Lectures 7:
Growth Model and the Data

ECO 503: Macroeconomic Theory I

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Fall 2014

Plan of Lecture

- The growth model and the data
  1. steady states and the data
  2. choosing parameter values, calibration
  3. transition dynamics and the data

Steady States and the Data

- In model steady state, we have

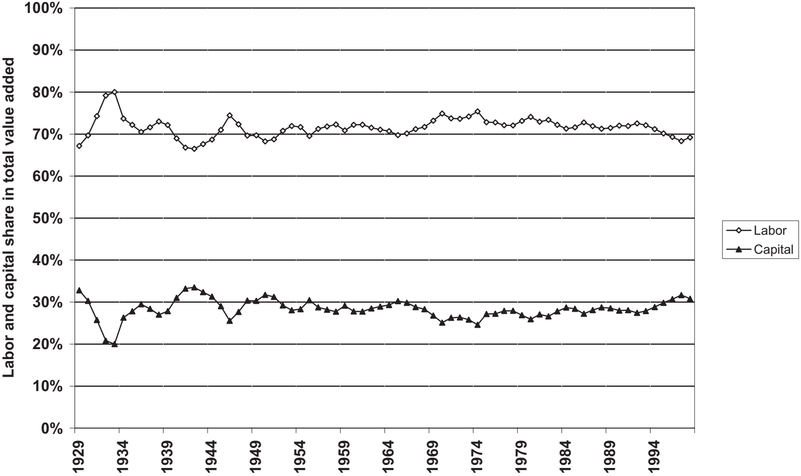
$$
\begin{aligned}
\frac{k_t}{y_t} &= \text{constant} \\
\frac{i_t}{y_t} &= \text{constant} \\
\frac{w_t h_t}{y_t} &= \text{constant} \quad \text{(labor share)} \\
\frac{R_t k_t}{y_t} &= \text{constant} \quad \text{(capital share)} \\
r_t &= \text{constant}
\end{aligned}
$$

- In fact, in version we examined so far, all of $k_t, i_t, \ldots$ constant. No growth in current version of growth model.
- But ratios above remain constant when we introduce growth (future lecture)

Steady States and the Data

- For U.S. economy post 1950, over longer time periods, we observe
  1. k_t/y_t roughly constant
  2. i_t/y_t roughly constant
  3. $R_t k_t/y_t$ roughly constant
  4. $w_t h_t/y_t$ roughly constant
- These observations are known as the "Kaldor Facts" (after economist Nicholas Kaldor)

U.S. Capital and Labor Shares



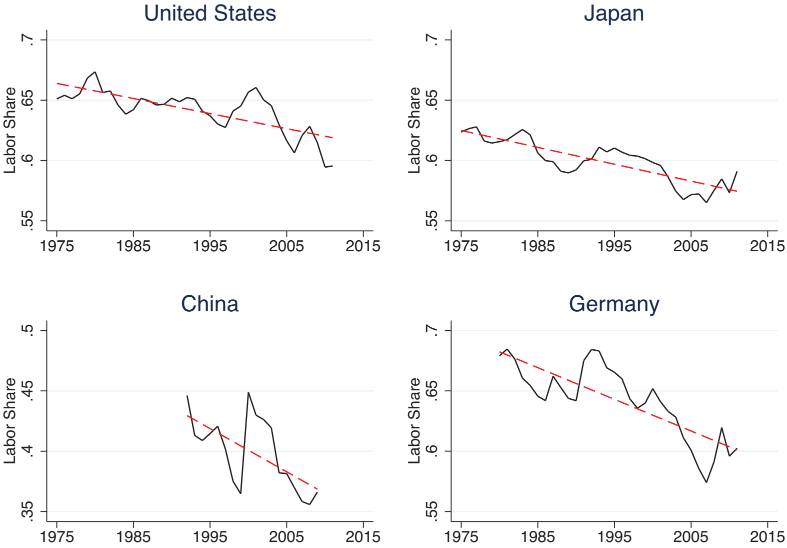
Steady States and the Data

- **Conclusion**: for the U.S. economy post 1950, one could interpret the data as fluctuating around a steady state

- Note: accuracy of "Kaldor Facts" recently questioned (see e.g. Piketty, 2014; Karabarbounis and Neiman, 2014)
    - people have proposed a variety of "fixes" to growth model
    - ongoing debate, let's ignore this for now

Accuracy of Kaldor Facts?

