When Inequality Matters for Macro and Macro Matters for Inequality

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(slides are on my website)

Motivation

- Last 30 years: a lot of progress developing macro models with rich heterogeneity in income, wealth, consumption in micro data
- Promising tool for macroeconomics
  - implications for policies differ: inequality matters for macro
  - distributional implications: macro matters for inequality

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  1. computational difficulties because distribution endogenous
  2. perception that aggregate dynamics similar to rep agent
- Our paper: these excuses less valid than you thought

These excuses are less valid than you thought

1. Efficient and easy-to-use computational method
   - open source `Matlab` toolbox online now
   - extension of linearization (Campbell 1998, Reiter 2009)
   - different slopes at each point in state space
   - exploit advantages of continuous time (Achdou et al. 2017)
2. Use methodology to illustrate interaction of macro + inequality
   - match micro behavior ⇒ realistic aggregate C + Y dynamics
   - aggregate shocks generate inequality dynamics...
   - ... and IRFs in HA model can differ dramatically from RA case

Outline

1. Explain methods in one-asset (Krusell-Smith) model
    - model description
    - linearization
    - dimensionality reduction
    - illustrative results
    - `https://sehyoun.com/EXAMPLE_PHACT_KS.html`
2. Two applications to illustrate macro + inequality interactions
    - richer two-asset (Kaplan-Moll-Violante) model
3. (Not in paper) a simple one-asset HANK model
    - `https://sehyoun.com/EXAMPLE_one_asset_HANK_web.html`

One-Asset Heterogeneous Agent Model with Aggregate Shocks (Krusell-Smith)

Households

$$\max_{\{c_{jt}\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{jt}) dt \qquad \text{such that}$$

$$\dot{a}_{jt} = w_t z_{jt} + r_t a_{jt} - c_{jt}$$

$$z_{jt} \in \{z_\ell, z_h\} \text{ Poisson with intensities } \lambda_\ell, \lambda_h$$

$$a_{jt} \geq 0$$

- c_{jt}: consumption
- u: utility function, $u' > 0$, $u'' < 0$.
- ρ: discount rate
- r_t : interest rate

Production

- Aggregate production function

$$Y_t = e^{Z_t} K_t^{\alpha} N_t^{1-\alpha} \text{ with } dZ_t = -\nu Z_t dt + \sigma dW_t$$

- Perfect competition in factor markets

$$w_t = (1-\alpha)\frac{Y_t}{N_t}, \qquad r_t = \alpha \frac{Y_t}{K_t} - \delta$$

- Market clearing

$$K_t = \int a g_t(a,z) da dz,$$
$$N_t = \int z g_t(a,z) da dz \equiv 1$$

Warm-Up: Stationary Eq without Aggregate Shocks

- This slide only: turn off aggregate shocks $Z_t \equiv 0$

$$\begin{aligned} \rho v(a,z) = &\max_c \; u(c) + \partial_a v(a,z)(wz + ra - c) \\ &+ \lambda_z (v(a,z') - v(a,z)) \end{aligned} \tag{HJB SS}$$

$$0 = -\partial_a[s(a,z)g(a,z)] - \lambda_z g(a,z) + \lambda_{z'} g(a,z') \tag{KF SS}$$

$$\begin{aligned} w &= (1-\alpha)K^\alpha, \quad r = \alpha K^{\alpha-1} - \delta, \\ K &= \int a g(a,z) da dz \end{aligned} \tag{PRICE SS}$$

Equilibrium with Aggregate Shocks

Aggregate state: $(g_t, Z_t) \Rightarrow$ absorb into time subscript t

- Recursive notation w.r.t. individual states only
- $\mathbb{E}_t$ is expectation w.r.t. aggregate states only

▸ fully recursive

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fully recursive

$$\begin{aligned}\rho v_t(a,z) = \max_c \; & u(c) + \partial_a v_t(a,z)(w_t z + r_t a - c) \\ & + \lambda_z(v_t(a,z') - v_t(a,z)) + \frac{1}{dt}\mathbb{E}_t\left[dv_t(a,z)\right],\end{aligned} \tag{HJB}$$

$$\partial_t g_t(a,z) = -\partial_a[s_t(a,z)g_t(a,z)] - \lambda_z g_t(a,z) + \lambda_{z'} g_t(a,z'), \tag{KF}$$

$$w_t = (1-\alpha)e^{Z_t}K_t^{\alpha}, \quad r_t = \alpha e^{Z_t}K_t^{\alpha-1} - \delta \tag{P}$$

$$K_t = \int a g_t(a,z)\,da\,dz$$

$$dZ_t = -\nu Z_t dt + \sigma dW_t$$

Note: $\frac{1}{dt}\mathbb{E}_t\left[dv_t\right]$ means $\lim_{s\downarrow 0}\mathbb{E}_t\left[v_{t+s} - v_t\right]/s$

