Lecture 3: Werning (2012)

“Managing a Liquidity Trap”

ECO 521: Advanced Macroeconomics I

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Outline

- Last time: New Keynesian 3 equation model, derived from micro foundations
- Ignored ZLB (or "liquidity trap"), $i(t) \geq 0$.
- This time: optimal policy at ZLB?
- Also optimal fiscal policy.

Advantages of Continuous Time

- Very nice and tractable
- Payoff: very clean results, even though complicated stuff.
  - Keep interest rate at zero past liquidity trap
  - Engineer output boom, not inflation
  - Lots more
- Policy involves optimal switching time, awkward in discrete time
- Graphical analysis using phase diagrams
- "Aerospace engineering approach" to optimal monetary policy: central bank controls trajectory of economy.

Model

- Last time: three equation model

$$\dot{x} = i - \pi - r \tag{IS'}$$

$$\dot{\pi} = \rho\pi - \kappa x \tag{PC'}$$

$$i = i^* + \phi\pi + \phi_x x \tag{TR'}$$

- Recall: $\kappa = (\varepsilon - 1)(1 + \varphi)/\theta =$ price flexibility
- This time: drop Taylor rule (TR'), replace with optimal monetary policy
- Also generalize to CRRA utility, $\sigma \neq 1$, impose ZLB

$$\dot{x} = \sigma^{-1}(i - \pi - r)$$

$$\dot{\pi} = \rho\pi - \kappa x$$

$$i \geq 0$$

The Natural Interest Rate

- Have shown last time: if $r(t) > 0$ for all t
  - first-best $(x(t), \pi(t)) = (0, 0)$ can be attained, e.g. with Taylor rule with $i^* = r$ and $\phi > 1$.
  - "Divine coincidence"
- This time: liquidity trap scenario:

$$r(t) = \begin{cases} \underline{r}, & t \in [0, T) \\ \bar{r}, & t \in [T, \infty) \end{cases}$$

where $\underline{r} < 0 < \bar{r}$.

- Why could natural interest rate go negative?
  - TFP growth down $r = \rho + \dot{A}/A$.
  - Anything that affects savings behavior ρ ("animal spirits",...)
  - Credit crunch (Guerrieri and Lorenzoni, 2011)

Liquidity Trap: No Commitment

- "No commitment" means central bank benevolent but cannot credibly announce plans for the future.
- Acts opportunistically at each point in time.
- Will see momentarily: this is a bad thing
- Time inconsistency problem. Classic article: Kydland and Prescott (1977) "Rules rather than Discretion"
- After the trap, $t \in [T, \infty)$: implement first best

$$(x(t), \pi(t)) = (0, 0)$$

- How do this? See last lecture. For example, Taylor rule with $i^* = r, \phi > 1$.

Liquidity Trap: No Commitment

- During the trap, $t \in [0, T)$: $i(t) = 0$, cannot attain first-best
- Dynamics governed by