Labor shares from Karabarbounis and Neiman (2014)



Using the Growth Model

- If we want to use growth model to provide **quantitative** assessments, need to choose functional forms and parameter values
- true for both “positive” and “normative” issues
    - this time: positive (capital accumulation in U.S.)
    - next time: normative (capital taxation)

Functional Forms

- Guideline
  1. parsimony
  2. choose functional forms in which parameters have clear economic interpretations

- Need to choose two functional forms

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

$$f(k_t) = Ak_t^{\alpha}$$

- Note: $\sigma = 1$ corresponds to log utility

$$\lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1 - \sigma} = \lim_{\sigma \to 1} \frac{e^{(1-\sigma)\log c} - 1}{1 - \sigma} = \frac{-\log c}{-1} \lim_{\sigma \to 1} e^{(1-\sigma)\log c} = \log c$$

- set $A = 1$: just choice of units

- now have 4 parameters: $\delta, \beta, \sigma, \alpha$

Choosing Parameter Values

- Literature suggests two distinct approaches
  1. estimation
  2. calibration
- Why not (always) estimate?
  - model is an abstraction, designed to capture some features and deliberately abstracting from others
  - (many) standard formal statistical procedures weight all aspects equally
- Alternative: choose the aspects of the data that your model was most aimed to capture
- Key idea of calibration: choosing parameters comes down to selecting moments to match. Sometimes you might want to use discretion in choosing moments.
- That being said, estimation and calibration are just variations on common theme
  - standard calibration is just exactly identified GMM estimation

Application: Calibration of Growth Model

- Model designed to capture capital accumulation process
- So let's use moments that relate to this process

$$\frac{k_t}{y_t}, \quad \frac{i_t}{y_t}, \quad r_t$$

- If we think post 1950, U.S. looks like fluctuations around st.st., can use average values in data to think about steady state values
- Intrepreting 1 time period = 1 year, this gives

$$\frac{k}{y} \approx 2.5, \quad \frac{i}{y} \approx 0.2, \quad r \approx 0.04 \;\; \text{(0.02-0.06)}$$

- recall 4 parameters $\delta, \beta, \sigma, \alpha$, but only 3 moments
- Note: σ doesn't influence st.st. so cannot identify it from st.st. $\Rightarrow$ 3 moments are sufficient to identify δ, β, α

Application: Calibration of Growth Model

1. $r = 0.04$: in steady state

$$1 + r = \frac{1}{\beta} \Rightarrow \quad \beta = \frac{1}{1+r} = \frac{1}{1.04} \approx 0.96$$

2. in steady state $i = \delta k$

$$\delta = \frac{i}{k} = \frac{i/y}{k/y} = \frac{0.2}{2.5} = 0.08$$

3. in steady state

$$\alpha k^{\alpha-1} = \alpha \frac{y}{k} = \frac{1}{\beta} - (1-\delta)$$
$$\alpha = \frac{k}{y}\left(\frac{1}{\beta} - (1-\delta)\right) = 2.5(1.04 - 0.92) = 0.3$$

4. σ: range of estimates in literature is $[1, 2.5]$. Will use $\sigma \to 1$ (Note: can show $\lim_{\sigma \to 1} \frac{c^{1-\sigma}-1}{1-\sigma} = \log c$)

- Recall from Lecture 4: half-life for convergence to steady state

$$t_{1/2} = \frac{\ln(2)}{|\lambda_1|}, \quad \lambda_1 = \frac{\rho - \sqrt{\rho^2 - 4\frac{1}{\sigma} f''(k) c}}{2}$$

$$= \frac{\rho - \sqrt{\rho^2 + 4\frac{1-\alpha}{\sigma}\alpha\frac{y}{k}\left(\frac{y}{k} - \delta\right)}}{2}$$