Linearization

Extension of standard linearization

1. Compute non-linear approximation to non-stochastic steady state

2. Compute first-order Taylor expansion around steady state

3. Solve linear stochastic differential equation

Warm Up: Linearizing a Representative Agent Model

- Optimality conditions in RBC model

$$\mathbb{E}_t \begin{bmatrix} dC_t \\ dK_t \\ dZ_t \end{bmatrix} = f(C_t, K_t, Z_t)dt, \qquad f : \mathbb{R}^3 \to \mathbb{R}^3$$

- C_t = consumption
- K_t = capital
- Z_t = productivity
- f_1 = Euler equation
- f_2 = resource constraint
- f_3 = productivity process

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- C_t = consumption = control variable
- K_t = capital = endogenous state variable
- Z_t = productivity = exogenous state variable
- f_1 = Euler equation
- f_2 = resource constraint
- f_3 = productivity process

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$$\mathbb{E}_t \begin{bmatrix} d\widehat{C}_t \\ d\widehat{K}_t \\ d\widehat{Z}_t \end{bmatrix} = \underbrace{\begin{bmatrix} B_{CC} & B_{CK} & B_{\Lambda Z} \\ B_{KC} & B_{KK} & B_{KZ} \\ 0 & 0 & B_{ZZ} \end{bmatrix}}_{\mathrm{B}} \begin{bmatrix} \widehat{C}_t \\ \widehat{K}_t \\ \widehat{Z}_t \end{bmatrix} dt$$

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3. Diagonalize matrix **B**, hope same number of stable eigenvalues as state variables (2 in this model)

   Set control variables $\perp$ to unstable eigenvectors $\Rightarrow$ policy function

$$\widehat{C}_t = D_K \widehat{K}_t + D_Z \widehat{Z}_t$$

Main Event: Linearizing a Heterogeneous Agent Model

1. **Compute non-linear approx. of non-stochastic steady state**

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   - Finite difference method from Achdou et al. (2017)
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Step 1: Compute non-stochastic steady state

$$\rho v(a,z) = \max_c \; u(c) + \partial_a v(a,z)(wz + ra - c) + \lambda_z (v(a,z') - v(a,z)) \qquad \text{(HJB SS)}$$

$$0 = -\partial_a[s(a,z)g(a,z)] - \lambda_z g(a,z) + \lambda_{z'} g(a,z') \qquad \text{(KF SS)}$$

$$w = (1-\alpha)K^{\alpha}, \quad r = \alpha K^{\alpha-1} - \delta, \qquad K = \int a g(a,z) da dz \qquad \text{(PRICE SS)}$$

Step 1: Compute non-stochastic steady state

$$\begin{aligned}\rho v_{i,j} =& u(c_{i,j}) + \partial_a v_{i,j}(wz_j + ra_i - c_{i,j}) \\ &+ \lambda_j(v_{i,-j} - v_{i,j}), \text{ with } c_{i,j} = u'^{-1}(\partial_a v_{i,j})\end{aligned} \qquad \text{(HJB SS)}$$

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$$\rho \mathbf{v} = \mathbf{u}(\mathbf{v}) + \mathbf{A}(\mathbf{v}; \mathbf{p})\,\mathbf{v} \qquad \text{(HJB SS)}$$

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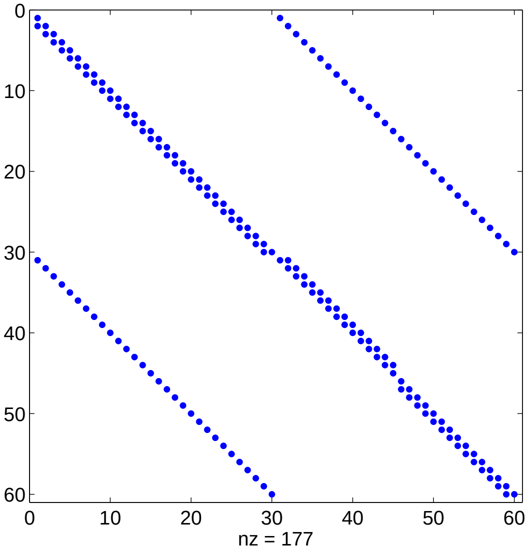
Visualization of **A** (output of `spy(A)` in Matlab)