$$\dot{x} = -\sigma^{-1}(\underline{r} + \pi)$$

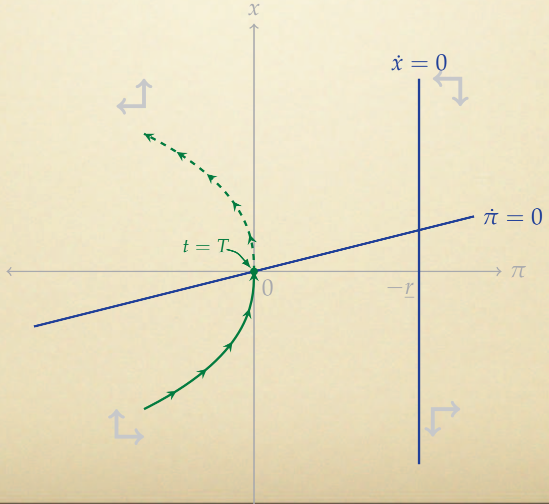
$$\dot{\pi} = \rho\pi - \kappa x$$

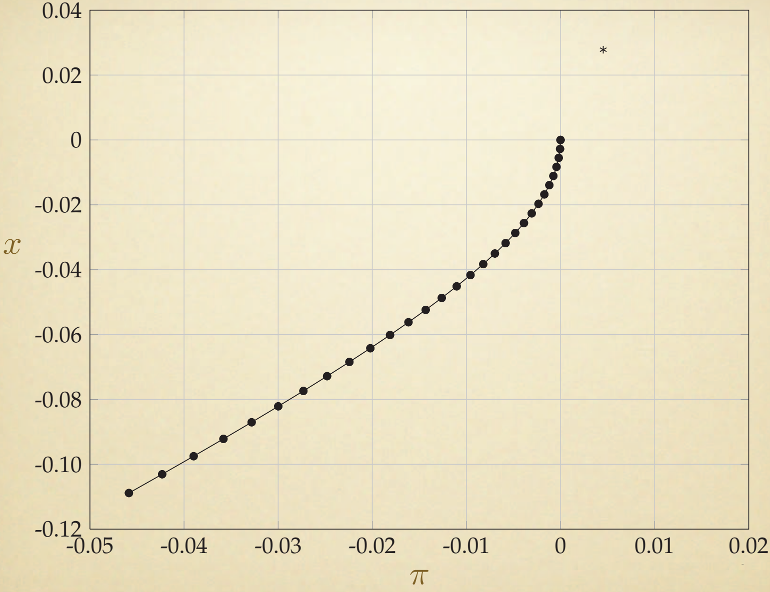
- Important: terminal condition

$$(x(T), \pi(T)) = (0, 0)$$

No Commitment

- at T: first best x(T)=π(T)=0
- before T: binding zero interest i(t)=0





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No Commitment

- Deflation and Depression...

Proposition.
Without commitment

$$x(t) < 0, \pi(t) < 0 \text{ for } t < T$$

As $T \to \infty$

$$x(0), \pi(0) \to -\infty$$

- intuition
  - too high real interest: too high growth
  - cumulative effect with T

Does Price Flexibility Help? No!

- Recall: $\kappa = (\varepsilon - 1)(1 + \varphi)/\theta$
- θ: scales price adjustment cost function
- **Proposition:** Higher κ (lower θ) leads to lower $x(t)$ and $\pi(t)$.

  In the limit as $\kappa \to \infty$ ($\theta \to 0$) we have

  $$x(t), \pi(t) \to -\infty$$

- Intuition: Phillips curve in integral form

  $$\pi(t) = \kappa \int_t^\infty e^{-\rho(s-t)} x(s) ds$$

- For given **negative** $x(s), s \geq t$, $\kappa \uparrow \Rightarrow \pi \downarrow$, i.e. more deflation.
- From Euler equation (IS curve)

  $$\dot{x} = -\sigma^{-1}(\underline{r} + \pi) \ \uparrow$$

- $\Rightarrow x(0) \downarrow$ since $x(T) = 0$ fixed.
- Discontinuity? No. $\pi(T) = 0$ suboptimal with $\kappa = \infty$.

Elbow Room

- No commitment $\Rightarrow$ deflation, depression
- Even simple, non-optimal policies make things better.
- Here's one. **For all** $t \geq 0$ **also after trap ends,** $t > T$, set

$$\pi(t) = -\underline{r} > 0, \quad x(t) = -\frac{1}{\kappa}\underline{r} > 0$$

- Problem: real interest rate too high.
- Partial fix: inflation forever
- Small positive output gap
- Can check $i(t) \geq 0$ for all t.
- Key: be able to commit to something other than

$$(\pi(T), x(T)) = (0, 0)$$

Optimal Monetary Policy with Commitment

- Planning problem

$$\min_{c,\pi,i} \frac{1}{2}\int_0^\infty e^{-\rho t}\left(x(t)^2 + \lambda\pi(t)^2\right) dt$$

$$\dot{x}(t) = \sigma^{-1}(i(t) - r(t) - \pi(t))$$

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

$$i(t) \geq 0$$

  and $x(0), \pi(0)$ free.

- Objective function: welfare **loss**, can be derived as second order approximation to welfare around zero inflation.
  See Gali (2008), Chapter 4, Appendix 1; Woodford (2003) Proposition 6.4.
- Note: high x bad because work too much ($MRS \neq MPL$)

Optimal Monetary Policy with Commitment

- Hamiltonian:

$$\mathcal{H} = \frac{1}{2}x^2 + \frac{1}{2}\lambda\pi^2 + \mu_x\sigma^{-1}(i - r - \pi) + \mu_\pi(\rho\pi - \kappa x) - \psi i$$

- (x, π): states (output gap, inflation)
- (μ_x, μ_π): co-states
- i: control (nominal interest rate)
- $\psi \geq 0$: Lagrange mult. on $i \geq 0$, compl. slackness $\psi i = 0$.
- Conditions for optimum:

$$\mu_x\sigma^{-1} = \psi \quad \Rightarrow \quad \mu_x \geq 0, \quad \mu_x i = 0$$

$$\dot{\mu}_x = \rho\mu_x - x + \kappa\mu_\pi$$

$$\dot{\mu}_\pi = \rho\mu_\pi - \lambda\pi + \sigma^{-1}\mu_x - \rho\mu_\pi$$

Optimal Monetary Policy with Commitment

- Massage a bit

$$\mu_x \geq 0, \quad \mu_x i = 0$$

$$\dot{\mu}_x = \rho\mu_x - x + \kappa\mu_\pi$$

$$\dot{\mu}_\pi = -\lambda\pi + \sigma^{-1}\mu_x$$

$$\dot{x} = \sigma^{-1}(i - \pi - r)$$

$$\dot{\pi} = \rho\pi - \kappa x$$

- Since $x(0), \pi(0)$ are free, two additional conditions

$$\mu_x(0) = 0, \quad \mu_\pi(0) = 0$$

- Recall: $\mu_x(0) =$ marginal value of one additional unit of $x(0)$ and similarly for $\pi(0)$.
- $+$ two transversality conditions.

