Handling Non-Convexities: Neoclassical Growth Model with Convex-Concave Production Function (Skiba, 1978)

1 Model Description

Consider the planning problem in the neoclassical growth model:

$$v(k_0) = \max_{\{c(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(c(t))dt \qquad \text{s.t.}$$
$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t), \quad k(0) = k_0.$$

But now assume that the production function is not strictly concave everywhere. In particular assume that

$$\begin{aligned} f(k) &= \max\{f_L(k), f_H(k)\}, \\ f_L(k) &= A_L k^\alpha, \\ f_H(k) &= A_H((k-\kappa)^+)^\alpha \end{aligned}$$

with $\kappa > 0$ and $A_H > A_L$. The idea is that the planner has costless access to a bad technology with productivity A_L, and that he can upgrade it to a good technology with productivity $A_H > A_L$ but only by paying a per-period fixed cost κ. This production function is plotted in Figure 1. Because of its look, some some people call this a "butterfly production function."

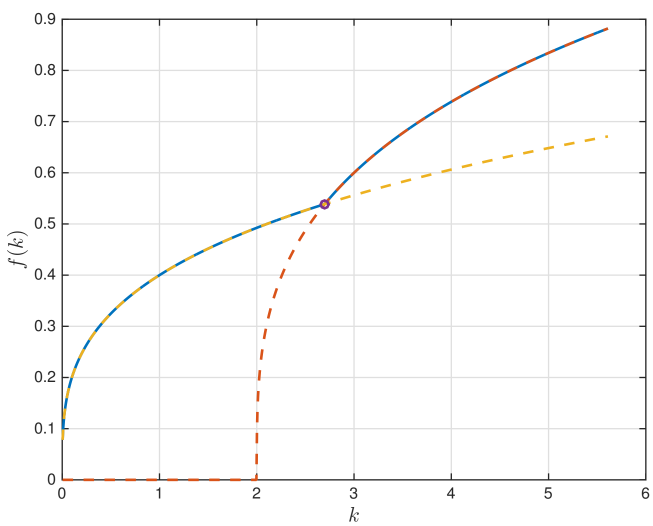


Figure 1: Convex-Concave Production

The HJB equation to be solved is:

$$\rho v(k) = \max_c \; u(c) + v'(k)(f(k) - \delta k - c).$$

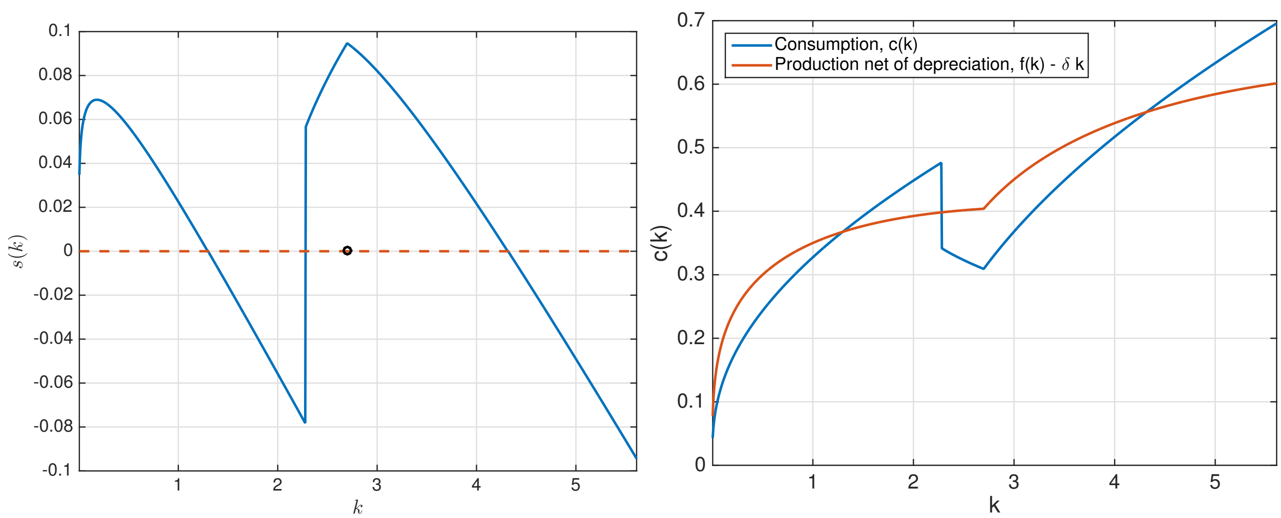
2 Algorithm

See `HJB_NGM_skiba.m`. The algorithm uses an implicit method and is exactly the same as in Section 2.1 of `http://www.princeton.edu/~moll/HACTproject/HACT_Numerical_Appendix.pdf`. Since the value function is not strictly concave (in fact, it has a convex kink) we use the upwind scheme described at the end of Section 2.1 and which here becomes (see `HACT_Numerical_Appendix.pdf` for an explanation of the notation):

$$\begin{aligned} v_i' = {} & v_{i,F}' \left(\mathbf{1}_{\{s_{i,F}>0\}} \mathbf{1}_i^{unique} + \mathbf{1}_{\{H_{i,F} \geq H_{i,B}\}} \mathbf{1}_i^{both} \right) \\ & + v_{i,B}' \left(\mathbf{1}_{\{s_{i,B}<0\}} \mathbf{1}_i^{unique} + \mathbf{1}_{\{H_{i,F} < H_{i,B}\}} \mathbf{1}_i^{both} \right) \\ & + \bar{v}_i' \mathbf{1}_{\{s_{i,F} \leq 0 \leq s_{i,B}\}} \end{aligned}$$

3 Results

Figure 2 plots the consumption and saving policy functions.



(a) Saving Policy Function

(b) Consumption Policy Function

Figure 2: Policy Functions in Skiba Model

References

Skiba, A K. 1978. "Optimal Growth with a Convex-Concave Production Function." *Econometrica*, 46(3): 527–39.