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$$\mathbf{p} = \mathbf{F}(\mathbf{g}) \qquad \text{(PRICE SS)}$$

- More generals models: (PRICE SS) becomes

$$0 = \mathbf{F}(\mathbf{g}, \mathbf{p})$$

Linearization: three steps

1. Compute non-linear approximation to non-stochastic steady state
    - Finite difference method from Achdou et al. (2017)
    - Steady state reduces to sparse matrix equations
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2. **Compute first-order Taylor expansion around steady state**
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   - Automatic differentiation: exact numerical derivatives
   - Efficient `Matlab` implementation for sparse systems
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Step 2: Linearize discretized system

- Discretized system with aggregate shocks

$$
\begin{aligned}
\rho \mathbf{v}_t &= \mathbf{u}(\mathbf{v}_t) + \mathbf{A}(\mathbf{v}_t; \mathbf{p}_t)\, \mathbf{v}_t + \frac{1}{dt}\mathbb{E}_t[d\mathbf{v}_t] \\
\frac{d\mathbf{g}_t}{dt} &= \mathbf{A}(\mathbf{v}_t; \mathbf{p}_t)^{\top} \mathbf{g}_t \\
\mathbf{p}_t &= \mathbf{F}(\mathbf{g}_{\mathrm{t}}; Z_t) \\
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- Write in general form

$$\mathbb{E}_t \begin{bmatrix} d\mathbf{v}_t \\ d\mathbf{g}_t \\ \mathbf{0} \\ dZ_t \end{bmatrix} = f(\mathbf{v}_t, \mathbf{g}_t, \mathbf{p}_t, Z_t)dt, \qquad \begin{bmatrix} \mathbf{v}_t \\ \mathbf{g}_t \\ \mathbf{p}_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \text{control} \\ \text{endog state} \\ \text{prices} \\ \text{exog state} \end{bmatrix}$$

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\end{aligned}
$$

- Linearize using automatic differentiation (code: `@myAD`)

$$
\mathbb{E}_t \begin{bmatrix} d\widehat{\mathbf{v}}_t \\ d\widehat{\mathbf{g}}_t \\ \mathbf{0} \\ dZ_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{B}_{vv} & \mathbf{0} & \mathbf{B}_{vp} & \mathbf{0} \\ \mathbf{B}_{gv} & \mathbf{B}_{gg} & \mathbf{B}_{gp} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{pg} & -\mathbf{I} & \mathbf{B}_{pZ} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\nu \end{bmatrix}}_{\mathrm{B}} \begin{bmatrix} \widehat{\mathbf{v}}_t \\ \widehat{\mathbf{g}}_t \\ \widehat{\mathbf{p}}_t \\ Z_t \end{bmatrix} dt
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3. **Solve linear stochastic differential equation**
   - Moderately-sized systems $\Longrightarrow$ standard methods OK
   - Large systems $\Longrightarrow$ dimensionality reduction

Model-Free Reduction Method

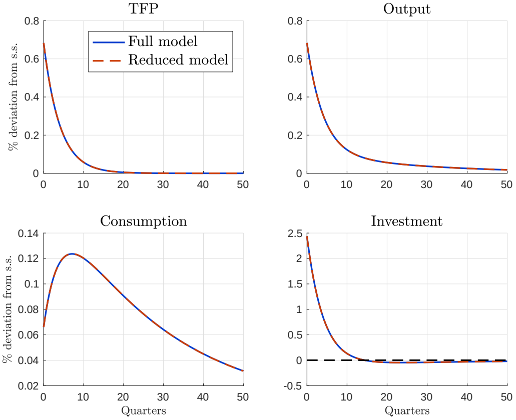
- Key insight: only need distribution $\mathbf{g}_t$ to forecast prices
  1. Krusell & Smith: guess moments ex-ante, check accuracy ex-post
  2. Our approach: computer chooses "moments", guarantee accuracy
- Approximate N-dimensional distribution with k-dimensional basis

$$\mathbf{g}_t \approx \gamma_{1t}\mathbf{x}_1 + \ldots + \gamma_{kt}\mathbf{x}_k$$

$\Rightarrow$ how to choose the basis $\mathbf{x}_1, \ldots, \mathbf{x}_k$?

