left-hand side) must equal total income in each period t (the right-hand side), which is the sum of dividend income, fixed income, and labor income.

Because of the infinite horizon setup, we also assume the following technicality conditions: a bound on asset holdings $N_{i,k,t} \in \Theta_k$ where Θ_k is compact, a lower bound on the price of one-period bonds $\liminf_{T \rightarrow \infty} Q_T > 0$, a no-bubble condition $\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} P_{k,T} = 0$, as well as a no-Ponzi condition $\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} (B_{i,T}Q_T + \sum_{k=1}^K N_{i,k,T}P_{k,T}) \geq 0$. Appendix E.3 discusses the implied present-value budget constraint whereas Appendix A.4.2 discusses the finite-horizon case. Finally, we assume that there exists a unique solution $\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}$, and that it is continuous with respect to asset prices.

Welfare effect. We are interested in the welfare effect of a small perturbation in the path of asset prices. Formally, we consider an infinitesimal deviation of the path of asset prices, denoted by $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^{\infty}$, holding everything else constant.¹⁵ We assume that the deviation does not explode over time, i.e. that it satisfies the no-bubble condition $\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} dQ_T = \lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} dP_{k,T} = 0$. As in the case of the two-period model above, we define the welfare gain of the deviation as the amount of money, received at $t = 0$, that would generate an equivalent change in individual welfare (equivalent variation). For an infinitesimal deviation, it corresponds to the deviation in welfare $dV_{i,0}$ scaled by the initial marginal utility of consumption $U'(C_{i,0})$.¹⁶

Proposition 1 (Welfare gain). *The welfare gain implied by a price deviation $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^{\infty}$ is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t \right). \quad (10)$$

The proposition, which we prove in Appendix 1, says the welfare gain corresponds to the

¹⁵This deviation can be seen as a comparative statics on the type of the economy the agent is born in, or, equivalently, as the realization of an unexpected ‘‘MIT’’ shock.

¹⁶In Proposition 17 (Appendix E.2), we state and prove a list of alternative interpretations of our concept of welfare gains. In particular, it also corresponds to the present value of the change in individual consumption in response to the deviation in asset prices.

present value of the deviation in trading profits induced by the deviation in the path of asset prices. As in the two-period model, the welfare gain of a deviation in asset prices depends on financial *transactions* rather than *holdings*. Note, however, that for the liquid asset, transactions and holdings coincide given that the asset must be continuously rolled over. Thus, declining interest rates (i.e., $dQ_t > 0$) benefit individuals holding short-term debt (i.e., $B_{i,t} < 0$) because lower debt payments relax their budget constraint. Finally, note that the adjustment cost function does not appear in the welfare formula, as a consequence of the envelope theorem.¹⁷

The thought experiment of Proposition 1 corresponds to a pure deviation in asset prices; that is, holding dividend and labor income fixed. In the financial literature, this is often described as a deviation in asset discount rates. We formalize the mapping between deviations in asset prices and deviations in asset discount rates in Appendix E.4. We also discuss the connection between our formula and the ones obtained in Auclert (2019) and Greenwald et al. (2021), who study the welfare effect of changes in interest rates, in Appendix E.5.

Aggregation. One implication of Proposition 1 is that, in an economy composed of households trading with each other, welfare gains aggregate to zero. Formally, indexing households by $i = 1, \dots, I$, $\sum_{i=1}^I (N_{i,k,t-1} - N_{i,k,t}) = 0$ for all k and $\sum_{i=1}^I B_{i,t} = 0$ imply $\sum_{i=1}^I dV_{i,0} / U'(C_{i,0}) = 0$. This reflects the fact that, for every seller that benefits from a rise in asset prices, there is a buyer that is equally hurt (in monetary terms), and thus, asset-price deviations are purely redistributive. While this result is important to keep in mind, two remarks are in order. First, the fact that welfare gains aggregate to zero says nothing about the desirability of asset price deviations from the point of view of a social planner, who may weigh differently the value of additional dollars to different individuals. More specifically, the effect of a price deviation on social welfare can be positive or negative, depending on whether the welfare weights assigned by the planner to individuals covary positively or negatively with individual welfare gains.¹⁸ Second, this result hinges on two important facts in our baseline economy: (i) agents equalize their marginal rates of substitutions across states/times and (ii) asset prices do not appear in the agent problems outside of their budget constraints. In Section D, we will relax these assumptions by considering economies with uninsurable shocks and/or borrowing constraints with collateral effects, in which cases welfare gains no longer aggregate to zero.¹⁹

Deviation in dividend and labor income. We can easily extend our proposition to compute the welfare effect of a joint deviation in asset prices, dividend income, and labor income.

¹⁷To apply the envelope theorem, we assumed that the solution of the optimization problem was locally continuous with respect to prices. While this does not rule out kinked adjustment costs (as in Kaplan et al., 2018), this does rule out adjustment cost functions that lead to discrete adjustments in response to infinitesimal changes in prices. Finally, while the particular functional form for χ_k does not matter for the first-order effect of asset price deviations on welfare, it would matter for higher-order effects, as discussed in Section 4.

¹⁸We will emphasize this point in Section 3 by aggregating individual welfare gains with different sets of welfare weights.

¹⁹In the language of Dávila and Korinek (2018), the welfare gains of a deviation in asset prices no longer aggregate to zero in the presence of distributive externalities (when agents do not equal their marginal rates of substitutions across dates or times) and/or collateral externalities (when asset prices matter for financial constraints).

Proposition 5, stated and proven in Appendix A, expresses the resulting welfare effect as

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\underbrace{\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t}_{\text{Effect of asset price changes}} + \underbrace{\sum_{k=1}^K N_{i,k,t-1} D_{k,t} + dY_{i,t}}_{\text{Effect of income changes}} \right). \quad (11)$$

Relative to our baseline formula, the expression for welfare gains is augmented with an additional term: the present-value of the deviation in income. This equation emphasizes the key distinction between a deviation in asset prices and a deviation in asset income: while only asset sellers benefit from a rising asset price, all asset holders benefit from a rise in dividend income. This formula is useful to quantify the redistributive effect of arbitrary shocks to the economy, which typically affect both income and asset prices in general equilibrium. To give a concrete example, in Appendix A.3, we use the formula to analyze the redistributive effect of productivity shocks in a general equilibrium production economy, through its effects on income and asset prices.²⁰

1.3 Extensions

The baseline model is deliberately stylized and abstracts from a number of potentially important features of the real world. Before we bring our theory to the data, we consider a number of model extensions. In the rest of this section, we briefly summarize how the extension affects our welfare gain formula (10) as well as its interpretation.

Stochastic environment. So far, we have focused on deterministic economies. In reality, individuals do not have perfect foresight over the future. Proposition 6 in the appendix shows that, in this case, the welfare gain of a deviation in asset prices (i.e., the amount of money that would generate the same increase in welfare from an ex-ante perspective at $t = 0$) is modified along two dimensions. First, what matters is the *expectation* of future financial transactions multiplied by the deviation in asset prices. Second, these trading profits need to be discounted using an *individual-specific* marginal rate of substitution $\beta^t U'(C_{i,t})/U'(C_{i,0})$, a random variable that no longer equals $R_{0 \rightarrow t}^{-1}$ in the presence of uninsurable idiosyncratic shocks. We will quantify the effect of these adjustments for welfare gains in Section 4.1, both theoretically and empirically.

While the version of the welfare gains formula in a stochastic environment differs from its deterministic counterpart as just discussed, in our baseline results, we will empirically implement the deterministic formula (10) which discounts *realized* transactions using a *constant* discount rates. One reason is simplicity. Another reason is that the statistic has a nice interpretation, even in stochastic environments: it corresponds to (minus) the amount of money, received at time $t = 0$, that would have allowed agents facing the deviation in asset prices to

²⁰More precisely, we focus on a tractable two-asset case with one long-lived asset in fixed supply (i.e., land) as well as physical capital with an AK technology (i.e., firms). Focusing on a two-period life-cycle to obtain closed-form solutions for prices, we then decompose the total welfare gain of the old and the young into the contribution of changes in land prices and changes in income.

maintain their original paths of consumption.²¹

Finally, our results remain valid in the case in which individuals can trade financial assets in some ex-ante stage to try to insure themselves against the deviation in asset-prices. Intuitively, while the ability to choose one's portfolio in anticipation of the deviation in asset prices affects individual trading patterns and the welfare effect of the deviation, it does not affect the formula for welfare gains *given* these trading patterns.²²

Borrowing constraints and collateral effects. In the baseline model, individuals can take unrestricted positions in the liquid asset (i.e., long and short). In reality, there are limits on how much individuals can borrow. These borrowing constraints affect our welfare gain formula via two distinct channels.²³ First, agents facing a borrowing constraint are not on their Euler equations, and they tend to discount future dollars by more than the rate of return on their debt (a “discount rate” channel). Second, in models where the borrowing constraint depends on collateral values (e.g., [Kiyotaki and Moore, 1997](#); [Miao and Wang, 2012](#)), higher asset prices have an additional effect on welfare by relaxing borrowing constraints (a “collateral” channel). Importantly, the strength of this collateral channel depends on asset holdings and not just asset sales. We will quantify the effect of these adjustments for welfare gains in Section 4.2, both theoretically and empirically.

Individual preferences. In the baseline model, we specified a utility function that depends only on consumption. In reality, individuals may also care about the quantity of assets they own. An important example is owning a house and living in it which generates a direct utility flow. In Appendix A.4.1, we consider an extension of the baseline model where assets enter the utility function directly. We show that, as long as the utility function depends on the *quantity* of assets owned, this “joy of ownership” channel does not affect our welfare gain formula.²⁴ Similarly, our sufficient statistic formula is robust to preferences for leisure and endogenous labor supply.

Finite lives and bequests. In the baseline model, we abstract from life-cycle considerations, inter-generational linkages, and bequests. In practice, bequests have been shown to be an important determinant of saving decisions ([De Nardi, 2004](#)). In Appendix A.4.2, we consider an extension of the baseline model where individuals have finite lives and give assets to their heirs (as well as, potentially, receive inheritance from their parents)

Finite lives by themselves do not change our formula for the welfare effect of asset prices.²⁵ We then study the welfare effect of asset prices when agents have altruistic preferences (i.e.,

²¹See Proposition 17 in Appendix E.2 for more results on the different interpretations of welfare gains, both in deterministic and stochastic economies.

²²We refer the reader to Appendix A.2 for the general analysis of welfare gains in a fully stochastic environment where labor income, dividends, asset prices, and asset price deviations themselves are stochastic.

²³For a formal statement, see Proposition 3 (case of soft borrowing constraints) and Proposition 15 (case of hard borrowing constraints).

²⁴It is only when individuals directly care about the market price of their assets per se that the welfare gain formula gains an additional term. We do not attempt to quantify such a channel in our empirical implementation.

²⁵This is consistent with our discussion of the two-period model.

agent i directly cares about some other agent j). When defining the welfare gain of individual i as the amount of money that makes him or her indifferent to the deviation in asset prices, assuming agent j is already compensated for it, our sufficient statistic formula remains the same. We also study the case in which agents have “warm glow” preferences instead (i.e. bequest in the utility function); we show that our formula for welfare gains remains the same as long as the bequest function depends on the quantity of assets bequeathed rather than their market prices per se.²⁶

Finally, we emphasize that, in the presence of inter vivos transfers, the welfare effect of a deviation in asset prices only depends on the number of shares sold by the individual, rather than on the overall change in the number of shares (that may come from bequests or inheritance).²⁷ This distinction is relatively easy to deal with in our empirical setting, since we directly observe housing transactions among individuals.

Businesses. In the baseline model, individuals directly own and trade financial assets. In reality, individuals typically own businesses that themselves own and trade financial assets (this includes, in particular, debt issued by businesses as well as share repurchases).

In Appendix A.4.3, we show that the sufficient statistic formula holds in the presence of a business sector once transactions done by businesses are allocated to their ultimate owners. Intuitively, it is irrelevant for welfare whether financial transactions are done directly by the individuals or indirectly through the businesses that they own. Similarly, it is irrelevant whether a business pays out dividends or repurchases its own shares; what matters instead is its cash flow stream (profits minus investment). In our empirical implementation, we will take into account the indirect financial transactions done by businesses owned by each individual when implementing our sufficient statistic.

Government. In Appendix A.4.4, we study an extension of the baseline model with a government that taxes and makes transfers and is allowed to run surpluses and deficits (subject to a no-Ponzi condition as in the individual problem). We do not assume that the government maximizes a social welfare function and instead make a weaker assumption on cost minimization (i.e., the marginal return of investing in the different assets is equalized). We obtain two main results.

First, relative to the individual welfare gain formula in the baseline model, there is an additional term that accounts for the present value of changes in net government transfers. The idea is that, in general, the government will adjust taxes and transfers in response to a change in asset prices. Second, summing over all individuals, we show that aggregate present

²⁶This result aligns with the extension for assets in the utility function discussed above. As in that case, we do not take into account the additional effect of having asset prices directly in the utility function in our empirical application. We discuss additional issues related to the use of “warm glow” preferences for welfare assessment in Appendix A.4.2.

²⁷In particular, for an individual that inherits a house and plans to live in it forever, there is no change in welfare from higher house prices. However, higher house prices do hurt individuals who do not inherit a house but are planning to buy one in the future. Thus, higher asset prices increase the relative difference between those that inherit and those that do not.

value of changes in net government transfers is precisely equal to the “welfare gain of the government” (i.e., equation (10) in the baseline model). This is intuitive and follows directly from the government budget constraint. For instance, if the government is a borrower and its cost of borrowing increases (i.e., negative government welfare gain), then it means that there are less resources available for making net transfers to individuals. Finally, we also examine the role of taxes that are indexed on asset prices in Appendix A.4.4.

2 Empirical framework

We now discuss how we implement our sufficient statistic formula to estimate the distribution of welfare gains due to the rise in asset valuations across individuals in Norway. We first define the counterfactual we use for asset prices. We then describe the combination of administrative and publicly-available data from Norway to quantify our sufficient statistic formula. A more detailed description can be found in Appendix B.

2.1 Implementation

We now discuss how we bring the theory to the data in order to estimate the distribution of welfare gains due to the secular rise in asset valuations in Norway.

First-order approximation. Proposition 1 gives a formula for the infinitesimal welfare gain associated with an arbitrary infinitesimal deviation in prices $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^\infty$. We use this formula to obtain a first-order approximation of the welfare effect of a non-infinitesimal deviation in the price of different assets $\{\Delta Q_t, \{\Delta P_{k,t}\}_k\}_{t=0}^\infty$:

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} - B_{i,t} \Delta Q_t \right). \quad (12)$$

We use the term sufficient statistic as this expression only depends on the observable path of financial transactions $(N_{i,k,t-1} - N_{i,k,t})P_{k,t}$ and $B_{i,t}Q_t$ — in particular, it does not require researchers to understand what drives these financial transactions over time or how they react to deviations in asset prices.

The latter is only true because we focus on first-order approximation of the welfare effect of a deviation in asset prices. The accuracy of this first-order approximation to measure the equivalent variation depends on the extent to which asset transactions respond to changes in asset prices. In our empirical settings, we will focus on asset price deviation across broad asset classes, in which case we can expect these responses to be low; for instance, [Gabaix and Koijen \(2021\)](#) provide evidence that demand elasticities at the asset class level (say, stocks versus bonds) is much lower than the demand elasticities within asset classes (say, stock A versus stock B). We will explore this topic more formally in 4.3.

Asset classes. One can rewrite this formula for welfare gains using a deviation of asset prices in relative terms:

$$\text{Welfare Gain}_i = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) P_{k,t} \times \frac{\Delta P_{k,t}}{P_{k,t}} - B_{i,t} Q_t \times \frac{\Delta Q_t}{Q_t} \right). \quad (13)$$

The term $(N_{i,k,t-1} - N_{i,k,t}) P_{k,t}$ corresponds to the financial transactions corresponding to asset k while $\Delta P_{k,t}/P_{k,t}$ corresponds to the percentage deviation in the asset price. Similarly, the term $B_{i,t} Q_t$ corresponds to the total amount of one-period bonds and $\Delta Q_t/Q_t$ corresponds to the percentage deviation in the price of these bonds.

We now specify our counterfactual for asset prices. First, we consider the same relative price deviations for all assets within a given asset class: equity, housing, and debt.²⁸ That is, we ask: what are the welfare gains associated with an $x\%$ deviation in the price of all assets within the same asset class? Because all financial transactions within a given asset class are multiplied by the same relative price deviation, we can aggregate financial transactions within asset class. Equivalently, our thought experiment allows us to re-interpret K as the number of asset *classes* rather than the number of assets.

Second, we take as the baseline a world in which asset prices grow at the same rate as dividends. Hence, our approach answers the following question: what are the welfare gains of the realized path of asset prices compared to a baseline scenario in which they grew proportionally to dividends? This is a natural question because, on a balanced growth path, asset prices grow at the same rate as asset dividends (i.e., price-dividend ratios are constant).²⁹ Put differently, our thought experiment can be understood as measuring the welfare effect of movements in the path of *de-trended* asset prices (where asset prices are de-trended by their cash-flows).

A large literature in finance argues that fluctuations in price-dividend ratios are mostly driven by fluctuations in future asset discount rates rather than in future expected dividend growth (see [Campbell and Shiller, 1988](#) for a seminal paper and [Kuvshinov, 2023](#) for a recent examination across asset classes and countries). If one is willing to assume that all of the rise in the price-dividend ratios in our sample comes from a decline in asset discount rates, rather than an increase in expected dividend growth, our approach can be interpreted as answering the following question: what are the welfare gains of the rise in asset prices due to declining discount rates?³⁰

Formally, we denote by $PD_{k,t} \equiv P_{k,t}/D_{k,t}$ the aggregate price-dividend ratio for asset class

²⁸Below we actually further split debt into mortgages and deposits. To be clear, we allow different households to earn heterogeneous returns within a given asset class. The only key assumption is that, in the counterfactual we examine, all assets within the same asset class experience the same deviation in relative prices.

²⁹In particular, price-dividend ratios are constant in models where asset discount rates and expected dividend growth rates are constant over time ([Campbell and Shiller, 1988](#)).

³⁰See Appendix B.1 for more details. Throughout the paper, we remain silent on what is the fundamental driver behind this decline in discount rates, which is a very much an open question. As discussed more precisely in Appendix A.3, under the assumption that the drivers of this decline did not directly impact the dividend or labor income of Norwegian households, our sufficient statistic formula (12) entirely captures the welfare effect of these drivers. If not, it only captures the effect operating through the deviation in asset prices.

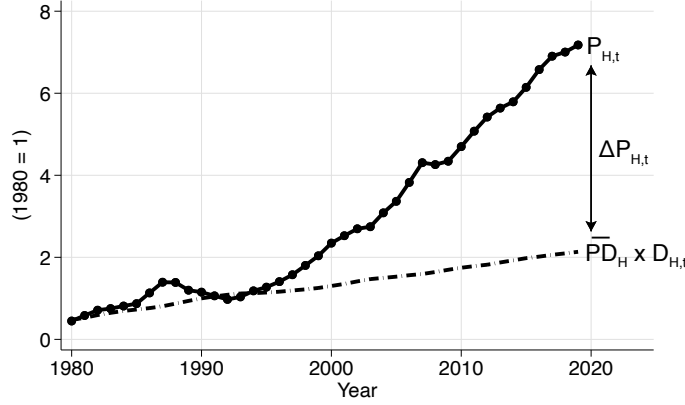


Figure 2: Graphical representation of the price deviation $\Delta P_{H,t}$

Notes. Figure 2 plots the house price index in Norway from Norges Bank's project on Historical Monetary Statistics (solid line) as well as the rental price index from *Statistics Norway* (dashed line). Both are normalized to one in 1980. The difference between the two can be interpreted as a deviation $\Delta P_{H,t}$ between the realized price path $P_{H,t}$ and a counterfactual price path with constant price-to-rent ratio $\overline{PD}_H \times D_{H,t}$.

k. Given a baseline value \overline{PD}_k , we consider the following price deviation for asset class k :

$$\Delta P_{k,t} = P_{k,t} - \overline{PD}_k \times D_{k,t} \implies \frac{\Delta P_{k,t}}{P_{k,t}} = \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}}. \quad (14)$$

This is equation (2) in the introduction. As a motivating example, Figure 2 plots the index of house prices in Norway together with the index of house rents. Starting around the mid-1990s, the price of housing has grown faster than rents. In this case, the price deviation corresponds to the difference between realized prices $\{P_{H,t}\}_{t=0}^T$ and the counterfactual price path associated with a constant price-to-rent ratio $\{\overline{PD}_H \times D_{H,t}\}_{t=0}^T$. For the liquid asset (i.e., the sequence of one-period bonds), we consider a deviation of the price of one-period bonds from a constant baseline value \overline{Q} (i.e., $\Delta Q_t / Q_t = (Q_t - \overline{Q}) / Q_t$).

Time horizon. While the formula (12) depends on of all transactions done by the individual, we only observe price deviations and financial transactions over a finite sample period.

Our simple solution to this issue will be to only do the summation from $t = 0$ to $t = T$, where T denotes the length of the sample period. In this case, the sufficient statistic should be interpreted as the welfare effect of asset price deviations up to time T . This truncation is inconsequential if either (i) the price deviation reverts to zero after T or (ii) if there is no trade after year T . More generally, if the price deviation remains positive after T , truncation overestimates the welfare gain for individuals who tend to buy financial assets after the truncation time T , while underestimating the welfare gain for individuals who tend to sell after T . Still, note that the bias due to truncation averages to zero in the full population, since there are as many sales as there are purchases after time T .³¹

³¹To fix ideas on the size of the bias, it is useful to consider the case of an individual who buys $N_{i,0}$ shares of some asset at time 0 and resells them at some time $t > T$. While the net welfare gain of these transactions is $N_{i,0} (R_{0 \rightarrow t}^{-1} dP_t - dP_0)$, a researcher observing transactions up to time T will estimate a welfare gain of $-N_{i,0} dP_0$

As an alternative to truncating the infinite sum (12), we also construct hypothetical price deviations and financial transactions after year T in Section 4. We show that these alternative measures give similar results to our truncated measure under a wide range of scenarios about the path of future asset prices. This comes from the fact that we observe a relatively long time sample ($T = 25$ years).

Sufficient statistic. Combining the first-order approximation of welfare gains (12) with the empirical price deviations (14) and truncating the formula at time horizon T , we obtain a sufficient statistic for the welfare gain of individual i due to the realized deviation of asset prices from balanced growth:

$$\text{Welfare Gain}_i = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) P_{k,t} \times \frac{PD_{k,t} - \overline{PD}_k}{PD_{k,t}} - B_{i,t} Q_t \times \frac{Q_t - \overline{Q}}{Q_t} \right). \quad (15)$$

This formula forms the core of our empirical implementation using administrative data.³²

We estimate equation (15) using data covering the 1994–2019 period. The reference year (i.e., $t = 0$) is 1994 and the sample length (T) is therefore 25 years. Our data cover the universe of individuals in Norway who were at least 18 years old for at least one year in the 1994–2019 period. We consider four asset classes: housing, debt, deposits, and equity, which correspond to the four main asset classes traded by Norwegian individuals. Note that we do not need to account for fully illiquid forms of wealth such as human wealth and defined-benefit pensions since they are not traded (i.e., they have no market price).

Given this, we estimate our sufficient statistic as follows:

$$\begin{aligned} \text{Welfare Gain}_i &= \sum_{k \in \{\text{housing, debt, deposit, equity}\}} \text{Welfare Gain}_{i,k}, \\ \text{Welfare Gain}_{i,\text{housing}} &= \sum_{t=0}^{25} R^{-t} (N_{i,H,t-1} - N_{i,H,t}) P_{H,t} \times \frac{PD_{H,t} - \overline{PD}_H}{PD_{H,t}}, \\ \text{Welfare Gain}_{i,\text{debt}} &= \sum_{t=0}^{25} R^{-t} (-B_{i,M,t} Q_{M,t}) \times \frac{Q_{M,t} - \overline{Q}_M}{Q_{M,t}}, \\ \text{Welfare Gain}_{i,\text{deposit}} &= \sum_{t=0}^{25} R^{-t} (-B_{i,D,t} Q_{D,t}) \times \frac{Q_{D,t} - \overline{Q}_D}{Q_{D,t}}, \\ \text{Welfare Gain}_{i,\text{equity}} &= \sum_{t=0}^{25} R^{-t} (N_{i,E,t-1} - N_{i,E,t}) P_{E,t} \times \frac{PD_{E,t} - \overline{PD}_E}{PD_{E,t}}, \end{aligned} \quad (16)$$

where \overline{PD}_H , \overline{Q}_M , \overline{Q}_D , and \overline{PD}_E represent the average valuation of housing, debt, deposits, and equity (respectively) over 1992–1996.^{33,34}

(i.e. a welfare loss) thereby underestimating the true welfare gains by $N_{i,0} R_{0 \rightarrow t}^{-1} dP_t$. Note that the bias depends on three distinct forces: (i) how large the truncation time T is (ii) how large the discount rate is relative to the baseline growth of house prices (i.e., how quickly $R_{0 \rightarrow t}^{-1} P_t$ decays to zero as $t \rightarrow \infty$, and (iii) how persistent are house price deviations after T (i.e., how large dP_t/P_t is for $t > T$).

³²This corresponds to the combination of formulas (1) and (2) in the introduction, generalized to multiple assets.

³³Relative to formula (15), we split the total amount of one-period bonds into two terms: $B_{D,t} Q_{D,t}$, the amount held in deposits, and $B_{M,t} Q_{M,t}$, the amounts held in debt, which is negative if individuals are net borrowers.

³⁴Note we use the same price deviation $(PD_{k,t} - \overline{PD}_k)/PD_{k,t}$ for all assets within an asset class. Hence, the

Our empirical implementation (16) also assumes that the discount rate in equation (15) is constant, $R_t = R$ and hence $R_{0 \rightarrow t}^{-1} = R^{-t}$. We set the discount rate to 5% (i.e., $R = 1.05$), which roughly corresponds to the average of the deposit and mortgage rates in a five-year window around the start of our sample.³⁵

Computing these welfare gains requires data on valuation ratios for each asset class (to compare the actual valuations to a baseline) as well as the market value of financial transactions (at the individual level). We now discuss each component separately.

2.2 Aggregate data on valuations

We rely on publicly available data sources for asset prices. For interest rates on debt and deposits (i.e., the inverse of the price of one-period bonds Q in the theory), we use *Statistics Norway's* database on interest rates on loans and deposits offered by banks and mortgage companies.³⁶ More than 90 percent of Norwegian mortgage debt in our sample has adjustable interest rates so that year-to-year variation in bank-level interest rates immediately affects individuals' interest costs.³⁷ Put differently, given that mortgage debt is mostly floating rate, we interpret the outstanding balance of the mortgage as a negative position in one-year bonds.

For the price-to-rent ratio in the Norwegian housing market (i.e., the price-dividend ratio $PD_{H,t} = P_{H,t}/D_{H,t}$ in the theory), we combine data from different sources. We combine two indices, one for house prices and one for housing rents, to obtain our price-to-rent series. The rental index comes from *Statistics Norway*, and is part of the official Consumer Price Index. The house price series comes from Norges Bank's project on Historical Monetary Statistics [Eitrheim and Erlandsen \(2005\)](#).³⁸ As these two series are indices, we scale their ratio so that in 2013, it equals the price-to-rent ratio for Norwegian residential real estate of 27 reported in [MSCI \(2016\)](#). We explore alternative constructions for the price-to-rent ratio in Appendix C.1.

We now turn to equity valuation (i.e., the price-dividend ratio for equity $PD_{E,t} = P_{E,t}/D_{E,t}$ in the theory). As explained in Appendix A.4.3, we focus on a valuation ratio for the overall corporate sector (i.e. unlevered equity). We measure it as the ratio between an aggregate measure of enterprise value (i.e., market value of equity plus debt) and the total cash flows

welfare gain for asset class k should be interpreted as the welfare gain due to a common deviation in the relative price of all assets within this asset class (the one given by the deviation in the aggregate price-dividend ratio of the asset class).

³⁵We pick a discount rate equal to the interest rate at the start of our sample $R = 5\%$, as a compromise between two opposite forces. On the one hand, to account for the effect of market incompleteness and borrowing constraints, Section 4.1 and 4.2 suggest that we use a discount rate that is higher than the rate of return on the liquid asset. On the other hand, to obtain an approximation of welfare gains that is valid at the second-order, Section 4.3 suggests that we use a discount rate equal to the average rate of return between the baseline and counterfactual economy, which would give a lower value $R \approx 2.5\%$. We explore the robustness of our results to these extensions in Section 4.

³⁶These data are available on *Statistics Norway's* website <https://www.ssb.no/en/statbank/table/08175/>.

³⁷Mortgage contracts in Norway typically are annuity loans with 25-year repayment schedules. When interest rates change, the payment schedule adjusts so that the sum of monthly debt repayment and interest costs remains constant at a new level throughout the remaining period of the contract. Such adjustments happen frequently, normally whenever the Central Bank policy rate changes.

³⁸This house price index is in turn obtained from combining data by the Norwegian Real Estate Broker's Association, the private consulting firm Econ Poyry, and listings at the main platform for house transactions Finn.no. Norges Bank updates these data regularly and provides them online, currently at <https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/>.

distributed to equity and debt holders among publicly-listed non-financial Norwegian firms using data from *Worldscope*.³⁹ Note that, unlike the price-dividend ratio, our equity valuation ratio is unaffected by the relative importance of dividend payouts versus share repurchases as well as firms' capital structure (i.e., debt versus equity financing). We account for the fact that firms have financial liabilities besides equity (such as debt for most firms and deposits for private banks) by allocating these indirectly-held assets to the equity holders (see Appendix A.4.3 for more details on the theoretical motivation and B.3.2 for more details on our implementation).

Figure 3 plots the yield of each asset class over time (i.e., $1/Q_t$ for debt and deposits and $D_{k,t}/P_{k,t}$ for long-lived assets $k \in \{H, E\}$), which are the inverse of the valuation ratios in Equation (16). The notches next to the vertical line marking the year 1993 correspond to our baseline values for each asset class. All yields decline substantially over time (i.e., valuations increase). On average, over our time sample, the housing yield fell by 5.7 pp., mortgage interest rates by 2.5 pp., deposit interest rates by 1.3 pp., and the equity yield by 0.5 pp. In particular, note that the equity yields have decreased a bit less in Norway relative to the rest of the world.

To compute the welfare gains of asset price deviations, Equation (16) requires a measure of the relative difference between valuations at time t and their average baseline value (i.e., their averages over the 1992–1996 period). Figure A1 in Appendix B visualizes these price deviations.

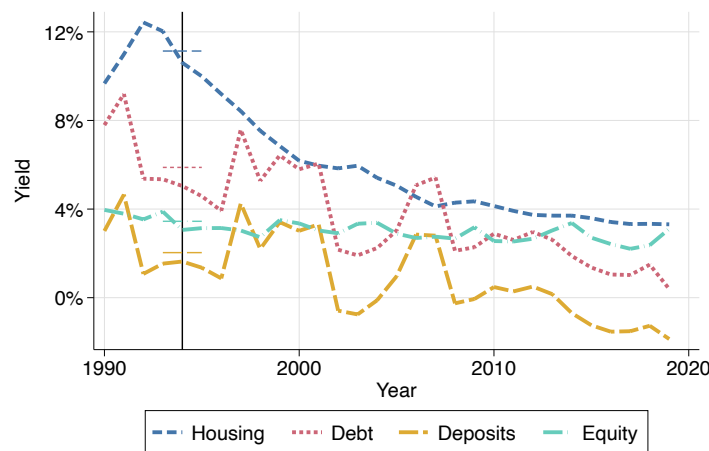


Figure 3: Evolution of yields in Norway

Notes. Figure 3 plots the yield of each asset class over time, i.e., the inverse of the valuation ratios in equation (16). For debt and deposit, the yield corresponds to the average real interest rate on mortgages and debt, respectively, as estimated by *Statistics Norway*. The housing yield corresponds to the rent-to-price ratio (see text for details). The equity yield corresponds to the aggregate ratio of cash flows to enterprise value amongst publicly-listed Norwegian firms from *Worldscope*.

³⁹We use a valuation ratio for Norwegian firms, as opposed to foreign firms, as Norwegians mostly own and sell domestic equity (more precisely, Norwegians' holdings of domestic equity account for 100% of their private equity holdings and 71.9% of their public equity holdings). Note that this contrasts with the Norwegian government, which mainly owns and buys foreign equity. In Appendix C.2, we will discuss how using separate price indices for domestic and foreign equity changes our estimates of welfare gains at the sectoral level.

2.3 Microdata on transactions

We combine data from a variety of Norwegian administrative registries that cover the universe of Norwegians from the end of 1993 to the end of 2019. These data come with identifiers at the individual, household, and firm level, as well as information on parent-children links. In particular, we use registries for individual tax payments, holdings of equity shares (listed and unlisted corporations), private business balance sheets, and housing transactions. Flow variables are measured annually, whereas assets and liabilities are valued at the end of the year. The data are uncensored (i.e., no top coding), and the only sources of attrition are mortality and emigration. The income and wealth data are largely third-party reported (i.e., employers and financial intermediaries) and scrutinized by the tax authority as they are used for income and wealth tax purposes.

Data on holdings. On individual balance sheets, we separately observe bank deposits, bond holdings (corporate, sovereign, mutual, and money market funds), debt, vehicles (cars and boats), stock mutual funds, publicly-listed and private businesses, housing and other forms of estate holdings. The values of the holdings of these asset classes are available starting from the end of 1993.

In principle, we observe each individual's holdings. However, while financial holdings are *registered* at the individual level, they are *taxed* at the household level. The reported allocation of assets between individuals within the household is therefore somewhat arbitrary and can vary substantially from year to year. To compute a consistent measure of individual holdings across time, we therefore aggregate holdings at the household-level and distribute it equally across adult household members.⁴⁰

We construct five main variables that cover most of financial wealth: “debt” (mortgages, student loans, and unsecured credit); “deposits” (bank deposits and bonds); “housing” (principal residence, secondary homes, and recreational estates); “private business equity” (equity in private businesses); “public business equity” (listed stocks and stock funds). All of these variables are recorded at market value at the end of the year, except for private business equity, which is a tax assessed value (i.e., the value reported to the tax authority, which is typically higher than the book value of equity, see Appendix B.3.2). For housing, we use a valuation approach that combines transaction data and registered housing characteristics to estimate a value for each house in every year (see Fagereng et al., 2020b for details on the valuation methodology). Note that this will only matter when reporting our welfare gains relative to total wealth.

Some individuals own private businesses. These firms hold assets and liabilities directly, but in many cases also own shares in other firms. To properly account for individuals' ownership, we must therefore include their indirect asset positions held through private businesses. Our procedure is as follows. For each individual, we compute their direct and indirect ownership of private businesses. For instance, if a individual owns 80% of firm A, which in turn

⁴⁰Our definition of a *household* is either a single individual, or a married or cohabitant (with children) couple. Each offspring older than 18 years living with its parents is a separate household.

holds 50% of firm B, then the individual owns 80% of firm A and 40% of firm B. Moreover, firm B might hold 25% of firm C, which then implies that the individual owns 10% of C. We compute each individual's indirect ownership by going through ten such layers of firm holdings. Equipped with these ownership shares and private firms' balance sheets, as well as publicly available data on public firms' balance sheets, we allocate holdings and transactions done by firms to their ultimate owners (see Appendix B.3.2 for details). This allows us to treat financial transactions conducted directly and those carried out indirectly (through firms one owns) in a similar manner. In Appendix A.4.3, we describe precisely how theory guides our consolidation of firms' financial transactions to individuals.

Our notion of welfare gain can be interpreted as the present value of the deviation in consumption due to the deviation in asset prices (see Equation 82). It would therefore be natural to express it as a share of the present value of consumption. However, we do not observe consumption directly in our sample. Instead, in some exercises, we will scale the welfare gain by "total wealth", which is defined as the sum of financial wealth (i.e., debt, deposits, housing, and equity) and human wealth (i.e., the present value of earned income, defined as future labor income plus net government transfers received between 1994 and 2019, discounted at 5% annually). We also set the minimum value of earned income to twice the base amount in the social security system.⁴¹

Appendix Table A2 summarizes the data. Throughout the paper, we express all values in real terms (2011 Norwegian Krone using the CPI) and then convert them to US dollars using a fixed exchange rate of 5.607. In Appendix B.3.1, we show that our aggregated microdata matches publicly-available data on individual wealth by asset very closely.

Data on transactions. Equation (16) highlights the fact that we need data on *holdings* for debt and deposits, and *net transactions* for housing and equity.

For housing, we observe the annual value of market transactions in the housing market at the individual level. Thus, net transactions in housing are directly observed. For public equities, we observe holdings at the beginning and end of the year and a price index. We then compute a measure of unrealized capital gains by assuming that all transactions are in the same direction and uniformly distributed within a year. Net transactions are thus constructed as the change in market value minus imputed capital gains. The price index used for imputation differs between assets. For listed stocks, the method differs depending on the available information. Starting in 2005, we have information on individual stock ownership and use market prices on individual stocks to impute capital gains. Before 2005, we lack information on individual stock ownership and use capital gains from the Financial Accounts to impute capital gains on listed stocks at the individual level. We also use capital gains from the Financial Accounts to impute individual capital gains for mutual funds.

For equity in private businesses, we impute the value of transactions using the data on ownership shares described earlier. In particular, if we see that a individual owns 50% of a

⁴¹As with financial holdings, an individual's human wealth is computed based on his or her household's human wealth.

private business in a given year and 25% the next year, this implies that the individual sold a 25% stake of the business.⁴² In Appendix B.3.2, we describe this methodology in detail. Private business equity transactions are extremely rare and not quantitatively important. As a result, private business owners are not meaningfully exposed to private equity valuation changes. It is worth stressing the fact that, even in a world in which business owners never sell their stakes in their businesses, they are still exposed to asset price changes via the financial transactions made by the firms that they own. For instance, if the interest rate on debt declines, then the owner of a private business that has a lot of debt will incur a positive welfare gain. This is particularly important for individuals at the top of the wealth distribution, as they hold a lot of assets through their private firms.

Bequest events pose two challenges when computing net transactions. First, housing transactions may be problematic at the time of death. In most cases, when an individual dies, the estate is transferred to the heirs. In this case, the heirs sell the property and net transactions are computed correctly. But in a few cases, parts of the estate is sold after death but before it is transferred to the heirs. In this case, we allocate the transaction to the living children of the deceased, in accordance with the Norwegian inheritance law.⁴³

Second, because our imputation of net transactions in equity is based on changes in holdings net of imputed capital gains, a bequest event may be problematic because transfers of wealth may be counted as transactions. For example, if one individual gives 100 equity shares to another individual, this should not be reported as a purchase by the recipient nor as a sale by the giver. To address this issue, we allocate all imputed equity transactions of givers to recipients when there is a bequest event. A bequest event is defined as any transfer reported in the inheritance tax registry (both inter vivo and at death).⁴⁴

3 Asset-price redistribution

We now estimate our sufficient statistic (16) for all Norwegians who were at least 18 years old at some point between 1994 and 2019. More precisely, we describe the heterogeneity in welfare gains across individuals in Section 3.1, across cohorts in Section 3.2, across the wealth distribution in Section 3.3, and across sectors (i.e., households, government, and foreigners) in Section 3.4.

3.1 Redistribution across individuals

Transactions. We start by documenting the heterogeneity in financial transactions. Table 1 reports summary statistics for transactions across the population, computing them every year

⁴²Alternatively, the business might have issued new equity, leading to a dilution of existing owners. In terms of welfare exposure to equity prices, those two scenarios are equivalent (see Appendix A.4.3).

⁴³By law, inheritance is split equally between all direct descendants unless explicitly specified otherwise in a will.

⁴⁴Before 2014, there was an inheritance tax in Norway and the tax authority collected information on sender, receiver, and the amount transacted. However, this register does not contain information on the types of assets transferred.

and averaging them across all years in our sample. Compared to Table A2, we also include indirect transactions via firms owned by individuals.

Housing transactions are very lumpy, while most people hold debt and deposits. The magnitude of equity transactions is much smaller than housing transactions, which reflects the fact that housing holdings dominate equity holdings for Norwegian individuals (see Table A2). Also, deposits are negative (and debt is positive) for a substantial fraction of the population. This comes from the fact that we report *consolidated* holdings and transactions: individuals that own equity in financial firms (e.g. banks) indirectly hold long positions in debt and short positions in deposits. Finally, financial transactions do not exactly average to zero: as we will discuss below, this reflects the fact that individuals in our sample also trade with the Norwegian government and the rest of the world.

Table 1: Summary statistics on transactions (net purchases in thousands of dollars)

Asset type	Average	S.D.	p1	p10	p25	p50	p75	p90	p99
Housing	0.93	116.33	-190.11	0.00	0.00	0.00	0.08	0.89	220.36
Debt	-73.63	3043.38	-602.58	-221.98	-127.49	-36.08	-0.03	9.53	348.22
Deposits	19.55	1849.23	-157.10	-1.80	1.01	7.56	28.73	76.67	339.56
Equity	-0.36	414.02	-27.06	-0.56	-0.03	0.00	0.04	1.35	30.34

Notes. All numbers are in thousands of 2011 US dollars.

Welfare gains. Figure 4 presents the histogram of total welfare gains. Note that the average welfare gain is close to zero, reflecting the fact that for every seller benefiting from higher asset prices, there is a seller equally harmed in monetary terms.⁴⁵ However, there is substantial heterogeneity: the welfare gain is $-\$184,000$ at the 1st percentile and $\$270,000$ at the 99th percentile, with an interquartile range of $\$31,000$. There is a large mass around zero, which reflects the fact that consumption remains close to income for a large fraction of individuals. As already mentioned, financial transactions within the household sector do not average to zero in our sample. As a result, welfare gains do not average to zero either: they average to $\$10,000$, which is slightly positive. In Appendix C.2, we will show that this positive welfare gain corresponds to a welfare loss of the Norwegian government as well as foreigners. The Kelly skewness of the distribution is fairly small, at 0.08, reflecting the fact that the distribution of welfare gains is fairly symmetrical around its mean.⁴⁶

To understand which asset class contributes the most to redistribution, Table 2 decomposes the average welfare gain into different percentile groups of the welfare gain distribution. More precisely, for each percentile group, the table reports the average welfare gain, as well as the average welfare gain due to each asset class. Housing is by far the asset class that generates the most redistribution. This comes from the fact that, even though housing transactions tend to be smaller than debt or deposits holdings (Table 1), the price deviations associated with

⁴⁵The reason our baseline statistic does not average to zero across individuals is that Norwegian households do not exclusively trade with other: they also trade with the government and foreign entities (see Section 3.4 below for more details).

⁴⁶Kelly skewness is defined as $(p90 + p10 - 2 \times p50) / (p90 - p10)$ where p10, p50, and p90 are the 10th, 50th and 90th percentiles of the distribution under consideration.