Graphical Representation

- Three Phases:
  - Phase I: During the Liquidity Trap, $t \in [0, T)$
  - Phase II: Just out of the Trap, $t \in [T, \hat{T})$
  - Phase III: After the Storm, $t \in [\hat{T}, \infty)$
- Go backwards in time. Draw phase diagrams III,II,I.
- Phase diagrams will be such that there is a unique $(x(0), \pi(0))$ that satisfies transversality conditions.
- By picking a time path for the nominal interest rate, $i(t)$, the central bank can pick these initial conditions and the trajectories for $(x(t), \pi(t)), t > 0$.
- Trick is stichting together three phase diagrams in right way.
- Note: it's all about whether $(x(t), \pi(t))$ are fixed or free at $t = 0, T, \hat{T}$.

Phase III: After the Storm

- $(\pi(\hat{T}), x(\hat{T}))$ inherited from the past, i.e. not free.
- First-best $(\pi(t), x(t)) = (0, 0)$ generally not feasible
- ZLB not binding: $\mu_x = \dot{\mu}_x = 0$.

$$\dot{\mu}_x = \rho\mu_x - x + \kappa\mu_\pi \quad \Rightarrow \quad x = \kappa\mu_\pi$$

$$\dot{\mu}_\pi = -\lambda\pi + \sigma^{-1}\mu_x \quad \Rightarrow \quad \dot{\mu}_\pi = -\lambda\pi$$

$$\Rightarrow \quad \dot{x} = \kappa\dot{\mu}_\pi = -\kappa\lambda\pi$$

- Combining with $\dot{x} = \sigma^{-1}(i - \pi - r)$

$$i = r + (1 - \kappa\sigma\lambda)\pi \equiv I(r, \pi)$$

- Same interest rate condition as in Clarida, Gali and Gertler (1999). Property: $\pi = 0 \Rightarrow i = I(r, 0) = r$.

Phase III: After the Storm

- System with optimal control $i = I(\pi, r) = r + (1 - \kappa\sigma\lambda)\pi$

$$\dot{x} = -\kappa\lambda\pi$$

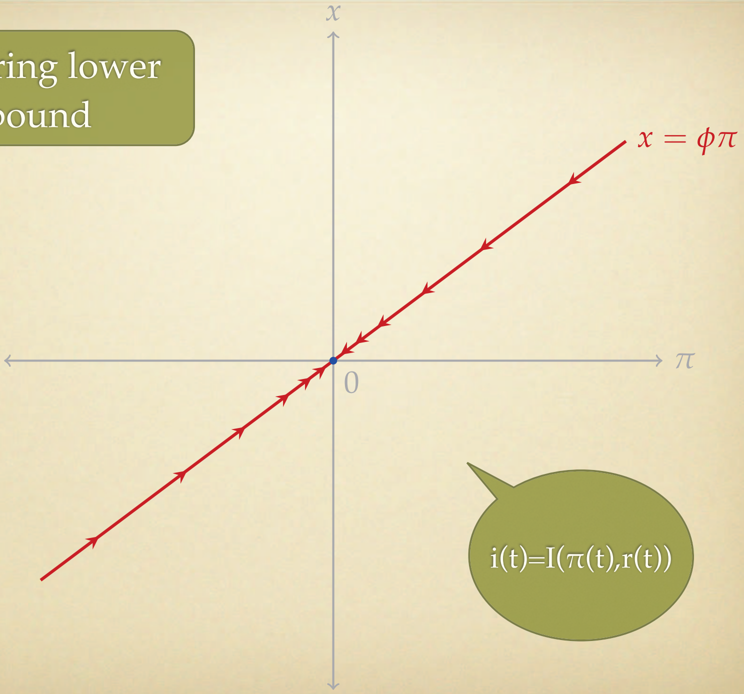
$$\dot{\pi} = \rho\pi - \kappa x$$

- Draw phase diagram $\Rightarrow$ saddle path
- **Claim:** In phase III (when the ZLB is slack)

$$x(t) = \phi\pi(t), \quad \phi \equiv \frac{\rho + \sqrt{\rho^2 + 4\lambda\kappa^2}}{2\kappa}$$

- That is, ϕ is **slope of the saddle path**.