- State-space reduction tools from engineering literature (Reiter 2010)
  - use "observability" criterion $\equiv$ matching impulse responses
  - adapt to problems with forward-looking decisions

Approximate Aggregation in Krusell & Smith Model



- Comparison of full distribution vs. $k = 1$ approximation

  $\Longrightarrow$ recovers Krusell & Smith's "approximate aggregation"

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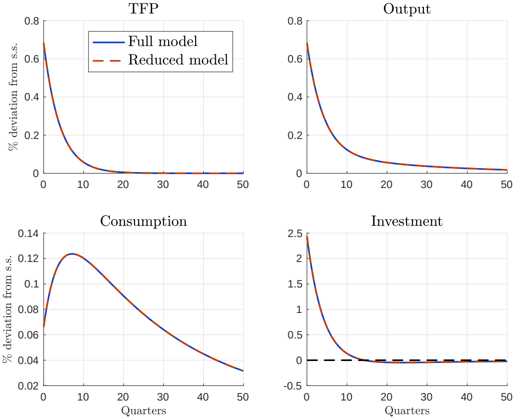

- Large-scale models in applications require $k = 300$

  $\Longrightarrow$ no approximate aggregation

Our Method Is Fast, Accurate in Krusell & Smith Model

Our method is fast

	w/o Reduction	**w/ Reduction**
Steady State	0.082 sec	0.082 sec
Linearize	0.021 sec	0.021 sec
Reduction	$\times$	0.007 sec
Solve	0.14 sec	0.002 sec
Total	0.243 sec	0.112 sec

- JEDC comparison project (2010): fastest alternative $\approx$ 7 minutes

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Our method is accurate

Agg Shock σ	0.01%	0.1%	0.7%	1%	5%
Den Haan Error	0.000%	0.002%	0.053%	0.135%	3.347%

- JEDC comparison project: most accurate alternative $\approx$ 0.16%

Applications

A Model of Distribution of Income, Wealth, and MPCs

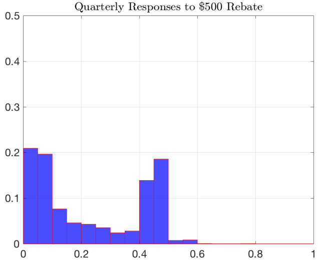
- Households: two-asset incomplete markets (Kaplan-Moll-Violante)
  - liquid asset
  - illiquid assets subject to transaction cost
- Aggregate production function with growth rate shocks

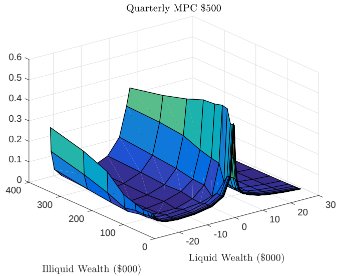
$$
\begin{aligned}
Y_t &= Q_t K_t^{\alpha} N_t^{1-\alpha} \\
\mathrm{d} \log Q_t &= Z_t \mathrm{d}t \\
\mathrm{d}Z_t &= -\eta Z_t \mathrm{d}t + \sigma \mathrm{d}W_t
\end{aligned}
$$

- Market clearing:
  - K_t = illiquid assets
  - B = liquid assets (fixed supply)

Application 1: Inequality Matters for C + Y Dynamics

- Campbell-Mankiw (1989): how match aggregate C + Y dynamics?
- Calibrate model to match
  1. Household side: distribution of income, wealth, and MPCs
  2. Firm side: dynamics of $\Delta \log Y_t$





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	Data	**Models**		
		Rep agent	Two-Asset	CM
Sensitivity to Income				
IV($\Delta \log C_t$ on $\Delta \log Y_t$ using $\Delta \log Y_{t-1}$)	0.503	0.247	0.656	
Smoothness				
$\frac{\sigma(\Delta \log C_t)}{\sigma(\Delta \log Y_t)}$	0.518	0.709	0.514	

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$\frac{\sigma(\Delta \log C_t)}{\sigma(\Delta \log Y_t)}$	0.518	0.709	0.514	0.676

Application 2: Agg Shocks Matter for Inequality

- With Cobb-Douglas prod'n, labor income inequality exogenous

$$\text{labor income } = w_t \times z_{jt}$$

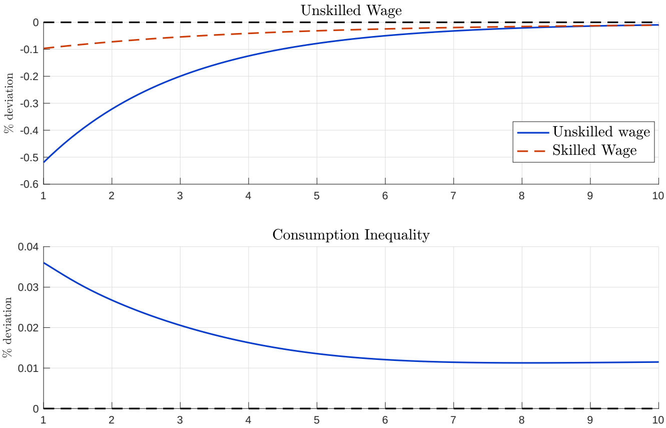
- Modify production function to generate endogenous inequality

$$Y_t = \left[\mu(Z_t^U N_t^U)^\sigma + (1-\mu)\left(\lambda K_t^\rho + (1-\lambda)(N_t^S)^\rho\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}}$$