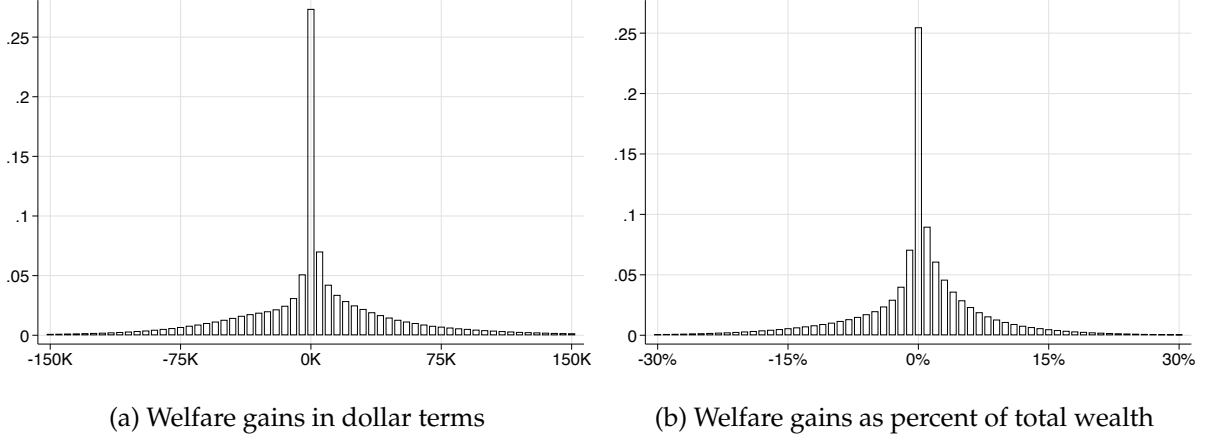


Figure 4: Distribution of welfare gains

Notes. This figure plots the density of individual welfare gains, as defined in (16), across individuals in Norway. More precisely, the figure plots the relative mass of individuals within equally spaced bins of welfare gains (width of \$1,000). Panel (a) plots welfare gains in levels (in 2011 US dollars) while panel (b) plots welfare gains as a percent of total wealth, which is defined as the sum of financial wealth and human capital at the end of 1993 (i.e., the present value of labor income and government benefits received between 1994 and 2019).

housing are much larger than the price deviations associated with debt and deposits (Figure A1). Nevertheless debt is also an important (and almost always positive) contributor, with a relatively large magnitude both at the top and at the bottom of the welfare gain distribution. Similarly, deposits make a very small and almost always negative contribution. Welfare gains due to equity are small, reflecting the fact that there are fewer equity transactions in our sample (Table 1) and that the run-up in equity prices was smaller than the run-up in house prices (see Figure A1).

Table 2: Decomposition of welfare gains by percentile groups

Asset	Average	Average by percentile groups of welfare gains					
		p0-1	p1-10	p10-50	p50-90	p90-99	p99-100
Housing	-4.7	-297.9	-99.9	-18.1	2.3	59.3	416.9
Debt	17.0	-226.9	18.3	8.7	16.3	39.9	206.5
Deposits	-2.4	71.3	-4.4	-3.0	-2.1	-4.4	-17.2
Equity	0.0	-53.2	-1.7	-0.2	-0.1	0.3	38.6
Total	9.9	-506.8	-87.7	-12.6	16.4	95.1	644.7

Notes. For each percentile group of welfare gains, the table reports the average welfare gain, as well as the average welfare gain due to each asset class, as defined in (16). All numbers are in thousands of 2011 US dollars.

Welfare gains as percent of total wealth. We now evaluate the dispersion of welfare gains relative to total wealth, defined as the sum of financial and human wealth (see Section 2.3). As discussed in Proposition 17 (Appendix E.2), welfare gains can be interpreted as the present value of the change in consumption due to the deviation in asset prices (see Equation 82). As a consequence, this *normalized* version of welfare gains can be interpreted as the relative change in consumption due to asset price deviations.⁴⁷ In this exercise and the ones below, we win-

⁴⁷Another way to interpret this number is that it corresponds to the relative increase in consumption every period that would be welfare equivalent to the change in asset prices (see Proposition 17 for more details).

normalize total wealth at the bottom 1% within each cohort to limit the influence of observations with very small total wealth.

Figure 4 shows significant heterogeneity in welfare gains, even after normalizing by initial wealth. The normalized welfare gain is -28% at the 1st percentile and 27% at the 99th percentile, with an interquartile range of 5.0% . While the Kelly skewness of the distribution is close to zero (-0.06), reflecting a symmetric distribution, the kurtosis of the distribution is 11 , reflecting a larger mass in the tails relative to the normal distribution.

Aggregation. Our notion of individual welfare gain represents the amount of cash, received in the baseline economy at time $t = 0$, that would make the individual indifferent between the baseline and perturbed paths of asset prices. As discussed in Saez and Stantcheva (2016), one can aggregate these individual welfare gains, together with a set of social marginal welfare weights, to compute the “social” welfare gain associated to the deviation in asset prices.⁴⁸ The key point is that, given a specific set of social marginal welfare weights that represent how much society value the marginal consumption of different individuals, our measures of individual welfare gains are the only inputs needed to compute the associated social welfare gain.

As an example, we plot in Figure 5 the social welfare gain obtained by social marginal welfare weights equal to individual total wealth at the power $-\sigma$, where σ can be interpreted as an index of social aversion for inequality.⁴⁹ When $\sigma = 0$, the social welfare gain is simply the average welfare gain in the population, which is roughly $\$10,000$. As σ increases, the social welfare gain decreases and ultimately becomes negative, reflecting the fact that the statistic weighs more and more the welfare gains of poorer individuals relative to richer individuals (and that, as we will see shortly, the rise in asset prices redistributed from the poor towards the wealthy).

Revaluation gains. We now compare *welfare* gains with *revaluation* gains, defined as the (present value of the) effect of the deviation in asset prices on wealth:

$$\text{Revaluation Gain} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \sum_{k=1}^K N_{k,t-1} P_{k,t-1} \Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right), \quad (17)$$

where we define $\Delta(P_{k,t}/P_{k,t-1}) \equiv (P_{k,t}/P_{k,t-1}) (\Delta P_{k,t}/P_{k,t} - \Delta P_{k,t-1}/P_{k,t-1})$ as the deviation in the capital gains component $P_{k,t}/P_{k,t-1}$ of asset returns caused by the price deviation $\{\Delta P_{k,t}\}_{t \geq 0}$.

Welfare gains are different from revaluation gains. This is because revaluation gains only capture the positive effect of rising valuations on returns through higher capital gains, while welfare gains also take into account the negative effects of higher valuations on returns through

⁴⁸More precisely, Saez and Stantcheva (2016) define the “social” welfare gain as $\sum_{i=1}^I g_i dV_{i,0}/U'(C_{i,0})$, where g_i corresponds to the social marginal welfare weight on individual i and $dV_{i,0}/U'(C_{i,0})$ corresponds to our (money-metric) notion of welfare gain of individual i . For the special case of a utilitarian social planner, g_i corresponds to the Pareto weight for individual i times the marginal utility of consumption.

⁴⁹Alternatively, this can be interpreted as the welfare change of a utilitarian social planner that aggregates equally the utility of individuals who have homothetic utility functions with parameter σ (since, in this case, consumption is proportional to total wealth).

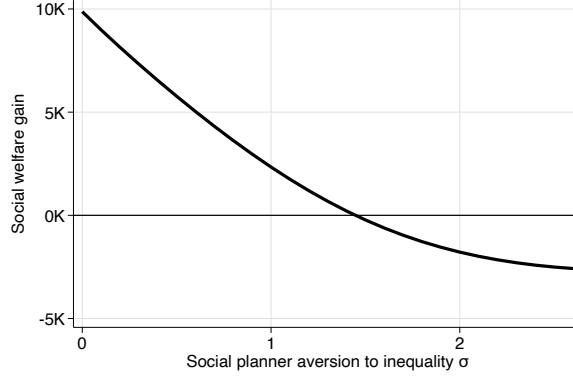


Figure 5: Social welfare gain as a function of inequality aversion

Notes. The figure plots the social welfare gain $\sum_{i=1}^I g_i \times \text{Welfare Gain}_i$ where $g_i \equiv \text{Total Wealth}_i^{-\sigma} / \sum_j \text{Total Wealth}_j^{-\sigma}$ denotes the social marginal welfare weight associated to individual i . Because our marginal social welfare weights sum up to one, we can interpret the result in dollar term: a social welfare gain of $\$X$ means that it is the same as giving $\$X$ to each individual.

lower dividend yields. In particular, revaluation gains systematically overestimate welfare gains in a time of inflated asset prices. We derive a formal expression for the difference between welfare and revaluation gains in Appendix E.6.

Figure 6a compares the density of welfare and revaluation gains, both as a percent of initial (total) wealth. As discussed above, welfare gains are centered around zero (0.1% on average). In contrast, revaluation gains are centered around a large positive value (15.9% on average). This reflects the fact that revaluation gains are positive for all asset holders while welfare gains are only positive for asset sellers.

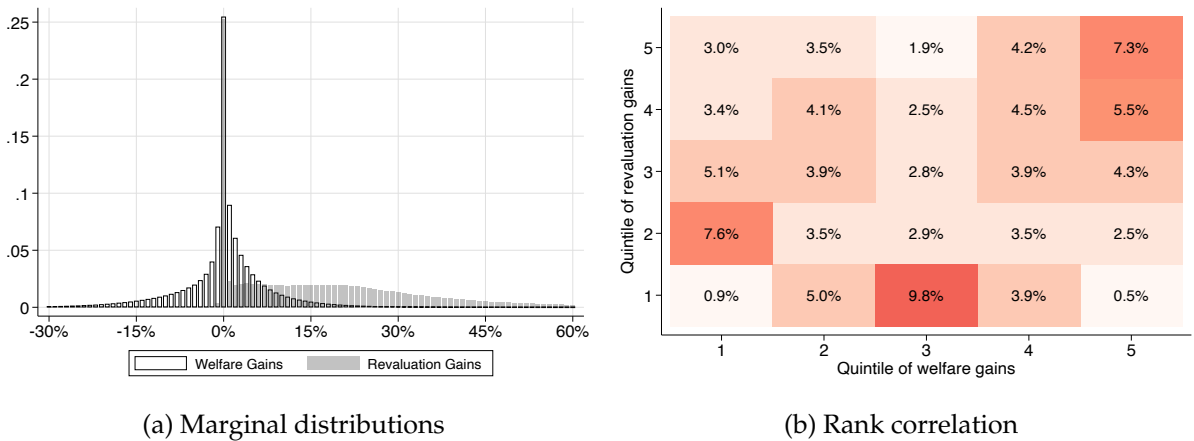


Figure 6: Welfare gains versus revaluation gains as percent of total wealth

Notes. Figure 6a plots the marginal distributions of welfare gains defined in (16), in black lines, and of revaluation gains defined in (17), in grey shading, across individuals in Norway. Figure 6b plots the joint density of the rank of welfare and revaluation gains; that is, the fraction of individuals within each quintile of welfare and revaluation gains. By definition of quintiles, numbers within each row (or column) aggregate to 1/5. Both welfare and revaluation gains are expressed as a percentage of initial total wealth, defined as the sum of financial wealth and human capital at the end of 1993 (i.e., the present value of labor income and government benefits received between 1994 and 2019).

Do individuals with higher revaluation gains also tend to have higher welfare gains? To answer this question, we now focus on the ordinal relationship between the two variables. Figure 6b plots an heatmap for the joint density of ranks of welfare gains and ranks of revaluation gains. Overall, we find that the Spearman's rank correlation between welfare gains and

revaluation gains is 0.16, which shows that there is a substantial difference between those who get richer from the rise in asset prices and those who truly benefit from it. Some individuals with large asset positions buy and hence lose in welfare terms; conversely, others with small positions sell and hence win.

3.2 Redistribution across cohorts

In the previous section, we documented a large amount of heterogeneity in welfare gains across individuals. We now focus on describing the heterogeneity in welfare gains across one observable characteristic: the age of each individual at the end of 1993 (or, alternatively, the cohort he or she belongs to). Indeed, the existing literature on household finance has documented large differences in portfolio holdings over the life cycle (e.g., [Flavin and Yamashita, 2011](#); [Cocco et al., 2005](#)). This heterogeneity may naturally generate heterogeneity in financial transactions, and, therefore, in welfare gains.

Transactions. Figure 7a plots the average (consolidated) financial transactions in equity and housing by age. Importantly (though unsurprisingly), younger individuals tend to be net buyers of housing and equity whereas older individuals tend to be net sellers. Figure 7b plots the average holdings of debt and deposits by age, as they also enter the sufficient statistic (16). Younger individuals hold a large amount of debt, primarily mortgage debt.

Welfare gains. Figure 7c plots the average welfare gain for different cohorts, indexed by individuals' age at the end of 1993. The main pattern is that welfare gains are negative for the young and positive for the old meaning that rising asset prices redistributed from the young towards the old. This is consistent with standard life cycle models of savings: the young save for retirement by purchasing financial assets while the old sell their financial assets to consume.

Quantitatively, the average welfare gain is approximately $-\$13,000$ for individuals below 15 years old in 1993 (Millennials), and around $\$23,000$ for individuals above 50 years old in 1993 (Baby boomers). The figure also decomposes welfare gains into the contribution of each asset class which reveals interesting patterns. On the one hand, higher house prices redistribute from young to old, as the young tend to buy houses from the old. On the other hand, lower mortgage rates redistribute from old to young, as the young tend to borrow from the old.⁵⁰ Overall, the effect of higher house prices dominates the effect of lower mortgage rates for two reasons. First, and most importantly, the housing yield decreased more than the interest rate on debt (see Figure 3). Second, as young people build equity in their houses, they decrease their mortgage balances over time, which means that they benefit relatively less from the decline in mortgage rates as they age.

⁵⁰As we discuss in Appendix C.2, the household sector as a whole is a net debtor. Therefore, the young do not borrow only from the old, but also from another sector of the economy, which turns out to be the government sector. Also note that, while lifecycle mortgage balances peak around age 30 (Figure 7b), the welfare effect of lower mortgage rates is highest for individuals who are 20 years old in 1993 (Figure 7c). This is due to two forces: (i) mortgage rates are mostly flat at the beginning of our sample and only start declining in 2001 (Figure 3), and (ii) this cohort spends a longer amount of time with mortgage debt than the older cohorts aged around 30 in 1993 (Figure 7b).

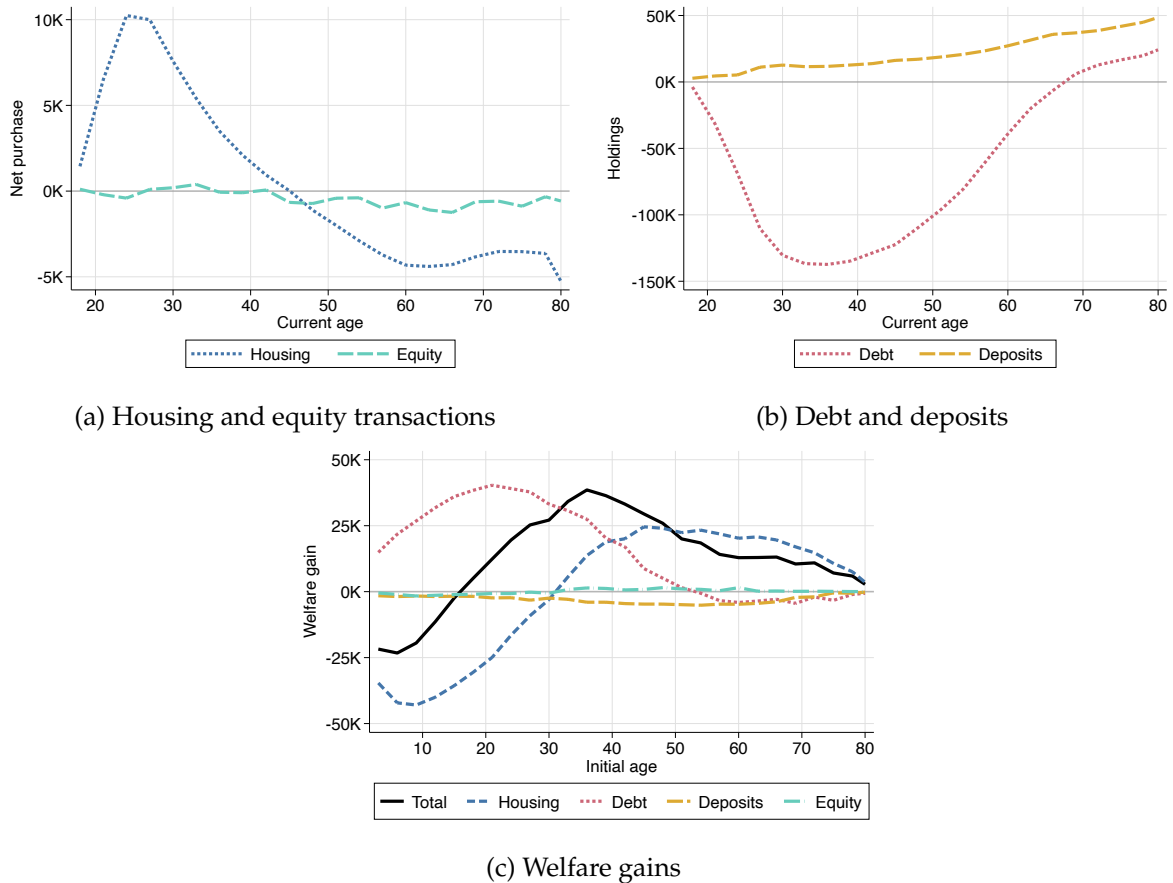


Figure 7: Financial transactions and welfare gains by age group

Notes. Figure 7a and 7b plot (consolidated) financial transactions per capita by age, averaged across all years in our sample. More precisely, for each asset class and year in our sample, we compute the average transaction within groups of individuals with a given age at the end of that year. We then average this quantity across all years in our sample. Figure 7c plots the average welfare gain (16) for individuals in each cohort. Cohorts are indexed by the age of individuals at the end of 1993. All numbers are in 2011 US dollars.

3.3 Redistribution across wealth percentiles

A growing literature has emphasized that rising asset valuations affect the distribution of wealth (e.g., Kuhn et al., 2020; Gomez, 2016; Greenwald et al., 2021). A natural question is: are these revaluation gains actually welfare gains? To answer this question, we compare revaluation and welfare gains across percentiles of the initial wealth distribution at the end of 1993. More precisely, we rank individuals according to their total initial wealth *within their cohort*. We then compare average revaluation and welfare gains at these different percentiles.

Transactions. Figure 8a plots the average consolidated transactions of equity and housing across different percentiles of the wealth distribution. To make it more easily comparable across different percentiles, we normalize average transactions by the total wealth at the end of 1993 at each percentile. The key observation is that richer individuals tend to be net sellers of equity while poorer individuals tend to be net buyers. In contrast, housing net purchases are mildly positive across most of the wealth distribution (consistent with the mildly positive aggregate housing net purchases by households – see Table A4).

Figure 8b plots the consolidated holdings of debt and deposits across the wealth distribu-

tion. As a proportion of financial wealth, the level of debt decreases (in absolute value) with the level of wealth while the level of deposits increases. The negative value of deposits at the top 1% reflects the fact that richer individuals tend to hold more equity, and, as a result, they indirectly hold negative positions in deposits through their ownership of Norwegian banks. Finally, the top 1% holds little debt on a consolidated basis.⁵¹

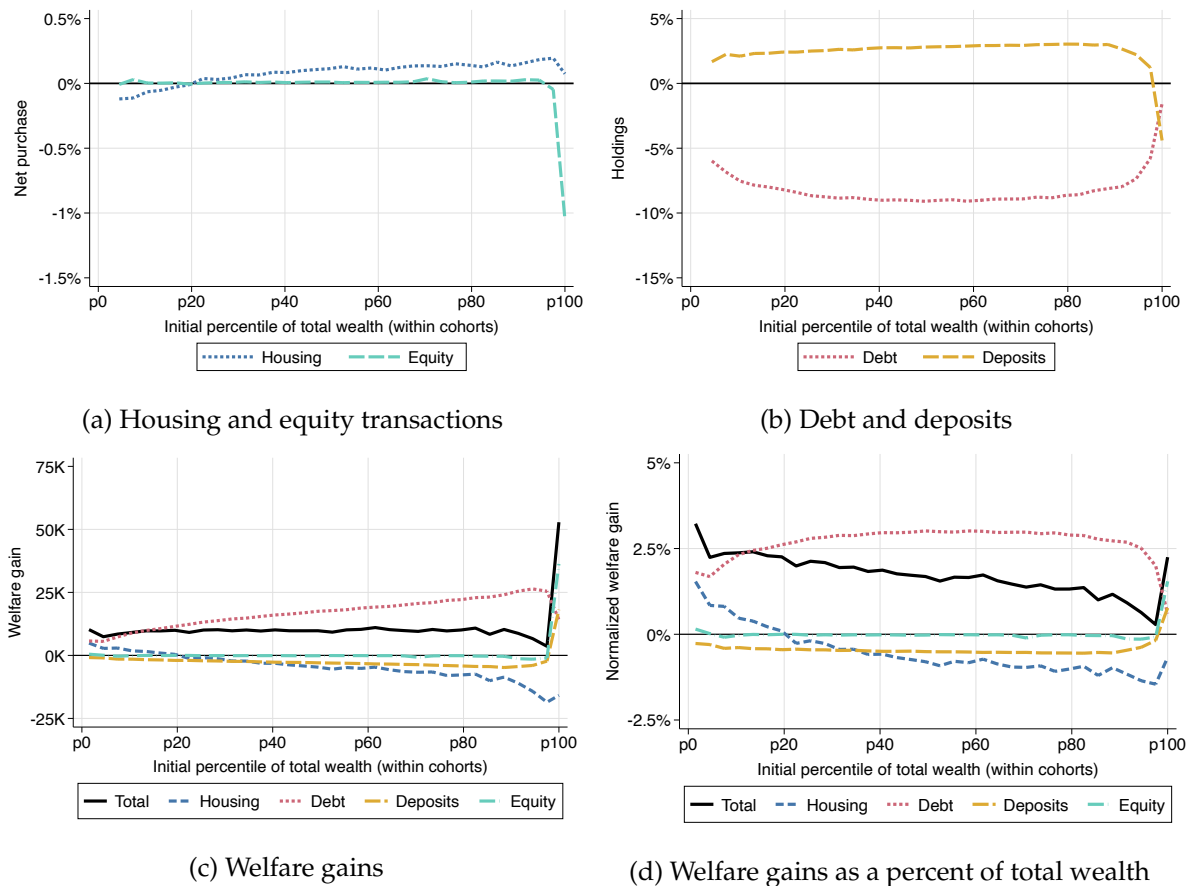


Figure 8: Financial transactions and welfare gains by wealth percentile

Notes. Figure 8a and 8b plot net transactions per capita data by initial wealth, averaged over across years, and divided by total wealth measured at the end of 1993. Figure 8c plots the average welfare gain, as defined in (16), for each wealth percentile in Norway. Figure 8c plots welfare gains as a percent of total wealth measured at the end of 1993 in each percentile. Wealth percentiles are constructed by ranking individuals within each cohort with respect to total wealth, defined as the sum of financial wealth and human capital at the end of 1993 (i.e., the present value of labor income earned and government benefits in our sample).

Welfare gains. Figure 8c plots the average welfare gains at different wealth percentiles. Welfare gains increase with total wealth: the top 1% experienced on average a \$53,000 welfare gain, while the corresponding number is \$10,000 at the bottom 1%. Figure 8d plots welfare gains as a percent of the average total wealth in each percentile. The main pattern is these “normalized” welfare gains tend to be stable across the wealth distribution, except for the top 1%. Individuals in the top 1% of their cohort experience a welfare gain of roughly 2.2% (as a percent of total wealth), which is higher than the population average of 1.5%. Moreover, most

⁵¹While richer individuals issue debt through their ownership in non-financial businesses, they also buy this debt through their ownership in financial businesses.

of the relatively higher welfare gains for the top 1% comes from equity, which reflects the fact that they tend to be net sellers in this asset class.

Revaluation gains. Finally, Figure 9 contrasts revaluation and welfare gains. Similarly to welfare gains, revaluation gains increase with top percentiles, which reflects the importance of revaluations for the rise in wealth inequality. However, the figures show that the magnitude of revaluation gains (39.0% of total wealth for the top 1%) is much bigger than the magnitude of welfare gains (2.2% of total wealth for the top 1%). Put differently, only a small part of these revaluation gains is relevant for welfare.

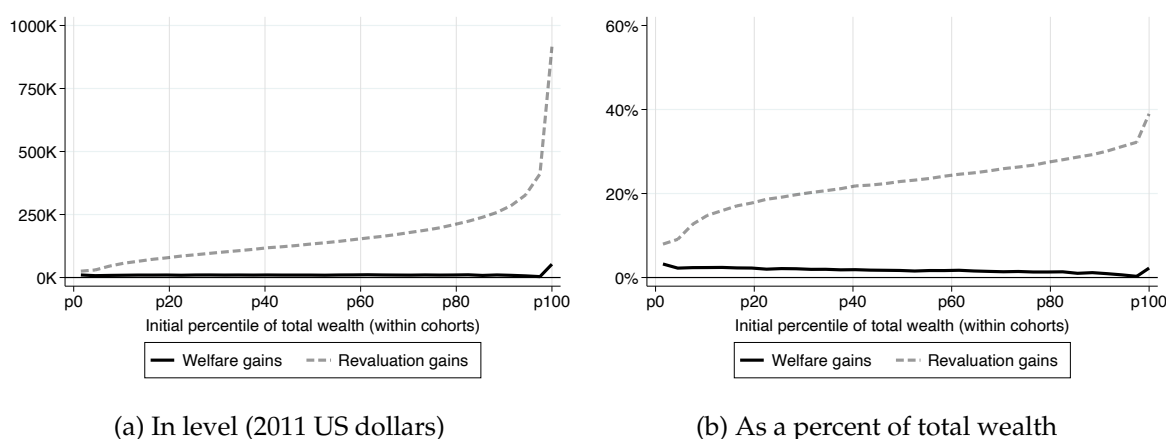


Figure 9: Welfare and revaluation gains across wealth percentiles

Notes. This figure plots the average welfare and revaluation gains, as defined in (16), for each wealth percentile in Norway. Figure (a) reports the two quantities in level (dollar terms) while Figure (b) reports the two quantities as a percent of total wealth, as measured at the end of 1993. Wealth percentiles are constructed by ranking individuals within each cohort with respect to total wealth, defined as the sum of financial wealth and human capital at the end of 1993 (i.e., the present value of labor income and government benefits received between 1994 and 2019).

3.4 Redistribution across sectors

As discussed in the previous sections, our baseline measure of welfare gains does not aggregate to zero across households. This is because Norwegian households do not exclusively trade with other: they also trade with the government and foreign entities.

In Appendix C.2, we use data on sectoral financial transactions from Norwegian national accounts to analyze the redistributive effects of asset prices across sectors. We show that the positive average welfare gain of Norwegian households is counter-balanced by a negative welfare gain of the Norwegian government: indeed, while Norwegian households are net debtors on average, the consolidated government (through Norway’s sovereign wealth fund) is a net saver.

As discussed in Section 1.3 (paragraph titled “Government sector”), a welfare loss for the government represents a loss of real resources available for net transfers to the household sector. While it is beyond the scope of this paper to quantify how the Norwegian government has adjusted (and will adjust) net transfers in response to persistently lower interest rates and higher asset prices, it is possible that the very individuals who experienced welfare losses in

our exercise (i.e., the young) will also be the ones to bear the brunt of future reductions in government transfers such as pension benefits.

4 Generalizations of the baseline sufficient statistic approach

We now implement a number of extensions and generalizations of our baseline sufficient statistic approach. In particular, we modify our sufficient statistic approach to take into account (i) uninsurable income risk, (ii) borrowing constraints with collateral effects, (iii) second-order effects, and (iv) extrapolation beyond the end of the sample. In each case, we discuss the theoretical difference relative to our baseline formula, our methodology to implement the correction, and its quantitative effect.

For the sake of transparency, we analyze each extension separately. As a preview of our results, Table 3 reports the effect of each generalization for the distribution of welfare gains across cohorts. Overall, we do find that each of these effects matter quantitatively. The last line of the table reports the effect of combining all these extensions together. We find that average welfare gains increase across the wealth distribution, with a more significant increase for the 20-40 cohorts.

Table 3: Welfare gains across cohorts: generalizations of our baseline approach

	Welfare gains by age group				
	Mean	0 – 20	20 – 40	40 – 60	60 – 80
Baseline	10.2	-10.7	27.1	24.8	9.4
Additional effect of ...					
Uninsurable income risk	+1.1	+2.1	+1.1	+0.2	-0.1
Borrowing constraints and collateral effects	+0.1	+1.5	-1.1	-0.7	+0.1
Second-order effects	+2.4	-5.9	+8.7	+7.8	+3.1
Extrapolation	+4.5	+3.7	+4.6	+0.1	+0.1
Combining all extensions	23.0	-2.6	45.6	35.4	9.5

Notes. The age group refers to the age of the cohort at the end of 1993. “Uninsurable income risk” reports the results obtained in Section 4.1 with $\gamma = 1$. “Borrowing constraints and collateral effects” reports the results obtained in Section 4.2 with $\zeta = 0.01$. “Second-order effects” reports the results obtained in Section 4.3. “Extrapolation” reports the results obtained in Section 4.4 with $\phi = 0.9$. “Combining all extensions” reports the results obtained in Appendix D.5. All numbers are in thousands of 2011 US dollars.

4.1 Uninsurable income risk

We have derived our sufficient statistic formula in a deterministic model. In reality, agents are exposed to both individual-specific and economy-wide shocks. In this section, we study theoretically and empirically the effect of uninsurable labor income risk on our sufficient statistic formula. We refer the reader to Appendix A.2 for a more general analysis of welfare gains in a fully stochastic environment where not just labor income but also dividends, asset prices, and asset price deviations themselves are stochastic.

Theory. The environment is the same as in the baseline model except that individual labor income $Y_{i,t}$ is now subject to idiosyncratic shocks. The individual chooses a stochastic path of consumption and asset holdings to maximize the expected utility of consumption:

$$V_{i,0} = \max_{\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_{k=1}^K\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right],$$

subject to initial asset holdings $B_{i,-1}$ and $\{N_{i,k,-1}\}$ and the usual sequence of budget constraints:

$$C_{i,t} + \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t + \sum_{k=1}^K \chi_k (N_{i,k,t} - N_{i,k,t-1}) = \sum_{k=1}^K N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t}.$$

The next proposition characterizes the welfare gain of a deviation in asset prices in this stochastic environment, defined as the individual's willingness to pay for the deviation in asset prices at $t = 0$.

Proposition 2. *In the presence of uninsurable income risk, the welfare gains from a deviation in asset prices for individual i is:*

$$dV_{i,0}/U'(C_{i,0}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t \right) \right]. \quad (18)$$

There are two differences with the baseline welfare gain formula. The first is that, in a stochastic environment, what matters for the ex-ante welfare effect of an asset-price deviation is the *expected* path of net asset sales. The second is that this expectation is under the individual's risk neutral measure, which tilts the objective measure by the growth of the individual's marginal utility of consumption $\beta^t U'(C_{i,t})/U'(C_{i,0})$ (i.e., the individual marginal rate of substitution). Because of uninsurable idiosyncratic shocks, this adjustment is individual-specific. To emphasize the role of this adjustment, we use the Euler equation $\mathbb{E}_0 [\beta^t U'(C_{i,t})/U'(C_{i,0})] = R_{0 \rightarrow t}^{-1}$ to rewrite the welfare gains formula as a sum of two terms:

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) = & \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \mathbb{E}_0 \left[\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t \right]}_{\text{Baseline}} \\ & + \underbrace{\sum_{t=0}^{\infty} \text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, \sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t \right)}_{\text{Covariance term}}. \end{aligned} \quad (19)$$

The first term captures the welfare gain due to the expected path of asset transactions (in the objective measure). The second term captures the welfare gain due to the covariance between the growth rate of marginal utility and net asset sales. In our context, we can expect this covariance term to be positive, as labor income shocks generate a positive comovement between the marginal utility of consumption and asset sales (e.g. individuals jointly reduce consumption and savings after a negative income shock). One implication of this covariance term is

that welfare gains no longer aggregate to zero in the population. While it is still the case that higher asset prices are purely redistributive from ex-post buyers to ex-post sellers (i.e., transactions sum up to zero in every state of the world), agents disproportionately weight the states in which they are sellers from an ex-ante perspective, meaning that welfare gains aggregate to a net positive sum across the population.

Finally, note that, even in the presence of uninsurable income risk, our baseline sufficient statistic (10), which discounts *realized* transactions using a *constant* discount rates, still has a valid interpretation as (minus) the amount of money, received at time $t = 0$, that would have allowed the individual facing the deviation in asset prices to maintain their original path of consumption.⁵²

Implementation. We now adjust our sufficient statistic approach to quantify the contribution of uninsurable labor income for ex-ante welfare. As seen in (19), the key empirical object that governs the effect of market incompleteness is the covariance between the growth of marginal utility of consumption and future asset sales at each horizon $t \geq 1$. To estimate this incomplete markets adjustment term in the data, we assume that individuals have CRRA utility with coefficient of relative risk aversion γ . The covariance term for asset k in (19) can be approximated as

$$\text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, (N_{i,k,t-1} - N_{k,t}) P_{k,t} \right) \approx R_{0 \rightarrow t}^{-1} \times \gamma \times \text{cov}_0 \left(\log \left(\frac{C_{i,t}}{C_{i,0}} \right), (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} \right) \quad (20)$$

using a log-linear approximation in consumption growth.⁵³ While this approximation is not strictly necessary, it makes the statistic more robust in the data, as consumption growth can have extreme outliers at the individual level, due to fat-tailed events or measurement errors (Toda and Walsh, 2015).

In our particular settings, we can construct a measure of individual consumption as a residual from the budget constraint (9), i.e., total net income minus net asset purchases. However, this measure has two limitations. First, measurement error in either income or asset purchases generates a mechanical negative correlation between consumption growth and asset purchases, leading us to underestimate the effect of incomplete markets (since we expect the covariance term to be positive). Second, our measure captures total spending rather than non-durable consumption, which would be the appropriate quantity in this context (see, for instance, Vissing-Jørgensen, 2002). To partially address these two issues, we substitute our measure of consumption growth by its projection on log income growth. See Appendix D.1 for

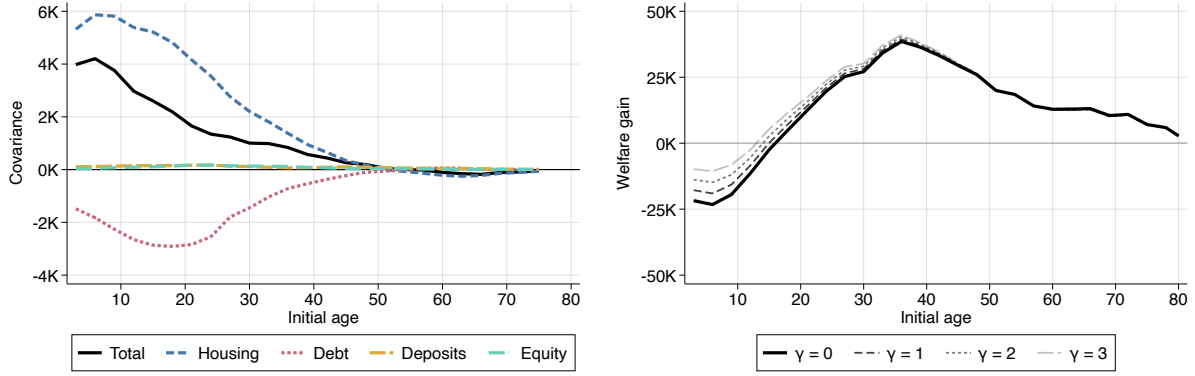
⁵²See Proposition 17 in Appendix E.2.

⁵³More precisely, we have

$$\begin{aligned} \text{cov}_0 \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}, (N_{i,k,t-1} - N_{k,t}) P_{k,t} \right) &= \text{cov}_0 \left(\beta^t \left(\frac{C_{i,t}}{C_{i,0}} \right)^{-\gamma}, (N_{i,k,t-1} - N_{k,t}) P_{k,t} \right) \\ &= R_{0 \rightarrow t}^{-1} \text{cov}_0 \left(\frac{(C_{i,t}/C_{i,0})^{-\gamma}}{\mathbb{E}_0 [(C_{i,t}/C_{i,0})^{-\gamma}]}, (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} \right). \end{aligned}$$

Approximating at the first-order in $c_{i,t} \equiv \log(C_{i,t}/C_{i,0})$ around $c^* \equiv -\frac{1}{\gamma} \log \mathbb{E}_0 [(C_{i,t}/C_0)^{-\gamma}]$ gives the result.

more details on our implementation.



(a) Covariance asset transactions and consumption

(b) Welfare gains

Figure 10: Accounting for uninsurable labor income risk

Notes. Figure 10a plots the covariances (20) aggregated over the horizons, for each cohort and asset class. Figure 10b plots the welfare gain including the incomplete-market adjustment term (19). The welfare gain for $\gamma = 0$ (risk neutrality) is the same as in Figure 7, since, in this case, the adjustment term is zero. Units are 2011 US dollars.

Results. Figure 10a reports the sum (over time) of the covariances (20) for each cohort and for each asset class. The positive “total” (i.e., the sum of the asset-specific covariances) reflects that households with a high consumption level—relative to others with the same observables in 1994—tend to purchase more housing and hold more debt. For any positive level of relative risk aversion γ , this is a force that will dampen the (ex-ante) welfare loss associated with rising house prices (and declining interest rates), given that housing purchases (and borrowing) disproportionately occur in idiosyncratic states in which individuals have high income and low marginal utility. Figure 10b reports average welfare gains across cohorts including the incomplete-market adjustment term (19) for different values of the risk aversion parameter γ . We find that the effect of uninsurable labor income risk is particularly important for younger cohorts, who face more uncertainty over their lifetimes. In particular, we find that the incomplete-market adjustment term offsets some of the welfare loss for the young: the average welfare gain for the cohort of individuals who are 10 years old in 1994 increases from $-\$19K$ when $\gamma = 0$ (baseline) to $-\$16K$ when $\gamma = 1$, up to $-\$8K$ when $\gamma = 3$.

Calibration approach. Overall, our results suggest that uninsurable labor income shocks only have a modest effect on our welfare gain formula in Norway. How general is this result? To answer this question, we take a more standard model-based approach and, in Appendix D.1.2, we study the welfare effect of asset-price deviations in a Bewley-type model in which agents face a realistic labor income process with both transitory and permanent labor income shocks. We show that market incompleteness generates a relatively small correction to the baseline sufficient statistic across a wide range of calibrations.

In Appendix D.1.2, we also consider a model in which individuals face idiosyncratic risk in *portfolio returns*, which is the dominant source of risk at the top of the wealth distribution (e.g. Fagereng et al., 2020a, Gomez, 2023). In this particular case, we are able to obtain simple

closed-form formulas for the effect of market incompleteness on welfare gains: return risk effectively increases individual discount rates by the product of their relative risk aversion and the variance of return shocks. This effect is small for realistic calibrations: for instance, with $\gamma = 1$ and $\sigma = 10\%$, the effective increase in discount rate is 1 pp. ($\approx 1 \times 0.1^2$). Moreover, this formula makes it easy to adjust our baseline sufficient statistic to take this effect into account, since it simply requires to adjust individual discount rates upwards.⁵⁴

4.2 Borrowing constraints and collateral effects

In the baseline model, individuals can take unrestricted positions in the liquid asset. In reality, individuals often face constraints on the amount of debt they can incur. More generally, the interest rate charged to an individual may increase with the debt level or decrease with the value of its assets. We now examine the effect of these borrowing constraints on our formula for welfare gains.

Theory. For simplicity, we consider a two-asset version of the baseline model. The agent maximizes

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \quad (21)$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_{i,t} + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + B_{i,t-1} + N_{i,t-1}D_t. \quad (22)$$

The key difference, relative to the baseline model, is that we allow the price of the liquid asset $Q_{i,t}$ to be individual-specific. More precisely, we assume that individuals face an interest rate schedule (or “credit surface” in the language of [Geanakoplos, 2016](#)):

$$Q_{i,t} = F(Q_t, B_{i,t}, N_{i,t}P_t), \quad (23)$$

where F is a smooth function of economy-wide reference price Q_t (e.g. “prime rate”), individual bond holdings $B_{i,t}$, and the market value of asset holdings $N_{i,t}P_t$. The dependence of $Q_{i,t}$ on bond holdings, $\partial Q_{i,t} / \partial B_{i,t}$, captures the idea that the interest rate faced by individuals may increase with individual debt balances, while the dependence of the interest rate on asset values, $\partial Q_{i,t} / \partial (N_{i,t}P_t)$, captures “collateral effects.” In particular, when $\partial Q_{i,t} / \partial (N_{i,t}P_t) > 0$, a higher value of asset holdings allows the individual to issue bonds at a higher price $Q_{i,t}$, i.e., to borrow at a lower interest rate $Q_{i,t}^{-1}$, thereby capturing the key idea in collateral constraint models that higher asset prices relax financial frictions. While we focus on smooth interest rate schedules in the main text, we also study the case where individuals face “hard” borrowing constraints in [Appendix D.2.3](#), which can be seen as a limiting case. The next proposition expresses the effect of borrowing constraints on the welfare gains of a deviation in asset prices.

⁵⁴Consistently with this idea, in our baseline approach, we use a relatively high discount rate of 5%, which is larger than the average rate of return on the deposits and debt over our time sample (see also the discussion in [Footnote 35](#)).

Proposition 3. *In the presence of the interest rate schedule (23), the welfare gain of individual i is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \tilde{R}_{i,0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right), \quad (24)$$

where $\tilde{R}_{i,0 \rightarrow t}^{-1} \equiv \prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right)$.

This proposition shows that borrowing constraints affect our welfare gain formula in two ways. First, when $\partial Q_{i,t}/\partial B_{i,t} > 0$ meaning that the interest rate increases with the amount of debt ($Q_{i,t}^{-1}$ increases as $B_{i,t}$ becomes more negative), any increase in individuals' debt level increases the interest rate on their *entire* debt balance so that they effectively face a higher marginal interest rate.⁵⁵ As a result, individuals discount more heavily the future, which dampens the welfare effect of future deviations in asset prices: we call this the “discount rate channel”. Second, when $\partial Q_{i,t}/\partial (N_{i,t} P_t) > 0$, agents who hold levered positions in the asset benefit from a rise in asset prices through lower debt payments: this is what we call the “collateral channel”.⁵⁶ Importantly, in the presence of the collateral channel, asset *holdings* $N_{i,t}$ matter for the welfare effects of asset-price changes (in contrast to our baseline results in which only asset *sales* $N_{i,t-1} - N_{i,t}$ mattered).