Ignoring lower bound



i(t)=I(π(t),r(t))

Phase III: After the Storm

- **Claim:** Saddle path is

$$x(t) = \phi\pi(t), \quad \phi \equiv \frac{\rho + \sqrt{\rho^2 + 4\lambda\kappa^2}}{2\kappa}$$

- Derivation (more general trick for finding slope of saddle path)
- With optimal control $i = I(r, \pi)$

$$dx/dt = -\kappa\lambda\pi$$

$$d\pi/dt = \rho\pi - \kappa x$$

- Saddle path: $x(\pi(t))$ such that this holds. Slope:

$$\frac{dx}{d\pi} = \frac{dx/dt}{d\pi/dt} = \frac{-\kappa\lambda\pi}{\rho\pi - \kappa x(\pi)}$$

- Guess $x(\pi) = \phi\pi$ (doesn't always work, if not use L'Hopital)

$$\phi = \frac{-\kappa\lambda}{\rho - \kappa\phi} \quad \Rightarrow \quad -\kappa\phi^2 + \phi\rho + \kappa\lambda = 0$$

- Solve quadratic, two roots ϕ, take positive one.

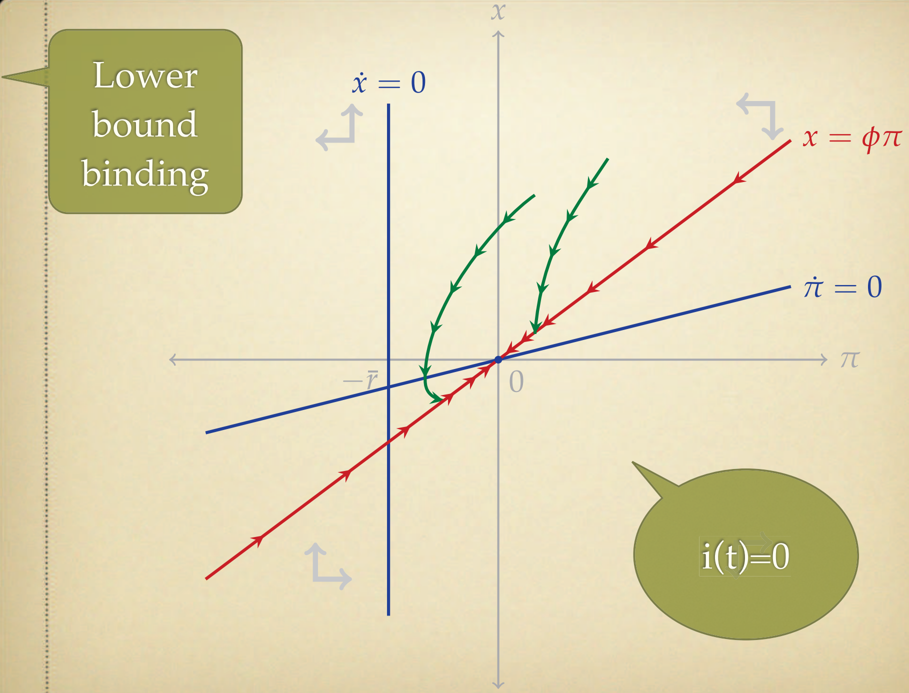
Phase II: Just out of the Trap

- Recall: $i(t) = 0$ in this phase
- Again $(\pi(T), x(T))$ inherited from the past, i.e. not free.
- System

$$\dot{x} = -\sigma^{-1}(\bar{r} + \pi)$$

$$\dot{\pi} = \rho\pi - \kappa x$$

- Same phase diagram as no-commitment case except that $\dot{x} = 0$ locus at $\pi = -\bar{r}$ rather than $\pi = -\underline{r}$


Lower bound binding
$\dot{x} = 0$
x
$x = \phi\pi$
$\dot{\pi} = 0$
π
$-\bar{r}$
0
i(t)=0