  - N_t^U: unskilled labor w/ low persistent productivity z_{jt}
  - N_t^S: skilled labor w/ high persistent productivity z_{jt}
  - Z_t^U: unskilled-specific productivity shock

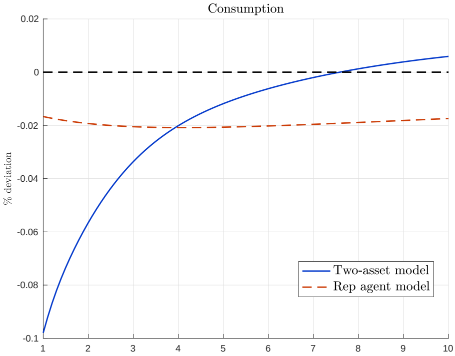
- Calibrate σ and ρ to generate capital-skill complementarity

Unskilled-Specific Shock Increases Inequality...



- Fluctuations in income inequality ≈ aggregate income

... And Generates Sharp Consumption Bust



- Many low-skill households hand-to-mouth

  $\Longrightarrow$ larger consumption drop than in rep agent model

One-Asset HANK Model

One-Asset HANK – Model Outline

- For details see `https://github.com/gregkaplan/phact/blob/master/examples/one_asset_HANK/docs/one_asset_hank_no_capital.pdf`
- Households:
  - as in Krusell-Smith model + endogenous labor supply
  - policy functions $c_t(a, z), \ell_t(a, z)$, distribution $g_t(a, z)$
- Firms:
  - monopolistic intermediate-good producers, labor demand L_t
  - quadratic price adjustment costs à la Rotemberg (1982)
  - $\Rightarrow$ New Keynesian Phillips curve
- Government: issues liquid debt B_t^g, spends, taxes/transfers
- Monetary authority: sets nominal rate based on a Taylor rule
- Equilibrium:

$$B_t^g = \int a g_t(a, z) da dz, \qquad L_t = \int \ell_t(a, z) g_t(a, z) da dz$$

Walking you through the code

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Macro With Inequality: No More Excuses!

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2. Use methodology to illustrate interaction of macro + inequality
   - match micro behavior ⇒ realistic aggregate C + Y dynamics
   - aggregate shocks generate inequality dynamics...
   - ... and IRFs in HA model can differ dramatically from RA case
3. Check out one-asset HANK model at
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- Estimating models w/ micro data on distributions within reach
- Lots of cool applications: come talk to us!

Fully recursive notation ▸ Back

$$w(g,Z) = (1-\alpha)e^Z K(g)^\alpha, \quad r(g,Z) = \alpha e^Z K(g)^{\alpha-1} - \delta \tag{P}$$

$$K(g) = \int a g(a,z) da dz \tag{K}$$

$$\begin{aligned}\rho V(a,z,g,Z) = \max_c \ & u(c) + \partial_a V(a,z,g,Z)(w(g,Z)z + r(g,Z)a - c) \\ & + \lambda_z (V(a,z',g,Z) - V(a,z,g,Z)) \\ & + \partial_Z V(a,z,g,Z)(-\nu Z) + \frac{1}{2}\partial_{ZZ} V(a,z,g,Z)\sigma^2 \\ & + \int \frac{\delta V(a,z,g,Z)}{\delta g(\tilde a,\tilde z)} (\mathcal{K}_Z g)(\tilde a,\tilde z) d\tilde a d\tilde z \end{aligned} \tag{∞d HJB}$$

$$(\mathcal{K}_Z g)(a,z) = -\partial_a[s(a,z,g,Z)g(a,z)] - \lambda_z g(a,z) + \lambda_{z'} g(a,z') \tag{KF operator}$$

$$s(a,z,g,Z) = w(g,Z)z + r(g,Z)a - c^*(a,z,g,Z)$$

- $\delta V/\delta g(a,z)$: functional derivative of V wrt g at point (a,z)