To formalize these two channels, we can re-write the expression for welfare gains in the presence of borrowing constraints, as given in Proposition 3, as a sum of three terms that capture, respectively, the welfare gains in the baseline model, the effect of the discount rate channel, and the effect of the collateral channel.

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \frac{\partial Q_{i,t}}{\partial Q_t} dQ_t \right)}_{\text{Baseline}} \\ &+ \underbrace{\sum_{t=0}^{\infty} \left(\tilde{R}_{i,0 \rightarrow t}^{-1} - R_{0 \rightarrow t}^{-1} \right) \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \frac{\partial Q_{i,t}}{\partial Q_t} dQ_t \right)}_{\text{Discount rate channel}} \\ &+ \underbrace{\sum_{t=0}^{\infty} \tilde{R}_{i,0 \rightarrow t}^{-1} \left(-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right)}_{\text{Collateral channel}}. \end{aligned} \quad (25)$$

Implementation. We now assume a specific parametric form for the interest rate schedule (23). More precisely, we assume that the individual-specific (log) interest rate increases linearly with the loan-to-value ratio

$$Q_{i,t} = Q_t e^{-\xi \times \text{LTV}_{i,t}}, \quad (26)$$

⁵⁵In fact, the schedule for the *average* interest payment $Q_{i,t}$ (23) implies the following schedule for the *marginal* interest payment $\partial_B (Q_{i,t} B_{i,t}) = Q_{i,t} + B_{i,t} \partial_B Q_{i,t} = Q_{i,t} \left(1 + \frac{B_{i,t}}{Q_{i,t}} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right)$.

⁵⁶While we do not discuss aggregation in the context of the collateral effects extension, banks charging lower mortgage interest rates in response to higher home values may also generate some losers, in particular bank shareholders who indirectly hold mortgage debt as an asset. An offsetting effect is that lower loan-to-value ratios may lower bank monitoring costs so that bank shareholders may not be impacted much overall.

where $LTV_{i,t} \equiv -B_{i,t}/(N_{i,t}P_{i,t})$. The parameter ξ governs the sensitivity of the interest rate to the loan-to-value ratio (and so the importance of borrowing constraints). The case $\xi = 0$ corresponds to the baseline model without borrowing constraints. Plugging the parametric form (26) into Proposition 3 gives the following simplified formula for welfare gains.

Corollary 4. *In the presence of a loan-to-value constraint represented by the interest rate schedule (26), the welfare gain of individual i is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \tilde{R}_{i,0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \left(\frac{dQ_t}{Q_t} + \xi \times LTV_{i,t} \times \frac{dP_t}{P_t} \right) \right), \quad (27)$$

where $\tilde{R}_{i,0 \rightarrow t}^{-1} \equiv \prod_{s=0}^{t-1} (Q_{i,s} (1 - \xi \times LTV_{i,s}))$.

This equation gives simple closed-form expressions for the effect of borrowing constraints on the welfare gains of a deviation in asset prices with two key modifications relative to the baseline: first, borrowing constraints increase the effective discount rate of agent i by $2\xi \times LTV_{i,t}$ (discount rate channel); second, they increase the welfare exposure of asset-holders to rising asset prices by an amount equivalent to an increase in their annual rate of asset sales of $\xi \times LTV_{i,t}^2$ (collateral channel).⁵⁷

Results. We estimate the parameter ξ by examining the relationship between individual mortgage interest rates and the ratio of mortgage debt to house value. Figure 11a presents a binned scatter plot of these two variables in the data. A clear positive relationship is visible: as loan-to-value ratios increase from 0 to 100%, mortgage interest rates increase by around 0.2 pp. from around 5% to 5.20%. Consistently with our parametric assumption (26), the relationship is approximately linear. In Appendix D.2.2, we estimate this relationship more formally using panel regressions and obtain values for ξ between 0.0025 and 0.005, depending on the controls included. The interpretation is that an increase of the loan-to-value ratio from zero to one is associated with a 0.25 pp. to 0.5 pp. (25 to 50 basis points) higher mortgage interest rate.

One potential concern, however, is that measurement error in the loan-to-value ratio or omitted variable bias may bias this coefficient downward. To deal with measurement errors, we also collect direct evidence of the interest rate schedule posted by one of the Norwegian banks (Bulder Bank), which indicates a higher value of $\xi \approx 0.01$ (i.e., an increase in the loan-to-value ratio from zero to one implies a 1 pp. rise in the interest rate).

We then implement the expression for welfare gains (27) in the data.⁵⁸ Figure 11b reports the average welfare gains in each cohort. Given the uncertainty regarding the value of ξ , we report results for a range of values $\xi \in \{0, 0.005, 0.01\}$, where the case $\xi = 0$ corresponds to the baseline welfare gain formula (i.e., same welfare gains as in Figure 7). We find that the

⁵⁷The first statement comes from the fact that the effective discount rate of agent i between t and $t+1$ is $Q_{i,t}(1 - \xi \times LTV_{i,t}) = Q_t e^{-\xi LTV_{i,t}} (1 - \xi \times LTV_{i,t}) \approx Q_t e^{1-2\xi \times LTV_{i,t}}$. The second statement comes from the fact that the effect of collateral constraints at time t in (27) is $(-B_{i,t} Q_{i,t}) \times \xi \times LTV_{i,t} \times (dP_t/P_t) \approx \xi \times LTV_{i,t}^2 \times N_{i,t} dP_t$. In words, collateral constraints mean that every asset holder gains an additional exposure to a rise in asset prices that is equivalent to a “shadow” increase in their annual selling rate by $\xi \times LTV_{i,t}^2$.

⁵⁸See Equation 68 in Appendix D.2.2 for more details.

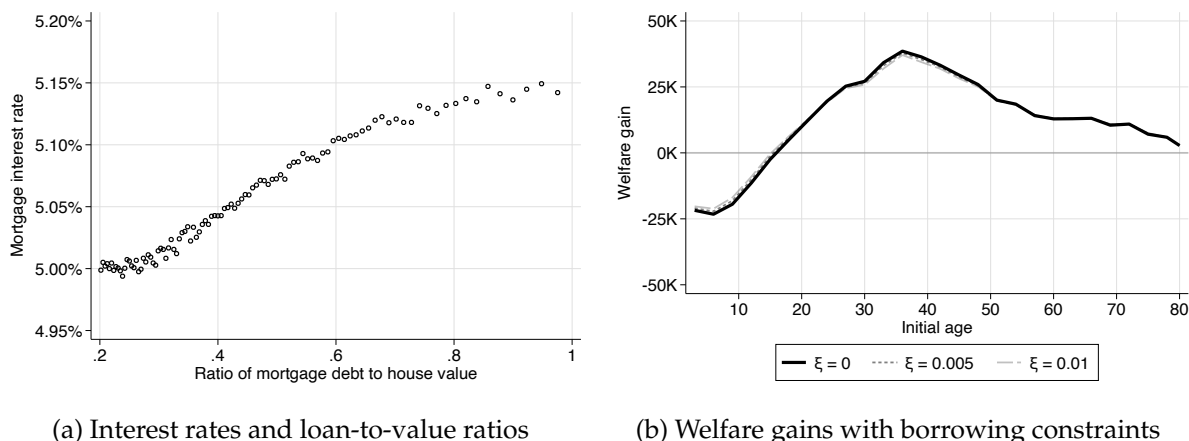


Figure 11: Accounting for borrowing constraints and collateral effects

Notes. Figure 11a contains a binned scatter plot of the interest rate on mortgages and the loan-to-value ratio across individuals over the years 1994–2019. To do this figure, we demean the individual-specific interest rate and loan-to-value ratios by their average values each year, adding back the average interest rate over 1994–2019. Each dot represents a percentile of the sample, ranked according to their loan-to-value ratio. Figure 11b plots the welfare gain including the collateral effect adjustment term (27). The welfare gain with $\xi = 0$ is the same as in Figure 7. Units are 2011 US dollars.

effect of borrowing constraints is small. Appendix Figure A9 plots separately the effects of the discount rate channel and of the collateral channel, the two effects tend to have opposite signs (the discount rate channel tends to be negative while the collateral channel is always positive), and so the two effects tend to cancel out.

In Appendix D.2.2, we also estimate the effect of borrowing constraints at the individual level. We document a sizable dispersion across individuals (in particular, within cohorts): while the average effect of borrowing constraints is close to zero, they can increase welfare by up to \$97,000 for the top 1% of individuals most impacted via these constraints. Still, despite this sizable dispersion, the correction due to borrowing constraints remains small relative to the dispersion in baseline welfare gains (Table 2).

Calibration approach. Overall, our results suggest that borrowing constraints only have a modest effect on our welfare gain formula in Norway. How general is this result? To answer this question, we use the closed-form formula (27) to evaluate the effect of changing ζ (the elasticity of interest rates to the loan-to-value ratio) for welfare. Consider, for example, an economy with ten times our baseline value for ζ ; that is, $\zeta = 0.1$ (an increase in the loan-to-value ratio by one increases the interest rate by 10%). In this case, borrowing constraints would effectively increase the discount rate of a borrower with $LTV_{i,t} = 0.25$ (the average loan-to-value ratio in our sample) by $2\zeta \times LTV_{i,t} = 5\text{pp}$; that is, from 5% to 10%. Moreover, due to the collateral channel, the same borrower would experience an additional welfare gain equivalent to a net increase in house sales by $\zeta \times LTV_{i,t}^2 \approx 0.6$, pp. which remains modest.⁵⁹ Overall, this type of computation makes it possible to assess the quantitative importance of borrowing constraints in different economic models or empirical settings.

How robust are these results to the assumption of a smooth interest rate schedule? To answer this question, we study the case where individuals face “hard” borrowing constraints

⁵⁹Figure 7a and Table A2 imply that the typical annual rate of home sales in each cohort is $\pm 4\%$.

in Appendix D.2.3. As in the case of our interest rate schedule (26), we derive simple closed-form formulas for the effect of these borrowing constraints on welfare gains in terms of three key parameters: (i) the proportion of individuals at the constraint, (ii) the wedge between the marginal rate of substitution across times and the interest rate for individuals at the constraint, and (iii) the loan-to-value ratio at the constraint. Overall, hard borrowing constraints lead to similar adjustments, both in spirit and in magnitude, to those derived in the baseline case where agents face a smooth interest rate schedule.⁶⁰

4.3 Second-order effects

Proposition 1 characterizes the welfare gains of an infinitesimal deviation in asset-prices. Hence, our sufficient statistic only captures the first-order effect of a non-infinitesimal deviation in asset prices. We now discuss theoretically and empirically the importance of higher-order effects.

Theory. We first derive a formula for the welfare gain corresponding to a non-infinitesimal deviation in asset prices. As in the proof of Proposition 1, we consider a non-infinitesimal deviation in the path of asset prices $\{\Delta Q_t, \{\Delta P_{k,t}\}_k\}_{t=0}^\infty$ and consider a continuum of intermediate economies where the deviation in asset prices is scaled by θ : $Q_t(\theta) = Q_t + \theta\Delta Q_t$ and $P_{k,t}(\theta) = P_{k,t} + \theta\Delta P_{k,t}$.

The money-metric welfare gain (i.e., the equivalent variation) of the deviation in asset prices indexed by θ is then the integral of infinitesimal welfare gain between 0 to θ :

$$EV_i(\theta) = \int_0^\theta \sum_{t=0}^\infty R_{0 \rightarrow t}^{-1}(u) \left(\sum_{k=1}^K (N_{i,k,t-1}(u) - N_{i,k,t}(u)) \Delta P_{k,t} - B_{i,t}(u) \Delta Q_t \right) du, \quad (28)$$

where $\{B_{i,t}(u), \{N_{i,k,t}(u)\}_k\}_{t=0}^\infty$ denote the path of asset holdings in the economy indexed by u after adjusting individual wealth at $t = 0$ to keep individual welfare fixed (i.e., Hicksian demands). For $u = 0$, this simply corresponds to the path of asset holdings in the baseline economy.

Using a trapezoidal approximation, we can then obtain a second-order approximation of welfare gains:⁶¹

$$EV_i(\theta) = \sum_{t=0}^\infty R_{0 \rightarrow t}^{-1}(\theta/2) \left\{ \sum_{k=1}^K \left(\frac{N_{i,k,t-1}(0) - N_{i,k,t}(0)}{2} + \frac{N_{i,k,t-1}(\theta) - N_{i,k,t}(\theta)}{2} \right) \Delta P_{k,t}(\theta) - \frac{B_{i,t}(0) + B_{i,t}(\theta)}{2} \Delta Q_t(\theta) \right\} + o(\theta^2). \quad (29)$$

⁶⁰The key difference between the two types of constraints is that soft borrowing constraints (interest rate schedule) affect the formula for welfare gains for everyone while hard borrowing constraints (strict borrowing limit) affects the formula for welfare gains for a limited number of agents — those hitting the limit. This stark distinction vanishes in a model with idiosyncratic risk, where all agents have some probability of hitting the constraint.

⁶¹We use the notation $o(\theta^2)$ to denote a term that converges to zero faster than θ^2 as $\theta \rightarrow 0$. Note that $\int_0^\theta f(u)g(u) du = f(\theta/2) \frac{g(0)+g(\theta)}{2} \theta + o(\theta^2)$ for any functions $f(\cdot), g(\cdot)$ (this can be proven formally by showing that both sides of the formula have the same first and second derivatives with respect to θ at zero). Setting $f(u) = R_{0 \rightarrow t}^{-1}(u)$, and $g(u) = N_{i,k,t-1}(u) - N_{i,k,t}(u)$, gives (29).

In comparison with the first-order approximation in (12), this second-order approximation requires to know how asset transactions respond to changes in asset prices (e.g., portfolio reshuffling). In particular, second-order effects are positive for individuals who respond to higher asset prices by selling more assets.⁶² One implication is that the accuracy of our baseline first-order approximation depends on the extent to which the financial transactions of individuals react to deviations in asset prices. Another difference with the baseline first-order approximation is that one should use the average interest rate between the baseline and counterfactual economy, $R_t(\theta/2)$, to discount future transactions.

Implementation. The empirical implementation of this second-order approximation requires additional assumptions: in contrast to the first-order approximation (12), we now need to specify what financial transactions would be if asset valuations had remained at their 1994 level. One way to do so would be to specify parametric forms for the utility function, the adjustment cost functions, as well as individuals' beliefs about future asset prices.

Instead, we make the simple assumption that, had valuations remained at their 1994 level, the quantity of transactions of a 30-year-old in each year would be the same as the transactions of a 30-year-old in 1994. Formally, we assume that the counterfactual transactions of individuals of age a are given by:

$$\begin{aligned}\bar{N}_{a,k,t}(\theta) - \bar{N}_{a,k,t-1}(\theta) &= \bar{N}_{a,k,0}(0) - \bar{N}_{a,k,-1}(0) \\ \bar{B}_{a,t}(\theta) &= G^t \bar{B}_{a,0}(0).\end{aligned}\tag{30}$$

where $\{\bar{B}_{a,t}(\theta), \{\bar{N}_{a,k,t}(\theta)\}_k\}_{t=0}^\infty$ denotes the average asset holdings of individuals of age a in year $1994 + t$ in the economy indexed by θ and $G = 1.01$ denotes the real per-capita growth rate of the economy in our sample period. Finally, we use the discount rate $R_t(\theta/2) = 1.025$, which reflects that the midpoint rate between the debt and deposit rates declined from 5% to 0% over our time sample (Figure 3).

Results. We now examine how these counterfactual transactions differ from actual transactions. Figure 12a compares the actual and counterfactual housing and equity transactions for different age groups. We see that the two quantities are very close, simply because real net housing and equity purchases have remained roughly constant over time. Figure 12b compares the actual and counterfactual debt balances. Net debt (debt minus deposits) has increased much more rapidly than one could expect from the growth of the economy. Intuitively, the young must now borrow more in order to finance the purchase of houses whose values have grown faster than the economy. Still, overall, we find that counterfactual transactions are relatively similar to actual transactions, which suggests that second-order effects are likely to be moderate in our settings.

Figure 12c plots the total second-order welfare gain computed using (29) while Figure 12d plots the second-order welfare gain class by asset class.⁶³ The figure confirms this intuition:

⁶²Martínez-Toledano (2022) empirically studies the effect of this type of market timing on wealth inequality.

⁶³See Appendix D.3 for more detail on the implementation.

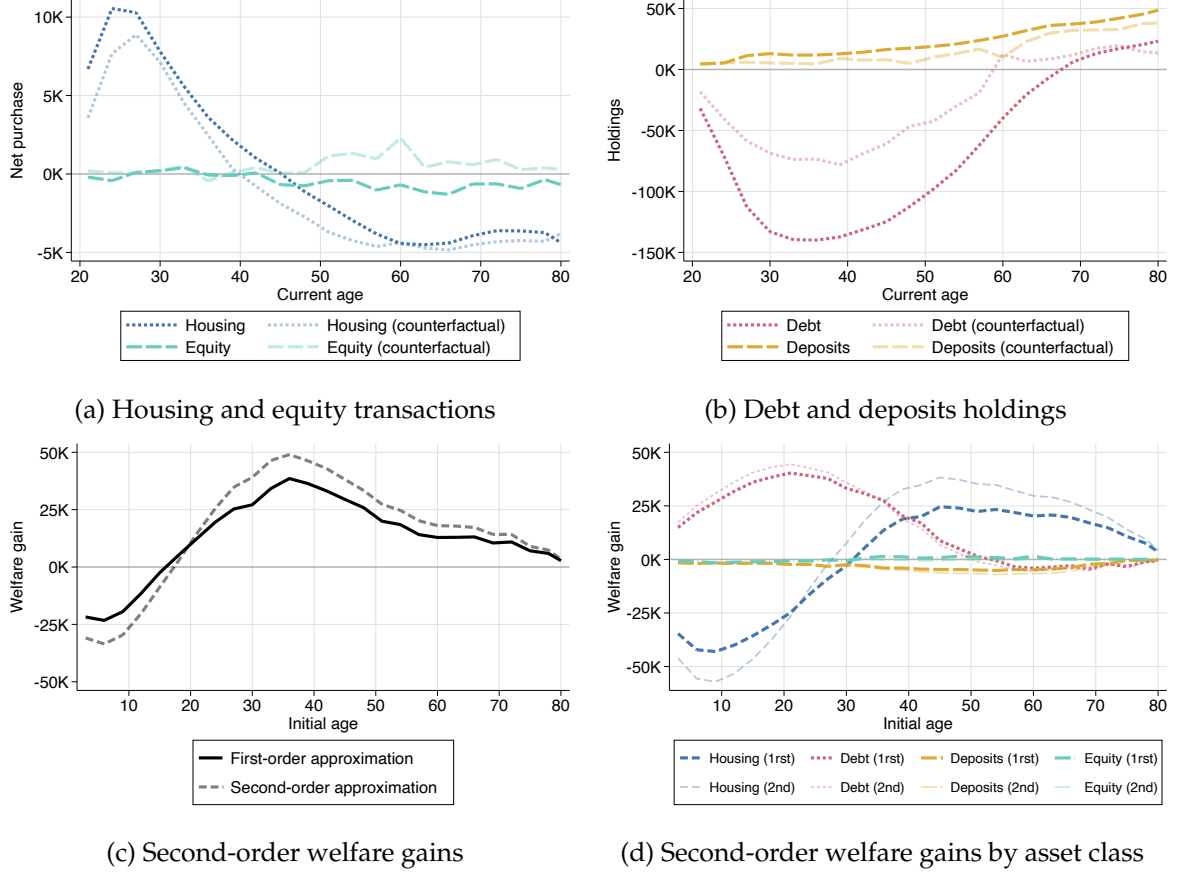


Figure 12: Accounting for second-order effects

Notes. Figure 12a and 12b compare actual versus counterfactual transactions (if valuations had remained at their baseline level). More precisely, Figure 12a plots $(\bar{N}_{a,k,t} - \bar{N}_{a,k,t-1})P_{k,t}$ and $(\bar{N}_{a,k,t}(\theta) - \bar{N}_{a,k,t-1}(\theta))P_{k,t}(\theta)$, averaged across years, for housing and equity. Figure 12b plots $\bar{B}_{a,t}Q_t$ and $\bar{B}_{a,t}(\theta)Q_t(\theta)$, averaged across years. Counterfactual asset transactions and bond holdings are estimated using (74). Figure 12c plots the average welfare gain at the first order and at the second order for individuals in each cohort (indexed by their age at the end of 1994) while Figure 12d plots it separately for each asset class. Units are 2011 US dollars.

the overall effect of the second-order adjustment is small and the results are quantitatively similar to those using our first-order approximation. Most of the effect is driven by the fact that we now use a much lower discount rate to discount the future (2.5% instead of 5%), which means that our second-order approximation tends to magnify the present-value of gains and losses obtained with our baseline (first-order) approach. One additional negative effect, for the young, is driven by the cross-elasticity of mortgage balance to house prices. As we have discussed, low mortgage rates provide an important offsetting effect for home buyers who are hurt by rising house prices. If house prices had remained at their initial values, the young would have had lower mortgage balances and, as a result, they would have benefited less from the decrease in mortgage rates (see Figure 12d for a plot of the second-order correction by asset class).

4.4 Extrapolation

Our measure of welfare gains in Proposition 1 expresses the welfare gains as the present value of all future transactions, multiplied by the path of future price deviations. However, as discussed in Section 2, we only apply our formula to a finite sample that ends in year 2019

($T = 25$). Therefore, our formula should be interpreted as the welfare gain associated with price deviations equal to zero after 2019 (i.e., assuming that valuations revert to the baseline in which asset prices grow at the same rate as dividends after 2019).

Implementation. How important is this truncation for our results? To examine this question, we recompute our welfare gains with different assumptions about the behavior of asset prices after 2019. More precisely, we assume that, after the end of the sample, valuations revert back to their baseline level according to a mean reversion parameter $\phi \in [0, 1]$. Formally, we assume that the valuation of asset class k at $t > T$ is given by:⁶⁴

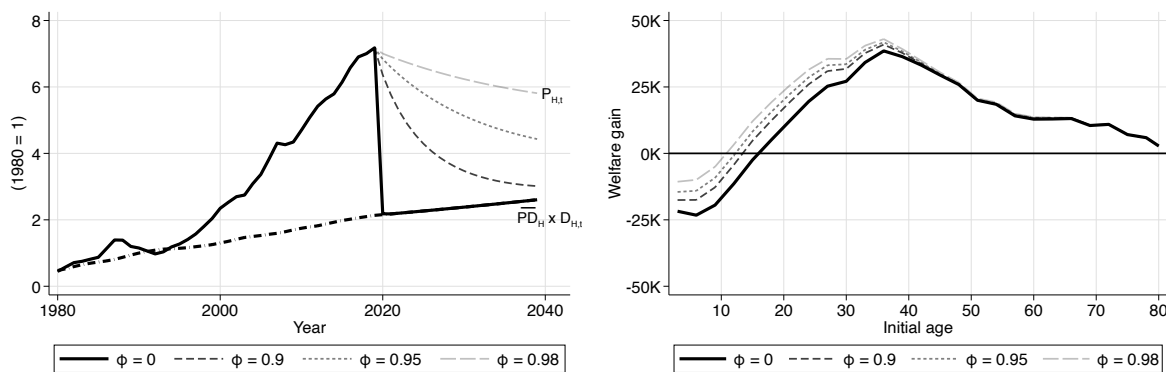
$$\log (PD_{k,t}/\overline{PD}_k) = \phi^{t-T} \log (PD_{k,T}/\overline{PD}_k), \quad \log (Q_t/\overline{Q}) = \phi^{t-T} \log (Q_T/\overline{Q}), \quad (31)$$

where $PD_{k,T}$ denotes the asset valuation in year 2019 and \overline{PD}_k denotes the baseline level of the asset valuation defined in Section 2. Our baseline summary statistic, which considers asset price deviations that stop after T , can be seen as the limit case $\phi = 0$. Figure 13a plots the series of house prices obtained using this methodology up to 2060, for values of ϕ between 0 and 1. Note that, in all scenarios, we assume that housing valuations ultimately revert back to their initial value ($\phi < 1$), consistent with the fact that asset valuations are stationary processes (Campbell and Shiller, 1988).

To implement the sufficient statistic formula, we also need to predict individuals' transactions in future years. To do so, we assume that the number of assets sold by a given cohort in a given year will equal the number of assets sold by the cohort with the same age in 2019, after adjusting for economic growth (see Appendix D.4 for details). This assumption is motivated by the fact that the quantity of transactions by age groups has remained remarkably stable over our sample period, as discussed above (Section 4.3).

Results. Figure 13 plots our estimated values for the average welfare gain in each cohort for different values of ϕ . As ϕ increases, two things happen. First, the graph of welfare gains is translated to the left. Intuitively, a high ϕ means that aging individuals sell more assets at elevated prices beyond the end of our sample period thereby increasing their welfare gains. However, this comes at the expense of young generations, unborn in 1994, who will ultimately purchase these assets. Second, the graph of welfare gains shifts up. This is because, as we showed in the sectoral analysis in Appendix C.2, individuals in Norway benefit on net from the rise in asset prices because they hold a positive amount of debt in the aggregate. As ϕ increases, higher valuations last for a longer time, which means that, on average, welfare gains increase. However, doing the same exercise for sectoral welfare gains would reveal that this comes at the cost of a decrease in the total welfare gains for the government. Appendix Figure A12 decomposes the welfare gains by asset class. The decomposition shows that, as ϕ increases, most of the higher welfare gains in the population comes from lower interest rates on debt.

⁶⁴See Campbell (2018) for an example of such a AR(1) specification for the logarithmic price-dividend ratio.



(a) Extrapolated house prices

(b) Welfare gains across cohorts

Figure 13: Accounting for extrapolated changes in asset prices beyond 2019

Notes. Figure 13a plots the path of future house prices for different values of ϕ , constructed using (31). Figure 13b plots the average welfare gain in each cohort with different assumptions about the future path of asset prices. Units are 2011 US dollars.

Conclusion

The main contribution of our paper is to provide a simple framework to quantify the welfare effects of fluctuations in asset prices. The core economic idea is that the welfare effect of changes in asset prices can be measured from the path of realized financial transactions: rising asset valuations benefit sellers and harm buyers. We implement our sufficient statistic formula using administrative data on financial transactions to quantify welfare gains and losses in Norway for the years 1994 to 2019.

Our empirical implementation generates four main findings. First, the rise in asset valuations had large redistributive effects, i.e., they resulted in significant welfare gains and losses. At the same time, welfare gains differed substantially from naïvely calculated revaluation gains; in particular, individuals with the highest revaluation gains were not necessarily the ones with the highest welfare gains. Second, rising asset prices redistributed across cohorts, with the old benefiting at the expense of the young. Third, they redistributed across the wealth distribution, from the poor toward the wealthy. Fourth, they also redistributed across sectors: declining interest rates benefited households at the expense of the government.

It is worth emphasizing the fact that, while our sufficient statistic approach is general, our empirical results are country-specific. Differences in institutions, regulations, and norms shape the exposure of household welfare to asset price changes. For instance, in Norway, public equities represent merely 3% share of household wealth (see Appendix Table A2), mortgages essentially all have floating interest rates, and the government is a net saver (through the Sovereign wealth fund, see Appendix C.2). One can expect the welfare effect of deviations in asset prices to be different in countries such as the United States, where public equities represent roughly 20% of household wealth, mortgages tend to have fixed interest rates where the government is a net debtor (see Greenwald et al., 2021).

Recent work building on our methods suggests that our sufficient statistic approach may also prove useful in other contexts. Del Canto et al. (2023) and Pallotti et al. (2023) study the money-metric welfare gains and losses from inflationary shocks of U.S. and Euro-area house-

holds and implement the corresponding welfare formulas using micro data. Similarly, [Crowley and Gamber \(2023\)](#) study the welfare consequences of the large asset-price and interest-rate changes on U.S. households over the time period 2021 to 2023 rather than the longer-run trends considered here. Another valuable exercise would be to systematically quantify the welfare consequences of higher-frequency asset-price booms and busts that the literature has emphasized as important drivers of wealth inequality dynamics ([Kuhn et al., 2020](#); [Martínez-Toledano, 2022](#); [Gomez, 2016](#); [Cioffi, 2021](#)).

Finally, our results on the redistributive effect of asset prices raise important questions for optimal capital gains and wealth taxation. Answering such questions requires studying environments with changing asset prices using the tools from public finance. [Aguiar et al. \(2024\)](#) take some steps in this direction.

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Appendix

A Appendix for Section 1

A.1 Welfare gains in a deterministic environment

We consider the deterministic economy described in Section 1.2. The following proposition characterizes the welfare effect of a joint deviation in the path of asset prices, labor income and dividend income. This effectively generalizes Proposition 1, which corresponds to the special case where the deviation in income is zero.

Proposition 5. *The welfare gain of a deviation in asset prices $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^\infty$ as well as labor and dividend income $\{dY_{i,t}, \{dD_{k,t}\}_k\}_{t=0}^\infty$ is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right). \quad (32)$$

Proof of Proposition 5. To provide some intuition, we first provide an heuristic derivation using the Lagrangian corresponding to the individual optimization problem and totally differentiating with respect to the sequence of asset prices $\{Q_t, \{P_{k,t}\}_{k=1}^K\}_{t=0}^\infty$ and income $\{dY_{i,t}, \{dD_{k,t}\}_{k=1}^K\}_{t=0}^\infty$. We then provide a more rigorous proof that uses a perturbation of this sequence indexed by a perturbation parameter θ and a version of the envelope theorem due to Oyama and Takenawa (2018).

Heuristic derivation. The Lagrangian associated with the optimization problem is

$$\begin{aligned} \mathcal{L}_i = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) + \sum_{t=0}^{\infty} \lambda_{i,t} \left(\sum_{k=1}^K N_{i,k,t-1} D_{k,t} + B_{i,t-1} + Y_{i,t} - C_{i,t} \right. \\ \left. - \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} - \sum_{k=1}^K \chi_k (N_{i,k,t} - N_{i,k,t-1}) - B_{i,t} Q_t \right). \end{aligned} \quad (33)$$

Assuming that the value function is differentiable, we can write the infinitesimal change in the value function in terms of the infinitesimal change in the Lagrangian:

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \left(\sum_{k=1}^K \frac{\partial \mathcal{L}_i}{\partial P_{k,t}} dP_{k,t} + \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t + \sum_{k=1}^K \frac{\partial \mathcal{L}_i}{\partial D_{k,t}} dD_{k,t} + \frac{\partial \mathcal{L}}{\partial Y_{i,t}} dY_{i,t} \right) \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right) \\ &= \lambda_{i,0} \sum_{t=0}^{\infty} (Q_0 \dots Q_{t-1}) \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right) \\ &= U'(C_{i,0}) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right). \end{aligned}$$

The third equality uses the first-order conditions for $B_{i,t}$, $\lambda_{i,t} Q_t = \lambda_{i,t+1}$, while the last equality uses the first-order condition for $C_{i,0}$, $U'(C_{i,0}) = \lambda_{i,0}$, as well as the definition of the cumulative return $R_{0 \rightarrow t}^{-1} = Q_0 \dots Q_{t-1}$.

Formal derivation. Consider a deviation in asset prices in the direction $\{\Delta Q_t, \{\Delta P_{k,t}\}_k\}_{t=0}^\infty$ and of income in the direction $\{\Delta Y_{i,t}, \{\Delta D_{k,t}\}_k\}_{t=0}^\infty$. Consider a parameter $\theta \in [0, 1]$ indexing the size of the perturba-

tion:

$$\begin{aligned} Q_t(\theta) &\equiv Q_t + \theta \Delta Q_t; & P_{k,t}(\theta) &\equiv P_{k,t} + \theta \Delta P_{k,t} \\ Y_{i,t}(\theta) &\equiv Y_{i,t} + \theta \Delta Y_{i,t}; & D_{k,t}(\theta) &\equiv D_{k,t} + \theta \Delta D_{k,t}. \end{aligned}$$

The optimization problem takes the form $V_{i,0}(\theta) = \max_x f(x, \theta)$ where $x = \{B_{i,t}, \{N_{i,k,t}\}_{t=0}^\infty\}$, and

$$f : (x, \theta) \mapsto \sum_{t=0}^{\infty} \beta^t U \left(\sum_{k=1}^K N_{i,k,t-1} D_{k,t} + B_{i,t-1} + Y_{i,t} - \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} - B_{i,t} Q_{i,t} - \sum_{k=1}^K \chi_k (N_{i,k,t} - N_{i,k,t-1}) \right).$$

Note that f is continuous in x , and that its derivative with respect to θ is

$$\partial_\theta f(x, \theta) = \sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} - B_{i,t} \Delta Q_t + \sum_{k=1}^K N_{i,k,t-1} \Delta D_{k,t} + \Delta Y_{i,t} \right),$$

which is continuous in x and θ . Under this set of assumptions, Proposition 2.1 in [Oyama and Takenawa \(2018\)](#) gives that V is differentiable at 0 and $V'(0) = \partial_\theta f(x^*, 0)$, where x^* denote the optimal solution of the maximization problem at $\theta = 0$. Using the expression for $\partial_\theta f$ above gives:

$$\begin{aligned} V'_{i,0}(0) &= \sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} - B_{i,t} \Delta Q_t + \sum_{k=1}^K N_{i,k,t-1} \Delta D_{k,t} + \Delta Y_{i,t} \right) \\ &= U'(C_{i,0}) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} - B_{i,t} \Delta Q_t + \sum_{k=1}^K N_{i,k,t-1} \Delta D_{k,t} + \Delta Y_{i,t} \right), \end{aligned}$$

where the second line uses the Euler equation. This concludes the proof as $dV_{i,0} = V'(0) d\theta$, $dQ_t = \Delta Q_t d\theta$, $dP_{k,t} = \Delta P_{k,t} d\theta$, $dY_{i,t} = \Delta Y_{i,t} d\theta$, and $dD_{k,t} = \Delta D_{k,t} d\theta$. \square

Proof of Proposition 1. The proposition obtains as a special case of Proposition 5 where the deviation in income is taken to be zero, i.e., $dY_{i,t} = 0$ and $dD_{k,t} = 0$ for $1 \leq k \leq K$. \square

A.2 Welfare gains in a stochastic environment

So far, we have focused on the case of a deterministic economy. We now extend our results to the case of a *stochastic* economy. For clarity, we distinguish between two different cases. In Appendix A.2.1, we first consider the case of a baseline stochastic economy that is subject to an unexpected deviation in the path of asset prices and income, i.e., the same comparative static exercise as in Proposition 1 but now in a stochastic environment. In Appendix A.2.2, we then consider the more complex case in which the realization of the deviation itself corresponds to a “shock” on which individuals can potentially contract ex-ante, i.e., individuals maximize expected values using a probability distribution over different realizations of this shock and can potentially insure against this shock by trading in financial markets.

A.2.1 Extension of Proposition 1 to stochastic environment (comparative statics exercise)

Setup. We first describe the stochastic economy. At each time period $t \geq 1$ there is a realization of a stochastic event $s_t \in \mathcal{S}$. We denote $s^t = (s_1, \dots, s_t)$ the history of these realizations up to time t . We allow asset prices and income to depend on the entire history of events s^t . More precisely, there is a sequence of one-period bonds with stochastic prices $Q_t(s^t)$, as well as K long-lived assets financial assets with stochastic prices $P_{kt}(s^t)$ and stochastic dividends $D_{kt}(s^t)$ for each asset k . Each individual

i receives some stochastic labor income every period $Y_{i,t}(s^t)$.⁶⁵ To minimize the notational burden, we omit the explicit dependence of each stochastic process on s^t when there is no risk of ambiguity.

Agents have the same preferences as in the baseline model. The problem of individual i is to choose a history-contingent path for consumption $C_{i,t}$ and asset holdings $B_{i,t}$, $\{N_{i,k,t}\}_k$ to maximize welfare:

$$V_{i,0} \equiv \max_{\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right],$$

subject to initial asset holdings $B_{i,-1}$ and $\{N_{i,k,-1}\}_k$, as well as a sequence of budget constraints for $t \geq 0$

$$C_{i,t} + \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} + B_{i,t} Q_t + \sum_{k=1}^K \chi_k (N_{i,k,t} - N_{i,k,t-1}) = \sum_{k=1}^K N_{i,k,t-1} D_{k,t} + B_{i,t-1} + Y_{i,t}. \quad (34)$$

As in the baseline model, we assume that there is a unique solution to the agent problem and that it is continuous with respect to asset prices, dividends, and income received in each state of nature.⁶⁶

Welfare gain. We now study the welfare effect of a deviation in the paths of asset prices and income. More precisely, we consider a history-contingent deviation in asset prices, $\{dQ_t(s^t), \{dP_{k,t}(s^t)\}_k\}_{t=0}^{\infty}$, as well as in dividend and labor income, $\{dY_{i,t}(s^t), \{dD_{k,t}(s^t)\}_k\}_{t=0}^{\infty}$.⁶⁷ As in the main text, this deviation can be interpreted either as a difference between two economies (e.g., comparative static) or as an unexpected MIT shock in the baseline economy.

We define the welfare gain of this deviation as the amount of money received at time $t = 0$ in the baseline economy that would have the same effect on individual welfare. The next proposition gives an expression for this welfare gain in terms of the path of future asset holdings.

Proposition 6. *The welfare gain of a deviation in asset prices $\{dQ_t, \{dP_{k,t}\}_k\}_{t=0}^{\infty}$ as well as labor and dividend income $\{dY_{i,t}, \{dD_{k,t}\}_k\}_{t=0}^{\infty}$ is*

$$dV_{i,0}/U'(C_{i,0}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right) \right]. \quad (35)$$

Relative to the formula obtained in a deterministic economy (Proposition 5), the baseline formula for the welfare gain changes in two ways. The first is that, in a stochastic world, what matters for (ex-ante) welfare is the expected path of net asset sales, not the realized one. The second is that this expectation is under the individual's risk neutral measure, which tilts the objective measure by the individual's marginal rate of substitution across states and times $\beta^t U'(C_{i,t})/U'(C_{i,0})$. This adjustment reflects the fact that individuals care about certain states of the world more than others. To be concrete in the context of idiosyncratic labor income risk, consider the ex-ante welfare gain of young individuals who face uncertainty over their future paths of labor income, and who plan to buy houses only if they are successful in the labor market. From today's perspective, these individuals "care more" about the states of the world in which their income is low, as their marginal utility of consumption will be higher

⁶⁵While the set of event realizations \mathcal{S} does not depend on i , the mapping from the history of stochastic events s^t to individual labor income $Y_{i,t}$ is individual specific, and so our framework includes models with purely idiosyncratic labor income shocks.

⁶⁶Note that, when the economy is stochastic, this assumption no longer requires the existence of adjustment cost, as different assets can have different risk profiles. Still, this assumption rules out the presence of arbitrage opportunities within the set of assets without adjustment costs.

⁶⁷Note that we allow this deviation to be state-contingent, although it does not play a key role for our results.

in these states. Hence, their expected housing purchases are lower under the risk-adjusted measure than under the objective measure.

Proof of Proposition 6. The Lagrangian associated with the individual problem at time $t = 0$

$$\begin{aligned} \mathcal{L}_i = & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right] \\ & + \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \lambda_{i,t} \left(Y_{i,t} + \sum_{k=1}^K N_{i,k,t-1} D_{k,t} + B_{i,t-1} - C_{i,t} - (N_{i,k,t} - N_{i,k,t-1}) P_{k,t} - B_{i,t} Q_t \right) \right], \end{aligned}$$

where $\lambda_{i,t}$ denotes the (stochastic) Lagrange multiplier associated to the budget constraint (34) at time t . One should understand the expectation \mathbb{E}_0 as a probability-weighted sum across different states of nature; and so this Lagrangian can be understood as the one obtained for a static optimization with as many constraints as there are states of nature.

As in the proof of Proposition 1, we simply apply the Lagrangian method to obtain:

$$\begin{aligned} dV_{i,0} = & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \lambda_{i,t} \left(dY_{i,t} + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} - (N_{i,k,t} - N_{i,k,t-1}) dP_{k,t} - B_{i,t} dQ_t \right) \right] \\ = & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) \left(dY_{i,t} + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} - (N_{i,k,t} - N_{i,k,t-1}) dP_{k,t} - B_{i,t} dQ_t \right) \right], \end{aligned}$$

where the second equality uses the first-order condition for $C_{i,t}$, $\beta^t U'(C_{i,t}) = \lambda_{i,t}$. Dividing each side by $U'(C_{i,0})$ gives the result. \square

Case of complete markets. We now add the assumption that markets are complete in the baseline economy.⁶⁸ In a dynamic environment such as ours, what this assumption means is that there is a subset of assets with zero adjustment cost that is large enough that individuals can replicate the Arrow-Debreu security associated to history s^t (i.e. a security that returns one dollar at time t if history s^t is realized and zero otherwise) with some trading strategy started at time $t = 0$ (for any time t and history s^t). In this case, there is a unique stochastic discount factor, $\Lambda_t(s^t)$ (up to a scaling factor). At each time and in each state, agents equalize their marginal rates of substitution across states and times to the growth of the stochastic discount factor:⁶⁹

$$\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} = \frac{\Lambda_t}{\Lambda_0}. \quad (36)$$

As a result, the welfare gain formula from Proposition 6 simplifies to:

$$dV_{i,0}/U'(C_{i,0}) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t + \sum_{k=1}^K N_{i,k,t-1} dD_{k,t} + dY_{i,t} \right) \right]. \quad (37)$$

Because of complete markets, all individuals use the same discount factor to value consumption in any future history relative to today.⁷⁰ One implication is that, for the case of a pure deviation in asset

⁶⁸Note that we are still considering the case of a comparative statics exercise between two economies (or, equivalently, an unexpected deviation at time $t = 0$; that is, an MIT shock). We will consider the case of markets that are complete with respect to the “realization” of the deviation itself in Appendix A.2.2 below.

⁶⁹Denoting $\pi(s^t)$ the probability of history s^t , $\pi(s^t)\Lambda_t(s^t)/\Lambda_0$ corresponds to the price at $t = 0$ of the Arrow-Debreu security associated to history s^t . Hence, this equality can be interpreted as the first-order condition reflecting that the individual i must be indifferent between using one marginal dollar at $t = 0$ to consume or to invest in the Arrow-Debreu security associated to history s^t .

⁷⁰In fact, we only need the markets to be complete starting from time $t = 0$; that is, we do not need agents to be able to insure against the deviation shock.

prices (i.e., no deviation in incomes $dD_{k,t} = dY_{i,t} = 0$), welfare gains aggregate to zero across a set of individuals trading with each other, since transactions aggregate to zero in every state in the world. Note that this critically hinges on the fact that markets are complete, which was de-facto the case in our baseline model (Proposition 1), as the presence of liquid one-period bonds in a deterministic economy is enough to make markets complete.