- in steady state of continuous time model: $r = \rho$

$$\lambda_1 = \frac{0.04 - \sqrt{(0.04)^2 + 4 \times 0.7 \times 0.3\frac{1}{2.5}\left(\frac{1}{2.5} - 0.08\right)}}{2}$$

$$\approx -0.15$$

$$\Rightarrow t_{1/2} \approx \frac{\ln(2)}{0.15} \approx 4.75 \text{ years}$$

Transition Dynamics and the Data

- Given our parameter values, model converges to steady state very quickly
  - suggests that it is reasonable that we are around steady state from 1950 to present
  - if instead half-life were 100 years, things would be different
- **Summary so far:** growth model does a good job at capturing some salient features of U.S. economy post 1950
  - also true once we extend it to actually feature growth
  - also true for other developed countries (e.g. entire OECD)
- **Can growth model also capture growth experience of poorer countries?**

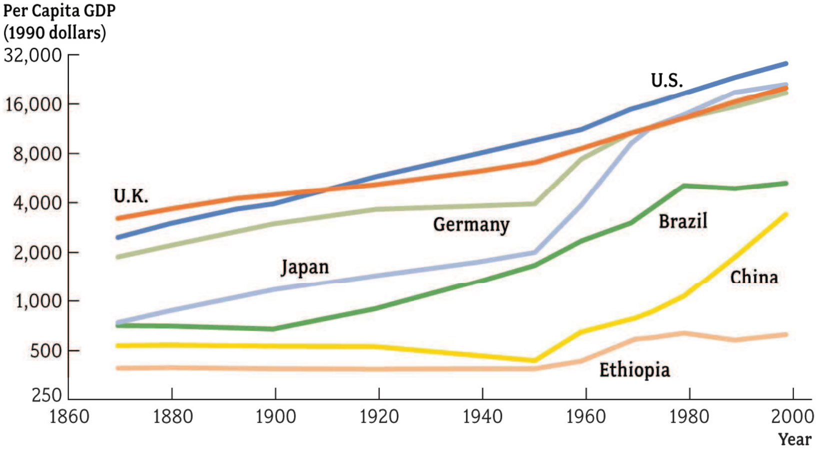
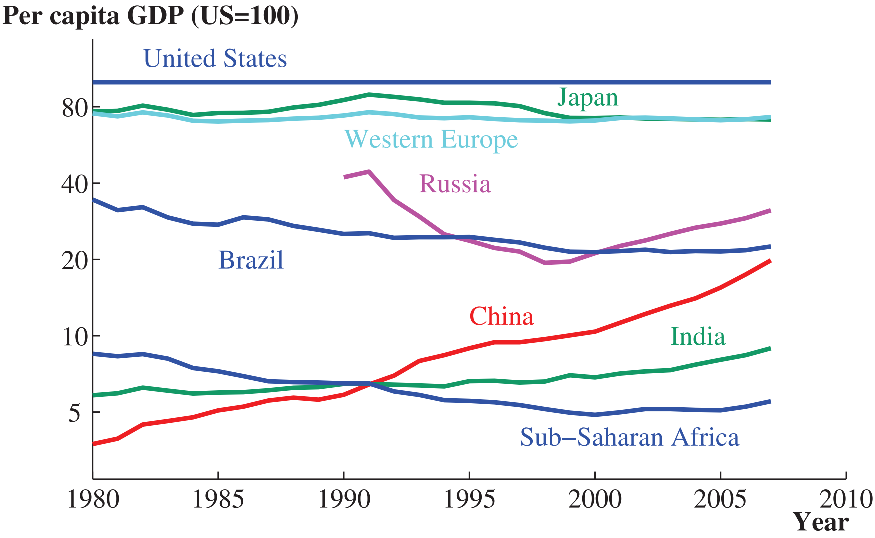


FIGURE 1.1 Per Capita GDP in Seven Countries, 1870–2000



Per capita GDP (US=100)
United States
Japan
Western Europe
Russia
Brazil
China
India
Sub–Saharan Africa
80
40
20
10
5
1980
1985
1990
1995
2000
2005
2010
Year