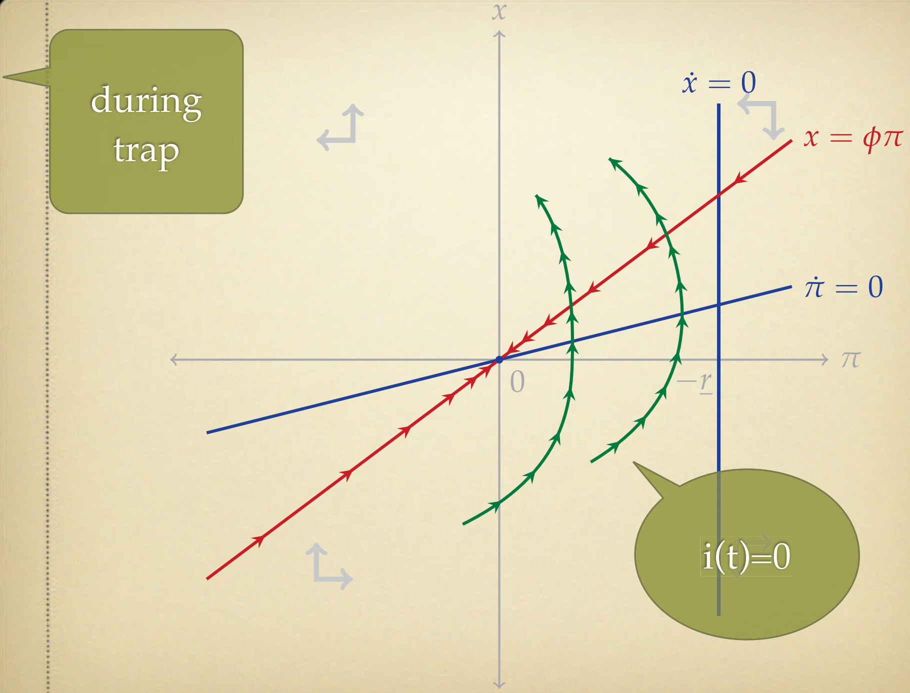
Phase I: During the Liquidity Trap

- System

$$\dot{x} = -\sigma^{-1}(\underline{r} + \pi)$$

$$\dot{\pi} = \rho\pi - \kappa x$$

- Same phase diagram as no-commitment case.
- $(x(0), \pi(0))$ free, but $(x(T), \pi(T))$ given.


during trap
x
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x = ϕπ
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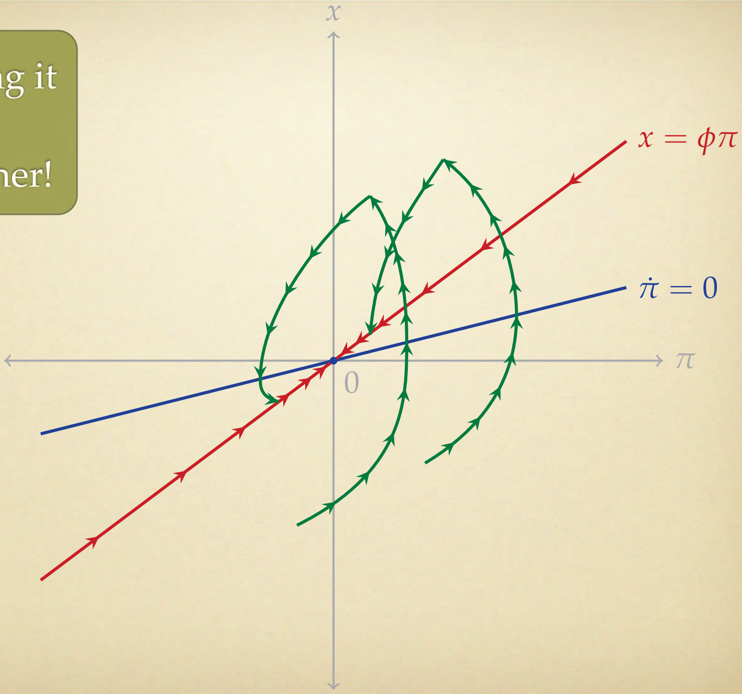
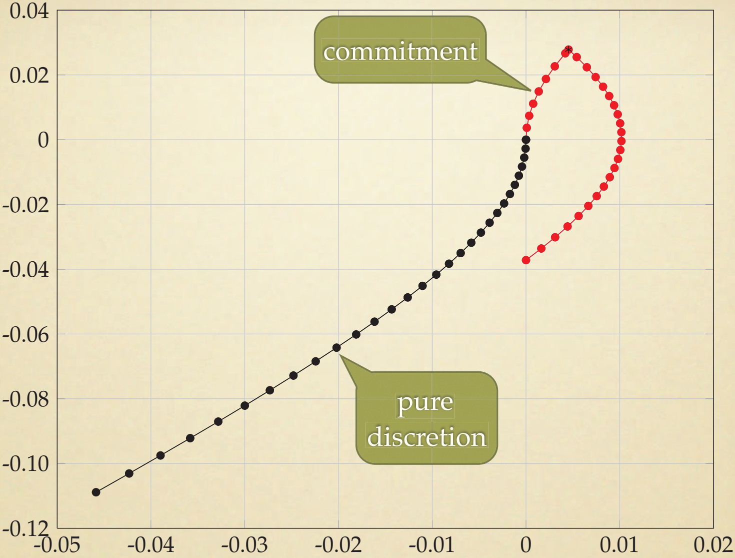
i(t)=0

Putting it all together!