A.2.2 Welfare effects of potentially insurable shocks

So far, we have considered the case of a stochastic economy with an unexpected deviation in the path of asset prices and income (i.e., a MIT shock). We now instead consider “deviation shocks” with individuals maximizing expected values using a probability distribution over different realizations of this shock. An important implication of this assumption is that individuals can potentially contract on the realization of the deviation itself. The presence of insurance markets affects the equilibrium trading pattern of individuals, and, therefore, their exposure to the “deviation shock”. Still, our sufficient statistic formula, which expresses individual welfare gains *given* individual trading patterns, remains unchanged.

Setup. The setup for the baseline economy is very similar to the previous section, except that we now add the realization of a stochastic event s_0 at time $t = 0$. To keep things simple, we assume that there are only two possible realizations for s_0 : a baseline scenario \bar{s} and a deviation \hat{s} .⁷¹ We assume that the realization of s_0 does not affect the probability distribution of events going forward, but it does affect the values of asset prices and income associated to each event history. More precisely, labor income, dividend income, and asset prices at time t are stochastic processes that depend on the entire history of shocks from 0 to t , $(s_0, s_1, \dots, s_t) = (s_0, s^t)$ where $s^t = (s_1, \dots, s_t)$ denotes the history from $t = 1$ as in the preceding section.

For any stochastic process $X_t(s_0, s_1, \dots, s_t)$, we denote by $\Delta X_t(s^t) \equiv X_t(\hat{s}, s^t) - X_t(\bar{s}, s^t)$ the difference in the realized value of X_t if state $s_0 = \hat{s}$ happens compared to state $s_0 = \bar{s}$. Finally, as in the proof of Proposition 5, we consider a sequence of economies in which the size of the difference in asset prices and income between \hat{s} and \bar{s} is scaled by θ and we will study the asymptotic limit $\theta \rightarrow 0$.⁷² To keep notations simple, we will leave implicit the fact that each stochastic process (e.g. asset prices, income, consumption...) is indexed by θ .

Agent preferences are the same as above. Individuals choose a history-contingent path of consumption $C_{i,t}$ and asset holdings $B_{i,t}, \{N_{i,k,t}\}_k$ to maximize expected utility

$$V_{i,-1} \equiv \max_{\{C_{i,t}, B_{i,t}, \{N_{i,k,t}\}_k\}_{t=0}^{\infty}} \mathbb{E}_{-1} \left[\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \right],$$

subject the following sequence of budget constraints

$$C_{i,t} + \sum_{k=1}^K (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t = \sum_{k=1}^K N_{i,k,t-1}D_{k,t} + B_{i,t-1} + Y_{i,t} \quad \text{for } t \geq 0 \quad (38)$$

$$\sum_{k=1}^K N_{i,k,-1}P_{k,-1} + B_{i,-1}Q_{-1} = W_{i,-1} \quad \text{for } t = -1. \quad (39)$$

⁷¹We could easily allow s_0 to have an arbitrary (finite) number of realizations, $s_0 \in \mathcal{S}$, and focus on the welfare gain of some realization relative to another. However, the notations become much more complex, as one needs to distinguish the ex-post realization of each stochastic process with respect to each possible realization of s_0 .

⁷²This small shock limit $\theta \rightarrow 0$ can be interpreted as the small time horizon limit $\Delta t \rightarrow 0$ in an economy in which asset prices and income follow Ito processes.

with initial financial wealth $W_{i,-1}$. The key difference with the earlier model is that we allow agents to trade before the realization of the deviation shock s_0 . Note that, for the sake of simplicity, we also assume away any adjustment costs in the assets, which allows us to get a simple formulation for the individual problem when markets are complete. Finally, we assume that the set of asset prices and dividends is such that there are no arbitrage opportunities, that there is a unique solution to the agent problem and that it is continuous with respect to θ .

Welfare gain. We now extend our notion of welfare gain (i.e., equivalent variation) to this stochastic economy. Formally, we define the welfare gain of the realization of $s_0 = \hat{s}$ relative to $s_0 = \bar{s}$ as the quantity of money, received at $t = 0$ that would equalize realized welfare across these two states. Denote by $\Delta V_{i,0} \equiv \mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) | s_0 = \hat{s}] - \mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t U(C_{i,t}) | s_0 = \bar{s}]$ the difference in the welfare of the individual at time $t = 0$ if state \hat{s} is realized relative to state \bar{s} (given our two-state assumption, $\Delta V_{i,0}$ is just a deterministic scalar). At the first-order in θ (i.e., when the difference between the two states is small), the welfare gain then corresponds to $\Delta V_{i,0}$ divided by the marginal utility of consumption. Using a similar derivation as in Proposition 6, the welfare gain is:⁷³

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} - B_{i,t} \Delta Q_t + \sum_{k=1}^K N_{i,k,t-1} \Delta D_{k,t} + \Delta Y_{i,t} \right) \right] + o(\theta). \quad (40)$$

The key takeaways are that (i) our sufficient statistic formula can be interpreted as the effect of a realized shock, and that (ii) it remains valid even when individuals can ex-ante insure against this shock. Put differently, while the ability to optimally choose portfolio at $t = -1$ may impact the trading patterns of the individual, it does not affect the welfare effect of a shock *given* these trading patterns.

Case of complete markets We now consider the case where markets are complete starting from time $t = -1$. As discussed above, this means that there is a unique (up to a scaling factor) stochastic discount factor, denoted Λ_t , for $t \geq -1$. Because we abstracted away from adjustment costs, the sequence of asset prices satisfies the following equations:

$$\begin{aligned} \Lambda_t P_t &= \mathbb{E}_t [\Lambda_{t+1} (D_{t+1} + P_{t+1})] & \text{for } 1 \leq k \leq K \\ \Lambda_t Q_t &= \mathbb{E}_t [\Lambda_{t+1}]. \end{aligned}$$

As a result, we can consolidate the sequence of budget constraints (38) for an individual starting with some initial wealth $W_{i,-1}$ to obtain the following present-value budget constraint:⁷⁴

$$W_{i,-1} = \mathbb{E}_{-1} \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_{-1}} (C_{i,t} - Y_{i,t}) \right]. \quad (41)$$

⁷³We use the notation $o(\theta)$ to denote a term that converges to zero faster than θ as $\theta \rightarrow 0$ (e.g., a second-order term in θ). Note that, while each side of this formula is a random variable contingent on the realization of the state s_0 , the difference between the values of these random variables between $s_0 = \hat{s}$ and $s_0 = \bar{s}$ is second-order in θ . Put differently, as in the deterministic case, welfare gains can be approximated at the first-order using our sufficient statistic with either the path of asset transactions in the baseline economy or the path of asset transactions in the perturbed economy.

⁷⁴The derivation is standard: multiply the period t budget constraint (38) by Λ_t , add the change in asset valuations between $t - 1$ and t , $\sum_k N_{i,k,t-1} (\Lambda_t P_{k,t} - \Lambda_{t-1} P_{k,t-1}) + B_{i,t-1} (\Lambda_t - \Lambda_{t-1} Q_{t-1})$, and take the expectation at time $t - 1$ to obtain $\Lambda_{t-1} W_{i,t-1} = \mathbb{E}_{t-1} [\Lambda_t (C_{i,t} - Y_{i,t}) + \Lambda_t W_{i,t}]$ where $W_{i,t} \equiv \sum_k N_{i,k,t} P_{k,t} + B_{i,t} Q_t$ denotes financial wealth at time t . Substituting out $W_{i,t}$ using the next-period budget constraint and solving forward gives the result.

Hence, because of complete markets, the problem of the agent can simply be expressed as choosing a history contingent consumption path to maximize utility subject to this present-value budget constraint. Taking the first-order condition of this problem with respect to $C_{i,t}$ pins down consumption in terms of the stochastic discount factor for any time $t \geq 0$.⁷⁵

$$\beta^t U'(C_{i,t}) = \mu_i \frac{\Lambda_t}{\Lambda_{-1}}, \quad (42)$$

where μ_i is the Lagrange multiplier corresponding to the present-value budget constraint (taking a value such that this constraint is satisfied). Differentiating with respect to the realization of $s_0 = \hat{s}$ relative to $s_0 = \bar{s}$ gives:

$$\frac{\Delta C_{i,t}}{C_{i,t}} = -\frac{1}{\gamma(C_{i,t})} \frac{\Delta \Lambda_t}{\Lambda_t} + o(\theta), \quad (43)$$

where $\gamma(C) \equiv -U''(C)C/U'(C)$ denotes the (local) relative risk aversion of the individual. Hence, market completeness pins down the effect of the realization of $s_0 = \hat{s}$, relative to the baseline $s_0 = \bar{s}$, on consumption: it depends on the relative prices of the Arrow-Debreu securities associated to these events (the sensitivity of the stochastic discount factor to the realization of s_0) divided by the agent relative risk aversion.

We can use this equation to pin down the welfare gain from the realization of $s_0 = \hat{s}$ relative to the baseline $s_0 = \bar{s}$. We use the fact that the welfare gain of a shock can be obtained as the present value of the effect of the shock on future consumption, which gives:⁷⁶

$$\begin{aligned} \frac{\Delta V_{i,0}}{U'(C_{i,0})} &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \Delta C_{i,t} \right] + o(\theta) \\ &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \Delta C_{i,t} \right] + o(\theta) \\ &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} C_{i,t} \left(-\frac{1}{\gamma(C_{i,t})} \frac{\Delta \Lambda_t}{\Lambda_t} \right) \right] + o(\theta), \end{aligned} \quad (40')$$

where the second equality uses (42) and the third equality uses (43). This equation expresses the welfare effect of the shock $s_0 = \hat{s}$ relative to the baseline $s_0 = \bar{s}$ in terms of its effect on the stochastic discount factor (i.e. the relative price of insurance for one state relative to the other) as well as individual preferences.

It is important to realize that complete markets do *not* imply that the welfare effect of a shock is equalized across agents. Instead, what complete markets imply is that agents equate the *marginal* benefit of being a bit more insured with its *marginal* cost, given by the price of the Arrow-Debreu security associated with the state (42). This does not necessarily imply people choose the same level of insurance in equilibrium. In particular, (40') implies that more risk-averse agents are typically less exposed to aggregate shocks in equilibrium.⁷⁷

This formula, which pins down the welfare gain in terms of its effect on the stochastic discount factor and individual preferences, is consistent with our earlier “sufficient statistic” formula (40), which expresses it in terms of endogenous portfolio choices. While the new formula helps to understand

⁷⁵To see why, think of $\mathbb{E}_{-1}[\cdot]$ in the present-value budget constraint as a sum over different states of nature.

⁷⁶See Proposition 17 in Appendix E.2 similar interpretations of our notion of welfare gains.

⁷⁷One analogy is an economy in which entrepreneurs with decreasing return to scale production function can borrow at some interest rate r . Entrepreneurs invest up until the marginal return on capital is equal to the interest rate, but that does not necessarily imply that the *average* return to capital is equalized across entrepreneurs.

the optimal amount of “welfare exposure” chosen by the individual when markets are complete, our “sufficient statistic” formula is still preferable in an empirical context because it is valid whether or not markets are complete.

While the equivalence between the two formulas is already implied by our results, it is helpful to derive it manually. Note that any portfolio strategy chosen by the agent must satisfy, at each time and in each state of nature:

$$\sum_k (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t = B_{i,t-1} + \sum_k N_{i,k,t-1}D_{k,t} + Y_{i,t} - C_{i,t},$$

where $C_{i,t}$ is given by (42). Differentiating with respect to the realization of $s_0 = \hat{s}$ relative to $s_0 = \bar{s}$ gives

$$\begin{aligned} \Delta C_{i,t} &= \sum_k (N_{i,k,t-1} - N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t} \\ &+ \sum_k (\Delta N_{i,k,t-1} - \Delta N_{i,k,t})P_{k,t} + \sum_k (\Delta N_{i,k,t-1})D_{k,t} - (\Delta B_{i,t})Q_t + \Delta B_{i,t-1} + o(\theta). \end{aligned}$$

Taking the present value conditional and rearranging gives

$$\begin{aligned} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \Delta C_{i,t} \right] &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\sum_k (N_{i,k,t-1} - N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t} \right) \right. \\ &+ \left. \sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\sum_k \left(\frac{\Lambda_{t+1}}{\Lambda_t} (D_{k,t+1} + P_{k,t+1}) - P_{k,t} \right) \Delta N_{i,k,t} + \left(\frac{\Lambda_{t+1}}{\Lambda_t} - Q_t \right) \Delta B_{i,t} \right) \right] + o(\theta) \\ &= \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\sum_k (N_{i,k,t-1} - N_{i,k,t})\Delta P_{k,t} - B_{i,t}\Delta Q_t + \sum_k N_{i,k,t-1}\Delta D_{k,t} + \Delta Y_{i,t} \right) \right] + o(\theta), \end{aligned} \quad (44)$$

where the second equality comes from the properties of the stochastic discount factor. Now, remember that the left-hand-side is pinned down by (40'). Hence, any trading strategy that implements the agent's optimal consumption plan (42) will return the same welfare gain, when used as an input to our sufficient statistic formula. Notice that, if an empirical researcher was to look only at how the realization of s_0 affects capital income $\Delta D_{k,t}$ and labor income $\Delta Y_{i,t}$, they would potentially miss an important component of welfare gains: the one that operates via changes in asset prices $\Delta P_{k,t}$ and interest rates ΔQ_t .

Ability to trade financial derivatives. We started this section by specifying an exogenous set of financial assets available to trade, and then assuming that this set was big enough to make markets complete. We now ask: what if agents can directly purchase financial derivatives? In this case, we now show that our sufficient statistic formula (40) still holds, but that one needs to take into account the effect of the shock on the synthetic dividends associated with these new derivatives. Put differently, even if the original shock corresponds to a pure change in the price of fundamental assets (i.e., no difference in dividends $\Delta D_{k,t}$ between the events $s_0 = \hat{s}$ and $s_0 = \bar{s}$), it may generate changes in the synthetic dividends exchanged by agents (as agents may trade derivatives with cash flows dependent on the realization of s_0).

One important example is the case in which the individual trades the entire set of Arrow-Debreu securities at time $t = -1$. That is, the agent decides at time $t = -1$ to buy a financial derivative that gives a flow of synthetic dividends $D_{i,t}(s_0, \dots, s_t) = C_{i,t}(s_0, \dots, s_t) - Y_{i,t}(s_0, \dots, s_t)$ at each time $t \geq 0$,

where $C_{i,t}$ is given by (36). Note that the fair value of this derivative exactly corresponds to the amount of financial wealth that the individual starts with, $W_{i,-1}$, given in (41). Applying our sufficient statistic formula (40) to this trading strategy yields:

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\underbrace{(\Delta C_{i,t} - \Delta Y_{i,t})}_{\text{effect of initial shock on dividends of derivative bought at } t=-1} + \underbrace{\Delta Y_{i,t}}_{\text{effect of initial shock on future labor income}} \right) \right] + o(\theta).$$

This formula, which captures welfare gains through the present value of changes in (synthetic) dividends of these derivatives, correctly recovers the welfare gain given in Formula (40'). More generally, any trading strategy used to sustain the optimal path of consumption pinned down by (36) will end up giving the same welfare gains through our sufficient statistic formula — the only difference is that, when agents trade financial derivatives, it is important to include changes in synthetic dividends to capture the full welfare gains. In practice, when implementing our sufficient statistic approach to the data, we do ignore welfare gains through changes in derivative cash flows as their importance in household portfolios is negligible.⁷⁸

Market clearing. For now, we have focused on the partial equilibrium setup of an individual i facing an exogenous set of financial assets that is big enough that markets are complete. We now use these results to study the welfare gain of a shock in a general equilibrium economy, in which markets are complete. Note that this is a strong assumption in our context, as, in particular, newborns cannot perfectly insure themselves with respect to the state of the economy they are born in.

More precisely, we now consider a general equilibrium endowment economy with I individuals, and an exogenous path for individual labor income $Y_{i,t}$ and dividend income $\{D_{k,t}\}_k$. For simplicity, we normalize the total number of shares of each asset to one. Equilibrium asset prices are such that total consumption across individuals equals total income:

$$\sum_{i=1}^I C_{i,t} = \sum_{i=1}^I Y_{i,t} + \sum_{k=1}^K D_{k,t}.$$

Differentiating with respect to the realization of $s_0 = \hat{s}$ relative to $s_0 = \bar{s}$, and combining with Equation (43) gives

$$\frac{\Delta \Lambda_t}{\Lambda_t} = - \frac{1}{\sum_{i=1}^I \frac{C_{i,t}}{\sum_{j=1}^I C_{j,t}} \frac{1}{\gamma(C_{i,t})}} \frac{\Delta \left(\sum_{i=1}^I Y_{i,t} + \sum_{k=1}^K D_{k,t} \right)}{\sum_{j=1}^I Y_{j,t} + \sum_{k=1}^K D_{k,t}} + o(\theta).$$

This equation expresses the (equilibrium) effect of the realization of $s_0 = \hat{s}$, relative to $s_0 = \bar{s}$, on the stochastic discount factor in terms of its (exogenous) effect on aggregate labor and dividend income, divided by the consumption-weighted harmonic mean of the relative risk aversion of individuals in the economy. Combining this formula with (40') gives the welfare gain from the realization of $s_0 = \hat{s}$, relative to $s_0 = \bar{s}$, for each individual:

$$\frac{\Delta V_{i,0}}{U'(C_{i,0})} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \frac{\Lambda_t}{\Lambda_0} \left(\frac{C_{i,t}}{\gamma(C_{i,t})} \right) \Delta \left(\sum_{i=1}^I Y_{i,t} + \sum_{k=1}^K D_{k,t} \right) \right] + o(\theta). \quad (40'')$$

⁷⁸According to data from Statistical Norway, the value of financial derivatives in 2024q1 represents less than 0.0004% of assets in the household sector.

This expression recovers the general idea that, with complete markets, the welfare of agents is only exposed to shocks that affect aggregate income.

To conclude this section, note that we have derived three distinct expressions for the welfare gain resulting from a small shock: (40), (40'), and (40''). The first expression (40), which serves as our sufficient statistic, is the most appropriate formula to take to the data, as it holds regardless of whether markets are complete; in contrast, (40') requires that the agent can trade a set of financial assets large enough to span all contingencies, while (40'') requires that *all* agents in the economy can do so.⁷⁹

A.3 Welfare gains in general equilibrium

We now discuss how to apply our sufficient statistic in a general equilibrium framework. Denote a set of fundamental parameters in the economy by a vector $\{z_t\}_{t=0}^{\infty}$, which could reflect technological productivity, policy, and so on. The thought experiment that we consider in this context is to perturb the path of z_t by $\theta \Delta z_t$, where θ scales the deviation of the shock, and compute its welfare effect, both through its effect on asset prices and on income (i.e., dividend and labor income): $dP_{k,t} = \partial_{\theta} P_{k,t} d\theta$, $dQ_t = \partial_{\theta} Q_t d\theta$, $dD_{k,t} = \partial_{\theta} D_{k,t} d\theta$, $dY_{i,t} = \partial_{\theta} Y_{i,t} d\theta$. An application of Proposition 5 then yields

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\underbrace{\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t}_{\text{Effect of } z_t \text{ through asset prices}} + \underbrace{\sum_{k=1}^K N_{i,k,t-1} D_{k,t} + dY_{i,t}}_{\text{Effect of } z_t \text{ through income}} \right). \quad (45)$$

This equation can be used to quantify the redistributive effect of a fundamental shock to the economy, which impacts both income and asset prices.

We now give a concrete example of asset-price redistribution in general equilibrium. We consider an environment with two assets: an asset in fixed supply (i.e., claim to land) and a reproducible asset (i.e., productive capital). We analyze the redistributive effects associated with perturbation of cash flows and productivity, disentangling the effects of asset prices versus income. We start with an extension of the baseline model with arbitrary household heterogeneity, and then specialize to a two-period OLG model with closed-form solutions.

A.3.1 General equilibrium model with production

We now consider a general-equilibrium extension of the baseline model with production motivated by the verbal discussion in Krugman (2021).⁸⁰ There are two assets: elastically supplied productive capital and an inelastically supplied long-lived asset (land). Households can convert the final good into physical capital K_{t+1} one-for-one. Capital depreciates at rate $0 \leq \delta \leq 1$. There is a representative firm that operates an AK technology $Y_t = \tilde{A}_t K_t$. Denoting by $A_t \equiv \tilde{A}_t + 1 - \delta$ the total return to capital, the equilibrium rental rate of capital at time t must equal A_t . The other asset (land) can be traded without adjustment costs. As in the baseline model, N denotes the number of shares, D the cash flows per share, and P the price per share. Shares are in fixed supply of shares normalized to one. The cash flows and productivity $\{D_t, A_t\}_{t=0}^{\infty}$ are exogenous, and land prices $\{P_t\}_{t=0}^{\infty}$ are endogenous.

⁷⁹Of course, the advantage of (40') and (40'') is that they give sharper predictions on welfare gains, as they express them in terms of the (exogenous) shocks in aggregate endowment and household preferences rather than their (endogenous) portfolio decision.

⁸⁰We thank the editor Andy Atkeson for suggesting the material in this section.

The household problem is

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t+1}, K_{i,t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

$$\text{s.t. } C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + K_{i,t+1} = Y_{i,t} + N_{i,t}D_t + A_t K_{i,t}.$$

The equilibrium price P_t is the solution to a system of two equations:

$$\frac{P_{t+1} + D_{t+1}}{P_t} = A_t, \quad \sum_{i=1}^I N_{i,t} = 1.$$

First, the land price P_t must make households indifferent between investing in land or capital. Second, the sum of land holdings must equal its fixed supply \bar{N} . Since, in equilibrium, households are indifferent between the two assets, and therefore the equilibrium portfolios are indeterminate, we assume that they all hold the same portfolio shares: $\frac{K_{i,t}}{K_{i,t} + N_{i,t}P_t} = \frac{K_t}{K_t + P_t}$. Solving forward and assuming no bubbles, we obtain a closed-form expression for land prices:

$$P_0 = \sum_{t=0}^{\infty} A_{0 \rightarrow t}^{-1} D_t,$$

where $A_{0 \rightarrow t} = A_1 \cdot A_2 \cdots A_t$ is the cumulative gross rental rate of capital. No arbitrage implies that the equilibrium discount rate $R_{0 \rightarrow t}^{-1} = A_{0 \rightarrow t}^{-1}$ is simply pinned down only by productivity.

Consider an arbitrary perturbation of productivity $\{dA_t\}_{t \geq 0}$. Even though the perturbation does not affect the cash flows generated by land, it does move the value of land due to its effect on equilibrium discount rates:

$$dP_0 = \sum_{t=0}^{\infty} d(A_{0 \rightarrow t}^{-1}) D_t.$$

A negative productivity shock therefore *increases* the value of land. A direct application of (45) yields

$$dV_{i,0}/U'(C_{i,0}) = \underbrace{\sum_{t=0}^{\infty} A_{0 \rightarrow t}^{-1} (N_{i,t-1} - N_{i,t}) dP_t}_{\text{Welfare gain through asset prices}} + \underbrace{\sum_{t=0}^{\infty} A_{0 \rightarrow t}^{-1} K_{i,t} dA_t}_{\text{Welfare gain through income}}.$$

The first term ‘‘Welfare gain through asset prices’’ sums up to zero across households because financial assets are in fixed supply. But the second term, which accounts for the increased capital income, does not sum to zero. The idea is that productivity shocks not only lead to *aggregate* changes in consumption, they also lead to redistribution between buyers and sellers of land, due to their effect on discount rates.

The cross-sectional correlation between the two components of welfare change will ultimately depend on the details of the model (i.e., the economic forces that determine household-level financial transactions). We now specialize to a simple household-side of the model, where we can sign the total welfare gains across old and young, both for productivity shocks and cash flow shocks.

A.3.2 An analytically tractable special case: two-period OLG

(Unless stated otherwise, we use the same notation as in the model above.) Consider an economy where at each year $t \geq -1$, a new cohort is born. Each cohort lives for two periods and has a subjective discount factor $\beta \in (0, 1)$. Households in cohort t are endowed with labor income Y_t when young and zero when old. The cohort born at $t = -1$ is endowed with all of the land ($N_{-1} = 1$). As before, denote the (ex-dividend) price of the asset at time t as P_t , and the one-period holding return on the asset by

$R_{t+1} = (D_{t+1} + P_{t+1})/P_t$, where D_t is the dividend from land at time t . Denote by N_t the share of land owned by cohort t at the end of period t .

Setup and equilibrium. The problem of the young in period $t \geq 0$ is

$$\begin{aligned} V_t &= \max_{C_t, C'_t, N_t, K_{t+1}} \log(C_t) + \beta \log(C'_t), \\ \text{s.t. } & C_t + N_t P_t + K_{t+1} = Y_t \\ & C'_t = N_t(D_{t+1} + P_{t+1}) + A_{t+1} K_{t+1}. \end{aligned}$$

where C_t and C'_t denote consumption when young and old, respectively.

The equilibrium is characterized by individual optimization and market clearing,⁸¹ with a familiar solution given by

$$\begin{aligned} C_t &= \frac{1}{1+\beta} Y_t, \quad C'_t = D_{t+1} + P_{t+1} + A_{t+1} K_{t+1} \\ K_{t+1} &= \frac{\beta}{1+\beta} Y_t - P_t, \quad P_t = \sum_{s \geq t+1} A_{t \rightarrow t+s}^{-1} D_s. \end{aligned}$$

In words, the young save a fraction $\frac{1}{1+\beta}$ of their income, purchase all of the land from the old, and invest the rest in physical capital. No arbitrage implies that the value of land is pinned down by future cash flows discounted using the return on capital.

Comparative static. Two things matter for equilibrium land prices: future productivity and future cash flows. We now do a comparative static on A_1 and D_1 at time $t = 0$ (i.e., transitory MIT shock). We call the generations born at $t = -1$ and $t = 0$ respectively “the old” and “the young”. The old’s value function is simply $V_{-1} = \beta \log C'_{-1}$. Applying (45), which decomposes total welfare gains into the contribution of asset prices and income, we have:

$$\begin{array}{rcl} \text{(Welfare gain of the old)} & dV'_{-1}/U'(C'_{-1}) & = \quad dP_0 + \quad 0 \\ \text{(Welfare gain of the young)} & dV_0/U'(C_0) & = \quad -dP_0 + A_1^{-1} \left(dD_1 + K_1 dA_1 \right) \\ \hline \text{(Sum of welfare gains)} & & = \quad 0 + A_1^{-1} \left(dD_1 + K_1 dA_1 \right) \end{array}$$

As in Proposition 5, welfare gains are the sum of two terms. The first term is the contribution of changes in asset prices, which has opposite sign for the young (who buy the asset) and the old (who sell the asset). The second term is the contribution of (capital) income.

Using the fact that $dP_t = 0$ for all $t \geq 1$, we have a closed form solution for the land price deviation: $dP_0 = (-P_0 dA_1 + dD_1)$. This allows us to immediately sign the total welfare gains in the model. Table A1 reports the results. For the productivity shock experiment, we consider $dA_1 < 0$ and, for the cash flow shock, we consider $dD_1 > 0$. In both cases, land prices increase (i.e., $dP_0 > 0$), which implies

⁸¹The equilibrium at time t is characterized by three equations in three unknowns (C_t, K_{t+1}, P_t):

$$\begin{aligned} \frac{1}{C_t} &= \beta \frac{R_{t+1}}{D_{t+1} + P_{t+1} + A_{t+1} K_{t+1}}, \\ \frac{1}{C_t} &= \beta \frac{A_{t+1}}{D_{t+1} + P_{t+1} + A_{t+1} K_{t+1}}, \\ C_t &= Y_t - P_t - K_{t+1}. \end{aligned}$$

In addition, we impose a restriction on the primitives $\{A_t, D_t\}_{t=0}^{\infty}$ to ensure no bubbles: $\lim_{T \rightarrow \infty} \frac{D_t}{P_t} = +\infty$ (see Hirano and Toda, *accepted*). As before, we assume that all households within a cohort hold the same portfolio.

Table A1: Welfare gain decomposition in general equilibrium

	Welfare gains		
	Through prices	Through income	Total
Productivity shock ($dA < 0$)			
Old	+	0	+
Young	-	-	-
Aggregate	0	-	-
Cash flow shock ($dD > 0$)			
Old	+	0	+
Young	-	+	0
Aggregate	0	+	+

Notes. "Total" is the sum of welfare effect within group; "Aggregate" is the sum of a welfare effect across groups; "+" means positive, "-" means negative.

a redistribution from young to old. However, the contribution of income depends on the shock. In the case of the productivity shock, income declines for the young. In the case of the cash flow shock, income increases for the young. Summing up, the total welfare gain associated with these shocks is always positive for the old, but can be negative or zero for the young.

Stochastic shock. So far, we have considered a comparative static experiment. Following the logic from Appendix A.2.2, we can instead consider the welfare effect of a stochastic shock between two different states.

Suppose that the economy is deterministic as before, except that, at time $t = 0$, the state $s \in \{\bar{s}, \hat{s}\}$ is drawn with probability $(1 - \pi, \pi)$, which determines the path of exogenous variables $\{A_t(s), D_t(s)\}_{t=0}^{\infty}$. We consider the same experiment as in the deterministic case (i.e., one-time shock). Denoting the size of the shocks to productivity and cash flows by $\Delta A_t \equiv A_t(\hat{s}) - A_t(\bar{s})$ and $\Delta D_t \equiv D_t(\hat{s}) - D_t(\bar{s})$, we consider the temporary shock.

$$(\Delta A_t, \Delta D_t) = \begin{cases} (\theta \Delta A, \theta \Delta D), & \text{if } t = 1 \\ 0, & \text{if } t \neq 1 \end{cases}$$

where the parameter θ indexes the size of the shock. Similarly, define the difference in welfare between the two states by $\Delta V_t = V_t(\hat{s}) - V_t(\bar{s})$.

There are only two possible states of the world and two assets. As a result, markets are complete from the point of view of agents alive at time -1 . However, because this cohort cannot trade securities with unborn agents (the young that will arrive at time $t = -1$), they cannot share risk across generations. Hence, the exposure of each type of agents remains the same as for the case of a MIT shock and the welfare gains associated with state $s = \hat{s}$ occurring, relative to $s = \bar{s}$ are given by:

$$\begin{array}{l} \text{(Welfare gain of the old)} \quad \Delta V_{-1}/U'(C'_{-1}) \approx \Delta P_0 + 0 \\ \text{(Welfare gain of the young)} \quad \Delta V_0/U'(C_0) \approx -\Delta P_0 + A_1^{-1}(\Delta D_1 + K_1 \Delta A_1) \\ \hline \text{(Sum of welfare gains)} \quad \approx 0 + A_1^{-1}(\Delta D_1 + K_1 \Delta A_1) \end{array}$$

The welfare gains formula are exactly the same as the MIT shock, except that the formula is approximate, due to the non-infinitesimal shocks. Therefore, the comparative statics from Table A1 remain correct.

Stochastic shocks with ex-ante risk-sharing. Suppose that the environment is exactly as before, except that we now allow young agents, who will only arrive in the economy at time $t = 0$, to trade with other agents at time $t = -1$; that is, before they are “born” — in other words, we are breaking the typical OLG incompleteness. Because there are two possible states and two assets (land and capital), markets are complete and so, applying the general results in Appendix A.2.2:

$$\frac{\Delta C_0}{C_0} = \frac{\Delta C'_{-1}}{C'_{-1}} = 0.$$

This equation says that the realization of the state $s \in \{\bar{s}, \hat{s}\}$ does not affect the consumption of either cohort at $t = 0$. This equation is derived as follows: complete markets imply that agents equate the relative growth of their consumption among each other (see Equation 43). Because aggregate income at $t = 0$ is not affected by the realization of the shock, market clearing then implies that the relative growth of consumption is zero for each agent.

How can agents implement this allocation? One way to do so is for the young to buy all the land from the old at $t = -1$, while selling them capital (or issuing riskless bonds to them) to finance their purchases. In this case, no land sale would be observed at $t = 0$ (i.e., after the realization of the shock) and so applying our sufficient statistic formula would give:

$$\begin{array}{rcl} \text{(Welfare gain of the old)} & \Delta V_{-1}/U'(C'_{-1}) \approx & 0 \times \Delta P_0 + 0 \\ \text{(Welfare gain of the young)} & \Delta V_0/U'(C_0) \approx & -0 \times \Delta P_0 + A_1^{-1}(\Delta D_1 + K_1 \Delta A_1) \\ \hline \text{(Sum of welfare gains)} & \approx & 0 + A_1^{-1}(\Delta D_1 + K_1 \Delta A_1) \end{array}$$

While this ex-ante trading stage affects trading patterns and the welfare gains of an aggregate shock, our sufficient statistic formula, which pins down welfare gains *given* trading patterns, remains valid (see the discussion in Appendix A.2.2). Finally, note that, another way to implement this allocation would be for agents to use future contracts instead of buying land early; that is, to use derivatives allowing them to “lock-in” the price of the transaction ex-ante. In this case too, our sufficient statistic formula would remain correct too, since there would be no deviation in the transaction price of the asset exchanged by individuals.⁸²

One key take away from this exercise is that, faced with a shock that affects asset prices at some time $t = 0$ (but that *do not* impact aggregate income at time $t = 0$), optimizing agents would prefer to trade assets *before* the shock takes place to avoid any type of redistributive effect ex-post. Of course, it is not always possible to do so, as, in reality, existing generations cannot contract with future (unborn) generations.

A.4 Extensions

A.4.1 Individual preferences

Assets in the utility function. We now examine the welfare effect of asset price deviations in the presence of assets in the utility function. Our main finding is that whether or not agents directly derive utility from asset ownership does not matter for our sufficient statistic: it is only when agents directly derive utility from the price of these assets that the welfare gains formula needs to be adjusted.

⁸²In the context of changes in interest rates, this could also be implemented by interest rate swaps.

For simplicity, we consider a two-asset version of the baseline model:

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + B_{i,t-1}.$$

The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = U(C_{i,t})$, coincides with the baseline model (i.e., assets ownership does not affect flow utility directly). The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = U(C_{i,t}, N_{i,t}, B_{i,t})$ means that individuals value the quantity of assets that they own directly (e.g. agents value owning a house relative to renting it). The case $U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = N_{i,t}P_t + B_{i,t}Q_t$ means that individuals value the market value of their wealth directly.

Proposition 7. *In the presence of assets in the utility function, the welfare gain implied by a price deviation $\{dQ_t, dP_t\}_{t=0}^{\infty}$ is:*

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) \\ &\quad + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\frac{\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} dP_t + \frac{\partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} dQ_t \right). \end{aligned}$$

Proof of Proposition 7. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) \\ &\quad + \sum_{t=0}^{\infty} \lambda_{i,t} (Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})). \end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} (- (N_{i,t} - N_{i,t-1}) dP_t - B_{i,t} dQ_t) + \sum_{t=0}^{\infty} \beta^t (\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) dP_t + \partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) dQ_t) \\ &= \partial_C U(C_{i,0}, N_{i,0}, P_0, B_{i,0}, Q_0) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) \\ &\quad + \partial_C U(C_{i,0}, N_{i,0}, P_0, B_{i,0}, Q_0) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\frac{\partial_P U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} dP_t + \frac{\partial_Q U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)}{\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t)} dQ_t \right), \end{aligned}$$

where the last line uses the FOC with respect to $B_{i,t}$, $\lambda_{i,t}Q_t = \lambda_{i,t+1}$, as well as the FOC with respect to $C_{i,t}$, $\partial_C U(C_{i,t}, N_{i,t}, P_t, B_{i,t}, Q_t) = \lambda_{i,t}$. \square

Endogenous labor supply. We now examine the welfare effect of price deviations in the case where labor supply is optimally chosen by the agent. Our main finding is that this extension does not affect the welfare gains of an infinitesimal deviation in asset prices, as a result of the envelop theorem.

Formally, we consider the following two-asset model. Agents maximize utility of consumption and

leisure

$$V_{i,0} = \max_{\{C_{i,t}, L_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1 - L_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = w_{i,t}L_{i,t} + B_{i,t-1}.$$

Note that, relative to the baseline model, labor income takes the form $w_{i,t}L_{i,t}$ where $w_{i,t}$ denotes the wage and $L_{i,t}$ denotes the quantity of labor, which is optimally chosen by the agent to maximize utility. We assume that $U(\cdot, \cdot)$ is increasing and concave with respect to each of its arguments.

Proposition 8. *The welfare gain implied by a price deviation $\{dQ_t, dP_t\}_{t=0}^{\infty}$ remains the same in the presence of endogenous labor supply; that is, the welfare gain is:*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t).$$

Proof of Proposition 8. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^{\infty} \beta^t U(C_{i,t}, 1 - L_{i,t}) \\ &+ \sum_{t=0}^{\infty} \lambda_{i,t} (w_t L_{i,t} + N_{i,t} D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1}) P_t - B_{i,t} Q_t - \chi(N_{i,t} - N_{i,t-1})). \end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} (-(N_{i,t} - N_{i,t-1}) dP_t - B_{i,t} dQ_t) \\ &= \partial_{C_{i,0}} U(C_{i,0}, 1 - L_{i,0}) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t), \end{aligned}$$

where the last line uses the FOC with respect to $B_{i,t}$, $\lambda_{i,t} Q_t = \lambda_{i,t+1}$ and the FOC with respect to $C_{i,0}$, $\partial_{C_{i,0}} U(C_{i,0}, 1 - L_{i,0}) = \lambda_{i,0}$. \square

A.4.2 Finite lives and bequests

We now examine the welfare effect of asset price deviations when individuals have finite lives. For pedagogical reasons, we first consider the case in which individuals do not care about future generations and hence die with exactly zero assets. We then move on to the more realistic case in which individuals care about future generations, distinguishing between the altruistic and the “warm-glow” bequest motives. Finally, we discuss the extent to which our notion of welfare gains still captures the present value of deviation of consumption with inter-generational linkages.

Finite lives. We consider an individual who will die with certainty at some horizon T with no offspring and hence no reason to leave bequests. For simplicity, we restrict ourselves to a two-asset version

of the baseline model. More precisely, the individual solves the following optimization problem:

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^T \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t = 0, 1, \dots, T$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1},$$

and with terminal holdings $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$. Note that these constraints will bind at the optimum, that is the individual will optimally die with zero assets, i.e. she will sell off her assets as she approaches the end of life.⁸³ These sales then show up in our welfare gains formula as the next proposition shows:

Proposition 9. *For an individual with a finite life of length T and no offspring, the welfare gain of an infinitesimal deviation in asset prices is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t). \quad (46)$$

This is exactly the same as formula (10) for an infinitely-lived individual, except for the T in the summation. Intuitively, the individual optimally sells off all of her assets before she dies. When asset valuations rise, this generates a welfare gain. While finite lives result in a different time path for optimal asset transactions, the way these asset transactions show up in our welfare gains formula is exactly the same as in the case with infinitely-lived individuals.

Proof of Proposition 9. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^T \beta^t U(C_{i,t}) \\ &+ \sum_{t=0}^T \lambda_{i,t} (Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})) \\ &+ \mu_{i,N}N_{i,T} + \mu_{i,B}B_{i,T}, \end{aligned}$$

where $\mu_{i,N}$ and $\mu_{i,B}$ are the Lagrange multipliers on the terminal conditions $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$. Totally differentiating the welfare function using the envelope theorem and following the same steps as

⁸³Alternatively, we could impose the weaker terminal condition that terminal wealth is non-negative; that is, $N_{i,T}P_T + B_{i,T}Q_T \geq 0$. In this alternative formulation, the individual is allowed to die with debt $B_{i,T}Q_T < 0$ in which case an asset sale $N_{i,T}P_T$ would happen right after the individual's death so as to pay off this debt (e.g. conducted by the bank or executor of the individual's will). Differentiating the corresponding Lagrangian

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^T \beta^t U(C_{i,t}) + \sum_{t=0}^T \lambda_{i,t} (Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})) \\ &+ \mu_i(N_{i,T}P_T + B_{i,T}Q_T) \end{aligned}$$

and using the first-order conditions for $N_{i,T}$ and $B_{i,T}$, $\lambda_{i,T}P_T = \mu_i P_T$ and $\lambda_{i,T}Q_T = \mu_i Q_T$, would lead to the following formula for welfare gains

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) + R_{0 \rightarrow T}^{-1} (N_{i,T} dP_T + B_{i,T} dQ_T).$$

This formula is the same as (46) in Proposition 9, although one needs to take into account the additional asset sales at time T right after the person's death. The intuition is that there is no economic difference whether the person makes these sales before she dies (which is the formulation in the main text) or the executor of the will doing it (which is the formulation in this footnote).

in the proof of Proposition 1

$$\begin{aligned}
dV_{i,0} &= \sum_{t=0}^T \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^T \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\
&= \sum_{t=0}^T \lambda_{i,t} (-(N_{i,t} - N_{i,t-1}) dP_t - B_{i,t} dQ_t) \\
&= U'(C_{i,0}) \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t),
\end{aligned}$$

where the second equality uses that the terms $\mu_{i,N}N_{i,T}$ and $\mu_{i,B}B_{i,T}$ in the Lagrangian do not depend on asset prices $\{P_t, Q_t\}_{t=0}^T$. \square

Finite lives with altruistic preferences. We now consider an individual with altruistic preferences. More precisely, we consider the following value function for individual i

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}, G_{N,i,t}, G_{B,i,t}\}} \sum_{t=0}^T \beta^t U(C_{i,t}) + \delta V_{j,0}, \quad (47)$$

which captures the fact that the individual cares about his or her consumption as well as the one of some agent j , which could represent an heir or a parent. We allow individual i to transfer assets to individual j at any point in time and so the sequence of budget constraint becomes

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t, \quad (48)$$

where $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ denote a path of transfers (through bequests or inter vivos transfers) from individual i to individual j . As in the finite live case, we assume that terminal holdings satisfy $N_{i,T} \geq 0$ and $B_{i,T} \geq 0$.

Consider a small deviation in the path of asset prices. Similarly to the baseline model, we define the welfare gain of the deviation as the amount of money, received at $t = 0$, that would generate an equivalent change in the welfare of individual i (i.e., equivalent variation). Because of altruistic preferences, we need to specify what happens to agent j across these two counterfactuals. Specifically, we assume that the value function of agent j is held constant across these two counterfactuals. This can be seen as the equivalent variation of individual i assuming that the deviation in asset prices only affects him or her, or, alternatively, as the equivalent variation of individual i assuming that agent j already receives his or her equivalent variation.