East Asian Miracles

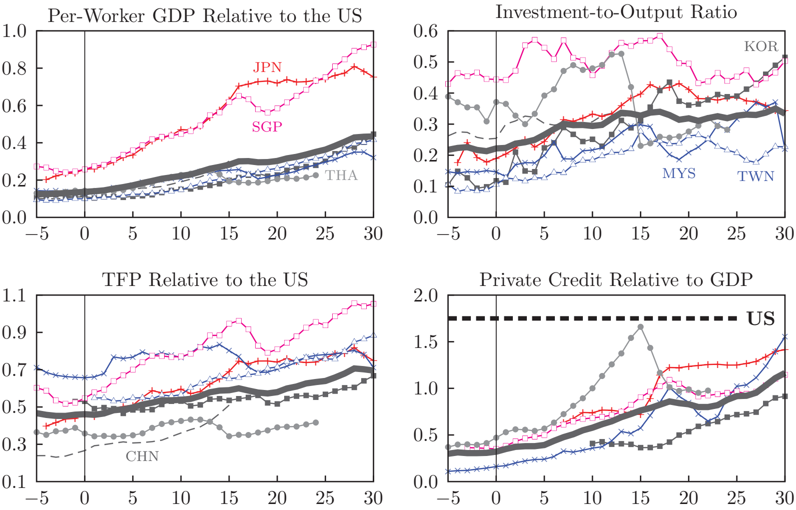


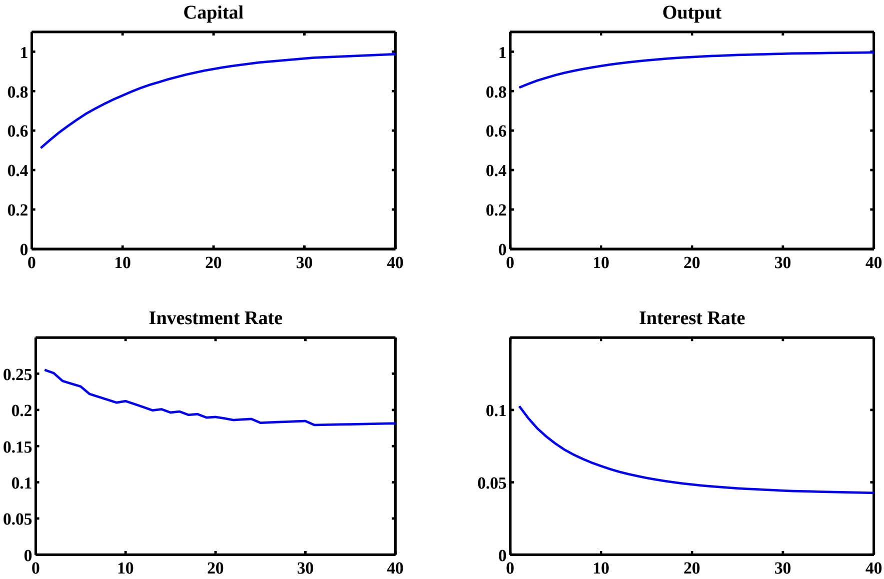
Fig. 1: Transitional Dynamics from the Economic Miracles

Development Dynamics in Data

1. Slow convergence
2. Rising investment-to-output ratio in early stages
3. Output growth partly explained by aggregate productivity (TFP) and reallocation dynamics

Can growth model capture these facts? **No!**

Neoclassical Transitions Get it Wrong

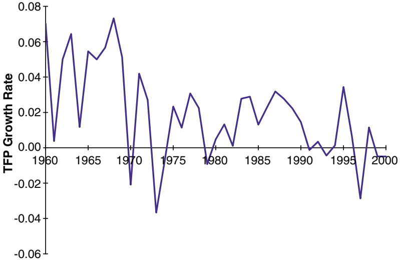


Neoclassical Transitions Get it Wrong

- King and Rebelo (1993): neoclassical growth model with constant (or no) exogenous TFP growth has no hope of explaining sustained growth as stemming from transitional dynamics.
- extremely counterfactual implications for the time path of the interest rate.
- According to their calculations for example, if the neoclassical growth model were to explain the postwar growth experience of Japan, the interest rate in 1950 should have been around 500 percent.

Needed: A Theory of TFP (Dynamics)

- In contrast, Chen, Imrohoroglu and Imrohoroglu (2006, 2007): the neoclassical growth model is, in fact, consistent with the Japanese postwar growth experience once one takes as given the time-varying TFP path measured in the data.
- TFP time path they feed into their model



- **Growth model fails as model of development dynamics**
- What we need instead: a theory of endogenous TFP dynamics
  - take my second year course!