# Credit crunch on a Huggett-Poisson-HACT economy

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Heterogeneous agent models in continuous time

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# 1 Credit crunch

The Huggett-Poisson-HACT model allows us to compute not only a snapshot of the economy's equilibrium but the transitional dynamics from an MIT shock as well. As shown in Achdou et al. (2015) we can see how the economy behaves when parameters such as $\lambda_i$ change. What can the model say about variations in $\underline{a}$? Can we evaluate the transitional dynamics of a tightening of the borrowing constraint? This parameter is a bit more tricky, because it governs the state constraint and space.

If we take the parameter values used in Achdou et al. (2015) we will have that low income households will have negative savings until they hit the borrowing constraint. Once they reach the debt limit they will have zero savings and this is where we use the state constraint $c_1(\underline{a}) = z_1 + r\underline{a}$. The upwind scheme and state constraints will push these households away from inadmissible values of the state space. Notice though that if we suddenly tighten the debt limit we will be instantaneously putting a mass of households into an inadmissible region. In such a case we would need to move households to the new borrowing constraint. They can achieve that by lowering consumption[1]. We will require households in the new inadmissible region to have positive period by period savings of $\Delta a$, that is, households move $\Delta a$ at each time step until they reach the new debt limit.

Besides the algorithm for the transitional dynamics we will need to modify the initial and terminal equilibrium conditions. It is important to keep the same asset grid throughout the experiment, otherwise we would be comparing apples with oranges. Given that we initially have a laxer borrowing constraint the first instinct will be to have a shorter asset grid in the terminal period. But by doing so we risk confusing points in the grid when computing the transitional dynamics. The trick will be to keep the initial larger state space and provide state constraints that will move the mass of households towards the new admissible region.

# 2 Initial equilibrium

Let's motivate the example with a few concrete values used in the Matlab codes. Let $-0.1692 = \underline{a}_{t_0} < \underline{a}_T = -0.15$ (a tightening of 11.35%). We can make a terminal asset grid of 800 points, going from $\underline{a}_T$ to $a_{max} = 5$. We would then have that $\Delta a = 0.0064$. Given that we need to evaluate all of the state space from the initial equilibrium, we will need to add a few points to the left of $\underline{a}_T$. As a consequence, in this particular example we add three $\Delta a = 0.0064$ points to the left of $\underline{a}_T$ and thus reach $-0.1692 = \underline{a}_{t_0}$. We will be carrying out the credit crunch

[1]Obviously they can only starve too far and will not be able to have negative consumption.

experiment with an asset grid of 803 points. Another way to say this is that the admissible region in the asset grid will shrink by $3\Delta a$. In the initial equilibrium we have the state constraint at $\underline{a}_{t_0} = -0.1692$. We then have to let households in the newly created inadmissible region be able to move to the new debt limit whilst still having positive consumption. In order to solve for the initial equilibrium run the Matlab file '`huggett_initial_creditcrunch.m`'.

# 3 New state constraints and up-wind scheme

When a credit crunch occurs, a household at $\underline{a}_{t_0}$ will suddenly find itself below the new limit. Taking the previous values mentioned above, it will have to meet not only interest payments but also reduce consumption by $\Delta a$ so that in the next instant it moves closer to $\underline{a}_T$. The new state constraints will thus look as follows. Remark that for this particular example $\underline{a}_{t_0} + 3\Delta a = \underline{a}_T$.

$$\begin{aligned}
\Delta a &= s_1(\underline{a}_{t_0}) = z_1 + r(\underline{a}_T - 3\Delta a) - c_1(\underline{a}_{t_0}) \\
\Delta a &= s_1(\underline{a}_{t_0} + \Delta a) = z_1 + r(\underline{a}_T - 2\Delta a) - c_1(\underline{a}_{t_0} + \Delta a) \\
\Delta a &= s_1(\underline{a}_{t_0} + 2\Delta a) = z_1 + r(\underline{a}_T - \Delta a) - c_1(\underline{a}_{t_0} + 2\Delta a) \\
0 &= s_1(\underline{a}_{t_0} + 3\Delta a) = z_1 + r(\underline{a}_T) - c_1(\underline{a}_{t_0} + 3\Delta a)
\end{aligned}$$

The last expression is the usual state constraint that was employed in the previous Huggett-HACT model. The new state constraints are just directing mass to the new admissible region. We need to modify the up-wind scheme so that it knows what to compute at those points where the indicator function does not choose either forward or backward differencing. Prior to this extension we used steady state consumption and the FOC at such points. We will now do the same except that we will have to reduce consumption by the amount required according to the state constraint, $\Delta a$. Shown below is an example of how this condition would be modified for a credit crunch where the debt limit tightens by $3\Delta a$.

$$\begin{gathered}
\bar{c}_{1,1} = z_1 + r\underline{a}_{t_0} - \Delta a \\
\bar{c}_{2,1} = z_1 + r(\underline{a}_{t_0} + \Delta a) - \Delta a \\
\bar{c}_{3,1} = z_1 + r(\underline{a}_{t_0} + 2\Delta a) - \Delta a \\
\bar{c}_{\underline{a}_T,1} = z_1 + r\underline{a}_T \\
\bar{v}'_{i,j} = u'(\bar{c}'_{i,j}) \\
v_{i,j} = v'_{i,j}\mathbb{1}_{S_F>0} + v'_{i,j}\mathbb{1}_{S_B<0} + \bar{v}'_{i,j}\mathbb{1}_{S_B>0>S_F}
\end{gathered}$$

Prior to this extension we would have only used the last three equations. Now, we have an extra condition for each point on the grid that is located to the left of the new debt limit.