Proposition 10. *With finite lives and altruistic preferences, the welfare gain of an infinitesimal deviation in asset prices is:*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t} - G_{N,i,t}) dP_t - (B_{i,t} - G_{B,i,t}) dQ_t).$$

Proof of Proposition 10. The Lagrangian associated with the optimization problem of individual i is

$$\begin{aligned}
\mathcal{L}_i &= \sum_{t=0}^T \beta^t U(C_{i,t}) + \delta V_{j,0} \\
&+ \sum_{t=0}^T \lambda_{i,t} (N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1})).
\end{aligned}$$

Totally differentiating the welfare function using the envelope theorem gives

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^T \lambda_{i,t} (- (N_{i,t} - N_{i,t-1} + G_{N,i,t}) dP_t - (B_{i,t} + G_{B,i,t}) dQ_t), \end{aligned}$$

where the second line uses the fact that, by in our thought experiment, we set the partial derivative of the value function of j with respect to asset prices to zero. Combining the FOCs for the liquid asset and for consumption at time $t = 0$ gives $\lambda_{i,t} = U'(C_{i,0})R_{0 \rightarrow t}^{-1}$. Plugging into the previous formula $dV_{i,0}$ gives the result. \square

Finite lives with warm glow preferences. We now discuss the alternative case where individuals have warm glow preferences; that is, where individuals care about their own consumption as well as the value of their bequests. More precisely, we assume that the individual value function is

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}, G_{N,i,t}, G_{B,i,t}\}} \sum_{t=0}^T \beta^t (U(C_{i,t}) + \mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_t)), \quad (49)$$

where $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ denote the path of asset transfers from individual i to other agents. Finally, we assume that the set of budget constraints is the same as the one with altruistic preferences (48). As in the baseline model, we define the welfare gain as the amount of money that would generate an equivalent change in individual welfare.

While this type of preferences is often used in the macro-literature (e.g. [De Nardi, 2004](#)), note that it is less adapted to welfare assessments than the altruistic model. To be more specific, these preferences imply that parents strictly value a deviation that allows them to increase the bequest to their children by one dollar, even if the deviation simultaneously costs their children $\$X$ dollars, where X is an arbitrary number.

Proposition 11. *With finite lives and warm glow preferences, the welfare gain of an infinitesimal deviation in asset prices is:*

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t} - G_{N,i,t}) dP_t - (B_{i,t} + G_{B,i,t}) dQ_t \right. \\ &\quad \left. + \frac{\frac{\partial \mathcal{U}}{\partial P_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dP_t + \frac{\frac{\partial \mathcal{U}}{\partial Q_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dQ_t \right). \end{aligned}$$

Relative to the sufficient statistic obtained in the baseline model, there is an additional term that represents the effect of warm glow preferences. This additional term can be interpreted as the additional amount that the agent could consume by decreasing the quantity of assets transferred to hold constant the utility value of bequests in every period. One important special case is when individuals care about the total market value of assets transferred to their heirs, $\mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_t) = \mathcal{U}(G_{i,N,t}P_t + G_{i,B,t}Q_t)$. In this case, the additional term due to warm glow preferences simplifies to $\sum_{t=0}^T R_{0 \rightarrow T}^{-1} (G_{i,N,t} dP_t + G_{i,B,t} dQ_t)$, which corresponds to the present value of the effect the deviation of asset prices on transferred assets. The key take away of this extension is the same as in [Appendix A.4.1](#) (which examines the case of asset-in-utility function): the welfare formula only gains an additional term when agents care about the value of asset prices per se.

Proof of Proposition 11. The Lagrangian associated with the optimization problem of individual i is

$$\begin{aligned}\mathcal{L}_i &= \sum_{t=0}^T \beta^t (U(C_{i,t}) + \mathcal{U}(G_{i,N,t}, P_t, G_{i,B,t}, Q_t)) \\ &+ \sum_{t=0}^T \lambda_{i,t} (N_{i,t-1}D_t + B_{i,t-1} + Y_{i,t} - G_{N,i,t}P_t - G_{B,i,t}Q_t - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - B_{i,t}Q_t - \chi(N_{i,t} - N_{i,t-1}))\end{aligned}$$

The envelope theorem gives

$$\begin{aligned}dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \beta^t \left(\frac{\partial \mathcal{U}}{\partial P_t} + \frac{\partial \mathcal{U}}{\partial Q_t} \right) + \sum_{t=0}^T \lambda_{i,t} \left(-(N_{i,t} - N_{i,t-1} + G_{N,i,t}) dP_t - (B_{i,t} + G_{i,t}) dQ_t \right).\end{aligned}$$

Using the FOC with respect to inter vivos transfers of the liquid asset at time t gives $\beta^t \frac{\partial \mathcal{U}}{\partial G_{i,B,t}} = \lambda_{i,t} Q_t$. Combining with the previous equation gives

$$dV_{i,0} = \sum_{t=0}^T \lambda_{i,t} \left(-(N_{i,t} - N_{i,t-1} + G_{N,i,t}) dP_t - (B_{i,t} + G_{i,t}) dQ_t + \frac{\frac{\partial \mathcal{U}}{\partial P_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dP_t + \frac{\frac{\partial \mathcal{U}}{\partial Q_t}}{\frac{1}{Q_t} \frac{\partial \mathcal{U}}{\partial G_{i,B,t}}} dQ_t \right).$$

Combining the FOCs for the liquid asset and for consumption at time $t = 0$ gives $\lambda_{i,t} = U'(C_{i,0}) R_{0 \rightarrow t}^{-1}$. Plugging into the previous formula $dV_{i,0}$ gives the result. \square

PV of consumption response with inter vivos transfers. In the baseline infinite-horizon model, our sufficient statistic formula for welfare gains (i.e., the present value of asset transactions interacted by deviations in asset prices) is equal to the present value of the actual response in individual consumption to the deviation in asset prices.⁸⁴ This equivalence no longer applies in a model with inter-generational links, since agents can use their trading profits to either increase consumption or increase transfers to other agents. We now show that, with inter vivos transfers, our sufficient statistic formula corresponds to the present value of the actual response in individual consumption *plus* in individual net transfers.

More precisely, we consider a model where individuals have *either* altruism or warm glow preferences (i.e. with a value function given by Equation 47 or Equation 49). Agents can give or receive assets over time and we denote $\{G_{N,i,t}\}_{t=0}^{\infty}$ and $\{G_{B,i,t}\}_{t=0}^{\infty}$ the path of net inter vivos gifts from individual i . We consider a deviation in the path of asset prices $\{dP_t, dQ_t\}_{t=0}^{\infty}$. Differentiating the individual budget constraint (48) gives

$$\begin{aligned}dC_{i,t} + P_t(dN_{i,t} - dN_{i,t-1}) + (N_{i,t} - N_{i,t-1}) dP_t + Q_t dB_{i,t} + B_{i,t} dQ_t + \chi'(N_{i,t} - N_{i,t-1})(dN_{i,t} - dN_{i,t-1}) \\ = D_t dN_{i,t-1} + dB_{i,t-1} - P_t dG_{N,i,t} - G_{N,i,t} dP_t - Q_t dG_{B,i,t} - G_{B,i,t} dQ_t,\end{aligned}$$

⁸⁴See Proposition 17 for a formal statement.

Taking the present value, aggregating across time periods, and re-arranging gives

$$\begin{aligned} \sum_0^T R_{0 \rightarrow t}^{-1} (dC_{i,t} + P_t dG_{N,i,t} + Q_t dG_{B,i,t}) &= \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t} - G_{N,i,t}) dP_t - (B_{i,t} + G_{B,i,t}) dQ_t) \\ &+ \sum_{t=0}^T R_{0 \rightarrow t}^{-1} (D_t dN_{i,t-1} - P_t (dN_{i,t} - dN_{i,t-1}) - \chi'(N_{i,t} - N_{i,t-1})(dN_{i,t} - dN_{i,t-1})) \\ &+ \sum_{t=0}^T R_{0 \rightarrow t}^{-1} (dB_{i,t-1} - Q_t dB_{i,t}) \end{aligned}$$

Rearranging the last two terms:

$$\begin{aligned} \sum_0^T R_{0 \rightarrow t}^{-1} (dC_{i,t} + P_t dG_{N,i,t} + Q_t dG_{B,i,t}) &= \sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t} - G_{N,i,t}) dP_t - (B_{i,t} + G_{B,i,t}) dQ_t) \\ &+ \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left(R_{t+1}^{-1} (D_{t+1} + P_{t+1} + \chi'(N_{i,t+1} - N_{i,t})) - \chi'(N_{i,t} - N_{i,t-1}) - P_t \right) dN_{i,t} \\ &+ \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left(R_{t+1}^{-1} - Q_t \right) dB_{i,t} \end{aligned}$$

The FOCs for asset holdings $N_{i,t}$ implies that the second line is zero while the definition of R_{t+1} implies that the third line is zero, and so we get:

$$\underbrace{\sum_0^T R_{0 \rightarrow T}^{-1} (dC_t + P_t dG_{N,i,t} + Q_t dG_{B,i,t})}_{\text{PV of response of consumption + net transfers}} = \underbrace{\sum_{t=0}^T R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t} - G_{N,i,t}) dP_t - (B_{i,t} + G_{B,i,t}) dQ_t)}_{\text{PV of trading profits (sufficient statistic)}}.$$

This equations shows that, in the presence of inter vivos transfers, our baseline sufficient statistic formula equals the present value of the response of consumption and net transfers to the deviation in asset prices. Intuitively, this equation says that, in a present-value term, any increase in trading profits due to the deviation in asset prices must be consumed or transferred to other agents.

One implication is that, in the presence of intergenerational transfers, our baseline sufficient statistic formula captures the present value of the effect of asset prices on consumption only under the additional assumption that the *quantity* of assets given through inter vivos transfers does not change in response to the deviation in asset prices. For instance, in the context of housing, this says that changes in asset prices do not affect the physical quantity of real estate (e.g., square meters) parents leave to their children.

In reality, this term could be positive or negative. On the one hand, agents may want to transfer more assets to their children in response to rising asset prices to compensate them for their welfare loss. On the other hand, agents may react to rising asset prices by borrowing more, leaving larger outstanding debt balances to their children (due to a pure substitution effect or due to a relaxation of borrowing constraints, as in Section 4.2).

A.4.3 Businesses

In the baseline model, we examined the welfare effect of changes in the path of the price of an asset $\{P_t\}_{t=0}^{\infty}$ holding constant its dividends $\{D_t\}_{t=0}^{\infty}$. However, this assumption does not seem adapted to businesses that themselves buy and sell financial assets, as changes in asset prices will typically affect their dividend payments. This appendix explains how we adapt our methodology to take into account such financial transactions by businesses. For example, it explains the reasoning behind our empirical measure for the equity valuation ratio used in Section 2.2 which is unaffected by share repurchases and

is capital-structure neutral.

The case of share repurchase. It is useful to start with an example in which a business can only make one type of financial transaction: repurchase its own shares. Formally, consider a business that produces an income stream (i.e., earnings minus investment) $\{\Pi_t\}_{t=0}^{\infty}$ from its fundamental (e.g., non-financial) operations. These cash flows are distributed to shareholders through both dividends and share repurchases:

$$\Pi_t = \mathcal{N}_{t-1}D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t)P_t \quad (50)$$

where D_t denotes the business dividends per share, P_t denotes the share price, and \mathcal{N}_t denotes the total amount of outstanding shares. When $\mathcal{N}_t < \mathcal{N}_{t-1}$ the business is repurchasing its own shares. From this equation it is already apparent that share repurchases and dividend payments are equivalent means of distributing cash flows $\{\Pi_t\}_{t=0}^{\infty}$ to shareholders as a whole (more on this shortly). As discussed above, the presence of share repurchases implies that changes in share prices will mechanically affect the path of dividends $\{D_t\}_{t=0}^{\infty}$, as higher share prices will force the firm to either spend more cash to buy the same amount of shares (which reduces dividends per share in the current period) or to buy fewer shares with the same amount of cash (which reduces dividends per share in future periods).

Let us consider the budget constraint of an individual i who, for simplicity, can only invest in the business:

$$(N_{i,t} - N_{i,t-1})P_t = N_{i,t-1}D_t + Y_{i,t} - C_{i,t}. \quad (51)$$

When the business repurchases its shares (i.e., $\mathcal{N}_t < \mathcal{N}_{t-1}$) this results in an income stream $(N_{i,t-1} - N_{i,t})P_t$ for those individuals selling their shares to the business. Denoting by $s_{i,t} \equiv N_{i,t}/\mathcal{N}_t$ the individual's ownership share of the business, we can combine the individual and business budget constraints, (51) and (50), to obtain:

$$(N_{i,t} - N_{i,t-1} + s_{i,t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t))P_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}.$$

Denoting by $M_t \equiv \mathcal{N}_t P_t$ the market value of the business, we obtain:

$$(s_{i,t} - s_{i,t-1})M_t = s_{i,t-1}\Pi_t + Y_{i,t} - C_{i,t}. \quad (52)$$

This budget constraint has the same form as (51), except that (i) the dividend per share D_t is replaced by the income stream from operations Π_t , (ii) the price per share P_t is replaced by the market value of the firm M_t , and (iii) the number of shares held by the individuals $N_{i,t}$ is replaced by the ownership share in the business $s_{i,t}$. An alternative viewpoint on this consolidated budget constraint is to consider the return to investing in the business. As usual, the return implied by the non-consolidated budget constraint is $R_{t+1} \equiv (D_{t+1} + P_{t+1})/P_t$, i.e., the return is the sum of dividend yield and capital gains. Multiplying and dividing by \mathcal{N}_t , we have

$$R_{t+1} \equiv \frac{\mathcal{N}_t D_{t+1} + \mathcal{N}_t P_{t+1}}{\mathcal{N}_t P_t} = \frac{\mathcal{N}_t D_{t+1} + (\mathcal{N}_t - \mathcal{N}_{t+1})P_{t+1} + \mathcal{N}_{t+1}P_{t+1}}{\mathcal{N}_t P_t} = \frac{\Pi_{t+1} + M_{t+1}}{M_t}$$

where the last equality uses (50) and the definition of the market value $M_t \equiv \mathcal{N}_t P_t$. Just like the consolidated budget constraint (52), writing the return as $R_{t+1} = (\Pi_{t+1} + M_{t+1})/M_t$ again makes clear that what ultimately matters are the business's cash flows $\{\Pi_t\}_{t=0}^{\infty}$ and its market value $\{M_t\}_{t=0}^{\infty}$ and not whether cash flows are distributed to shareholders via dividend payouts or share repurchases.

In our baseline model we examined the welfare effect of changes in the path of the price of an

asset $\{P_t\}_{t=0}^\infty$ holding constant its dividends $\{D_t\}_{t=0}^\infty$. The consolidated budget constraint (52) makes clear that, in the presence of share repurchases, the correct analogous experiment is instead to consider deviations in the market value of the business, $\{M_t\}_{t=0}^\infty$, holding constant its income stream $\{\Pi_t\}_{t=0}^\infty$. In particular, for investors as a whole, it is irrelevant whether the business increases its dividend payments or share repurchases; what matters instead is whether the firm's income stream changes $\{\Pi_t\}_{t=0}^\infty$.

Using a similar reasoning as in Proposition 1, we get that the welfare gain of the individual is

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (s_{i,t-1} - s_{i,t}) dM_t, \quad (53)$$

Hence, in the presence of share repurchases, what matters for welfare is not the number of shares N_t directly traded by the individual, but the overall change in his/her ownership share $s_{i,t}$ in the business. In particular, note that individual welfare gains still aggregate to zero, as ownership shares always aggregate to one in the population. Similarly, and as already noted, what matters is not deviations in the share price P_t holding constant dividends D_t but deviations in the market value $M_t = N_t P_t$ holding constant the income stream Π_t .

One way to understand expression (53) is to consider the case of a business that repurchases a given fraction of its shares every period. A rise in valuations benefits individuals who sell shares to the business while hurting the owners of the business as the business needs to spend more cash to purchase the same number of shares. The two effects compensate exactly for individuals who sell a fraction of their holdings equal to the fraction of outstanding shares purchased by the business, i.e., who have $s_{i,t} = s_{i,t-1}$. On the other hand, for individuals who do not sell any of their shares to the business, $N_{i,t} = N_{i,t-1}$ so that $N_{t-1} - N_t > 0$ implies $s_{i,t} = N_{i,t}/N_t < N_{i,t-1}/N_{t-1} = s_{i,t-1}$, only the second effect is operational and hence those individuals lose from higher valuations.

The case of arbitrary financial transactions. We now consider the more general case where, every period, the business can (i) repurchase its own shares (ii) buy and sell one-period bonds, and (iii) buy and sell K financial assets. The business budget constraint is:

$$\Pi_t + \sum_k \mathcal{N}_{k,t-1} D_{k,t} + \mathcal{B}_{t-1} = \mathcal{N}_{t-1} D_t + (\mathcal{N}_{t-1} - \mathcal{N}_t) P_t + \sum_k (\mathcal{N}_{k,t} - \mathcal{N}_{k,t-1}) P_{k,t} + \mathcal{B}_t Q_t, \quad (54)$$

where, as above, Π_t denotes the income stream of a business from its non-financial operations, D_t denotes dividends per share and \mathcal{N}_t denotes the total amount of outstanding shares. The new part is $\mathcal{N}_{k,t}$, which denotes asset holdings in asset k , and \mathcal{B}_t , which denotes bond holdings.

Let us consider an individual investing in K financial assets, one-period bonds, as well as in the business. The individual budget constraint is

$$(N_{i,t} - N_{i,t-1})P_t + \sum_k (N_{i,k,t} - N_{i,k,t-1})P_{k,t} + B_{i,t}Q_t = N_{i,t-1}D_t + \sum_k N_{i,k,t-1}D_{k,t-1} + B_{i,t-1} + Y_{i,t} - C_{i,t}. \quad (55)$$

Combining it with the business budget constraint (54) gives the following consolidated budget constraint:

$$\begin{aligned} & (N_{i,t} - N_{i,t-1} + s_{i,t-1}(\mathcal{N}_{t-1} - \mathcal{N}_t))P_t + \sum_k (N_{i,k,t} - N_{i,k,t-1} + s_{i,t-1}(\mathcal{N}_{k,t-1} - \mathcal{N}_{k,t}))P_{k,t} + (B_{i,t} + s_{i,t-1}\mathcal{B}_t)Q_t \\ & = s_{i,t-1}\Pi_t + \sum_k (N_{i,k,t-1} + s_{i,t-1}\mathcal{N}_{k,t-1})D_{k,t} + (B_{i,t-1} + s_{i,t-1}\mathcal{B}_{i,t-1}) + Y_{i,t} - C_{i,t}, \end{aligned}$$

where, as above, $s_{i,t} \equiv N_{i,t}/\mathcal{N}_t$ denotes the individual ownership share in the business.

We can simplify this expression after denoting $\tilde{N}_{i,k,t} \equiv N_{i,k,t} + s_{i,t}\mathcal{N}_{k,t}$ the individual's consolidated shares in asset k of the individuals through its ownership of the business, $\tilde{B}_{i,t} \equiv B_{i,t} + s_{i,t}\mathcal{B}_t$ the individual's consolidated bond holdings, and $\tilde{M}_{i,t} \equiv \mathcal{N}_{i,t}P_t - \mathcal{B}_{i,t}Q_t - \sum_k \mathcal{N}_{i,k,t}P_{k,t}$ the market value of the firm exclusive of financial assets:

$$(s_{i,t} - s_{i,t-1})\tilde{M}_t + \sum_k (\tilde{N}_{i,k,t} - \tilde{N}_{i,k,t-1})P_{k,t} + \tilde{B}_{i,t}Q_t = s_{i,t-1}\Pi_t + \sum_k \tilde{N}_{i,k,t-1}D_{k,t} + \tilde{B}_{i,t-1} + Y_{i,t} - C_{i,t}. \quad (56)$$

This has the same form as (55), except that (i) D_t , the business dividend per share of the business, is replaced by Π_t , the business income stream from its non-financial operations, (ii) P_t , the business price per share, is replaced by \tilde{M}_t , the market value of its fundamental (non-financial) component, (iii) $N_{i,t}$, the number of shares held by the individual, is replaced by $s_{i,t}$, their ownership share in the business, and (iv) individual asset holdings in financial assets and one-period bonds, $\{N_{i,k,t}\}_k$ and $B_{i,t}$, are replaced by their consolidated ones, $\{\tilde{N}_{i,k,t}\}_k$ and $\tilde{B}_{i,t}$.

This budget constraint allows us to consider the welfare gain of a deviation in the market value of the fundamental component of a business, \tilde{M}_t , holding constant its income stream Π_t , together with our usual deviations in asset prices $\{P_{k,t}\}_{k=0}^K, Q_t$:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left((s_{i,t-1} - s_{i,t}) d\tilde{M}_t + \sum_k (\tilde{N}_{i,t-1} - \tilde{N}_{i,t}) dP_{k,t} - \tilde{B}_{i,t} dQ_t \right)$$

There are two main takeaways of this formula relative to (53). First, when measuring individual financial transactions, we should also account for all of the indirect transactions done through the businesses that they own (i.e. $\tilde{N}_{i,t-1} - \tilde{N}_{i,t}$ instead of $N_{i,t-1} - N_{i,t}$). Second, when measuring deviations in business valuations, we should only consider deviations in the market value of their non-financial components (i.e. $d\tilde{M}_t$ instead of dM_t). Put differently, this formula tells us to split businesses between their financial and non-financial components, and assign their financial components to the individuals who ultimately own them.

A.4.4 Government

We now examine the welfare effect of asset price deviations in the presence of government transfers. For simplicity, we consider a two-asset version of the baseline model. Suppose that the government makes targeted transfers to individuals $i \in \{1, \dots, I\}$, where $T_{i,t}$ denotes the net amount of resources transferred from the government to individual i at time t . The individual problem is now given by

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$C_{i,t} + (N_{i,t} - N_{i,t-1})P_t + B_{i,t}Q_t + \chi(N_{i,t} - N_{i,t-1}) = Y_{i,t} + T_{i,t} + N_{i,t-1}D_t + B_{i,t-1}.$$

We assume that the government can trade both assets and thus faces, at each period $t \geq 0$, the following budget constraint:

$$(N_{G,t} - N_{G,t-1})P_t + B_{G,t}Q_t = N_{G,t-1}D_t + B_{G,t-1} - \sum_{i=1}^I T_{i,t} - \chi(N_{G,t} - N_{G,t-1}), \quad (57)$$

where, for simplify, χ is assumed to be differentiable. We do not fully specify the government problem, but we assume that the government's portfolio choice satisfies the following cost-minimization condition

$$Q_t^{-1} = \frac{D_{t+1} + P_{t+1} - \chi'(N_{G,t+1} - N_{G,t})}{P_t + \chi'(N_{G,t} - N_{G,t-1})}, \quad (58)$$

at every $t \geq 0$. The idea is that the government minimizes the cost of borrowing (or alternatively maximizes the return on saving) by adjusting portfolio shares until the marginal return on the long-lived asset (net of adjustment costs) is equalized with the bond return.

The following proposition characterizes the welfare gain in the presence of government transfers.

Proposition 12. *In the presence of government transfers, the welfare gain of individual i is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dT_{i,t}.$$

Moreover, the aggregate contribution of deviations in government transfers $dT_{i,t}$ to individual welfare is

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \sum_{i=1}^I dT_{i,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t).$$

Proof of Proposition 12. The welfare gain formula follows immediately from the envelope theorem, as in the baseline model. This proof focuses on the second equation. Differentiating the government budget constraint (57), we obtain

$$\begin{aligned} \sum_{i=1}^I dT_{i,t} &= (N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t \\ &\quad - (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} + (D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t-1} - Q_t dB_{G,t} + dB_{G,t-1}. \end{aligned}$$

The sum of aggregate net transfer deviations discounted using the liquid asset return is

$$\begin{aligned}
\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \sum_{i=1}^I dT_{i,t} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} \\
&\quad + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (D_t + \chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t-1} \\
&\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Q_t dB_{G,t} + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dB_{G,t-1} \\
&= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (\chi'(N_{G,t} - N_{G,t-1}) + P_t) dN_{G,t} \\
&\quad + \sum_{t'=-1}^{\infty} R_{0 \rightarrow t'+1}^{-1} (D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1}) dN_{G,t'} \\
&\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Q_t dB_{G,t} + \sum_{t'=-1}^{\infty} R_{0 \rightarrow t'+1}^{-1} dB_{G,t'} \\
&= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t) \\
&\quad - \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left(\chi'(N_{G,t} - N_{G,t-1}) + P_t - Q_t (D_{t'+1} + \chi'(N_{G,t'+1} - N_{G,t'}) + P_{t'+1}) \right) dN_{G,t} \\
&= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} ((N_{G,t-1} - N_{G,t}) dP_t - B_{G,t} dQ_t).
\end{aligned}$$

The second equality uses a change of variables $t' \equiv t - 1$. The third equality uses the fact that $R_{0 \rightarrow t+1}^{-1} = R_{0 \rightarrow t}^{-1} Q_t$ as well as $dN_{G,-1} = dB_{G,-1} = 0$. The fourth equality uses the cost-minimization assumption (58). \square

The formula for the welfare gain of individual i differs from the one in the baseline model since it includes the present-value of deviations in net government transfers. The reason is that the government might respond to a change in asset prices by adjusting net transfers. Moreover, the second part of Proposition 12 states that the discounted sum of aggregate net transfers to the household sector is equal to the “welfare gain of the government”. Note that we obtain this result without making assumptions on the objective of the government. It is merely a consequence of the budget constraint of the government.

Taxes on assets In the baseline model, individuals pay no taxes on either their asset holdings, asset transactions or income generated by these assets. We now consider an extension with four types of taxes: wealth taxes, asset transaction taxes, taxes on dividend income, and taxes on interest income.

Formally, we consider: (i) a non-linear wealth tax $\tau_{W,t}$ on the market value of wealth $N_{i,t-1}P_t$, (ii) a non-linear transaction tax $\tau_{\chi,t}$ on the market value of asset sales $(N_{i,t-1} - N_{i,t})P_t$, (iii) a dividend income tax $\tau_{D,t}$ on dividend income $N_{i,t-1}D_t$, and (iv) a linear tax $\tau_{Q,t}$ on interest income or equivalently on the cost of buying bonds B_tQ_t . Individuals maximize

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

subject to budget constraints at each period $t \geq 0$

$$\begin{aligned}
(N_{i,t} - N_{i,t-1})P_t + \tau_{\chi,t}((N_{i,t-1} - N_{i,t})P_t) + \chi(N_{i,t} - N_{i,t-1}) + B_{i,t}Q_t(1 + \tau_{Q,t}) \\
= Y_{i,t} + N_{i,t-1}D_t - \tau_{D,t}(N_{i,t-1}D_t) - \tau_{W,t}(N_{i,t-1}P_t) + B_{i,t-1} - C_{i,t}.
\end{aligned}$$

Here the functions $\tau_{\chi,t}(\cdot)$, $\tau_{W,t}(\cdot)$, $\tau_{D,t}(\cdot)$ are non-linear and potentially time-dependent tax functions. This allows us to capture a number of features of real-world tax systems. For example, transaction taxes often apply on both sales and purchases (i.e. $\tau_{\chi,t}(\cdot)$ may be positive and increasing when $N_{i,t-1} - N_{i,t} > 0$, positive and decreasing when $N_{i,t-1} - N_{i,t} < 0$ and zero when $N_{i,t-1} - N_{i,t} = 0$). Similarly, there are often large exemption levels, in particular for wealth taxes $\tau_{W,t}(\cdot)$. In contrast, we restrict the tax on interest income to be linear with tax rate $\tau_{Q,t}$ so as to preserve an Euler equation that is independent of bond holdings $B_{i,t}$. Finally, we assume that the tax functions $\tau_{\chi,t}(\cdot)$, $\tau_{W,t}(\cdot)$ are differentiable.

Proposition 13. *In the presence of taxes on wealth, asset sales, and interest income, $\tau_{W,t}$, $\tau_{\chi,t}$ and $\tau_{Q,t}$, the welfare gain is*

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) = & \sum_{t=0}^{\infty} \tilde{R}_{0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t) \right) dP_t \right. \\ & \left. - \tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1} dP_t - B_{i,t} (1 + \tau_{Q,t}) dQ_t \right). \end{aligned}$$

The presence of taxes changes our baseline formula in Proposition 1 in three noteworthy ways. First, whereas Proposition 1 implied that it is asset transactions and not asset holdings that matter for welfare gains from asset price changes, holdings do matter whenever there is a wealth tax (i.e., a tax on the market value of asset holdings). In particular, whenever asset prices increase, $dP_t > 0$, asset holders experience a welfare loss $\tau'_{W,t}(N_{i,t-1}P_t)N_{i,t-1} dP_t$.

Second, a transaction tax reduces asset sellers' welfare gains from rising asset prices because the after-tax asset price faced by sellers increases by less than the pre-tax price

$$0 < \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t) \right) dP_t < dP_t \quad \text{when} \quad N_{i,t} - N_{i,t-1} > 0 \text{ and } dP_t > 0.$$

However, it also *increases* asset buyers' welfare losses from rising asset prices because the after-tax asset price faced by buyers increases by *more* than the pre-tax price

$$0 < dP_t < \left(1 - \tau'_{\chi,t}((N_{i,t} - N_{i,t-1})P_t) \right) dP_t \quad \text{when} \quad N_{i,t} - N_{i,t-1} < 0 \text{ and } dP_t > 0.$$

Third and related, both transaction and wealth taxes introduce aggregate welfare losses for the household sector as a whole. Finally, though unsurprisingly, the presence of dividend income taxes leave welfare gains from asset-price changes unaffected.

Proof of Proposition 13. The Lagrangian is

$$\begin{aligned} \mathcal{L}_i = & \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) + \sum_{t=0}^{\infty} \lambda_{i,t} (Y_{i,t} + N_{i,t-1}D_t + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1})P_t - \chi(N_{i,t} - N_{i,t-1}) \\ & - \tau_{\chi,t}((N_{i,t-1} - N_{i,t})P_t) - B_{i,t}Q_t(1 + \tau_{Q,t}) - \tau_{W,t}(N_{i,t-1}P_t) - \tau_{D,t}(N_{i,t-1}D_t)). \end{aligned}$$

The first-order condition for $B_{i,t}$ is

$$\lambda_{i,t+1} = \lambda_{i,t} \tilde{Q}_t \quad \text{where} \quad \tilde{Q}_t = Q_t(1 + \tau_{Q,t})$$

is the after-tax bond price. The infinitesimal change in the value function is given by the infinitesimal

change in the Lagrangian:

$$\begin{aligned}
dV_{i,0} &= \sum_{t=0}^{\infty} \left(\frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t \right) \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t}) dP_t - \tau'_{\chi,t} ((N_{i,t-1} - N_{i,t}) P_t) (N_{i,t-1} - N_{i,t}) dP_t \right. \\
&\quad \left. - \tau'_{W,t} (N_{i,t-1} P_t) N_{i,t-1} dP_t - B_{i,t} (1 + \tau_{Q,t}) dQ_t \right) \\
&= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t}) \left(1 - \tau'_{\chi,t} ((N_{i,t} - N_{i,t-1}) P_t) \right) dP_t - \tau'_{W,t} (N_{i,t-1} P_t) N_{i,t-1} dP_t - B_{i,t} (1 + \tau_{Q,t}) dQ_t \right) \\
&= U'(C_{i,0}) \sum_{t=0}^{\infty} \tilde{R}_{0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) \left(1 - \tau'_{\chi,t} ((N_{i,t} - N_{i,t-1}) P_t) \right) dP_t \right. \\
&\quad \left. - \tau'_{W,t} (N_{i,t-1} P_t) N_{i,t-1} dP_t - B_{i,t} (1 + \tau_{Q,t}) dQ_t \right).
\end{aligned}$$

where the third equality uses the Euler equation for B_t which implies $\lambda_{i,t} = U'(C_{i,0}) \tilde{R}_{0 \rightarrow t}^{-1}$ with $\tilde{R}_{0 \rightarrow t} = (\tilde{Q}_0 \dots \tilde{Q}_{t-1})^{-1}$. \square

B Appendix for Section 2

B.1 Interpreting price deviations away from constant price-dividend ratio

As explained in Section 1.3 when implementing Proposition 1, we construct the empirical price deviations $\Delta P_{k,t}$ as deviations of asset prices away from a world in which the price-dividend ratio was constant within each asset class (see equation 14 and Figure 2). We now briefly explain why, under the assumption that dividends grow at a constant rate, price deviations around a constant price-dividend ratio can be interpreted as deviations around a constant value for discount rates.

For simplicity, we focus on the case in which dividends are deterministic and grow at a constant rate:

$$D_{t+s} = D_t G^s. \quad (59)$$

Under this constant-growth assumption, the price of an asset is

$$\begin{aligned}
P_t &= \sum_{s=1}^{\infty} R_{t \rightarrow t+s}^{-1} D_{t+s} \\
&= D_t \sum_{s=1}^{\infty} R_{t \rightarrow t+s}^{-1} G^s.
\end{aligned} \quad (60)$$

When discount rates are constant, $R_t = \bar{R}$ for all t with $\bar{R} > G$, this simplifies to

$$P_t = D_t \times \overline{PD} \quad \text{with} \quad \overline{PD} = \frac{G}{\bar{R} - G}, \quad (61)$$

i.e., the price-dividend ratio is constant and the price grows at the same rate as dividends. This is the original ‘‘Gordon growth model’’, studied in [Gordon and Shapiro \(1956\)](#).

In our exercise, we construct price deviations as deviations of asset prices from a baseline with a constant price-dividend ratio, $\Delta P_t = (PD_t - \overline{PD}) \times D_t$. Combining (60) and (61), the difference in prices is:

$$\Delta P_t = \left(\sum_{s=1}^{\infty} (R_{t \rightarrow t+s}^{-1} - R^{-s}) G^s \right) D_t. \quad (62)$$

Table A2: Individual wealth at the end of 1993

Asset type	Average	S.D.	p10	p25	p50	p75	p90	p99
Total wealth	781.07	727.57	386.96	529.47	723.43	938.59	1197.68	2075.54
Financial wealth	116.30	616.65	-17.63	5.52	79.87	166.08	280.46	729.60
Housing	131.49	247.77	0.00	0.00	104.62	185.19	289.22	664.71
Debt	-51.00	367.76	-116.53	-67.16	-22.47	-0.17	0.00	21.82
Deposits	23.19	144.80	0.05	1.23	6.59	22.37	55.09	215.14
Human wealth	664.77	365.91	299.82	427.77	621.75	824.85	1045.41	1776.12

Notes. The table displays the summary statistics for individual wealth as of December 31st 1993. The total number of observations is 3,268,017. Values are reported in thousands of 2011 US dollars. Each statistic is computed for each variable separately.

This formula shows that, under the assumption that dividend grows at a constant rate, variations of the price-dividend ratio over time correspond to variations in discount rates. The same ideas hold in a stochastic environment (Campbell and Shiller, 1988).

B.2 Data on asset prices

Figure A1 plots the relative price deviations (defined in equation 14) for our four asset classes: housing, equity, debt, and deposits.

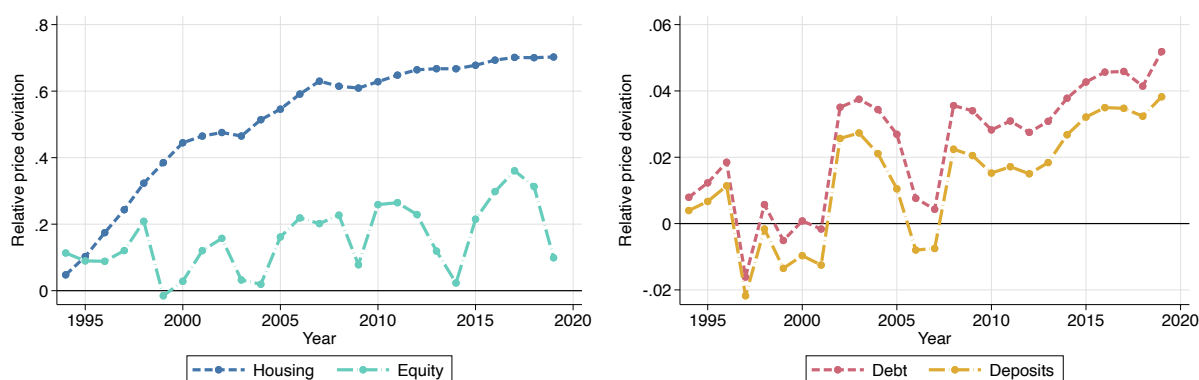


Figure A1: Relative price deviations for four asset classes in Norway

Notes. Note that the scale of the price deviation for debt (or deposits) is much smaller than the one for housing. Still, these two asset classes will end up having comparable effects on welfare, as the size of debt holdings (i.e. issuance of one-period bonds) is much higher than the size of housing transactions, as seen in Figure 7.

B.3 Microdata on holdings and transactions

B.3.1 Summary statistics and validation

Table A2 reports summary statistics on the balance sheet of Norwegian individuals at the end of 1993 (start of our sample).

Figure A2 compares the aggregate value of individuals' net assets for each asset category in the microdata as well as the ones reported in the Financial Accounts. Overall, the microdata aligns closely with the Financial Account data. The only notable discrepancies are public equity which is higher in the microdata than in the National accounts after 2010, and mutual fund equity which is higher in the Financial Accounts than in our microdata throughout our sample period.

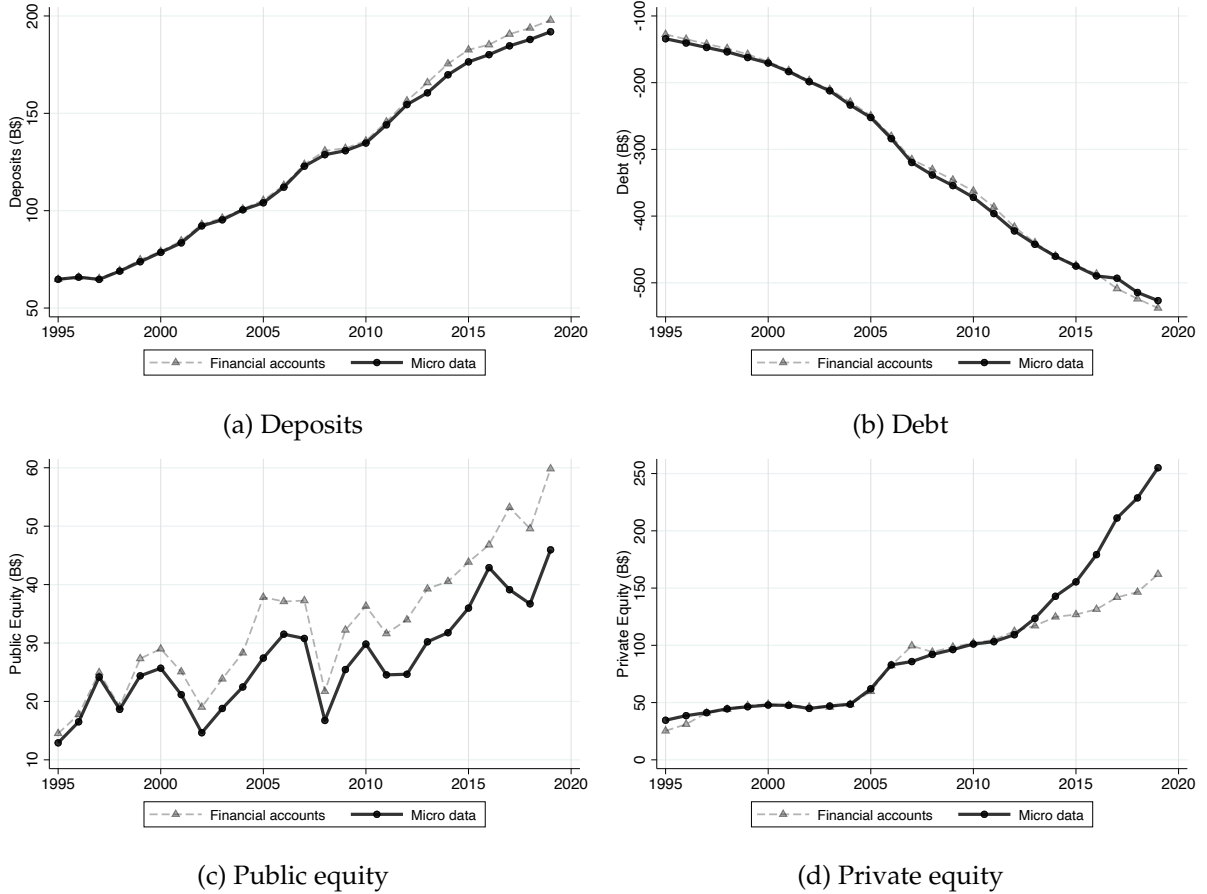


Figure A2: Aggregated administrative microdata versus the Financial Accounts (Holdings)

B.3.2 Imputing indirect holdings and transactions

Individuals who own firms are indirectly exposed to asset price changes through the asset holdings and transactions of the firms they own. We now describe how we impute these indirect holdings and transactions.

Private businesses. Starting in 2005, our data contains information on the ownership of limited liability businesses. The data contains information on the number of shares owned by an individual or a firm, and the market price if that exists. In addition, we observe the total number of shares issued by a company.

We first compute the direct ownership share of firm j by an individual or firm i . We obtain this number by dividing the number of shares held by owner i by the number of shares outstanding in firm j (i.e., the total number of shares issued by the firm minus the shares held by the firm itself). More precisely, the direct ownership share of a owner i in firm $j \neq i$ is

$$s_{ij} \equiv \frac{N_{ij}}{\sum_{i \neq j} N_{ij}},$$

where N_{ij} denotes the number of shares held by an owner i in firm j .

In our sample, a substantial fraction of businesses are owned by other businesses. For example, a common structure among wealthy individuals is to have one umbrella private holding company that owns several holding companies operating in different sectors. Our goal is to allocate the financial transactions done by all of these businesses to their ultimate owner. Formally, denote s_{ij}^n the ownership

share of individual i in firm j through n (and exactly n) intermediate firm layers. When $n = 0$, this corresponds to our direct ownership share $s_{ij}^0 = s_{ij}$. For $n > 0$, we can compute the ownership shares of individual i in firm j at level n recursively:

$$s_{ij}^n \equiv \sum_k s_{ik}^{n-1} s_{kj}.$$

Finally, we obtain the consolidated ownership share of an individual by aggregating the ownership shares at all levels $n \geq 0$.⁸⁵

$$\tilde{s}_{ij} = \sum_{n=0}^{\infty} s_{ij}^n.$$

In practice, we only compute indirect ownership shares up to $n = 10$ as indirect ownership shares are close to zero past that point.

Using these ownership shares, we construct an individual-level measure of private business book equity, which we define as the book value of a firm's assets minus net financial assets.⁸⁶ We only use book equity to compute the value of private businesses transactions, which we describe shortly. More generally, we rely on the tax assessed value of private business equity, which we observe over the full sample (i.e., starting in 1994).