commitment
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Main Results

$$I(\pi, r) \equiv r + (1 - \kappa\sigma\lambda)\pi$$

- result 1: slack ZLB $i(t) = I(\pi(t), r(t))$
- result 2:

$$I(\pi(t), r(t)) < 0 \text{ for } t \in [t_0, t_1)$$

$$\Rightarrow \quad i(t) = 0 \text{ for } t \in [t_0, t_2) \text{ with } t_1 < t_2$$

- result 3: inflation must be positive at some point
- result 4: output takes both signs
- result 5: inflation may be positive throughout

Communication

- What kind of commitment?
  - <u>needed</u>: policy commitments for t>T...
    - 1. promised targets

$$x(T), \pi(T)$$

    - 2. interest rate and exit inflation $\hat{T} > T$

$$i(t) = 0 \text{ for } t < \hat{T}$$

$$\pi(\hat{T})$$

  - <u>irrelevant</u>: policy commitments for t<T

Inflation or Boom?

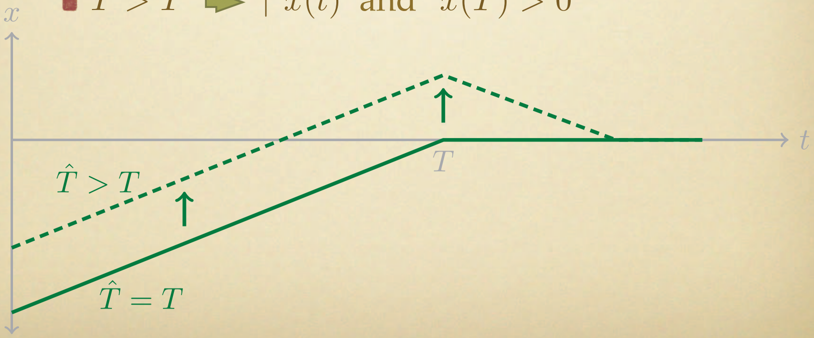
- Literature: purpose of monetary policy in liquidity trap = promote inflation.
- Werning: not true, real objective = engineer consumption boom.
- Paper: three special cases
- Special case 1: if process fully rigid so that inflation is zero, still want to set $i(t) = 0$ past trap.
- Special case 3: arbitrary no inflation constraint, $\pi \leq 0$.

#2 Rigid Prices

- completely rigid prices

$$x(t) = \sigma^{-1} \int_t^{\hat{T}} r(s)ds$$

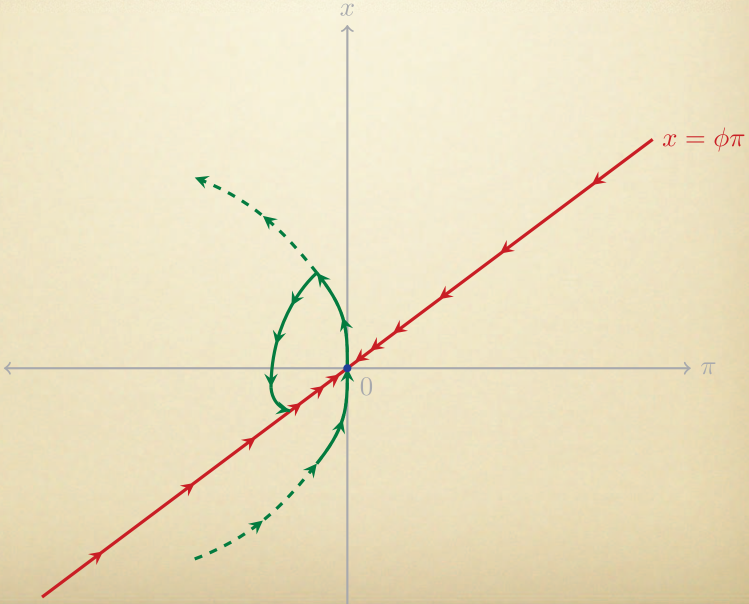
  - $\hat{T} = T$ ➡ $x(t) < 0$ (no commitment)
  - $\hat{T} > T$ ➡ $\uparrow x(t)$ and $x(T) > 0$



Inflation or Boom?

- Want to set
$$\int_0^\infty e^{-\rho t} x(t) dt = 0$$
- Current recession and subsequent boom should average out.
- Intuition: lower future interest rates to discourage savings

Avoiding Inflation



Government Spending

- Utility $U(C, N, G)$
- Public goods, G, valued in U, resources: $C + G = Y$.
- Planning problem

$$\min_{c,\pi,i,g} \frac{1}{2}\int_0^\infty e^{-\rho t}\left((c(t) + (1-\Gamma)g(t))^2 + \lambda\pi(t)^2 + \eta g(t)^2\right) dt$$

$$\dot{c}(t) = \sigma^{-1}(i(t) - r(t) - \pi(t))$$

$$\dot{\pi}(t) = \rho\pi(t) - \kappa(c(t) + (1-\Gamma)g(t))$$

$$i(t) \geq 0$$

  and $x(0), \pi(0)$ free.