# 4 Terminal equilibrium

The only modifications for the terminal state code are keeping the same asset grid and making sure that there is no mass of households to the left of $\underline{a}_T$ on this last period. The new state-constraints will be a bit 'harsher' on the terminal equilibrium. This is because it is the last period to adjust and we cannot have households that are still adapting to the new limit. If a household happens to be $3\Delta a$ points to the left of the new limit, it will be forced to have savings of $3\Delta a$, and so on. The state constraints will force all households to be at or to the right of the new limit, either by saving $\Delta a$ or more. The up-wind scheme will be modified accordingly. The terminal equilibrium is solved with the Matlab file '`huggett_terminal_creditcrunch.m`'.

# 5 Transitional dynamics

Besides the adjustments for the up-wind scheme and new state constraints, the transitional dynamics algorithm will not change much. As before, we have to make sure that the interest rate is dependent on the time step. We will also need to activate the indicator function for the right ends of the asset grid: $Ib(I,:) = 1$ and $If(I,:) = 0$ (this makes the code run better). The algorithm should run with the same configuration as in the other transitional dynamics for a Huggett-Poisson economy. However, I found that for the parameter setting I used (slightly higher endowments and different $\lambda$'s) setting $x_i = 5$ works the best. Nonetheless, this is not necessary to make the algorithm run and clear the asset market at each point in time. Use the Matlab file '`huggett_transition_creditcrunch.m`' for the transitional dynamics (you need to run the two previous codes first).

# 6 Results

The savings functions at the initial equilibrium will be just as before. In the terminal state, however, they will look different. The decision rules will now tell us that agents must have positive savings if they find themselves in the inadmissible region. High income households will have positive savings regardless and since the forward differencing applies for their case they will always move to the right of the new debt limit, without any additional state constraints required. The changes in the savings functions will affect the transitional dynamics (see below). Figure 1 shows the transitional dynamics of an 11.35% tightening of the debt limit.

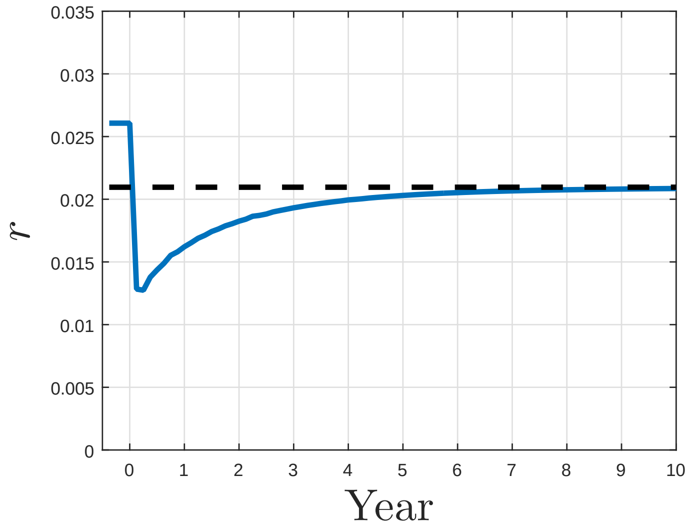


Figure 1: Response of $r(t)$ after a credit crunch from $\underline{a}_{t_0} = -0.1692$ to $\underline{a}_T = -0.15$.

The dynamics will be smoother if $\lambda_1$ is larger than $\lambda_2$, that is, if it is relatively more likely that we will not be stuck at the low income state while being forced to quickly reduce debt.

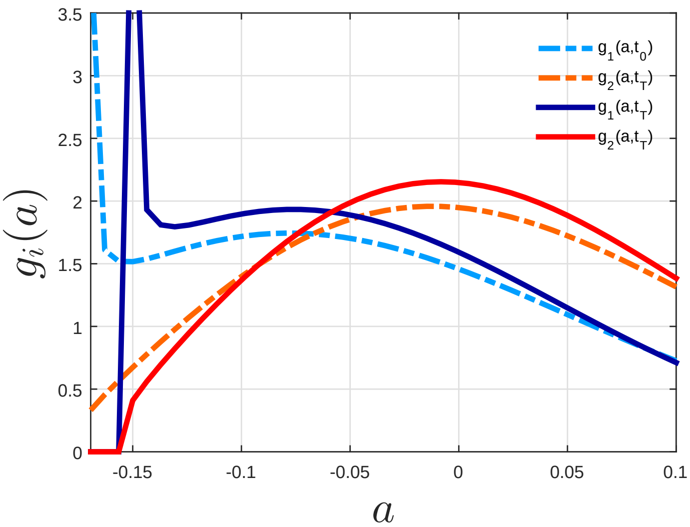


Figure 2: Initial and terminal distributions of wealth.

The transition GIF for the wealth distributions may be seen here: http://giphy.com/gifs/26BRAci8OvWqmiiSk. Only the first 50 periods are shown since the distributions barely move afterwards[2].

## References

Yves Achdou, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. Heterogeneous agent models in continuous time. 2015.

[2]The Matlab code for creating a GIF of the transitional dynamics of a credit crunch is 'gifmaker_transition.m'.