Table A3 reports the average value of indirect holdings and transactions as a fraction of the tax assessed value of the equity in the firm over the 2005-2019 period. Private firms have, on average, positive net leverage (i.e., debt exceeds deposits). Moreover, private firms hold a significant amount of housing and (publicly-traded) stocks on their balance sheet, with a small amount of transactions every year. Before 2005, we do not observe the balance sheet of private firms, hence we do not have data on indirect holdings and transactions. From 1994 to 2004, we therefore impute indirect holdings and transactions by using the values in Table A3 multiplied by the tax assessed value of equity.

Table A3: Indirect holdings through private businesses (share of tax assessed value, 2005–2019 average)

Asset class	Holdings	Transactions
Deposits	0.40	–
Debt	1.11	–
Housing	0.65	–0.03
Stocks	0.16	–0.00

To measure the net transactions in private business equity for individual i in firm j in year t , we use the formula

$$\text{private equity transaction}_{ijt} = (\tilde{s}_{ij,t+1} - \tilde{s}_{ij,t}) \times \text{book equity}_{jt} \times Q,$$

where, as above, $\tilde{s}_{ij,t}$ denotes the ownership share of individual i in firm j at time t . If the firm does not exist at time t and enters at time $t + 1$, we set the net transactions in private business equity to zero. Note that this formula automatically accounts for equity issuance. For instance, when a firm issues

⁸⁵ Formally, denote Ω the matrix of ownership within firms, that is, $\Omega_{ij} = s_{ij}$ for $i \neq j$ and $\Omega_{ij} = 0$ for $i = j$. Then, the vector of consolidated ownership of an individual i with direct ownership shares $\mathbf{s}_i = (s_{ij})_j$ is given by $(I - \Omega')^{-1} \mathbf{s}_i = \sum_{n=0}^{\infty} (\Omega')^n \mathbf{s}_i$.

⁸⁶ For instance, suppose that a firm has \$2 of assets, which includes \$1 of stocks, and \$0.25 of debt outstanding. The net financial assets of the firm is then $\$1 - \$0.25 = \$0.75$. Book equity is then $\$2 - \$0.75 = \$1.25$.

equity to finance its growth, the existing owners get diluted (i.e., their ownership share declines). In terms of exposure to asset price changes, this is equivalent to the owners selling equity shares.

The term Q represents the ratio between the market value of private business equity and its book value. While we do not observe Q directly, we set it to a value of 0.80, which corresponds to the aggregate share of the tax assessed value of private business equity to book value of private business equity, averaged over the 2005-2019 period. Before 2005, we do not observe ownership shares and therefore set private equity transactions to zero.

Public businesses. Finally, we impute indirect holdings and transactions due to the ownership of publicly-traded stocks. We start from the indirect holdings and transactions of individuals through their ownership of the aggregate corporate sector, as reported in the Financial Accounts (see Appendix C.2 for more details). We then subtract the aggregate indirect holdings and transactions due to their ownership of private businesses, as computed above. We therefore obtain residually the indirect aggregate holdings and transactions of public businesses that must be allocated to individuals. We then allocate these indirect holdings and transactions to individuals, for every year in our sample, in proportion to their equity holdings of public firms.

C Appendix for Section 3

C.1 Alternative measures of asset price deviations

We now explore the sensitivity of our results to the construction of the price-dividend ratio for equity and housing.

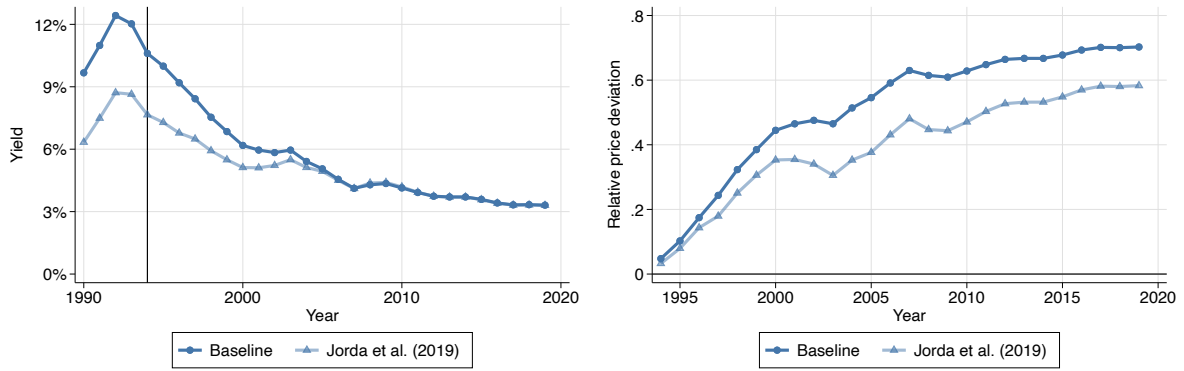
In particular, [Jordà et al. \(2019\)](#) use a similar construction methodology for the price-to-rent ratio. However, they rescale the growth in the rental index reported by SSB to account for potential bias.⁸⁷ This adjustment ends up decreasing the average price-to-rent ratio from before 2005 relative to our baseline time series, thereby implying a lower rise in the price-dividend ratio.

By construction, all ratios are equal in 2013. Due to the gradual rescaling done by [Jordà et al. \(2019\)](#), their series gradually diverge from our baseline series before 2005. Using these alternative series lead to a lower increase in the price-dividend ratio, and therefore, a lower price deviation relative to our 1992-1996 baseline. Figure A3b reports the implied deviation in the price of housing relative to 1992-1996 (as defined in equation (14)) for our baseline and for the alternative series. In average, the housing price deviation implied by [Jordà et al. \(2019\)](#) represents 0.76 of our baseline. Hence, as a rule of thumb, one can expect the welfare gains due to housing to be scaled by 0.76 if one were to reproduce all our figures with this alternative ratio. As an example, we recompute the welfare gains across generation using this alternative series in Figure A3c

C.2 Redistribution across sectors

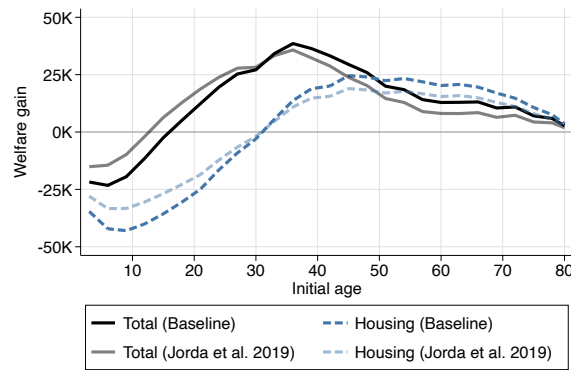
As discussed in Section 3, welfare gains do not aggregate to zero within the household sector. The reason is that individuals trade with other non-household entities, such as the government and foreigners. We now conduct a systematic investigation of welfare gains across sectors. This is particularly important in Norway given the scale of the sovereign wealth fund, which purchases domestic and foreign assets on behalf of Norwegian households.

⁸⁷More precisely, [Jordà et al. \(2019\)](#) says “we stipulate that the rental index during late 1990s and early 2000s—a period when house prices increased substantially—understated the growth of rents relative to prices, leading the rent-price approach to overstate the historical rental yields. To correct for this presumed bias, we adjust the growth in rents up by a factor of 1.5 for the years 1990 to 2005.”



(a) Housing yield (ratio of rents to house prices)

(b) Price deviation



(c) Welfare gains across cohorts

Figure A3: Comparing our series for house-to-rent ratio with [Jordà et al. \(2019\)](#)

More precisely, we group all entities in the economy into three sectors: households (H), the government (G), and foreigners (F). The key accounting identity that we use is that every asset bought by one sector must be sold by another sector. With this in mind, it is immediate that in a multisector economy, our measurement of welfare gains implies:

$$\text{Welfare Gain}_H + \text{Welfare Gain}_G + \text{Welfare Gain}_F = 0, \quad (63)$$

where the sector-level welfare gain is defined analogously to Equation (16). In words, a positive welfare gain for the household sector must be exactly offset by a welfare loss in another sector. We first present the data in Section C.2.1 and we discuss the results in Section C.2.2.

C.2.1 Data sources

We use publicly available data from the *Financial Accounts*, which cover all holdings and transactions of financial assets in the Norwegian economy starting from 1995. For our analysis, we combine the government sector with the central bank and the non-profit sector. Importantly, this means that our government sector includes the *Government Pension Fund of Norway*, which is financed by income taxes on the energy (oil and gas) sector. It is composed of the Government Pension Fund Global — which invests in foreign assets — and the Government Pension Fund Norway — which is smaller and invests in domestic and Scandinavian assets.⁸⁸

⁸⁸Over our sample period, the Government Pension Fund Global fund's value grew from approximately zero in 1997 to approximately one 1B\$ in 2019. Its portfolio mandate first prescribed 40 percent equities and 60 percent fixed income assets. In 2007 this was changed to 60 percent equities. In 2010, the fund's portfolio was extended

We consider the following sectors of the economy:

1. Households (14);
2. Government (121, 13, 15);
3. Foreigners (2).
4. Corporations
 - 4.1 Non financial corporations (11)
 - 4.2 Monetary financial institutions (122-123)
 - 4.3 Non-MM investment funds (124)
 - 4.4 Other financial institutions (125-127)
 - 4.5 Insurance corporations and pension funds (128-129)

The numbers in parentheses denote the sector codes from the Financial Accounts that we aggregate. Note that our definition of “Government” includes the central bank as well as the non-profit sector (i.e., institutions that serve the domestic household sector). We consolidate the different sectors constituting the corporate sector to their ultimate owner (i.e., either households, the government, or foreigners) by using the exact formula provided in Footnote 85. The consolidation process therefore adjusts the measures of holdings and transactions by households, the government, and foreigners by accounting for their indirect holdings and transactions through their ownership of the corporate sector. Note that this consolidation maintains the Financial Accounts identities, and, in particular, financial transactions remain in zero sum.

We consider the following asset categories:

1. Deposits (22);
2. Loans and debt securities (30, 40);
3. Public equity shares (511);
4. Private equity shares (512);
5. Fund equity shares (520);
6. Other (10, 21, 519, 610–800).

The numbers in parentheses denote the line items from the Financial Accounts that we aggregate. The category “other” contains assets that are either quantitatively unimportant or illiquid. We can further decompose each asset category using the identity of the sector issuing the security (e.g., public equity shares issued by the corporate sector versus the foreign sector). Real estate is a real asset rather than a financial asset, which means that housing holdings and transactions are not recorded in the Financial Accounts. We therefore augment the Financial Accounts with between-sector housing holdings and transactions, which we construct by aggregating the housing transaction registry data described in Section 2.3.

The resulting dataset covers the total amount of asset holdings and transactions for three sectors (households, government, and foreigners) and four asset classes (housing, deposits, debt, equity) over the 1995–2019 period.

to real estate with a 5 percent weight, and the fixed income share was cut to 35. A fiscal policy rule states that the expected real rate of return, first 4% and since 2017 3%, of the current fund value can be spent over the national budget each year. As the fund grew over our sample period, so did government spending. Details regarding the fund’s mandate and investment strategy are provided at <https://www.nbim.no/en/the-fund/how-we-invest>.

C.2.2 Results

Sectoral transactions. Before we quantify the welfare gains by sector, we briefly discuss the main pattern of housing and equity transactions as well as debt and deposit holdings across sectors, as reported in Table A4.

The annual levels of net housing purchases across sectors are very low (less than \$1,000 per capita in absolute value). The reason is that most housing transactions are within the household sector, with minimal transactions between sectors. Regarding equity purchases, households have a positive but small level of net equity purchase on average. In contrast, the government is a net buyer of foreign equities via the sovereign wealth fund described above. Those transactions are quite large, and amount to more than \$5,000 per capita per year.

Table A4: Transactions (net purchases) across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	0.92	-0.38	-0.54	0.00
Debt	-74.39	65.12	9.28	0.00
Household debt	-65.22	24.82	40.40	0.00
Corporate debt	-26.81	-9.95	36.76	0.00
Government debt	6.38	-38.63	32.25	0.00
Foreign debt	11.26	88.87	-100.14	0.00
Deposits	20.38	-7.95	-12.44	0.00
Corporate deposits	15.58	-11.23	-4.35	0.00
Government deposits	0.67	-1.95	1.28	0.00
Foreign deposits	4.14	5.23	-9.37	0.00
Equity	0.63	6.98	-7.61	0.00
Corporate equity	-0.62	-0.65	1.27	0.00
Foreign equity	1.25	7.63	-8.88	0.00

Notes. All numbers are in thousands of 2011 US dollars, and divided by the population of Norway. Averages over 1995-2019. "Household debt" is debt taken by households (mostly mortgages); "Corporate debt" is debt issued by the corporate sector (i.e., bonds and bank loans); "Foreign debt" contains all debt issued by foreigners (e.g. foreign corporate entities, foreign households, and foreign governments); "Corporate deposits" is deposits issued by private banks; "Government deposits" is central bank reserves; "Corporate equity" is equity issued by corporations; "Foreign equity" is equity issued by foreign corporations.

Table A4 reveals that the household sector has a large amount of debt. Most of it is household debt (mainly mortgages), but some of it is corporate debt, which individuals indirectly hold through their ownership of businesses. While households, on net, hold debt securities as liabilities (i.e., they are indebted), the government, on net, holds debt securities as assets (i.e., they are lenders). In fact, the debt level of households is approximately equal to the government's net holding of debt securities (roughly \$50,000 per capita). The foreign sector only holds a small amount of debt on net. While households do not borrow directly from the government, the effect is the same in terms of welfare redistribution: a decline in interest rates redistribute from the government towards households.⁸⁹

A similar pattern holds for deposits, although the magnitudes are much smaller. The household sector is a net holder of deposits (including indirectly via its ownership of businesses), while the gov-

⁸⁹Most of household debt is mortgages, who are then securitized into mortgage bonds by private banks. Then, these bonds are for the most part sold to domestic pension funds as well as foreigners. However, foreigners also issue a large amount of debt that is held by the sovereign wealth fund. This explains why the net foreign debt position is close to zero in Table A4. The sovereign wealth fund's holding of foreign bonds then account for most of the government's net holding of debt securities, while a small fraction are held by other public pension funds that invest domestically. The main domestic public pension funds are *Folketrygdfondet* and *Kommunenes Landspensjonskasse* (see Bank, 2021 for an overview of Norway's financial system).

ernment and foreign sector hold these deposits as liabilities. The reason is that deposits are a liability for the financial sector, and since the government includes the central bank, and foreigners are important holders of financial business equity, they are ultimately liable for interest payments on these deposits.

Sectoral welfare gains. Table A5 reports the detailed welfare gains asset class by asset class, including a breakdown within asset class (i.e., equity is the sum of domestic corporate equity and foreign corporate equity). All welfare gains are scaled by the number of individuals in Norway in 1994. Note

Table A5: Welfare gains across sectors

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	-4.4	1.8	2.6	0.0
Debt	16.4	-15.6	-0.8	0.0
Household debt	14.7	-5.5	-9.2	0.0
Corporate debt	5.8	2.8	-8.6	0.0
Government debt	-1.3	8.4	-7.1	0.0
Foreign debt	-2.7	-21.4	24.1	0.0
Deposits	-2.5	1.0	1.5	0.0
Corporate deposits	-1.8	1.4	0.4	0.0
Government deposits	-0.1	0.3	-0.2	0.0
Foreign deposits	-0.6	-0.7	1.2	0.0
Equity	-0.9	-10.8	11.7	0.0
Corporate equity	0.8	0.7	-1.5	0.0
Foreign equity	-1.7	-11.4	13.2	0.0
Total	8.7	-23.5	14.9	0.0

that welfare gains sum up to zero within each asset class, by construction, and that the welfare gain per capita in the household sector is very similar to the one estimated in our microdata (see Table 2). The small difference is due to the fact that our microdata does not aggregate exactly to the Norwegian Financial Accounts (see Appendix B.3.1), as well as the fact that our microdata starts in 1994 while the Norwegian Financial Accounts only start in 1995.

The household sector has a positive welfare gain of roughly \$9,000 per capita. Breaking down the welfare gain by asset class, we find a large positive contribution of debt (\$18,000) and a small contribution of deposits (-\$3,000). Equity transactions make a negligible contribution (-\$1,000) and housing transactions a more important one (-\$5,000). The positive welfare gain of the household sector is therefore mostly due to declining interest rates, which have been beneficial to households since they are net debtors (i.e., their debt exceeds their bank deposits).

If the household sector has experienced a positive welfare gain, who is the counterparty that experienced a welfare loss? For the most part, it was the government. As discussed earlier, the government is a net saver and is thus hurt by declining interest rates. Overall, the welfare gain of the government is negative, with a large contribution of debt and equity. As reported in Table A4, this comes from the fact that the government is a net holder of debt and a net purchaser of equity. In contrast, the contributions of deposits and housing for welfare gains are negligible (<\$2,000 in absolute value).

The fact that the Norwegian government is hurt by rising asset prices and declining interest rates can seem surprising from a U.S. perspective. In the U.S., the government is a net debt issuer, and so it tends to benefit from a rise in asset prices at the expense of households and foreigners, who hold its debt. The same effect holds true in Norway: as shown in Table A5, if we restrict ourselves to the debt



Figure A4: Welfare gains across sectors

Notes. Figure A4 contains the welfare gain for each sector of the economy, as well as the contribution of each asset class. To make it comparable to the other figures in our paper, the aggregate welfare gain of each sector is divided by the number of individuals in Norway. Units are 2011 US dollars.

issued by the government (i.e. the row “Government debt”), the rise in asset prices does benefit the government at the expense of households and foreigners. However, this effect is swamped by the fact that the Norwegian government holds a large amount of debt issued by households and foreigners: this is why the government is ultimately hurt by rising asset prices and declining interest rates.

As discussed in Section 1.3, the loss of the government represents a loss of real resources available for net transfers to the household sector. While it is beyond our paper’s scope to quantify how the Norwegian government has adjusted (and will adjust) net transfers in response to persistently lower interest rates and higher asset prices, it is entirely possible that the very individuals who experienced welfare losses (i.e., the young) will also be the ones to bear the brunt of future reductions in government transfers such as pension benefits.

C.2.3 Domestic versus foreign price indices

In all of these exercises we use the same price deviation for foreign assets as for domestic assets (that from Section 2). This assumption was innocuous when computing the average welfare gain within the household sector as most of the financial transactions between Norwegians are transactions of domestic assets. However, this assumption becomes more restrictive when discussing welfare gains across sectors, as the Norwegian government buys a large amount of foreign assets. Price deviations for foreign equity and foreign debt may differ from the ones for Norwegian (domestic) assets.

For the sake of robustness, we now re-estimate sectoral welfare gains using different price indices for domestic versus foreign assets. For holdings of foreign debt, we use as a measure of yield the OECD average 3-year government bond yield (series from Global Financial Data). For transactions of foreign equity, we use as a measure of yield the ratio of total firm payout to total enterprise value in the universe of firms from Worldscope. Figure A6 plots the evolution of these two quantities, relative to the ones used for domestic equity and debt. One can see that, while yields follow similar dynamics in Norway and in the rest of the world, the yield of equity decreased less in Norway relative to the rest of the world. Figure A6 plots the resulting relative price deviation (14) used in the welfare gains formula.

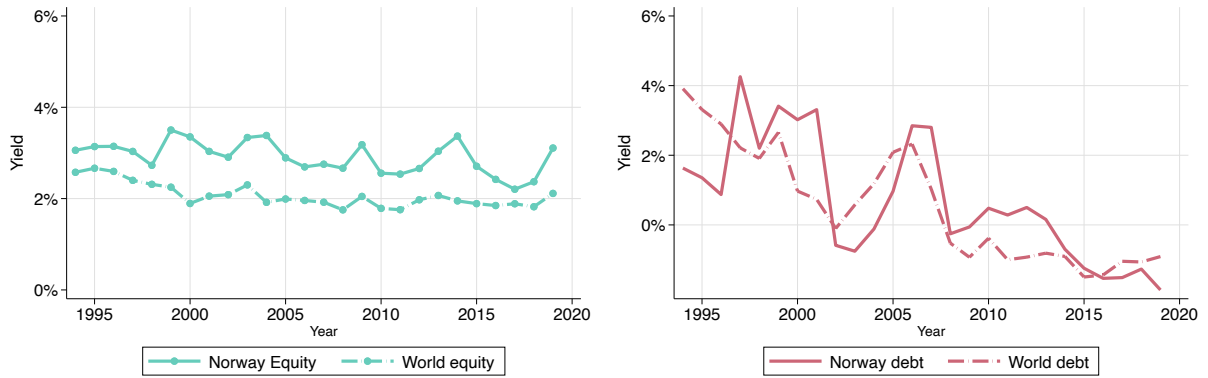


Figure A5: Yields for domestic and foreign assets

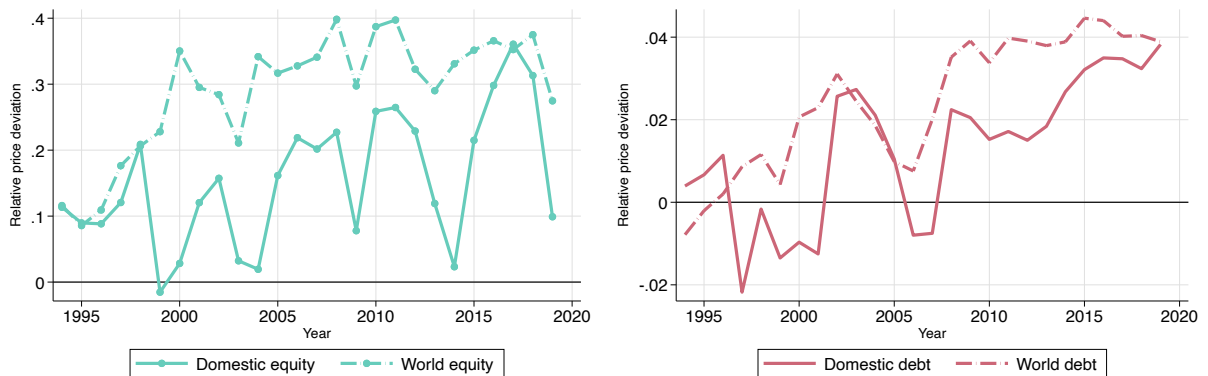


Figure A6: Price deviations for domestic and foreign assets

Table A6 reports the welfare gains using this foreign price indices for debt, deposit, and equity. Compared to Table A5, the rows that change are indicated by a dagger † sign. Overall, one can see that we obtain very similar result. The main difference is that the welfare gains of the Norwegian government are more negative in magnitude, which reflects the fact that they disproportionately purchased foreign equity, whose valuation increased more than the valuation of domestic equity.

D Appendix for Section 4

D.1 Uninsurable income risk

Proof of Proposition 2. The proposition obtains as a special case of Proposition 6 in the case without debt in labor or dividend income; that is, $dY_{i,t} = 0$ and $dD_{k,t} = 0$ for $1 \leq k \leq K$. \square

D.1.1 Empirical implementation of Equation 20

We now detail how we estimate the incomplete market adjustment term (i.e., the covariance term given in Equation 20). We defined income as net non-financial income (i.e., labor income plus net government transfers, exactly as we do when computing human wealth). We measure consumption as the residual of income minus net asset purchases, consistently with the budget constraint (9). We ensure that our measure of income and consumption is always higher than the basic amount (grunnbeløp) used in the Norwegian social security system and we winsorize the top 1% of observations every year. Finally, we construct asset savings for each individual and for each asset class: for housing and equity, it is

Table A6: Welfare gains across sectors using heterogeneous price indices

Asset type	Sector			Total
	Households	Government	Foreign	
Housing	-4.4	1.8	2.6	0.0
Debt	16.7	-14.9	-1.8	0.0
Household debt	14.7	-5.5	-9.2	0.0
Corporate debt	5.8	2.8	-8.6	0.0
Government debt	-1.3	8.4	-7.1	0.0
Foreign debt [†]	-2.5	-20.7	23.1	0.0
Deposits	-3.0	0.4	2.5	0.0
Corporate deposits	-1.8	1.4	0.4	0.0
Government deposits	-0.1	0.3	-0.2	0.0
Foreign deposits [†]	-1.0	-1.3	2.3	0.0
Equity	-0.6	0.8	-0.1	0.0
Corporate equity	0.8	0.7	-1.5	0.0
Foreign equity [†]	-1.5	0.1	1.4	0.0
Total	8.7	-11.9	3.2	0.0

transactions-based — that is, $S_{i,k,t} = (N_{i,k,t} - N_{i,k,t-1})P_{k,t}$, while, for debt and deposits, it is holdings-based — that is, $S_{i,k,t} = B_{i,t}Q_t$.

Remember that our covariance term (20) should capture the uncertainty at the individual level. Hence, to account for this, we residualize our variables (asset savings, log income, and log consumption) on individual characteristics known at $t = 0$ for each cohort c and horizon t . The set of controls is: (i) highest lifetime education achievement (i.e., “less than high school”, “high school”, “college” dummies), (ii) deciles of within-cohort financial wealth at the beginning of our sample (i.e., ten dummies), and (iii) the average of income in the first three years of our sample.

After residualizing all of these quantities, we then regress our measure of consumption (or spending) on labor income for each cohort c and horizon t :

$$\log C_{i,t} = \alpha_{c,t} + \beta_{c,t} \log Y_{i,t} + u_{i,c,t}. \quad (64)$$

We then compute, for each cohort c and horizon t and asset class k , the covariance between asset sales $S_{i,k,t}$ and our predicted measure of consumption $\widehat{\log C_{i,t}} = \beta_{c,t} \log Y_{i,t}$. Alternatively, this methodology can be interpreted as a regression of asset sales on consumption instrumented by labor income for each cohort and horizon, using initial individual characteristics as controls.

D.1.2 Quantifying welfare gains in models of incomplete markets

We now study the welfare effect of deviation in asset prices in heterogeneous-agent incomplete market models. The goal is to complement our sufficient statistic approach with a more standard model-based approach, based on external calibration. We focus on two classes of incomplete market models. First, we consider the welfare effect of asset price deviations in a Bewley-type model, in which individual labor income is subject to both permanent and transitory shocks. We then study the welfare effect of asset price deviations in a random growth model of wealth accumulation, for instance where entrepreneurs face idiosyncratic return risk.

Quantifying the effect of stochastic labor income We consider a model in which agents live for 65 years. They spend 40 years as workers and 25 as retired. The permanent labor income of workers Y_t evolves as a random walk (in log) with normal innovations with mean μ and volatility σ . Moreover, workers switch from being employed to unemployed, with transition probabilities given by $\lambda_{U \rightarrow E}$ and $\lambda_{E \rightarrow U}$; when unemployed, workers only earn a fraction χ_U of their permanent income. Finally, during retirement, agents earn a fraction χ_R of their permanent income. Agents can save in an asset that delivers a constant flow of dividends. The price of the asset is such that investing in the asset returns an interest rate R . We denote $s_t \in \{E, U, R\}$ the current state of a household with age t , corresponding to employment, unemployment, and retired.

Households have homothetic utilities with relative risk aversion γ , impatience parameter β . Moreover, agents have a preference over the size of bequest they leave $bW^{1-\gamma}/(1-\gamma)$. Formally, the problem of an agent with age t is to maximize

$$V(Y_{i,t}, W_{i,t}, x_{i,t}) = \mathbb{E}_t \left[\sum_{s=t}^{65} \beta^{s-t} \frac{C_{i,s}^{1-\gamma}}{1-\gamma} + \beta^{65-t} b \frac{W_{i,T}^{1-\gamma}}{1-\gamma} \right],$$

subject to

$$\begin{aligned} W_{i,t+1} &= R_{i,t+1} W_{i,t} + \chi_{s,t} Y_{i,t} - C_{i,t} \\ \log Y_{i,t+1} &= \log Y_{i,t} + \epsilon_{t+1} 1_{x_{it} \in \{E, U\}}, \quad \text{where } \epsilon_{i,t+1} \sim N(\mu, \sigma^2). \end{aligned}$$

and $x_{i,t}$ evolves as a Markov chain on $\{E, U\}$ with transition probabilities $\lambda_{U \rightarrow E}$ and $\lambda_{E \rightarrow U}$ for $t \leq 40$, and then switches to R for retirement at $t = 40$. Initial wealth is log normally distributed and, on average, equals the terminal wealth of deceased households. Initial income is set to one, as a normalization. Because the lower bound on labor income is zero, the natural borrowing constraint is $W_{i,t} \geq 0$.

We calibrate the growth and volatility of the innovations of log labor income $\mu = 0.01$ and $\sigma = 0.10$ following Wang et al. (2016). We set the annual probability of switching from employment to unemployment to 5%, the annual probability of switching from unemployment to employment to 80%, and the income multiplier when unemployed to $\chi_U = 0.6$ following Krueger et al. (2016). Finally, we pick the impatience parameter β and the bequest motive parameter b to match a ratio of average wealth to average labor income of 6 and a ratio of terminal wealth (at 85) to average wealth of 0.7, which gives $\beta = 0.95$ and $b = 0.02$.

Table A7: Parameters

Description	Symbol	Value
<i>Income process</i>		
Labor income growth	μ	0.01
Labor income volatility	σ	0.10
Income multiplier when retired	χ_R	0.70
Transition probability to unemployment	λ_{EU}	0.05
Transition probability to employment	λ_{UE}	0.80
Income multiplier when unemployed	χ_U	0.60
<i>Asset prices</i>		
Interest rate	$\log R$	0.05
<i>Household preferences</i>		
Relative risk aversion	γ	1.50
Impatience parameter	β	0.95
Bequest preference	b	0.02

As in Wang et al. (2016), the problem is homogeneous in the amount of assets $W_{i,t}$ and permanent labor income $Y_{i,w}$ and so one only needs to solve the value function as a function of $w_{i,t} = W_{i,t}/Y_{i,t}$. We plot the solution of the model in Figure A7. Figure A7a reports the annual purchases of households as a function of their financial wealth and states $x \in \{E, U, R\}$ (averaged across ages). One can see that unemployed agents are net asset sellers, while employed agents tend to be net asset buyers — they only start selling if their financial wealth is 15 times their labor income. At retirement, agents start selling until their financial wealth equals approximately 3 times their labor income pre-retirement. Figure A7b reports the overall density of assets scaled by permanent labor income within employed, unemployed, and retired. Note that the density of assets is even more dispersed as the permanent labor income is itself log-normally distributed across agents, with a variance increasing linearly with age for workers. Finally, figure A7c plots the ratio of marginal utility between employed and unemployed. Naturally, this ratio tends towards $\chi_R^{-\gamma}$ (the ratio of labor income) as assets tend to zero and 1 (perfect insurance) as assets tend to infinity.

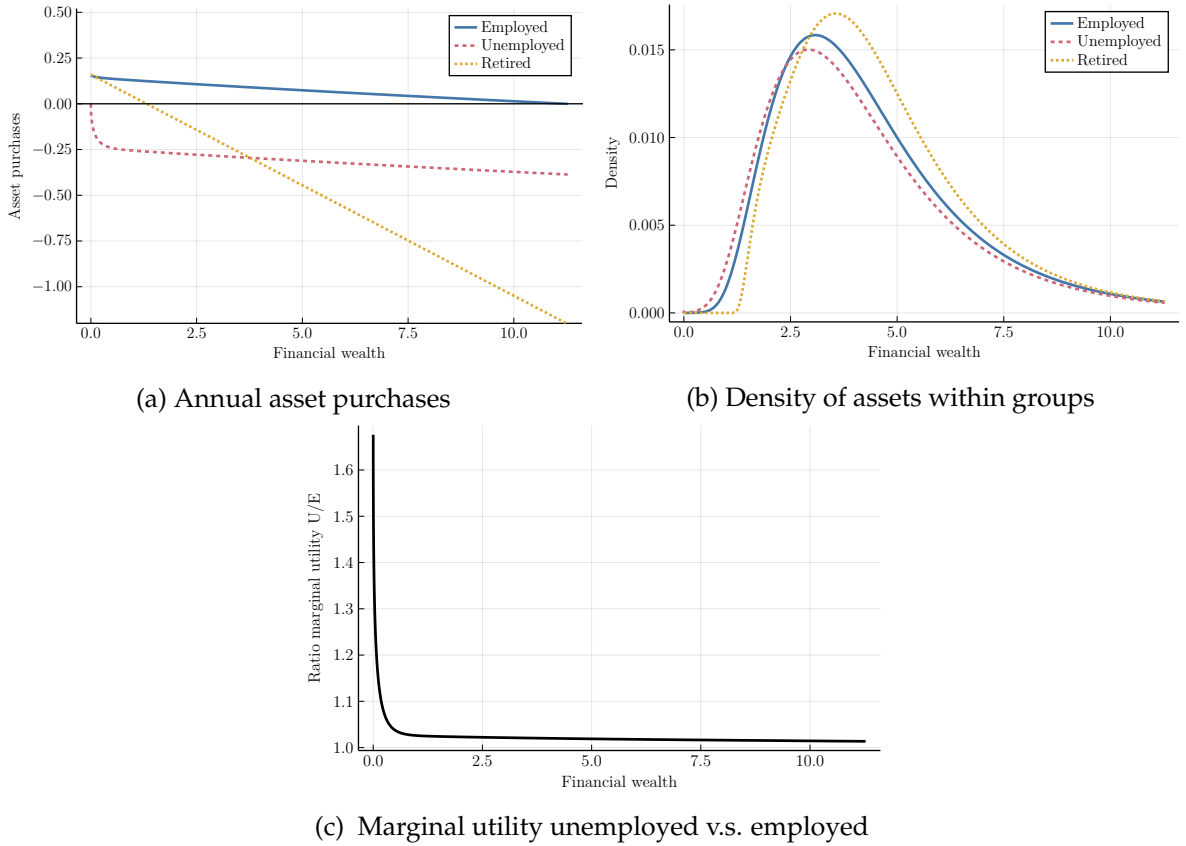


Figure A7: Model solution

Notes. These figures plot important economic quantities in the model calibrated using the parameter values in Table A7. Asset purchases and financial wealth are scaled by permanent labor income.

We now examine the difference between our baseline sufficient statistic formula 10 and the actual welfare gains of a deviation in asset prices in an incomplete market environment (18) through the lens of the calibrated model. More precisely, we consider a mean-reverting deviation in the interest rate $dR_t/R_t = \phi^t dR_0/R_0$ with $dR_0/R_0 = -5\%$ and $\phi = 0.95$, as plotted in Figure A8a. Since the price of the asset is given by $P_t = \sum_{t=1}^{\infty} R_{0 \rightarrow t}^{-1} D$, this implies a deviation in the price of the asset given by $dP_t/P_t = -(1 - \phi/R)^{-1} dR_t/R_t$. For this deviation, we compute two quantities: the actual welfare gain given by (18), where transactions are discounted by the individual-specific marginal rate of substitution $\beta^t U'(C_{i,t})/U'(C_{i,0})$ across states and times, and the one approximated by our sufficient statistics,

where transactions are simply discounted by the interest rate.

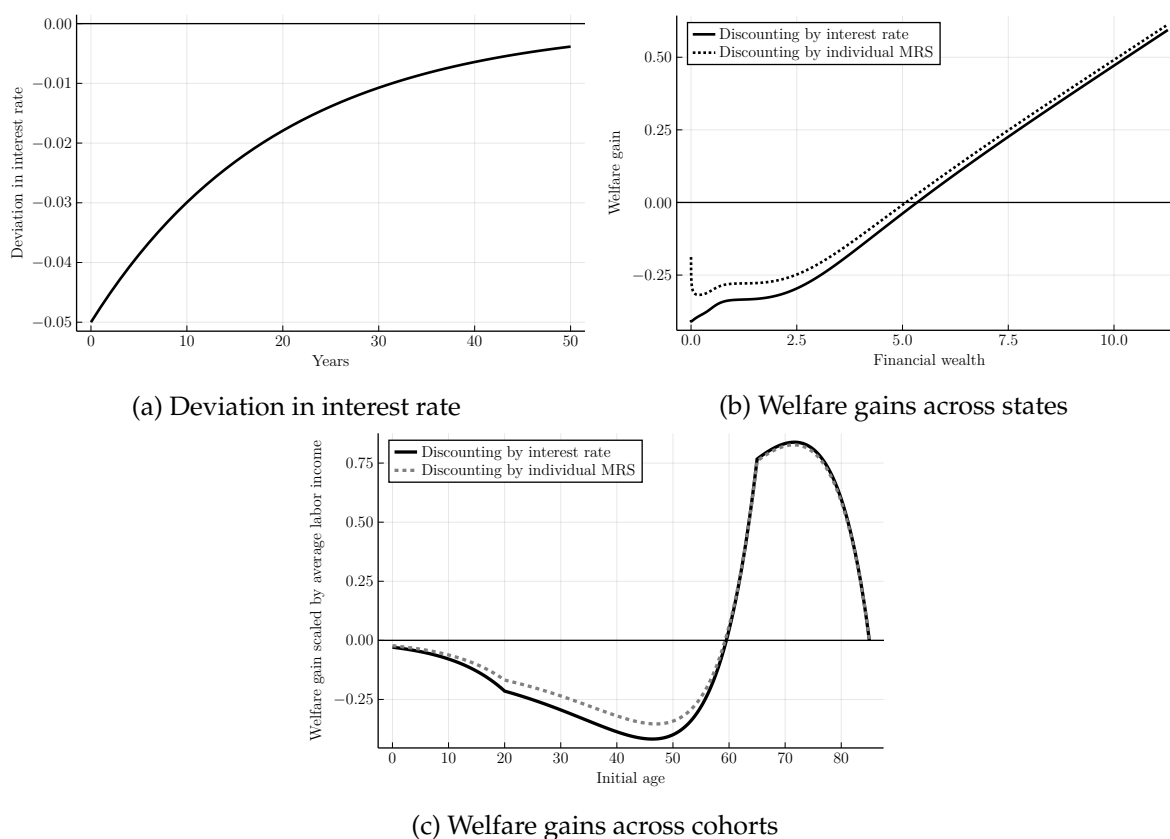


Figure A8: Welfare effect of a deviation in interest rates in calibrated model

Notes. These figures plot the welfare effect of a mean-reverting 5% drop in interest rates in the model calibrated using the parameter values in Table A7. Unless specified otherwise, financial wealth and welfare gains are scaled by permanent labor income. Welfare gains discounting by interest rate correspond to the first term in (19) (i.e., the output of our baseline sufficient statistic approach). The difference between discounting by individual marginal rate of substitution (MRS) versus the interest rate corresponds to the covariance term in (19).

Figure A8b compares these two quantities for workers as a function of their financial wealth. One can see that the “true” welfare gains (discounted by the individual specific marginal rate of substitution or MRS) is always above the one discounted by the interest rate. This positive covariance between individual-specific marginal rates of substitutions and asset sales is driven by two distinct forces. The first force is due to transitory income shocks (the transition between employment and unemployment). As shown in (A7), workers tend to sell assets when they are unemployed, i.e. when their marginal utility of consumption is high. This effect is particularly high for agents in the left tail of the wealth distribution, as the ratio of their marginal utility between being employed and unemployed is particularly high (Figure A7c). The second force is due to permanent income shocks: young workers who experience a positive permanent income shock scale up their asset purchases, as they now target a higher level of savings. Hence, like transitory income shocks, permanent income shocks generate a positive covariance between the growth of marginal utility and asset sales.

Finally, Figure A8c plots the average welfare gains across cohorts. Similarly to our empirical results obtained plotted in Figure 10b, the wedge between the two quantities is particularly important for younger generations: this reflects the fact that, in the model as in the data, younger households face more idiosyncratic risk (through labor income risk) than older households. Finally, note that the model yields substantially higher welfare gains for older households relative to the data — this reflects the fact that, in the data, asset prices only gradually increase over time, whereas they jump to their new values in our experiment.

The key advantage a model, relative to our reduced-form approach, is that we can examine how different calibrations would affect the wedge between our baseline statistic and actual welfare gains. This is useful to understand whether we should expect the wedge between the two quantities to matter in settings outside Norway (external validity). To examine this question, we re-solve the models after successively increasing the degree of income risk (through permanent innovations σ or transitory ones χ_U), the degree of risk aversion for household (relative risk aversion γ), and the persistence of the price deviation (ϕ). For each alternative calibration, we report the average absolute value of the welfare gain across households, the average absolute value of our sufficient statistic, the root mean squared error between the two quantities, as well as the correlation between the two (across all ages and households states). We report the result in Table A8. Intuitively, increasing the degree of income risk or the risk aversion of households increases the wedge between our sufficient statistic formula and the actual welfare gains. Still, note that the difference between the two notions of welfare gains (the theoretical one and our empirical proxy) remain relatively small across the calibrations – in particular, the correlation between the two measures across individuals is very close to one. Overall, these results suggest that the effect of uninsurable labor income risk is likely to be small not just for Norway, but also for other countries.

Table A8: Welfare gains with alternative parameters

	Welfare gains discounted by ...		RMSE	Correlation
	Interest rate (abs. value)	Individual MRS (abs. value)		
Baseline	0.22	0.20	0.05	0.99
σ : 0.10 \rightarrow 0.20	0.56	0.61	0.24	0.97
χ_U : 0.60 \rightarrow 0.30	0.22	0.19	0.05	0.99
γ : 1.50 \rightarrow 3.00	0.29	0.28	0.09	0.99
ϕ : 0.95 \rightarrow 0.99	0.27	0.25	0.10	0.97

Quantifying the effect of idiosyncratic returns The previous model covered the case of idiosyncratic shocks in labor income, an important source of market incompleteness in the left tail of the distribution. We now discuss the case of idiosyncratic shocks in wealth shocks, which is the relevant source of market incompleteness in the right tail of the distribution. We treat this case separately from the previous model for the sake of clarity and because, in this case, we are able to get closed-form formula for the effect of idiosyncratic shocks in the limit of high wealth (i.e., in the limit where labor income is zero).

Formally, consider an individual with CRRA utility that can invest in a risk-free asset with constant interest rate $R = Q^{-1}$ or in a linear technology with i.i.d. stochastic return $A_{i,t}$. Because of the linearity of investment opportunities and of the homotheticity of preferences, it is well known that an agent maximizing his or her expected utility chooses to invest a constant fraction of his or her wealth in the risky technology, α , and to consume a constant fraction of his or her wealth, ρ . As a result, the wealth of the agent evolves as

$$W_{i,t+1} = (R + \alpha (A_{i,t+1} - R) - \rho) W_{i,t}.$$

Such models where wealth evolves with random innovations (“random-growth” models) are particularly important to generate realistic concentration of wealth at the very top. It turns out that, in these models, one can also characterize the effect of incomplete markets in closed form.