- $c = (C - C^*)/C^* \approx \log C - \log C^*$, $g = (G - G^*)/C^*$
- Γ is neoclassical multiplier
- Flexible prices: optimal spending $c = -(1-\Gamma)g$

Spending

- Gap x=c+(1-Γ)g transformation

$$\min_{x,\pi,i,g} \frac{1}{2}\int_0^\infty e^{-\rho t}\left(x(t)^2+\lambda\pi(t)^2+\eta g(t)^2\right)dt$$

$$\dot{x}(t)=(1-\Gamma)\dot{g}(t)+\sigma^{-1}(i(t)-r(t)-\pi(t))$$

$$\dot{\pi}(t)=\rho\pi(t)-\kappa x(t)$$

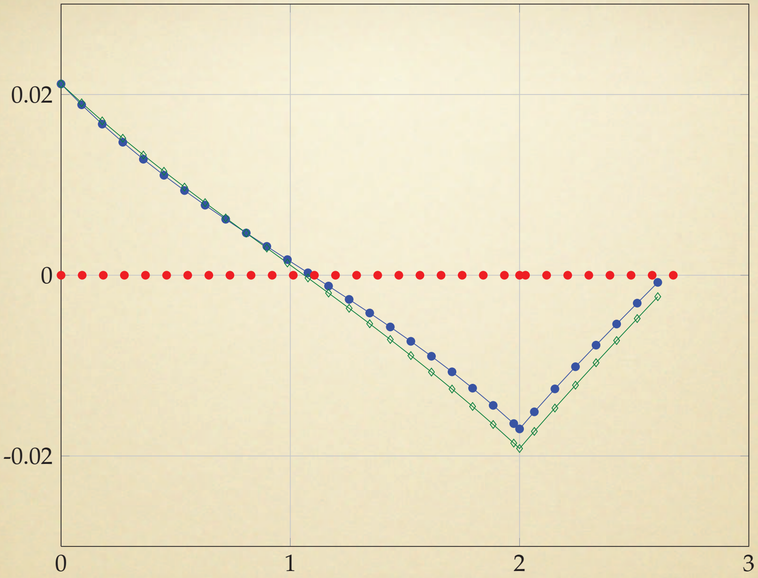
$$i(t)\geq 0$$

$$x(0),\pi(0)\text{ free.}$$

- spending loosens Euler equation

Proposition. Spending is initially positive. But falls over time, and becomes negative.

- front-loading



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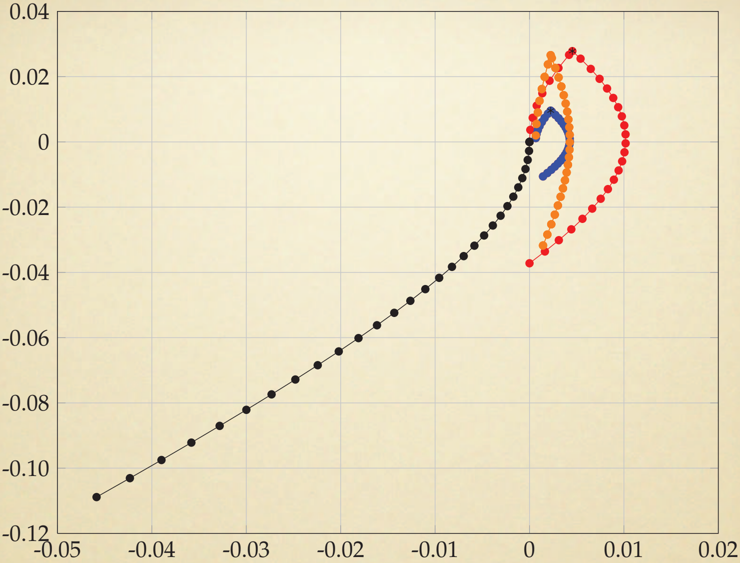
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Stimulus

- Decomposition
  - "opportunistic": static cost-benefit...

