Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?

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Online Appendix: Closed Form Solution of a Neoclassical Growth Model with Capitalists and Workers

The purpose of this appendix is to explain why my model with optimizing households à la Ramsey, aggregates to something that looks like a Solow model, equations (20) and (21) in the paper. I show that this is the consequence of three assumptions: (i) the separation of individuals into "capitalists" (or "entrepreneurs") and "workers", (ii) that workers cannot save, and (iii) log-utility for the capitalists. I show this in the most stripped down version of the model that delivers this result: an almost standard neoclassical growth model (with no heterogeneity as in the paper).

A well-known result for the neoclassical growth model is that a linear or AK technology and log utility deliver a closed form solution with a constant savings rate.[1] For some applications, a drawback is that this model does not have a steady state. I here show that a slightly different setup with "capitalists" and "workers" also yields closed form solutions but has a steady state. This is relatively general and therefore also useful as building block for other more complicated models. The main "trick" is that the presence of workers together with constant returns to scale in capital and labor means that capitalists face *individual constant* returns. This yields constant savings rate for capitalists. However, the presence of workers who cannot save means that there are *aggregate decreasing* returns: when the capital stock increases this increases wages and hence decreases capitalists' returns to capital.

Capitalists and Workers. Time is discrete. There is a representative capitalist who solves

$$\max_{c_t, \ell_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.}$$

$$k_{t+1} = A k_t^\alpha \ell_t^{1-\alpha} - w_t \ell_t + (1-\delta)k_t - c_t.$$

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[1] Alternatively, a constant savings rate can be obtained with decreasing returns but log-utility and full depreciation.

where k_t is capital, ℓ_t is labor, w_t is the wage, δ the depreciation rate and β the discount rate. There is a unit mass of hand-to-mouth workers who inelastically supply one unit of labor each period. The labor market clearing condition is

$$\ell_t = 1.$$

Individual Behavior. After maximizing out over labor, ℓ_t, the representative capitalist's profits become linear in capital

$$\Pi_t = \max_{\ell_t} \left\{ A k_t^\alpha \ell_t^{1-\alpha} - w_t \ell_t \right\}$$

$$\ell_t = \left(\frac{1-\alpha}{w_t} \right)^{1/\alpha} A^{1/\alpha} k_t$$

$$\Pi_t = A^{1/\alpha} \pi_t k_t, \quad \pi_t \equiv \alpha \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)/\alpha}$$

The problem of the capitalist therefore becomes

$$\max_{c_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.}$$

$$k_{t+1} = A^{1/\alpha} \pi_t k_t + (1-\delta) k_t - c_t$$

It is easy to show that – as in an AK model – with log-utility, the optimal decision rule takes the form

$$k_{t+1} = \beta [A^{1/\alpha} \pi_t k_t + (1-\delta) k_t]. \tag{1}$$

Equilibrium. Labor market clearing is

$$1 = \ell_t = \left(\frac{\pi_t}{\alpha} \right)^{\frac{1}{1-\alpha}} A^{\frac{1}{\alpha}} k_t \quad \Leftrightarrow \quad \pi_t = \alpha A^{\frac{\alpha-1}{\alpha}} k_t^{\alpha-1}$$

Substituting into (1), the equilibrium law of motion for the capital stock is

$$k_{t+1} = \beta [\alpha A k_t^\alpha + (1-\delta) k_t]. \tag{2}$$

This completely summarizes the dynamics of the economy. As in the main text, the capital stock evolves as in a Solow model:

$$k_{t+1} = \hat{s}Ak_t^\alpha + (1 - \hat{\delta})k_t, \quad \text{where} \quad \hat{s} \equiv \alpha\beta, \quad \hat{\delta} \equiv 1 - \beta(1 - \delta)$$

are constant savings and depreciation rates. From (2) the steady state capital stock of the model solves

$$1 = \beta[\alpha Ak^{\alpha-1} + 1 - \delta] \tag{3}$$

Comparison with neoclassical growth model   It can be seen from (3) that the steady state of the present model is *the same* as that in a standard neoclassical growth model with decreasing returns to scale to capital and an optimizing representative agent. However, the characterization of the transition dynamics (2) is much simpler. In particular, there is no need to draw phase diagrams or the like.