Proposition 14. Under the assumptions discussed above, the welfare gain of a deviation in the risk free rate dQ_t is

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \left(R e^{c(1)+c(-\gamma)-c(1-\gamma)} \right)^{-t} (-\mathbb{E}_0[B_{i,t}] dQ_t), \quad (65)$$

where $c(\xi) \equiv \log \mathbb{E} \left[(R + \alpha(A_{i,t} - R) - \rho)^\xi \right]$ denotes the Cumulating Generating Function (CGF) of log wealth growth and where $B_{i,t} = (1 - \lambda)W_{i,t}/Q_t$ denotes the agent holdings in one-period bonds.

This is the same formula as in the baseline model, except that, to adjust for idiosyncratic wealth returns, one needs to adjust the baseline discount rate, R , by a quantity dependent on the dispersion of wealth growth, $e^{c(1)+c(-\gamma)-c(1-\gamma)}$. In the presence of stochastic wealth growth, this adjustment is higher than one as the function $c(\cdot)$ is convex.⁹⁰

This implies that the presence of idiosyncratic wealth shocks systematically decreases the magnitude of welfare gains. The intuition is that, due to the linearity of policy functions, individuals jointly increase their consumption and their financial transactions after a positive wealth shock. As a result, the growth of marginal utility covaries negatively with financial transactions, which dampens welfare gains (in magnitude) relative to the baseline formula. Such a formula would still hold in a model with multiple types of assets: the key requirement is that the agent policy decisions (including financial transactions) scale linearly with the level of wealth. Finally, note that there is a key difference between the effect of transitory labor income shocks and the effect of wealth shocks. As discussed above, the presence of labor income shocks tends to *increase* the welfare gains of rising asset prices (relative to the baseline formula), as agents tend to purchase assets in good times and sell assets in bad times. In contrast, the presence of idiosyncratic return shocks tends to *dampen* welfare gains relative to the baseline formula, as agents tend to scale up their bigger financial transactions (sales or purchases) in good times and scale them down in bad times.

We now discuss how big this adjustment is quantitatively. For instance, consider the case in which R_t^K is log normally distributed with mean μ and variance σ^2 . In this case, using a Taylor approximation, the log growth in wealth is also log normally distributed with mean $\log R + \alpha(\mu - \log R) + \frac{1}{2}\alpha(1 - \alpha)\sigma^2$ and variance $\alpha^2\sigma^2$.⁹¹ For a log normal variable X , we have $\log \mathbb{E} [X^\xi] = \xi\mu + \frac{1}{2}\xi^2\mu^2$. As a result, the formula for welfare gains (65) simplifies to:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \left(R e^{\gamma\alpha^2\sigma^2} \right)^{-t} (-\mathbb{E}_0[B_{i,t}] dQ_t).$$

This formula tells us that, to adjust for idiosyncratic wealth shocks, one simply needs to adjust discount rates by $\gamma\alpha^2\sigma^2$. To take a realistic example, consider the case $\alpha = 1.5$, $\sigma = 10\%$, and $\gamma = 1$, in which case the adjustment is $\gamma\alpha^2\sigma^2 \approx 2$ pp. This justifies our choice of using a relatively high discount rate in our empirical exercise (we use $\log R \approx 0.05$, which is higher than the average midpoint rate between deposit and debt rates in our time sample).

Proof of Proposition 14. We start from the formula for welfare gains in a stochastic environment given in Proposition 6:

$$dV_{i,0}/U'(C_{i,0}) = -\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{U'(C_{i,t})}{U'(C_{i,0})} B_{i,t} \right] dQ_t.$$

⁹⁰See Martin (2013) for similar results on discount rates in the context of aggregate shocks as well as an geometric interpretation for the discount rate adjustment.

⁹¹As shown in Campbell and Viceira (2002), we have

$$\log (R + \alpha(A_{i,t} - R) - c) \approx \log R + \alpha(\log A_{i,t} - \log R) - \rho + \frac{1}{2}\alpha(1 - \alpha)\sigma^2.$$

For an agent with CRRA preferences, we have $U'(C_{i,t})/U'(C_{i,0}) = (C_{i,t}/C_{i,0})^{-\gamma} = (W_{i,t}/W_{i,0})^{-\gamma}$. Now, we have $B_{i,t} = ((1 - \alpha)/Q_t) W_{i,t}$ and so

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= -\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{W_{i,t}^{-\gamma}}{W_{i,0}^{-\gamma}} ((1 - \alpha)/Q_t) W_{i,t} \right] dQ_t \\ &= -\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{W_{i,t}^{1-\gamma}}{W_{i,0}^{1-\gamma}} \right] \mathbb{E}_0 \left[\frac{W_{i,t}}{W_{i,0}} \right]^{-1} \mathbb{E}_0[B_{i,t}] dQ_t \\ &= -\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[\frac{W_{i,1}^{1-\gamma}}{W_{i,0}^{1-\gamma}} \right]^t \mathbb{E}_0 \left[\frac{W_{i,1}}{W_{i,0}} \right]^{-t} \mathbb{E}_0[B_{i,t}] dQ_t, \end{aligned}$$

where the last equation results from the fact that returns (and therefore wealth growth) are i.i.d. Now, Euler equation implies $R^{-1} = \beta \mathbb{E}_0 \left[\frac{W_{i,1}^{-\gamma}}{W_{i,0}^{-\gamma}} \right]$. Plugging this into the previous equation gives

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= -\sum_{t=0}^{\infty} \left(R \frac{\mathbb{E}_0 \left[\frac{W_{i,1}^{1-\gamma}}{W_{i,0}^{1-\gamma}} \right]}{\mathbb{E}_0 \left[\frac{W_{i,1}^{-\gamma}}{W_{i,0}^{-\gamma}} \right] \mathbb{E}_0 \left[\frac{W_{i,1}}{W_{i,0}} \right]} \right)^{-t} \mathbb{E}_0[B_{i,t}] dQ_t \\ &= -\sum_{t=0}^{\infty} \left(R e^{c(1-\gamma) - c(1) - c(-\gamma)} \right)^{-t} \mathbb{E}_0[B_{i,t}] dQ_t, \end{aligned}$$

which concludes the proof. \square

D.2 Borrowing constraints and collateral effects

D.2.1 Proof of Proposition 3

Proof of Proposition 3. The Lagrangian associated with the individual problem is

$$\begin{aligned} \mathcal{L}_i &= \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) \\ &\quad + \sum_{t=0}^{\infty} \lambda_{i,t} (Y_{i,t} + N_{i,t-1} D_{i,t} + B_{i,t-1} - C_{i,t} - (N_{i,t} - N_{i,t-1}) P_t - B_{i,t} Q_{i,t} - \chi(N_{i,t} - N_{i,t-1})). \end{aligned}$$

Totally differentiating the welfare function using the envelope theorem, we obtain

$$\begin{aligned} dV_{i,0} &= \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial P_t} dP_t + \sum_{t=0}^{\infty} \frac{\partial \mathcal{L}_i}{\partial Q_t} dQ_t, \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t) \\ &= \sum_{t=0}^{\infty} \lambda_{i,t} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right). \end{aligned}$$

where the last equality uses the definition of the interest schedule (23). Combining this equation with the first-order condition with respect to consumption $C_{i,t}$, $\beta^t U'(C_{i,t}) = \lambda_{i,t}$, gives:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right). \quad (66)$$

We can go further by using the first-order condition with respect to bond holdings $B_{i,t}$, $\lambda_{i,t} \left(Q_{i,t} + B_{i,t} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right) = \lambda_{i,t+1}$ for $t \geq 0$. Combined with the first-order for consumption, we obtain a modified Euler equation for $t \geq 0$

$$\frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} = Q_{i,t} \left(1 + \frac{B_{i,t}}{Q_{i,t}} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right). \quad (67)$$

This equation says that agents equalize their marginal rate of substitutions between t and $t + 1$ to the effective *marginal* interest rate that they face given the schedule (23). Plugging this equation into (66) gives

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right) \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t \right) \right),$$

which concludes the proof after defining $\tilde{R}_{i,0 \rightarrow t}^{-1} \equiv \prod_{s=0}^{t-1} Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right)$. \square

D.2.2 Empirical implementation

Proof of Corollary 4. The parametric form (26) $Q_{i,t} = Q_t e^{-\zeta \text{LTV}_{i,t}}$ with $\text{LTV}_{i,t} \equiv -B_{i,t}/(N_{i,t} P_t)$ implies the following expressions for the derivatives of the individual-specific bond price $Q_{i,t}$ with respect to asset prices and asset holdings:

$$\frac{\partial Q_{i,t}}{\partial Q_t} = \frac{Q_{i,t}}{Q_t} \quad ; \quad \frac{\partial Q_{i,t}}{\partial B_{i,t}} = \zeta \frac{Q_{i,t}}{N_{i,t} P_t} \quad ; \quad \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} = \zeta \frac{Q_{i,t} \text{LTV}_{i,t}}{N_{i,t} P_t}.$$

Plugging these derivatives into the expression for welfare gains obtained in Proposition 3 gives

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \tilde{R}_{i,0 \rightarrow t}^{-1} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \left(\frac{dQ_t}{Q_t} + \zeta \times \text{LTV}_{i,t} \times \frac{dP_t}{P_t} \right) \right),$$

with $\tilde{R}_{i,0 \rightarrow t}^{-1} = \prod_{s=0}^{t-1} Q_{i,s} (1 - \zeta \times \text{LTV}_{i,t})$. This concludes the proof. \square

Reduced-form evidence on ζ . We now describe how we estimate the parameter ζ in (26). We start from the full sample of individuals over the 1994–2019 period. To compute the implied mortgage interest rate, we first compute the interest costs by outstanding debt, both of which are readily available in our data. We compute the interest rate and loan-to-value of individual i at time t as

$$\text{Interest rate}_{i,t} = \frac{\text{Interest costs}_{i,t}}{\frac{1}{2} \text{Debt}_{i,t-1} + \frac{1}{2} \text{Debt}_{i,t}}, \quad \text{LTV}_{i,t} = \frac{\text{Debt}_{i,t}}{\text{Housing value}_{i,t}},$$

where $\text{Debt}_{i,t} \equiv -B_{i,M,t} Q_{i,M,t}$.

To estimate ζ in (26), we use the approximation $\text{Interest rate}_{i,t} \approx -\log Q_{i,t}$ and estimate a regression of the form

$$\text{Interest rate}_{i,t} = \alpha_t + \zeta \times \text{LTV}_{i,t-1} + u_{i,t},$$

where α_t is a year fixed-effect, and $u_{i,t}$ is an error term. We remove observations with a loan-to-value lower than 0.2 as (i) the interest rate estimate is imprecise for low debt levels and (ii) these low values are more likely to be driven by consumer debt rather than mortgage debt (empirically, we actually find that interest rates decrease between 0 and 0.2). Specification (1) in Table A9 reports the results. The implied value of ζ is approximately 0.0025: a 10 pp. increase in the loan-to-value ratio implies a 0.025 pp. (2.5 basis point) increase in the interest rate.

One potential concern with the regression evidence is the presence of omitted variable bias. For

instance, if some households are deemed safer by banks, they may face an interest rate schedule that is shifted downward and they may endogenously choose a higher loan-to-value ratio. This would dampen the relationship between interest rates and loan-to-value ratios. Therefore, we also estimate a specification with age dummies and education groups as controls. Specification (2) in Table A9 reports the results. The implied value of ζ is approximately 0.005: a 10 pp. increase in the loan-to-value ratio implies a 0.05 pp. (5 basis point) increase in the interest rate.

Table A9: Regression of mortgage interest rate on loan-to-value

Mortgage interest rate	(1)	(2)
Loan-to-value	0.00239*** (0.00003)	0.00459*** (0.00002)
Year fixed effects	✓	✓
Age and education controls		✓
Sample size	26,876,068	28,876,068
R^2	0.507	0.520

Notes. Standard errors in parentheses (***) $p < 0.01$. The education groups are: “less than high school”, “high school”, “college”.

Another potential concern with the regression evidence is that the presence of measurement error in the loan-to-value variable will generate an attenuation bias (i.e., bias the estimate of ζ towards zero). We therefore provide external evidence on the relationship between loan-to-value and mortgage interest rate using posted interest rate schedules published by banks. In Norway, it appears that the largest banks do not provide this data on their websites. However, some smaller banks present the interest rate they charge on a mortgage as a function of the loan-to-value ratio. Table A10 presents an example of such a schedule from *Bulder Bank*.

Table A10: Example of interest rate schedule

Loan-to-value	Interest rate
< 50%	3.33%
50 – 55%	3.41%
55 – 60%	3.51%
60 – 65%	3.56%
65 – 70%	3.60%
70 – 75%	3.64%

Notes. Extracted on October 26 2022 from Bulder Bank’s website (<https://www.bulderbank.no/priser>).

Using the last four rows of Table A10 and using the midpoints of the loan-to-value range, we obtain a linear slope of $\zeta = 0.011$; that is, a 10 pp. increase in the loan-to-value ratio implies a 11 basis point rise in the interest rate. This is roughly four times as large as the regression evidence without controls and twice as large as the regression evidence with controls. For robustness, we focus on estimating the effect of borrowing constraints using two potential estimates $\zeta \in \{0.005, 0.01\}$.

Implementation. Finally, to implement our formula for welfare gains that takes into account borrowing constraints (27), we use the following modified version of our sufficient statistic formula (16):

$$\begin{aligned}
\text{Welfare Gain}_i^{BC} &= \sum_{k \in \{\text{housing,debt,deposit,equity}\}} \text{Welfare Gain}_{i,k}^{BC}, \\
\text{Welfare Gain}_{i,\text{housing}}^{BC} &= \sum_{t=0}^{25} \tilde{R}_{i,0 \rightarrow t}^{-1} \left((N_{i,H,t-1} - N_{i,H,t}) P_{H,t} + \zeta \times \text{LTV}_{i,t}^2 \times N_{i,H,t} P_{H,t} \right) \frac{PD_{H,t} - \overline{PD}_H}{PD_{H,t}}, \\
\text{Welfare Gain}_{i,\text{debt}}^{BC} &= \sum_{t=0}^{25} \tilde{R}_{i,0 \rightarrow t}^{-1} (-B_{i,M,t} Q_{M,t}) \times \frac{Q_{M,t} - \overline{Q}_M}{Q_{M,t}} \\
\text{Welfare Gain}_{i,\text{deposit}}^{BC} &= \sum_{t=0}^{25} \tilde{R}_{i,0 \rightarrow t}^{-1} (-B_{i,D,t} Q_{D,t}) \times \frac{Q_{D,t} - \overline{Q}_D}{Q_{D,t}}, \\
\text{Welfare Gain}_{i,\text{equity}}^{BC} &= \sum_{t=0}^{25} \tilde{R}_{i,0 \rightarrow t}^{-1} (N_{i,E,t-1} - N_{i,E,t}) P_{E,t} \times \frac{PD_{E,t} - \overline{PD}_E}{PD_{E,t}},
\end{aligned} \tag{68}$$

where the individual-specific discount factor $\tilde{R}_{i,0 \rightarrow t}^{-1}$ and loan-to-value ratios $\text{LTV}_{i,t}$ are given by:

$$\begin{aligned}
\tilde{R}_{i,0 \rightarrow t}^{-1} &\equiv R^{-t} e^{-\zeta \sum_{s=0}^{t-1} \text{LTV}_{i,s}} \prod_{s=0}^t (1 - \zeta \times \text{LTV}_{i,s}) \approx R^{-t} e^{-2\zeta \sum_{s=0}^{t-1} \text{LTV}_{i,s}}, \\
\text{LTV}_{i,t} &\equiv \max \left(0, \min \left(-\frac{B_{i,M,t} Q_{M,t}}{N_{i,H,t} P_{H,t}}, 1 \right) \right).
\end{aligned}$$

As in the baseline sufficient statistic formula (16), our measure of debt holdings $B_{i,M,t} Q_{M,t}$ include both directly-held debt and indirectly-held debt (i.e. the debt on the balance sheet of businesses owned by individuals). Similarly, our measure of housing wealth $N_{i,H,t} P_{H,t}$ include both directly-held and indirectly-held housing. Hence this modification of the sufficient statistic formula affects not only individuals who buy a home via a mortgage but also entrepreneurs who use real estate assets as a collateral (as in Chaney et al., 2012).

Figure A9 decomposes the average wealth gain in each cohort into three terms: the baseline welfare gains (the one obtained in the baseline case without borrowing constraints), a term due to the discount rate channel, and a term due to the collateral channel, as defined by (25). Note that the term due to the discount rate channel tends to be negative while the term due to the collateral channel is always positive. Hence, the two effects tend to cancel out, which explains why the overall effect of borrowing constraints is small in our setting.

Histograms. In the main text, we focus on the *average* effect of borrowing constraints on welfare gains within cohorts. However, there is also an important heterogeneity in the effect of this correction within cohorts. In particular, we can expect individuals with a higher amount of debt to disproportionately benefit from the collateral channel.

Table A11 reports the average effect of borrowing constraints on welfare gains in the population for each of our $\zeta \in \{0.005, 0.01\}$ (as the sum of the discount rate and collateral channels). We also report it in six percentile bins. Looking at the row with $\zeta = 0.01$, the average correction due to borrowing constraints is approximately \$2,000 in the population. This is a bit lower than the average baseline welfare gain in the population, which is \$10,000 (Table 2).

Similarly to debt holdings, the effect of borrowing constraints on welfare gains are right-skewed. As shown in Table A11, they become as high as \$30,000 for the top 1% of most affected individuals. Note that this remains much smaller than the top 1% of baseline welfare gains, which is \$645,000 (Table 2).

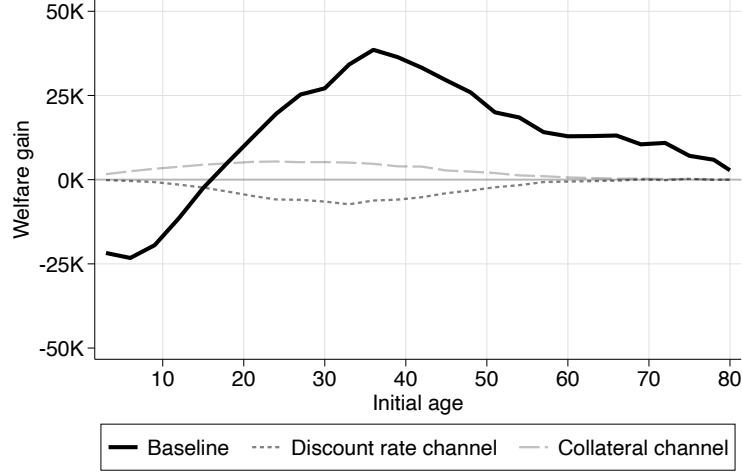


Figure A9: Decomposing the effect of borrowing constraints on welfare gains

Notes. This figure implements the decomposition of welfare gains in the presence of borrowing constraints (25) as a sum of three terms: our baseline formula, a discount rate channel, and a collateral channel. The discount rate channel corresponds to the effect of increasing individual discount rates by $\xi \frac{1}{i} \sum_{s=0}^{t-1} LTV_{i,s}$. The collateral channel corresponds to the effect of increasing the annual home sales of individual by $\xi \times LTV_{i,t}^2 \times N_{i,H,t} P_{H,t} (PD_{H,t} - \overline{PD}_H) / PD_{H,t}$. See (68) for more detail.

Table A11: Distribution of the effect of borrowing constraints on welfare gains

Value for ξ	Average	Average by percentile groups					
		p0-1	p1-10	p10-50	p50-90	p90-99	p99-100
$\xi = 0.005$	0.1	-71.7	-3.5	-0.1	0.6	4.5	52.2
$\xi = 0.01$	0.1	-137.3	-6.9	-0.2	1.1	8.6	97.4

Notes. We compute for each individual the effect of borrowing constraints on welfare gains, computed as the difference between welfare gains taking into account borrowing constraints (68) and the baseline formula (16). As seen in Equation 25, this difference can be interpreted as the sum of the terms corresponding to the discount rate channel and to the collateral channel. The table reports the average effect of borrowing constraints within percentile. All numbers are in thousands of 2011 US dollars.

Figure A10 plots the distribution of the effect of welfare gains across individuals in Norway. To avoid scaling issue, we do not plot the density at zero (i.e., observations with zero debt), as they account for roughly 50% of our observations (see Table A11).

D.2.3 Case of hard borrowing constraints

In the main text, we have focused on quantifying theoretically and empirically the effect of a smooth borrowing constraint (i.e. an interest rate schedule). This methodology choice is guided by our empirical setting, as the gradient of mortgage rate to loan-to-value appears to be fairly linear in Norway. For the sake of external validity, we now discuss how to quantify theoretically and empirically the effect of hard borrowing constraints. We obtain simple formulas that are very similar to the ones obtained in the case of our interest rate schedule (26).

Theory. We consider an economy in which individual i faces a hard borrowing constraint; that is, when there is a strict upper bound on the amount of borrowing:

$$-B_{i,t} \leq \Phi(N_{i,t}P_t), \quad (69)$$

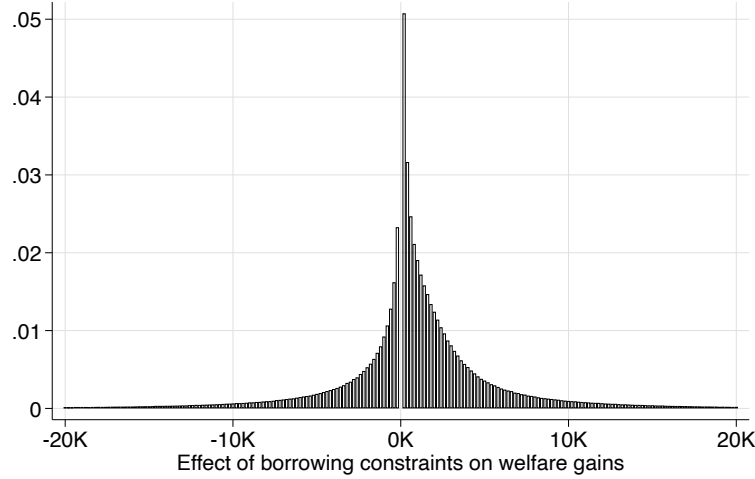


Figure A10: Distribution of the effect of borrowing constraints on welfare gains

Notes. We compute for each individual the effect of borrowing constraints on welfare gains, computed as the difference between welfare gains taking into account borrowing constraints (68) and the baseline formula (16). As seen in Equation 25, this difference can be interpreted as the sum of the terms corresponding to the discount rate channel and to the collateral channel. The figure plots the density of this object across individuals in Norway for $\xi = 0.01$. More precisely, the figure plots the relative mass of individuals within equally-spaced bins of welfare gains (width of \$100). For the sake of legibility, we do not report the relative mass of individuals with a welfare gain lower than 100 in absolute value, as approximately half of the population has no debt. All numbers are in thousands of 2011 US dollars.

where $\Phi(\cdot)$ is a smooth function. This general specification includes both ad-hoc borrowing limits of the form $-B_{i,t} \leq \phi$, where $\phi > 0$ is a borrowing limit, as well as collateral constraints of the form $-B_{i,t} \leq \lambda N_{i,t} P_t$ where $\lambda > 0$ is a maximum ratio of loan to asset value.

Collateral constraints may impact even wealthy individuals. One particular reason is asset illiquidity; that is, the case where adjustment costs $\chi(N_{i,t} - N_{i,t-1})$ are large (the “wealthy hand-to-mouth” of Kaplan and Violante, 2014). Another reason is tax avoidance: in particular, individuals may borrow against their assets to consume as part of a “buy, borrow, die” tax avoidance strategy.⁹² Just like in the case of asset illiquidity, individuals may want to borrow as much as possible and may therefore run into collateral constraints precisely *because* of the differential tax treatment of borrowing and asset sales. Another reason is the fact that rich individuals may get direct utility from ownership of the asset (as in Appendix A.4.1). In this case, individuals may not want to sell their assets precisely *because* of the utility benefit from ownership and may instead be pushed into collateral constraint.⁹³

Note that the hard borrowing constraint (69) can be seen as a limiting case of our general interest schedule framework. To see this formally, consider a particular interest rate schedule (23) of the form:⁹⁴

$$Q_{i,t} = Q_t \left(1 + e^{-\theta(B_{i,t} + \Phi(N_{i,t} P_t) + 1/\sqrt{\theta})} \right)^{-1}. \quad (70)$$

⁹²One main reason individuals use a “buy, borrow, die” strategy is step-up in basis at death. This feature of the U.S. tax system (and some other countries) means that dying without ever having sold an asset and passing it on to an heir greatly reduces the heir’s capital gains tax bill if he or she sells the inherited asset.

⁹³Note that, in the case of tax avoidance, rising asset prices will also have an effect on the *relative* welfare of asset sellers who use the strategy relative to those who do not. When asset prices rise, asset sellers who do not use the “buy, borrow, die” strategy pay higher capital gains taxes which attenuates their welfare gain (as in Appendix A.4.4). In contrast, this attenuation effect is smaller (or non-existent) for individuals who use the strategy because they pay less (or no) capital gains taxes in the first place.

⁹⁴The term in $1/\sqrt{\theta}$ is to ensure that $Q_{i,t} \rightarrow Q_t$ as $\theta \rightarrow \infty$ at the upper bound $-B_{i,t} = \Phi(N_{i,t} P_t)$.

As $\theta \rightarrow \infty$, this interest rate schedule converges to

$$\lim_{\theta \rightarrow \infty} Q_{i,t} = \begin{cases} Q_t & \text{if } -B_{i,t} \leq \Phi(N_{i,t}P_t) \\ 0 & \text{otherwise.} \end{cases}$$

In other words, individuals can borrow at the reference interest rate Q_t^{-1} whenever the borrowing limit $-B_{i,t} \leq \Phi(N_{i,t}P_t)$ is satisfied but they face an interest rate of infinity otherwise; hence, this limiting economy corresponds to the hard borrowing constraint (69). See Figure A11 for a graphical representation.

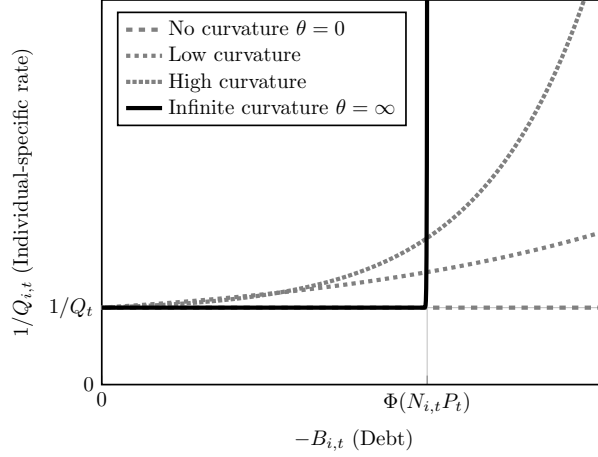


Figure A11: Interest rate schedule (70) for different values of θ

The following proposition gives an expression for the welfare gains of a deviation in asset prices in the presence of a hard borrowing constraint.

Proposition 15. *When agent i faces a hard borrowing constraint (69), the welfare gain of a deviation in asset prices is*

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t + \left(Q_t - \beta \frac{U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t}P_t) N_{i,t} dP_t \right).$$

This proposition says that the presence of a hard borrowing constraint affects our welfare gain formula through two channels: a discount rate channel and a collateral channel. First, when the borrowing constraint binds, the Euler equation is not satisfied and $\beta U'(C_{i,t+1})/U'(C_{i,t}) < 1/Q_t$, which means that agents discount more future dollars compared to the interest rate on the liquid asset (“discount rate” channel). Second, when $\Phi'(N_{i,t}P_t) > 0$, a rise in asset prices relaxes the borrowing constraint, which allows the individual to consume more today at the expense of tomorrow. This increases welfare as long as the borrowing constraint binds because, in this case, the benefit of increasing consumption today at time t , $U'(C_{i,t})$, is strictly higher than the cost of decreasing consumption tomorrow $\beta U'(C_{i,t+1})/Q_t$ (“collateral” channel).

One implication of this proposition is that, in this setting, the welfare gains of a deviation in asset prices no longer aggregate to zero. First, there is an effect due to the discount rate channel. With $\Phi(\cdot) > 0$, agents at the borrowing constraint tend to be net borrowers, and so they disproportionately benefit from the welfare effect of lower interest rates. Because they are also the ones with higher discount rates, this tends to reduce the sum of welfare gains due to lower rates across individuals. Second, there is an effect due to the collateral channel: when $\Phi'(N_{i,t}P_t) > 0$, the formula for welfare gains has an additional

term that is positive for every asset holder. While the first effect has an ambiguous sign in general, the second effect is always positive.

We now briefly compare Proposition 15, which focuses on the effect of a hard borrowing constraint, to Proposition 3 (Section 4.2), which focuses on the effect of a smooth interest rate schedule. Both propositions say that borrowing constraints affect welfare gains through two channels: a discount rate channel and a collateral channel. However, the intuition for the collateral channel is slightly different across the two propositions. Proposition 3 says that, in the presence of an interest rate schedule, collateral constraints affect welfare gains because higher asset prices allow agents to borrow at a lower interest rate. In comparison, Proposition 15 says that, in the case of a hard borrowing constraint, collateral constraints affect welfare gains because higher asset prices allow constrained individuals to better smooth their consumption over time. To understand the connection between the two results, note that $U'(C_{i,t})/(\beta U'(C_{i,t+1}))$ can be seen as the shadow interest rate of individual i ; that is, to the interest rate at which the individual is indifferent to lend or to borrow a dollar. Seen this way, as in the case of the interest rate schedule of Proposition 3, Proposition 15 says that a rise in asset prices increases welfare because it allows individuals at the borrowing constraint to decrease their borrowing costs, as they can borrow an infinitesimal amount $\Phi'(N_{i,t}P_t)N_{i,t}dP_t$ at rate $1/Q_t$ rather than at the shadow rate $U'(C_{i,t})/(\beta U'(C_{i,t+1}))$.

Finally, as in the case of the interest rate schedule (Proposition 3), we can formally write the expression for welfare gains in the presence of a hard borrowing constraint as a sum of three terms, which capture, respectively, welfare gains in the baseline (frictionless) model, a term capturing the discount rate channel, and a term capturing the collateral channel.

$$\begin{aligned}
dV_{i,0}/U'(C_{i,0}) &= \underbrace{\sum_{t=0}^{\infty} R_{0,t}^{-1} ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t)}_{\text{Baseline}} \\
&\quad + \underbrace{\sum_{t=0}^{\infty} \left(\frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} - Q_0 \dots Q_{t-1} \right) ((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} dQ_t)}_{\text{Discount rate channel}} \\
&\quad + \underbrace{\sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \left(Q_t - \beta \frac{U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t}P_t) N_{i,t} dP_t}_{\text{Collateral channel}}. \tag{71}
\end{aligned}$$

Proof of Proposition 15. One way to derive this expression is to apply the usual Lagrangian method augmented with the hard borrowing constraint (69). Instead, for the sake of intuition, we use a derivation that relies on the expression for welfare gains obtained in the case of an interest rate schedule and passes to the limit of a “hard” borrowing constraint. As stated in Proposition 3 the welfare gain of a deviation in asset prices when the agent faces an interest rate schedule is:

$$\begin{aligned}
dV_{i,0}/U'(C_{i,0}) &= \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right) \right) \\
&\quad \times \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} \left(\frac{\partial Q_{i,t}}{\partial Q_t} dQ_t + \frac{\partial Q_{i,t}}{\partial (N_{i,t}P_t)} N_{i,t} dP_t \right) \right).
\end{aligned}$$

Remember that the modified Euler equation (67) in the case of an interest rate schedule is

$$Q_{i,t} \left(1 + \frac{B_{i,t}}{Q_{i,t}} \frac{\partial Q_{i,t}}{\partial B_{i,t}} \right) = \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})}.$$

This implies that the discount rate in the expression for welfare gains can be rewritten as:

$$\prod_{s=0}^{t-1} \left(Q_{i,s} \left(1 + \frac{B_{i,s}}{Q_{i,s}} \frac{\partial Q_{i,s}}{\partial B_{i,s}} \right) \right) = \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})}. \quad (72)$$

This also implies that the term capturing the effect of collateral constraints can be rewritten as:

$$-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t = \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \frac{\partial Q_{i,t} / \partial (N_{i,t} P_t)}{\partial Q_{i,t} / \partial B_{i,t}} N_{i,t} dP_t,$$

where the quantity $\frac{\partial Q_{i,t} / \partial (N_{i,t} P_t)}{\partial Q_{i,t} / \partial B_{i,t}} N_{i,t} dP_t$ can be interpreted as the additional amount of cash that agent i can borrow as a result of the change in asset prices while keeping the interest rate constant. Hence, this expression says that the collateral channel in an interest schedule model can be seen in two equivalent ways: as the monetary value of lower interest costs in the left-hand-side (corresponding to the case where the agent holds the amount of debt fixed), or as the welfare value of better consumption smoothing in the right-hand-side (obtained when the agent borrows more in response to the rise in asset prices, keeping the individual-specific interest rate fixed).

In the particular case of the interest rate schedule (70), the last equation simplifies to

$$-B_{i,t} \frac{\partial Q_{i,t}}{\partial (N_{i,t} P_t)} N_{i,t} dP_t = \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t} P_t) N_{i,t} dP_t. \quad (73)$$

Plugging (72) and (73) into the expression for welfare gains given above yields

$$dV_{i,0} / U'(C_{i,0}) = \sum_{t=0}^{\infty} \frac{\beta^t U'(C_{i,t})}{U'(C_{i,0})} \times \left((N_{i,t-1} - N_{i,t}) dP_t - B_{i,t} Q_{i,t} \frac{dQ_t}{Q_t} + \left(Q_{i,t} - \frac{\beta U'(C_{i,t+1})}{U'(C_{i,t})} \right) \Phi'(N_{i,t} P_t) N_{i,t} dP_t \right).$$

Since it is true for any θ , it is also valid in the limit $\theta \rightarrow \infty$ (i.e., in the limit of a hard borrowing constraint), which concludes the proof. \square

Calibration approach. We now turn to the quantification of the effect of a hard borrowing constraint on our welfare gains formula. As seen in (71), the effects of the discount rate and collateral channels depend on two important statistics: (i) the difference between the shadow rate at which the individual is willing to borrow and the market rate, as represented by $Q_t - \beta U'(C_{i,t+1}) / U'(C_{i,t})$, and (ii) the extent to which higher asset prices actually relax the borrowing limit, $\Phi'(N_{i,t} P_t)$.

We first discuss the quantification of $Q_t - \beta U'(C_{i,t+1}) / U'(C_{i,t})$. This term is zero for individuals outside the borrowing constraint but positive for individuals at the borrowing constraint. As discussed above, this term can be interpreted as the difference between the highest rate at which the individual would be willing to borrow and the cost of debt. As a specific example, it is useful to think of an entrepreneur (or an investor) who has access to a decreasing-return-to scale technology (or an investment opportunity), and who cannot equalize the marginal return of the technology with the marginal cost of funds because of collateral constraint.

We now turn to the quantification of $\Phi'(N_{i,t} P_t)$. Many models assume that borrowing constraints take the form $\Phi(N_{i,t} P_t) = \lambda_i N_{i,t} P_t$, where λ_i corresponds to the limiting debt to asset ratio, which implies $\Phi'(N_{i,t} P_t) = \lambda_i$. Alternatively, other models assume that borrowing limits depend on earnings rather than collateral values (e.g. Lian and Ma, 2021, Drechsel, 2023), in which case $\Phi'(N_{i,t} P_t) = 0$.⁹⁵

⁹⁵We would also obtain a derivative of zero in a model where the borrowing limit only depends on the “book-

Given an estimate for these two objects, we can use (71) to quantify the effect of borrowing constraints on welfare gains. For the sake of calibration, we now assume that the wedge between the shadow interest rate and the cost of funding averages to $Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t}) = 5\%$ for agents at the constraint, that $\pi = 10\%$ of agents are at the borrowing constraint, and that the maximum loan-to-value ratio averages to $\lambda_i = 0.5$.⁹⁶ In this case, (71) tells us that, on average, borrowing constraints increase the discount rate of households by $\pi(Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t})) \approx 0.5\text{pp}$ (discount rate channel) and that collateral constraints increase the welfare exposure of home owners to a rise in asset prices by an amount equivalent to an increase in their annual rate of home sales by $\pi(Q_t - \beta U'(C_{i,t+1})/U'(C_{i,t})) \lambda_i \approx 0.25\text{pp}$. (collateral channel).

Note that this quantification exercise ends up being very similar, in spirit and in magnitude, to the quantification exercise done for interest rate schedules in the main text (Section 4.3). Indeed, we estimated that, for the interest rate schedule (26) with $\zeta = 0.01$ and an agent loan-to-value ratio $\text{LTV}_{i,t} = 0.5$, borrowing constraints increased the agent discount rate of agent by $2\zeta \times \text{LTV}_{i,t} \approx 1\text{pp}$. (discount rate channel) and that collateral constraints increased the welfare exposure of home owners to a rise in asset prices by an amount equivalent to an increase in their annual rate of home sales by $\zeta \times \text{LTV}_{i,t}^2 \approx 0.25\text{pp}$. (collateral channel). Hence, the two models end up giving similar results. The main difference between the two models, however, is that, with an interest rate schedule, borrowing constraints affect all individuals in the economy (as long as they have some outstanding debt) while, with a hard borrowing constraint, borrowing constraints purely affect the subset of individuals for which the borrowing constraint is binding.⁹⁷

D.3 Second-order

We now detail how we implement our second-order approximation (29) under the assumption (30). Multiplying and dividing each term in (29) by the price of the corresponding asset gives:

$$\text{EV}_i(\theta) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(\theta/2) \left\{ \sum_{k=1}^K \left(\frac{(N_{i,k,t-1}(0) - N_{i,k,t}(0))P_{k,t}}{2} + \frac{(N_{i,k,t-1}(\theta) - N_{i,k,t}(\theta))P_{k,t}}{2} \right) \frac{\Delta P_{k,t}}{P_{k,t}} - \frac{B_{i,t}(0)Q_t + B_{i,t}(\theta)Q_t}{2} \frac{\Delta Q_t}{Q_t} \right\} + o(\theta^2).$$

Averaging across cohorts gives

$$\overline{\text{EV}}_a(\theta) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1}(\theta/2) \left\{ \sum_{k=1}^K \left(\frac{(\overline{N}_{i,k,t-1}(0) - \overline{N}_{i,k,t}(0))P_{k,t}}{2} + \frac{(\overline{N}_{i,k,t-1}(\theta) - \overline{N}_{i,k,t}(\theta))P_{k,t}}{2} \right) \frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\overline{B}_{a,t}(0)Q_t + \overline{B}_{a,t}(\theta)Q_t}{2} \frac{\Delta Q_t}{Q_t} \right\} + o(\theta^2).$$

value" of asset holdings $N_{i,t}$ rather their market value $N_{i,t}P_t$.

⁹⁶These last two numbers are roughly consistent with Chaney et al. (2012), who estimate that firm investment increases by \$0.03 in response to a \$1 rise in house prices. Direct evidence on the number of firms that are borrowing constrained is more scarce, Champagne and Gouin-Bonenfant (2023) estimate that 4% of Canadian SMEs have been constrained within the last year.

⁹⁷This distinction would not be as stark in a stochastic environment, as most individuals would have some probability of hitting the constraint in the future.

Our assumption on counterfactual asset transactions (30) implies

$$\begin{aligned} (\bar{N}_{a,k,t}(\theta) - \bar{N}_{a,k,t-1}(\theta)) P_{k,t} &= \frac{G^t PD_{k,t}}{PD_{k,0}} (\bar{N}_{a,k,0}(0) - \bar{N}_{a,k,-1}(0)) P_{k,0} \\ \bar{B}_{a,t}(\theta) Q_t &= G^t \frac{Q_t}{Q_0} \bar{B}_{a,0}(0) Q_0. \end{aligned} \quad (74)$$

Plugging into the formula for welfare gains above gives

$$\begin{aligned} \overline{EV}_a(\theta) &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (\theta/2) \left\{ \sum_{k=1}^K \left(\frac{(\bar{N}_{i,k,t-1}(0) - \bar{N}_{i,k,t}(0)) P_{k,t}}{2} + \frac{G^t PD_{k,t}}{PD_{k,0}} \frac{(\bar{N}_{a,k,0}(0) - \bar{N}_{a,k,-1}(0)) P_{k,0}}{2} \right) \frac{\Delta P_{k,t}}{P_{k,t}} \right. \\ &\quad \left. - \frac{\bar{B}_{a,t}(0) Q_t + G^t \frac{Q_t}{Q_0} \bar{B}_{a,0}(0) Q_0}{2} \frac{\Delta Q_t}{Q_t} \right\} + o(\theta^2). \end{aligned}$$

Note that all terms are now observable empirically, and so we can implement this formula in the data.

D.4 Valuation changes beyond the end of our sample period

We now explain how we constructed individuals' transactions in the future. As discussed in the main text, we assume that the average quantity of assets sold by a given cohort in a given year will equal the average quantity of assets sold by the cohort with the same age in 2019, after adjusting for economic growth. Formally, we assume that the average transactions of individuals of age a at time $t > T$ is given by:

$$\begin{aligned} \bar{N}_{a,k,t} - \bar{N}_{a,k,t-1} &= \bar{N}_{a,k,T} - \bar{N}_{a,k,T-1}, \\ \bar{B}_{a,t} &= \bar{B}_{a,T}. \end{aligned} \quad (75)$$

We predict that real per-capita growth rate of the economy will be $G = 1.01$ going forward (which corresponds to the per-capita growth rate of Norway's GDP over our time sample). Under this assumption, we have, for $t > T$, $P_{k,t} = PD_{k,t} D_t = PD_{k,t} G^{t-T} D_T = \frac{G^{t-T} PD_{k,t}}{PD_{k,T}} P_{k,T}$, where the price-dividend ratio $PD_{k,t}$ is given by (31). Hence, we can construct future transaction values as:

$$\begin{aligned} (\bar{N}_{a,k,t} - \bar{N}_{a,k,t-1}) P_{k,t} &= \frac{G^{t-T} PD_{k,t}}{PD_{k,T}} (\bar{N}_{a,k,T} - \bar{N}_{a,k,T-1}) P_{k,T}, \\ \bar{B}_{a,t} &= \bar{B}_{a,T}, \end{aligned}$$

where the right-hand-side is observable. Finally, we predict the population size in each cohort using the average death rate by age group in our sample (and that the initial size of future cohorts is the same as the size of the youngest cohort in our sample).

Figure A12 decomposes the welfare gains asset class by asset class. Interestingly, one can see that most of the effect of extrapolating price deviation on total welfare is driven by the effect of lower interest rates on debt.