$$g^*(c) \equiv \arg\max_g U(c, c+g, g)$$

    - low c ➡ low (shadow) wage ➡ higher g

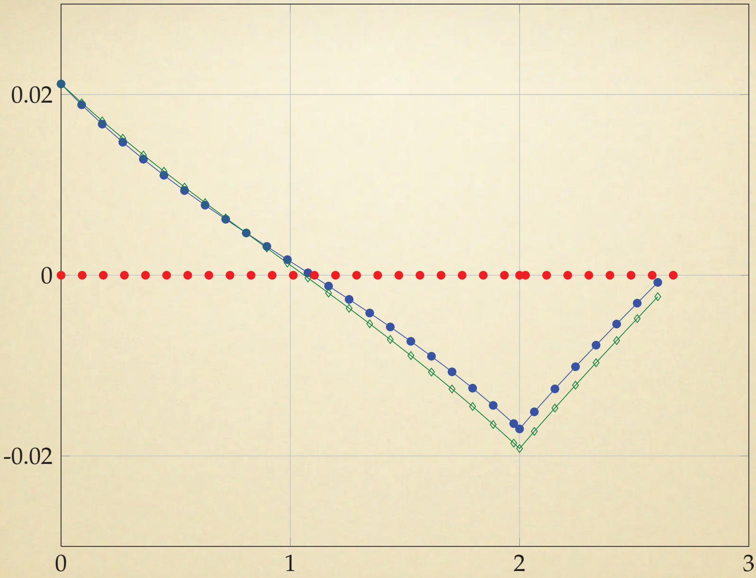
  - "stimulus"...

$$\hat{g}(t) = g(t) - g^*(c(t))$$

    - attempt to manipulate consumption

Opportunistic vs. Stimulus Spending

- Decomposition
  - Total spending (blue)
  - Opportunistic spending (green)
  - Stimulus spending (red)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Stimulus

- Planning Problem

$$\min_{\hat{x},\pi,i,\hat{g}} \frac{1}{2}\int_0^\infty e^{-\rho t}\left(c(t)^2 + \hat{\lambda}\pi(t)^2 + \hat{\eta}\hat{g}(t)^2\right)dt$$
$$\dot{c}(t) = \sigma^{-1}(i(t) - r(t) - \pi(t))$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa\left(\psi c(t) + (1-\Gamma)\hat{g}(t)\right)$$
$$i(t) \geq 0,$$
$$c(0), \pi(0) \text{ free.}$$

- stimulus: loosens Phillips Curve

Stimulus

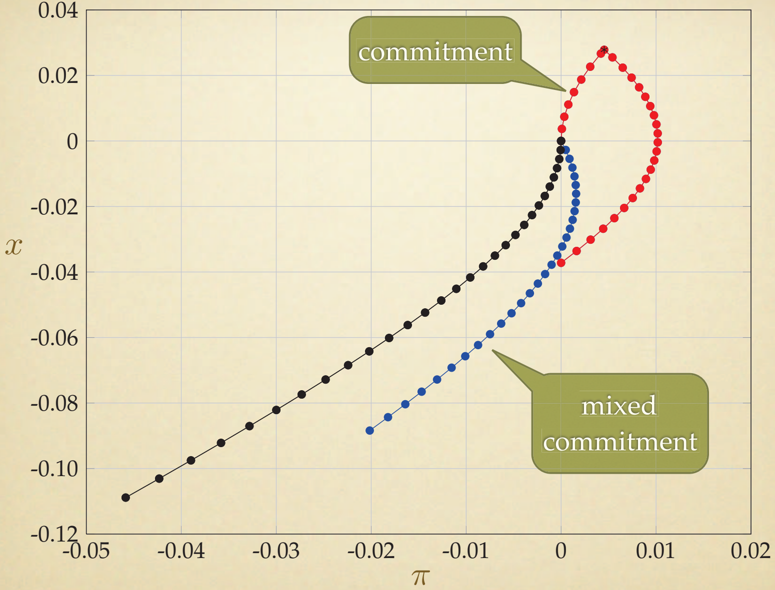
Proposition. Stimulus:
(a) initially zero;
(b) may be zero throughout;
(c) switches signs: starts positive, then negative, or vice versa

Optimal Fiscal Policy: Summary

- Almost all spending opportunistic, stimulus = very small component
- Opportunistic spending does affect private consumption, by affecting inflation. "Leaning against the wind" mitigates both deflations and inflations.
- However, effects are incidental, would have been obtained by completely myopic policy maker.
- Model **not** screaming for stimulus

Mixed Commitment

- Monetary: discretionary
- Spending: Commitment up to $t = T$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commitment
mixed commitment

Conclusions

- **Liquidity trap**
  - no commitment: deflation and depression
  - worse with flexible prices
- **Monetary policy**
  - avoids deflation
  - commitment important
- **Fiscal policy**
  - countercyclical
  - all opportunistic
- **Mixed commitment**
  - role for extra stimulus
  - larger if prices are more flexible

Related Literature

- Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011), "When Is the Government Spending Multiplier Large?," Journal of Political Economy
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- Fernandez-Villaverde, Gordon, Guerron and Rubio-Ramirez (2012), "Nonlinear Adventures at the Zero Lower Bound", Working Paper
- Woodford (2011),"Simple Analytics of the Government Expenditure Multiplier," American Economic Journal: Macroeconomics