D.5 Combining all extensions

For the sake of transparency, we have analyzed the effect of each extension separately. We conclude this section by analyzing the effect of combining all extensions at the same time. We start by a simple exercise in which we simply add the effect of each extension linearly. Figure A13 plots the result. Overall, the combined effect of extensions is to increase welfare gains across the distribution, especially

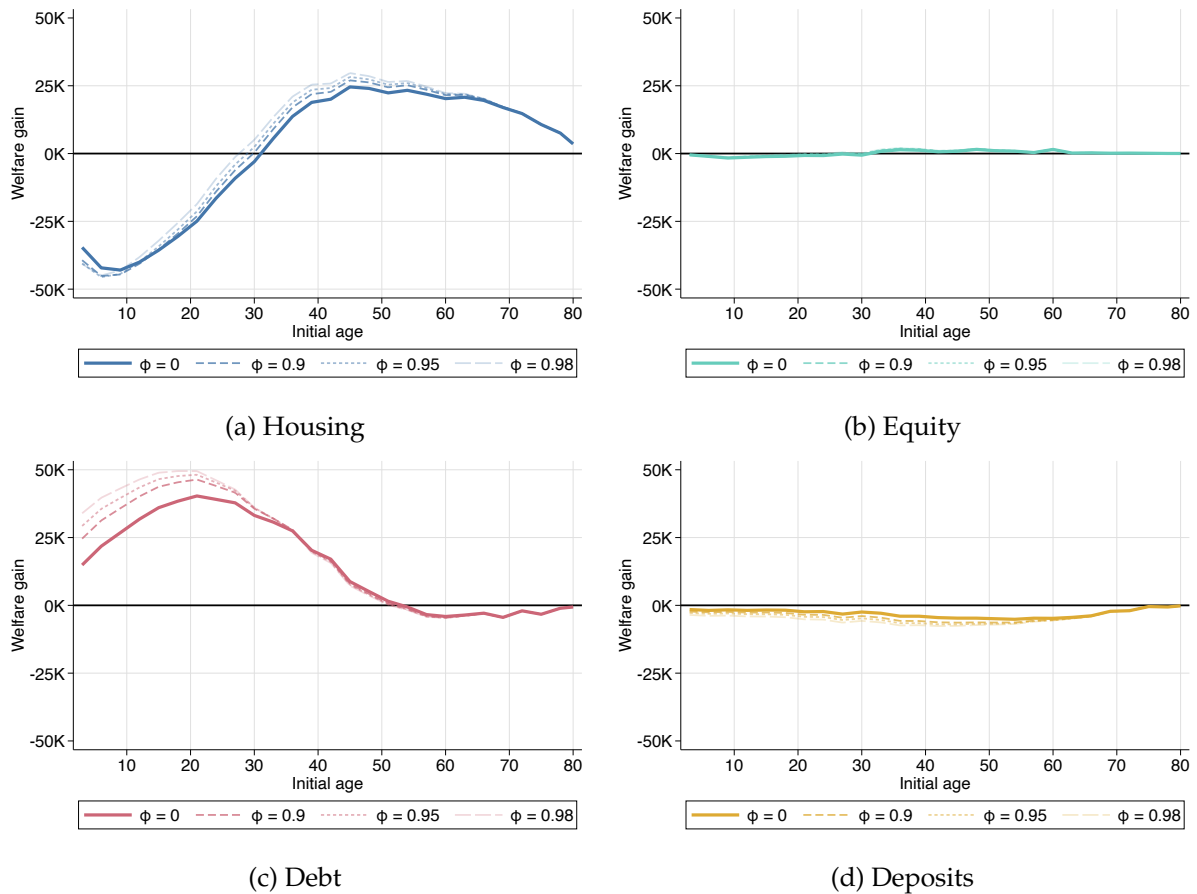


Figure A12: Welfare gain depending on the behavior of asset prices in the future, asset class by asset class

for middle-aged cohorts.

Additionally, we do a second exercise where we combine all extensions at the same time, taking into account the interaction terms between the different extensions (e.g., using the individual-specific discount rate from the borrowing constraints to discount the covariance terms between consumption and asset transactions). One significant challenge is that we need to extrapolate the covariance terms between asset transactions and consumption (computed in Section 4.1) beyond our time sample. To do so, for cohorts born before 1974 (i.e. older than 20 years old at the start of our sample), we use a log linear extrapolation of the covariance terms estimated in the last five years of the sample. For cohorts born after 1974, we use the life-cycle pattern of covariance terms estimated for the 1970-1974 cohorts (including the log-linear extrapolation mentioned above). Figure A13 plots the result of combining all extensions together. We find that the result is relatively close to the effect of adding extensions linearly, which validates our decision to consider each extension separately in the main text (for the sake of simplicity).

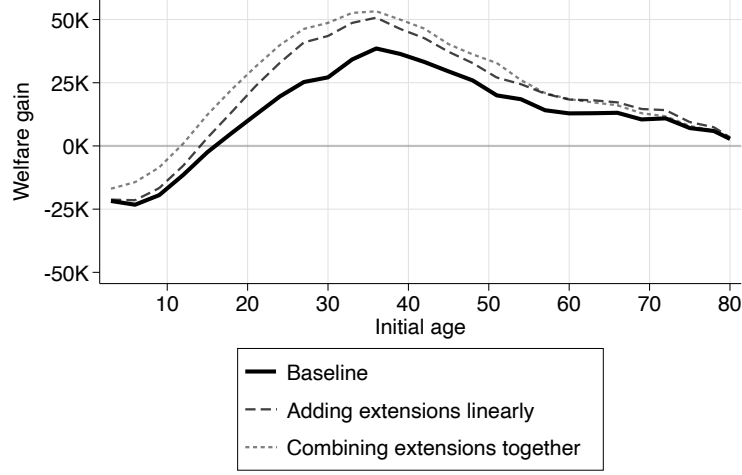


Figure A13: Welfare gains across cohorts after combining all extensions

E Additional theoretical results

E.1 Two-periods model

Deriving Equation 5. We first provide a derivation of Equation (5). The Lagrangian associated with the agent optimization problem is

$$\mathcal{L}_i = U(C_{i,0}) + \beta U(C_{i,1}) + \lambda_{i,0}(Y_{i,0} + (N_{i,-1} - N_{i,0})P_0 - C_{i,0}) + \lambda_{i,1}(N_{i,0}D_1 + Y_{i,1} - C_{i,1}).$$

The envelop theorem gives

$$\frac{\partial V_{i,0}}{\partial P_0} = \frac{\partial \mathcal{L}_i}{\partial P_0} = \lambda_{i,0}(N_{i,-1} - N_{i,0})P_0.$$

The first-order with respect to consumption at time $t = 0$ gives $\lambda_{i,0} = U'(C_{i,0})$ which concludes the proof.

Solving for the response of consumption As discussed in Proposition 17, our notion of welfare gains can be interpreted as the present-value of the deviation in consumption in response to the deviation in asset prices; indeed, by definition, we have

$$dV_{i,0} = dU(C_{i,0}) + \beta dU(C_{i,1}) = U'(C_{i,0}) dC_{i,0} + \beta U'(C_{i,1}) dC_{i,1}.$$

Dividing by $U'(C_{i,0})$ and using the Euler equation implies

$$dV_{i,0}/U'(C_{i,0}) = dC_{i,0} + R_1^{-1} dC_{i,1}.$$

where $R_1 \equiv D_1/P_0$. Hence, what this equation says is that the present-value of the deviation in consumption in response to asset prices is pinned down by our sufficient statistic formula (Equation 5). However, our results are silent on how much of the consumption response happens at time $t = 0$ relative to time $t = 1$. We now derive the “term structure” of the consumption response in the context of the two-periods model and emphasize that, in contrast with our expression for welfare gains, it depends critically on individual preferences and, in particular, on the individual elasticity of intertemporal substitutions.

Lemma 16. *In the two-period model, the response of consumption to an asset-price deviation is:*

$$\begin{aligned} dC_{i,0} &= \underbrace{\text{MPC}_{i,0} \times (N_{i,-1} - N_{i,0}) dP_0}_{dC_{i,0}^{\text{income}}} & \underbrace{-\psi(C_{i,1}) \times \text{MPC}_{i,0} \times C_{i,1} R_1^{-1} d \log R_1}_{dC_{i,0}^{\text{substitution}}} \\ dC_{i,1} &= \underbrace{(1 - \text{MPC}_{i,0}) \times R_1 \times (N_{i,-1} - N_{i,0}) dP_0}_{dC_{i,1}^{\text{income}}} & \underbrace{+\psi(C_{i,1}) \times \text{MPC}_{i,0} \times C_{i,1} d \log R_1}_{dC_{i,1}^{\text{substitution}}} \end{aligned} \quad (76)$$

where $\text{MPC}_{i,0} \equiv \left(1 + R_1^{-1} \frac{C_{i,1} \psi(C_{i,1})}{C_{i,0} \psi(C_{i,0})}\right)^{-1}$ is the marginal propensity to consume out of income at time $t = 0$ and $\psi(C) \equiv -U'(C)/(U''(C)C)$ denotes the local elasticity of intertemporal substitution.

The expressions in obtained for $dC_{i,0}$ and $dC_{i,1}$ are similar to those in [Auclert \(2019\)](#). Classically, the response of consumption to a change in asset prices can be decomposed into an income effect and a substitution effect. The income effect, denoted $dC_{i,t}^{\text{income}}$, corresponds to the response of consumption to a change in initial income that is welfare-equivalent to the change in prices, $(N_{i,-1} - N_{i,0}) dP_0$. In comparison, the substitution effect, denoted $dC_{i,t}^{\text{substitution}}$, corresponds to the response of consumption to the change in prices holding welfare constant. Note that the substitution effect of a higher asset price (i.e., a lower asset return) is positive at $t = 0$ and negative at $t = 1$. Consistently with these definitions, we have the present value of the income response equates the welfare gain while the present value of the substitution effect equates zero

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= dC_{i,0}^{\text{income}} + R_1^{-1} dC_{i,1}^{\text{income}}, \\ 0 &= dC_{i,0}^{\text{substitution}} + R_1^{-1} dC_{i,1}^{\text{substitution}}. \end{aligned} \quad (77)$$

Proof of Lemma 16. Differentiating the budget constraints at $t = 0$ and $t = 1$ gives

$$\begin{aligned} dC_{i,0} + (N_{i,0} - N_{i,-1}) dP_0 + dN_{i,0} P_0 &= 0 \\ dC_{i,1} &= D_1 dN_{i,0}. \end{aligned}$$

Substituting out $dN_{i,0}$ gives the consolidated (differentiated) budget constraint:

$$dC_{i,0} + \frac{P_0}{D_1} dC_{i,1} = (N_{i,-1} - N_0) dP_0. \quad (78)$$

Now, differentiating the Euler equation $\beta U'(C_{i,1})/U'(C_{i,0}) = P_0/D_1$ gives

$$\frac{U''(C_{i,1})}{U'(C_{i,1})} dC_{i,1} - \frac{U''(C_{i,0})}{U'(C_{i,0})} dC_{i,0} = \frac{dP_0}{P_0}. \quad (79)$$

Hence, we obtain a system of two equations, (78) and (79), and two unknowns, $dC_{i,0}$ and $dC_{i,1}$. This system can be represented as a matrix equation:

$$\begin{pmatrix} 1 & \frac{P_0}{D_1} \\ -\frac{U''(C_{i,0})}{U'(C_{i,0})} & \frac{U''(C_{i,1})}{U'(C_{i,1})} \end{pmatrix} \begin{pmatrix} dC_{i,0} \\ dC_{i,1} \end{pmatrix} = \begin{pmatrix} (N_{i,-1} - N_{i,0}) dP_0 \\ \frac{U''(C_{i,1})}{U''(C_{i,1})} \frac{dP_0}{P_0} \end{pmatrix}.$$

Solving this system gives

$$\begin{pmatrix} dC_{i,0} \\ dC_{i,1} \end{pmatrix} = \frac{1}{1 + \frac{U''(C_{i,0})}{U'(C_{i,0})} \frac{U''(C_{i,1})}{U'(C_{i,1})} \frac{P_0}{D_1}} \begin{pmatrix} 1 & -\frac{P_0}{D_1} \\ \frac{U''(C_{i,0})}{U'(C_{i,0})} & \frac{U''(C_{i,1})}{U'(C_{i,1})} \end{pmatrix} \begin{pmatrix} (N_{i,-1} - N_{i,0}) dP_0 \\ \frac{U''(C_{i,1})}{U''(C_{i,1})} \frac{dP_0}{P_0} \end{pmatrix}.$$

Using the definition $R_1 = D_1/P_0$ and $\psi(C) = -U'(C)/(U''(C)C)$, this simplifies to

$$\begin{pmatrix} dC_{i,0} \\ dC_{i,1} \end{pmatrix} = \frac{1}{1 + R_1^{-1} \frac{C_{i,1}\psi(C_{i,1})}{C_{i,0}\psi(C_{i,0})}} \begin{pmatrix} 1 & -R_1^{-1} \\ \frac{C_{i,1}\psi(C_{i,1})}{C_{i,0}\psi(C_{i,0})} & 1 \end{pmatrix} \begin{pmatrix} (N_{i,-1} - N_{i,0}) dP_0 \\ \psi(C_{i,1}) C_{i,1} d \log R_1 \end{pmatrix}. \quad (80)$$

To conclude the proof, it remains to separate the total consumption response into an income and substitution effect. By definition, the income effect is the response of consumption to a change in income at $t = 0$ that is welfare-equivalent to the change in prices; that is, $dY_{i,0} = (N_{i,-1} - N_{i,0}) dP_0$. It can be obtained by setting $d \log R_1$ to zero in (80). Conversely, the substitution effect is defined as the response of consumption to a change in price dP_0 holding welfare constant. It can be obtained by setting dP_0 to zero in (80). This concludes the proof. \square

E.2 Equivalent interpretations of welfare gains

In our main text, we defined the welfare gain of a deviation in asset prices as the amount of money, received in the baseline economy, that would have the same impact on welfare as the deviation in asset prices (equivalent variation). As long as the deviation in asset prices is infinitesimal, it is well known that this quantity is the same as (minus) the amount of money, received in the perturbed economy, that would return the agent to their original welfare level (compensating variation). We now list five other equivalent ways of thinking about our notion of welfare gains in our baseline model.

Proposition 17. *Consider the baseline economy described in Section 1.2 with an infinitesimal deviation in asset prices. The following objects are equal:*

- (i) *Willingness to pay for the deviation in asset prices (equivalent variation)*
- (ii) *Deviation in welfare divided by the marginal utility of consumption*
- (iii) *Present value of the deviation in consumption*
- (iv) *Present value of the deviation of consumption if the individual were to maintain their original path of asset holdings*
- (v) *Present value of consumption times the relative deviation in consumption in every period that would generate the same increase in welfare (Lucas's notion of welfare gain)*
- (vi) *Minus the deviation in initial wealth necessary for individuals to maintain their original paths of consumption and asset holdings (Slutsky's wealth compensation)*

Proof of Proposition 17. We show each equivalence in succession. We first show the equality between (i) and (ii) Denote $V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t, \{P_{k,t}\}_k\}_{t=0}^\infty)$ the value function (or indirect utility) of the individual problem maximizing (8) subject to (9). Define the equivalent variation of a non-infinitesimal deviation in asset prices $\{\theta \Delta Q_t, \{\theta \Delta P_{k,t}\}_k\}_{t=0}^\infty$, denoted $EV_i(\theta)$, as the amount of money, at time $t = 0$, which would have the equivalent change in welfare as the deviation in asset prices; that is,

$$V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1} + EV_i(\theta), \{Q_t, \{P_{k,t}\}_k\}_{t=0}^\infty) = V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta \Delta Q_t, \{P_{k,t} + \theta \Delta P_{k,t}\}_k\}_{t=0}^\infty).$$

The implicit function theorem gives

$$\begin{aligned} \partial_{\theta=0} EV_i(\theta) &= \frac{\partial_{\theta=0} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta \Delta Q_t, \{P_{k,t} + \theta \Delta P_{k,t}\}_k\}_{t=0}^\infty)}{\partial_{B_{i,-1}} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t, \{P_{k,t}\}_k\}_{t=0}^\infty)} \\ &= \frac{\partial_{\theta=0} V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta \Delta Q_t, \{P_{k,t} + \theta \Delta P_{k,t}\}_k\}_{t=0}^\infty)}{U'(C_{i,0})} \end{aligned}$$

where the second line uses the first-order condition for consumption. This implies

$$dEV_i = dV_{i,0}/U'(C_{i,0}),$$

using the notations $dEV_i \equiv (\partial_{\theta=0}EV_i) d\theta$ and $dV_{i,0} \equiv (\partial_{\theta=0}V_{i,0}(\{N_{i,k,-1}\}, B_{i,-1}, \{Q_t + \theta\Delta Q_t, \{P_{k,t} + \theta\Delta P_{k,t}\}_{k=1}^{\infty}\})) d\theta$.

We now show the equality between (ii) and (iii). Totally differentiating the definition of welfare (8) gives

$$dV_{i,0} = \sum_{t=0}^{\infty} \beta^t dU(C_{i,t}) = \sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) dC_{i,t}. \quad (81)$$

Combining with the Euler equation $U'(C_{i,0}) = \beta^t R_{0 \rightarrow t} U'(C_{i,t})$ gives

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_{i,t}, \quad (82)$$

which proves the statement.

We now show the equality between (ii) and (iv). Consider an hypothetical individual that would not change asset holdings in response to the deviation in asset prices and denote $dC_{i,t}^{\text{fixed holdings}}$ the resulting deviation in consumption. Differentiating the budget constraint (9) gives:

$$dC_{i,t}^{\text{fixed holdings}} \equiv \sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t.$$

Taking the present value of both terms over multiple periods and using Proposition 1 gives:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} dC_{i,t}^{\text{fixed holdings}},$$

which proves the statement. Note that this is really the result of the envelope theorem: re-optimizing asset holdings in response to a change in asset prices does not have a (first-order) effect on welfare. This equivalence, however, breaks in more complex cases, such as when the individual faces a binding borrowing constraint (see Proposition 15).

We now show the equality between (ii) and (v). Consider a non-infinitesimal deviation in asset prices $\{\theta\Delta Q_t, \{\theta\Delta P_{k,t}\}_{k=1}^{\infty}\}$ indexed by a scale θ . Denote $c_i(\theta)$ the relative increase in consumption in each period that would generate the same increase in welfare as the deviation for individual i . By definition, this quantity satisfies

$$\Delta V_{i,0}(\theta) = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}(1 + c_i(\theta))).$$

Differentiating around $\theta = 0$ gives

$$dV_{i,0} = \sum_{t=0}^{\infty} \beta^t U'(C_{i,t}) C_{i,t} dc_i,$$

where $dV_{i,0} \equiv (\partial_{\theta=0}\Delta V_{i,0}) d\theta$ and $dc_i \equiv c_i'(0) d\theta$. Combining with Euler equation $U'(C_{i,0}) = \beta^t U'(C_{i,t}) R_{0 \rightarrow t}$

gives

$$\begin{aligned} dV_{i,0} &= U'(C_{i,0}) \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_{i,t} dc_i \\ \implies dV_{i,0}/U'(C_{i,0}) &= \left(\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_{i,t} \right) dc_i, \end{aligned}$$

which proves the statement. Note that this derivation allows us to interpret our notion of welfare gain dividend by the present value of consumption (or “total wealth”) as the percentage increase in consumption in every period that would have the same effect on welfare as the deviation, which corresponds to the notion of welfare gain in [Lucas \(2000\)](#).

We now show the equality between (ii) and (vi). Denote $dB_{i,t}^{\text{fixed consumption and holdings}}$ the deviation in bond holdings that would allow an hypothetical individual to maintain his or her original path of consumption and asset holdings (beyond bonds) despite the deviation in asset prices. Differentiating the budget constraint (9) gives:

$$Q_t dB_{i,t}^{\text{fixed consumption and holdings}} - dB_{i,t-1}^{\text{fixed consumption and holdings}} = \sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t,$$

Solving this forward and using the no-Ponzi condition gives

$$\begin{aligned} dB_{i,-1}^{\text{fixed consumption and holdings}} &= - \sum_{t=0}^{\infty} (Q_0 \dots Q_{t-1}) \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} - B_{i,t} dQ_t \right) \\ &= - \sum_{t=0}^{\infty} R_{0 \rightarrow T}^{-1} \left(\sum_{k=1}^K (N_{i,k,t-1} - N_{i,k,t-1}) dP_{k,t} - B_{i,t} dQ_t \right). \end{aligned}$$

Combining with Proposition 1 gives

$$dV_{i,0}/U'(C_{i,0}) = - dB_{i,-1}^{\text{fixed consumption and holdings}},$$

which proves the statement. □

Generalization to a stochastic environment. Proposition 17 shows the equivalence between our notion of welfare gains to five other concepts in the context of the baseline (deterministic) economy described in Section 1.2. We now discuss how this result would generalize to stochastic economies. First, as long as markets are complete, the proposition would still hold. Indeed, all the derivations in the proof of Proposition 17 would remain the same after replacing the return of the risk-free rate asset $R_{0,t}^{-1}$ by the growth of the stochastic discount factor Λ_t/Λ_0 (see Appendix A.2).

In the more realistic case where markets are incomplete, however, the proposition is no longer true. More precisely, the equivalence between (i) and (ii) still holds. The equivalence between (i)/(ii) and (iii)/(iv)/(v) only holds if one understand the term “present value” as meaning “discounted by the marginal rate of substitution of the individual” (which may differ from the economy-wide stochastic discount factor when markets are incomplete).

More importantly, the equivalence between (i)/(ii) and (vi) no longer holds: when markets are incomplete, the agent cannot invest some initial cash in a way that ensures that the cash-flow from the trading strategy exactly covers the deviation in the budget constraint due to the deviation in asset prices in each state of the world. As discussed in the main text, however, our baseline implementation of the sufficient statistic formula (16), which simply discounts realized transactions using an homogeneous

discount rate $R_{0 \rightarrow t}$, is still equivalent to (vi), since it corresponds to the amount of wealth at $t = 0$ that, invested in the liquid asset, would have been enough, *ex-post*, to cover the realized deviations in the budget constraint due to the deviation in asset prices.

E.3 Present-value budget constraint

Present-value budget constraint. We now derive the present-value budget constraint implied by the period budget constraints (9) and the no-Ponzi condition stated in the main text. At the optimum, the no-Ponzi condition holds with equality and is therefore given by

$$\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} \left(B_T Q_T + \sum_{k=1}^K N_{i,k,T} P_{i,k,T} \right) = 0. \quad (83)$$

To derive the present-value budget constraint, it is useful to express a period budget constraint in terms of the individual's financial wealth

$$W_{i,t} \equiv B_{i,t} Q_t + \sum_{k=1}^K N_{i,k,t} P_{k,t}.$$

Using this definition, (9) can be rewritten as

$$C_{i,t} + W_{i,t} = Y_{i,t} + R_t W_{i,t-1} + \sum_{k=1}^K (R_{k,t} - R_t) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{i,k,t} - N_{i,k,t-1}), \quad (84)$$

where the reader should recall that $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ is the return of asset k and $R_{t+1} = 1/Q_t$ is the return on the one-period bond. The right-hand side of this constraint says that the individual's capital income equals total wealth times the bond return $R_t W_{i,t-1}$ plus the excess return from holding long-lived assets $\sum_{k=1}^K (R_{k,t} - R_t) N_{i,k,t-1} P_{k,t-1}$ minus any adjustment costs from transacting these. The no-Ponzi condition (83) becomes

$$\lim_{T \rightarrow \infty} R_{0 \rightarrow T}^{-1} W_{i,T} = 0. \quad (85)$$

We then have:

Lemma 18. *The present-value budget constraint implied by the sequence of budget constraints (9) and the no-Ponzi condition (83) is*

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_{i,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} \left[Y_{i,t} + \sum_{k=1}^K (R_{k,t} - R_t) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{i,k,t} - N_{i,k,t-1}) \right] + R_0 W_{i,-1}, \quad (86)$$

where $W_{i,-1} \equiv \sum_{k=1}^K N_{i,k,-1} P_{k,-1} + B_{i,-1} Q_{-1}$ denotes initial financial wealth.

Proof of Lemma 18. Recursive forward substitution of the sequence of budget constraints (84) (eliminating $W_{i,0}$, $W_{i,1}$ and so on) yields

$$R_{0 \rightarrow T}^{-1} W_{i,T} + \sum_{t=0}^T R_{0 \rightarrow t}^{-1} C_{i,t} = \sum_{t=0}^T R_{0 \rightarrow t}^{-1} \left[Y_{i,t} + \sum_{k=1}^K (R_{k,t} - R_t) N_{i,k,t-1} P_{k,t-1} - \sum_{k=1}^K \chi(N_{i,k,t} - N_{i,k,t-1}) \right] + R_0 W_{i,-1},$$

where we have used the definition of the cumulative return $R_{0 \rightarrow t} = R_1 \cdot R_2 \cdot \dots \cdot R_t$. Taking the limit as $T \rightarrow \infty$ and imposing the no-Ponzi condition (85) yields (86). \square

Alternative expression for welfare gains. While the proof of Proposition 1 uses the sequence of period budget constraints, our welfare gains formula can also be derived from the corresponding present-value constraint. We here explain the connection between the two proofs. To explain the logic in the most transparent fashion and avoid cluttered notation, we here consider a version of the baseline model without bonds and only one long-lived asset N_t without adjustment costs. Everything generalizes to the case covered in our baseline model, i.e. a bond and K long-lived assets that are potentially subject to adjustment costs.

In this one-asset version, the problem of the agent can be reformulated as maximizing the discounted value of utility $\sum_{t=0}^{\infty} \beta^t U(C_{i,t})$ subject to the following present-value budget constraint.

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} C_{i,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} Y_{i,t} + N_{i,-1}(P_0 + D_0).$$

where the asset's return is $R_{t+1} = (D_{t+1} + P_{t+1})/P_t$, and the no-Ponzi condition is $\lim_{T \rightarrow \infty} R_{0 \rightarrow T} N_{i,T} P_T = 0$. Proposition 1 implies that the welfare gain is:

$$dV_{i,0}/U'(C_{i,0}) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,t-1} - N_{i,t}) dP_t. \quad (87)$$

We now show how to derive our expression for the welfare effect of asset prices from this alternative maximization problem. Along the way, we derive two alternative expressions for the welfare gains formula (equations (88) and (88) below). Both are instructive and carry important economic intuition. The Lagrangian corresponding to the maximization problem is

$$\mathcal{L}_i = \sum_{t=0}^{\infty} \beta^t U(C_{i,t}) + \lambda_i \left(N_{i,-1}(P_0 + D_0) + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (Y_{i,t} - C_{i,t}) \right).$$

where λ_i is the Lagrange multiplier on the present-value budget constraint. Using the envelope condition to compute $dV_{i,0}$ and that $\lambda_i = U'(C_{i,0})$, the welfare gain from a deviation in asset prices $\{dP_t\}_{t=0}^{\infty}$ and the associated deviation in asset returns $\{dR_{t+1}\}_{t=0}^{\infty}$ equals

$$dV_{i,0}/U'(C_{i,0}) = N_{i,-1} dP_0 + \sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t}) dR_{0 \rightarrow t}^{-1}. \quad (88)$$

This equation says that the welfare gain from an asset-price change is the change in initial wealth, $N_{i,-1} dP_0$, plus an extra term $\sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t}) dR_{0 \rightarrow t}^{-1}$. This formulation of the welfare-gains formula clarifies that, when asset discount rates change, price deviations not only result in a revaluation of initial wealth $N_{i,-1} dP_0$ but also affect the discounting in the present-value budget constraint. This is exactly this intuition which is represented graphically in the two-period models in Figure 1.

To derive the second alternative formulation of our welfare gains formula, we now write this extra

term as:

$$\begin{aligned}
\sum_{t=0}^{\infty} (Y_{i,t} - C_{i,t}) dR_{0 \rightarrow t}^{-1} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (Y_{i,t} - C_{i,t}) \left(- \sum_{s=1}^t d \log R_s \right) \\
&= - \sum_{t=1}^{\infty} \left(\sum_{s=t}^{\infty} R_{0 \rightarrow s}^{-1} (Y_{i,s} - C_{i,s}) \right) d \log R_t \\
&= \sum_{t=1}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{i,t-1} P_{t-1} d \log R_t \\
&= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,t} P_t d \log R_{t+1},
\end{aligned}$$

where the third equality uses the consolidated budget constraint from the point of view of time t . Plugging this expression into (88) gives

$$dV_{i,0}/U'(C_{i,0}) = N_{i,-1} dP_0 + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,t} P_t d \log R_{t+1}. \quad (89)$$

The intuition is that welfare gains depend on prices today but also returns going forward. This is exactly this intuition we have for the difference between welfare gains and revaluation gains discussed in Section 1.1.

Finally, to get back our main welfare formula (87), we need to express the deviation in interest rates in terms of the deviation in asset prices. Differentiating the definition of the asset return gives

$$d \log R_{t+1} = R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t}.$$

Plugging into (89) gives

$$dV_{i,0}/U'(C_{i,0}) = N_{i,-1} dP_0 + \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,t} P_t \left(R_{t+1}^{-1} \frac{dP_{t+1}}{P_t} - \frac{dP_t}{P_t} \right) = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,t-1} - N_{i,t}) dP_t,$$

where the second equality uses the assumption that $R_{0 \rightarrow t+1}^{-1} dP_{t+1} \rightarrow 0$ as $t \rightarrow \infty$, and we have therefore recovered our main welfare gains formula (87).

E.4 Equivalence between deviations in asset prices and discount rates

We mentioned in the main text that deviation in asset prices, holding dividends constant, are equivalent to deviations in discount rates. We now express this idea more formally. Consider a sequence of dividends $\{D_{k,t}\}_{t=0}^{\infty}$ and a sequence of discount rates $\{R_{k,t}\}_{t=0}^{\infty}$. Assuming the no-bubble condition, the price of the asset is then given by

$$P_{k,t} = \sum_{s=t+1}^{\infty} \frac{D_{k,s}}{R_{k,t+1} \cdots R_{k,s}}.$$

This formula makes clear that, in the absence of dividend changes, price changes must come from changes in discount rates. The following proposition gives a formal mapping between the two.⁹⁸

Proposition 19. *Holding dividends constant, a given deviation in the path of asset discount rates $\{dR_{k,t}\}_{t=0}^{\infty}$*

⁹⁸See also Knox and Vissing-Jorgensen (2022) for a similar result.

generates the following deviation in asset prices $\{dP_{k,t}\}_{t=-1}^{\infty}$:

$$\frac{dP_{k,t}}{P_{k,t}} = - \sum_{s=t}^{\infty} \left(\prod_{u=t}^s \left(1 + \frac{D_{k,u}}{P_{k,u}} \right)^{-1} \right) \frac{dR_{k,s+1}}{R_{k,s+1}}. \quad (90)$$

This equation gives the deviation in asset prices generated by a given deviation path in discount rates. In spirit, this equation is very similar to [Campbell and Shiller \(1988\)](#)'s log linearized present value identity: indeed, in the particular case where the price-dividend ratio is constant on the baseline path, Equation (90) simplifies to

$$d \log P_{k,t} = - \sum_{s=t}^{\infty} \rho^{s-t} d \log R_{k,s+1},$$

where $\rho \equiv 1/(1 + D_{k,t}/P_{k,t})$.

Proof of proposition 19. Differentiating the definition of returns $R_{k,t+1} = (D_{k,t+1} + P_{k,t+1})/P_{k,t}$ gives

$$dR_{k,t+1} = \frac{dP_{k,t+1} - R_{k,t+1} dP_{k,t}}{P_{k,t}}.$$

Rearranging this equation gives the deviation in prices today as a function of the deviation in returns and prices tomorrow:

$$dP_{k,t} = R_{k,t+1}^{-1} P_{k,t} dR_{k,t+1} + R_{k,t+1}^{-1} dP_{k,t+1}.$$

Iterating forward,

$$dP_{k,t} = \sum_{s=t}^{\infty} (R_{k,t+1} \dots R_{k,s})^{-1} P_{k,s} \frac{dR_{k,s+1}}{R_{k,s+1}}.$$

Dividing by $P_{k,t}$,

$$\frac{dP_{k,t}}{P_{k,t}} = \sum_{s=t}^{\infty} (R_{k,t+1} \dots R_{k,s})^{-1} \frac{P_{k,s}}{P_{k,t}} \frac{dR_{k,s+1}}{R_{k,s+1}}. \quad (91)$$

Now, we have

$$R_{k,s+1} = \frac{D_{k,s+1} + P_{k,s+1}}{P_{k,s}} = \left(1 + \frac{D_{k,s}}{P_{k,s}} \right) \frac{P_{k,s+1}}{P_{k,s}},$$

which implies

$$(R_{k,t+1} \dots R_{k,s})^{-1} \frac{P_{k,s}}{P_{k,t}} = \prod_{u=t}^s \left(1 + \frac{D_{k,u}}{P_{k,u}} \right).$$

Plugging this equality into (91) gives the result. □

E.5 Connection with [Auclert \(2019\)](#)

[Auclert \(2019\)](#) examines the effect of a one-time perturbation in the path of interest rates on consumption and welfare. We now discuss how this result relates to our Proposition 1. Consider an economy where, at time $t = 0$, individuals can trade bonds of all maturities. Denote Q_h the price of the bond with maturity $h \geq 1$. That is, the long-term interest rate between 0 and h is $R_{0 \rightarrow h} = 1/Q_h$.

As in the baseline model, the individual receives labor income $Y_{i,t}$ at time t and they initially own $N_{i,-1}$ shares of a long lived asset that pays a sequence of dividends $\{D_t\}_{t=0}^{\infty}$. The individual i chooses consumption and holdings to maximize utility

$$V_{i,0} = \max_{\{C_{i,t}, N_{i,t}, B_{i,t}\}_0^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{i,t}),$$

with the following sequence of budget constraints

$$\begin{aligned} C_{i,0} + \sum_{h=1}^{\infty} B_{i,h} Q_{h,0} &= N_{i,-1} D_0 + Y_{i,0} && \text{for } t = 0, \\ C_{i,t} &= N_{i,-1} D_t + B_{i,h} + Y_{i,t} && \text{for } t \geq 1, \end{aligned}$$

where $B_{i,h}$ denotes the number of bonds with maturity h bought at time $t = 0$. Proposition 1 states that the welfare gain of a perturbation in the price of bonds with different maturities depends on transactions:

$$\begin{aligned} dV_{i,0}/U'(C_{i,0}) &= - \sum_{h=1}^{\infty} B_{i,h} dQ_{h0} \\ &= \sum_{h=1}^{\infty} (N_{i,-1} D_h + Y_{i,h} - C_{i,h}) dQ_{h0} \\ &= \sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (C_{i,h} - Y_{i,h} - N_{i,-1} D_h) d \log R_{0 \rightarrow h}. \end{aligned}$$

This corresponds to Appendix formula (A.37) in Auclert (2019).

In the special case in which the perturbation is a level shift in the yield curve (i.e., $d \log R_{0 \rightarrow h} = h d \log R$ for $h > 1$), the formula simplifies to

$$dV_{i,0}/U'(C_{i,0}) = \left(\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (C_{i,h} - Y_{i,h} - N_{i,-1} D_h) h \right) d \log R.$$

This formula expresses the welfare gain of a permanent rise in interest rate on welfare, as a share of total wealth, as the difference between the duration of consumption and the duration of income, where “duration” is defined as the value-weighted time to maturity of a sequence of cash flows (see Greenwald, Leombroni, Lustig and Van Nieuwerburgh, 2021).⁹⁹

E.6 Comparing welfare and revaluation gains

We now explain the revaluation gains defined in (17) in the main text and how these differ from our baseline welfare gains formula. We first consider infinitesimal price deviations $\{dP_{k,t}\}_{t \geq 0}$ and then discuss non-infinitesimal deviations $\{\Delta P_{k,t}\}_{t \geq 0}$ as in equation (17).

Infinitesimal Deviations. We now discuss the relationship between welfare and revaluation gains due to infinitesimal price deviations.

⁹⁹More specifically, the duration of consumption is $\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} \frac{C_h}{\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} C_h} h$ while the duration of income is $\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} \frac{Y_{i,h} + N_{i,-1} D_h}{\sum_{h=1}^{\infty} R_{0 \rightarrow h}^{-1} (Y_{i,h} + N_{i,-1} D_h)} h$.

Proposition 20. Consider an asset $1 \leq k \leq K$ and a sequence of price deviations $(dP_{k,t})_{t \geq 0}$. We have:

$$\begin{aligned} \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t}}_{\text{Welfare gain}} &= \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} P_{k,t-1} d \left(\frac{P_{k,t}}{P_{k,t-1}} \right)}_{\text{Revaluation gain}} \\ &+ \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{i,k,t-1} P_{k,t-1} \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} d \left(\frac{D_{k,t}}{P_{k,t-1}} \right)}_{\text{Effect of price deviations on dividend yields}}. \end{aligned} \quad (92)$$

The proposition decomposes the welfare effect of the deviation in asset prices (the left-hand side in Equation 92) into two terms. The first term (“revaluation gains”) corresponds to the positive effect of a rise in asset prices on returns through higher capital gains. The second term corresponds to the negative effect of higher prices on returns through lower dividend yields.

This generalizes the intuition of the two-period model in a model with infinite horizon and multiple assets. The key message is that, following a rise in asset prices, revaluation gains overestimate welfare gains because they only take into account the positive effect of rising prices on capital gains without taking into account their negative effects going forward through lower dividend yields.

Finally note that the capital gains deviation that enters the revaluation gain in (92) can also be written as

$$d \left(\frac{P_{k,t}}{P_{k,t-1}} \right) = \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{dP_{k,t}}{P_{k,t}} - \frac{dP_{k,t-1}}{P_{k,t-1}} \right). \quad (93)$$

Proof of Proposition 20. Using summation by parts on the sequence $(R_{0 \rightarrow t}^{-1} dP_{k,t})_{t \geq 0}$ and $(N_{i,k,t})_{t \geq 0}$, the welfare gain for asset k can be rewritten as:

$$\begin{aligned} \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,k,t-1} - N_{i,k,t}) dP_{k,t} &= \sum_{t=0}^{\infty} N_{i,k,t-1} \left(R_{0 \rightarrow t}^{-1} dP_{k,t} - R_{0 \rightarrow t-1}^{-1} dP_{k,t-1} \right) \\ &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} P_{k,t-1} \frac{dP_{k,t} - R_t dP_{k,t-1}}{P_{k,t-1}}. \end{aligned} \quad (94)$$

This equation highlights a duality between measuring welfare gains as the present value of sales interacted with price deviations (the left-hand-side) and the present value of asset holdings interacted with return deviations (the right-hand side).

To see why $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ can be interpreted as the deviation in returns, note that we have:

$$\begin{aligned} \frac{dP_{k,t} - R_t dP_{k,t-1}}{P_{k,t-1}} &= \frac{dP_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} dP_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}} dP_{k,t-1} - R_t dP_{k,t-1}}{P_{k,t-1}} \\ &= \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{dP_{k,t}}{P_{k,t}} - \frac{dP_{k,t-1}}{P_{k,t-1}} \right) + \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t \right) \frac{dP_{k,t-1}}{P_{k,t-1}} \\ &= d \left(\frac{P_{k,t}}{P_{k,t-1}} \right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} \left(-\frac{D_{k,t}}{P_{k,t-1}} \right) \frac{dP_{k,t-1}}{P_{k,t-1}} \\ &= d \left(\frac{P_{k,t}}{P_{k,t-1}} \right) + \frac{R_t - \frac{P_{k,t}}{P_{k,t-1}}}{R_{k,t} - \frac{P_{k,t}}{P_{k,t-1}}} d \left(\frac{D_{k,t}}{P_{k,t-1}} \right), \end{aligned} \quad (95)$$

where the third line uses the definition of the return of asset k at time t $R_{k,t} \equiv (D_{k,t} + P_{k,t})/P_{k,t-1}$. This decomposes $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ into a part due to the deviation in capital gains (the first

term in the RHS) and a part due to the deviation in dividend yields (the second part in the RHS). In the particular case where $R_{k,t} = R_t$ (no adjustment costs), we have $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1} = d(P_{k,t}/P_{k,t-1}) + d(D_{k,t}/P_{k,t-1})$; that is, $(dP_{k,t} - R_t dP_{k,t-1})/P_{k,t-1}$ corresponds exactly to the deviation in the return of asset k . The proposition obtains by combining (94) with (95). \square

Non-infinitesimal deviations. In our empirical application, we measure welfare and revaluation gains using non-infinitesimal price changes. We now derive give a counterpart of Proposition 20 above for non-infinitesimal price deviations.

Corollary 21. Consider an asset $1 \leq k \leq K$ and a sequence of non-infinitesimal changes in prices $(\Delta P_{k,t})_{t \geq 0}$. We have:

$$\begin{aligned} \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t}}_{\text{Welfare gain}} &= \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} P_{k,t-1} \Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right)}_{\text{Revaluation gain}} \\ &+ \underbrace{\sum_{t=0}^{\infty} R_{0 \rightarrow t-1}^{-1} N_{i,k,t-1} P_{k,t-1} \left(\frac{P_{k,t}}{P_{k,t-1}} - R_t \right) \frac{\Delta P_{k,t-1}}{P_{k,t-1}}}_{\text{Effect of price deviations on dividend yields}}. \end{aligned} \quad (96)$$

where we define

$$\Delta \left(\frac{P_{k,t}}{P_{k,t-1}} \right) \equiv \frac{P_{k,t}}{P_{k,t-1}} \left(\frac{\Delta P_{k,t}}{P_{k,t}} - \frac{\Delta P_{k,t-1}}{P_{k,t-1}} \right) \quad (97)$$

as the deviation in the capital gains component $P_{k,t}/P_{k,t-1}$ of asset returns caused by the price deviation $\{\Delta P_{k,t}\}_{t \geq 0}$.

Note that $\Delta(P_{k,t}/P_{k,t-1})$ defined in (97) is the natural discrete counterpart to $d(P_{k,t}/P_{k,t-1})$ in (93).

Proof of Corollary 21. The proof follows the same steps as the proof of Proposition 20, except with non-infinitesimal price deviations. Using summation by parts:

$$\sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} = \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} P_{k,t-1} \frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

In turn, we can write

$$\frac{\Delta P_{k,t} - R_t \Delta P_{k,t-1}}{P_{k,t-1}} = \frac{\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1}}{P_{k,t-1}} + \frac{\frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} - R_t \Delta P_{k,t-1}}{P_{k,t-1}}.$$

Plugging into the previous equation gives

$$\begin{aligned} \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} (N_{i,k,t-1} - N_{i,k,t}) \Delta P_{k,t} &= \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} \left(\Delta P_{k,t} - \frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} \right) \\ &+ \sum_{t=0}^{\infty} R_{0 \rightarrow t}^{-1} N_{i,k,t-1} \left(\frac{P_{k,t}}{P_{k,t-1}} \Delta P_{k,t-1} - R_t \Delta P_{k,t-1} \right). \end{aligned}$$

Rearranging gives the result. Finally, note that, as price deviations become infinitesimal, each term in the formula converges to the respective term in Proposition 20. \square