Lecture 1:

Background and Overview
Hamiltonians and Phase Diagrams

Distributional Macroeconomics

Part II of ECON 2149

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Plan

1. Admin
2. Overview – what do I mean by “Distributional Macroeconomics”?
3. Hamiltonians and Phase Diagrams

Course Structure

Two Parts:

(1) Substance: distributional macroeconomics

(2) Tools: continuous time methods

- Everything is flexible, feedback very useful!

Lectures

- Tools lectures: you don't need to do anything
- Substance lectures: 1 core paper which you all read in advance. I present. All of you prepare a 3 slide discussion, focusing on:
  - Good stuff – what the paper did well
  - Bad stuff – what the paper didn't do well
  - Extensions – what you could do extending this
- At the end of substance lectures, I will summarize some related literature
- So you only have to read 1 paper in advance – but please read this in detail so we can discuss and deconstruct this

Research Proposal or Take-Home Final (You Choose)

Research proposal:

- purpose: get you started with your research
- should be consistent with your interests, an original research idea
- must be related to the course's topic, anything in either Jesus' or my part
- due on day before take-home final handed out (TBD)
- if you want feedback on an idea, shoot us an email

Take-home final:

- in late April, exact date TBD
- will cover both Jesus' and my parts

Admin Summary

To pass the class you have to

(1) Write either research proposal or take-home final

(2) Solve a few problem sets

(3) Every “substance lecture” have a 3 slide discussion prepared for the class paper

- Good stuff – what the paper did well
- Bad stuff – what the paper didn’t do well
- Extensions – what you could do extending this

(4) Turn up, keep awake, and ask the occasional question

What do I mean by “Distributional Macroeconomics”?

- Study of macroeconomic questions in terms of distributions rather than just aggregates

    - typical example: distributions of income and wealth

- More technically: macroeconomic theories in which relevant state variable is a distribution (or: “heterogeneous agent models”)

Main Message

- Hard to coherently think about macro if ignore distribution

- Instead, rich interaction:

$$\text{distribution} \quad \Longleftrightarrow \quad \text{macroeconomy}$$

- Or perhaps more precisely:

macroeconomy **is** a distribution

Inequality in Macro: A History of Thought

I find it useful to categorize macroeconomic theories as follows:

- before modern macro: 1930 to 1970
- 1st generation modern macro: 1970 to 1990
- 2nd generation modern macro: 1990 to financial crisis
- 3rd generation modern macro: after the financial crisis

Main drivers of evolution in modern macro era

1. better data
2. better computers & algorithms
3. current events (rising inequality, financial crisis)

Before Modern Macro: 1930 to 1970

1. Keynesian IS/LM: about aggregates, no role for inequality/distribution by design

2. Distribution does play role in growth theory

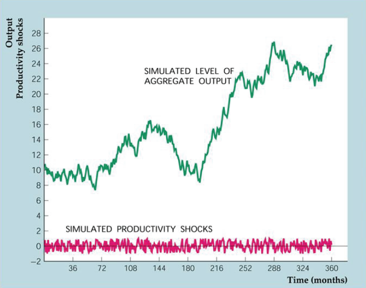
    - mostly factor income distribution: Kaldor, Pasinetti and other Cambridge UK theorists

    - rarely personal income distribution: e.g. Stiglitz, Blinder

3. Disconnected empirical work on inequality (Kuznets)

First Generation Macro Theories: 1970 to 1990

Representative agent models, e.g. RBC & New Keynesian models



About aggregates, no role for inequality/distribution by design

Advertised as "microfounded" but representative agent assumption cuts 1st generation modern macro from much of micro research

First Generation Macro Theories: 1970 to 1990

What's wrong with that?

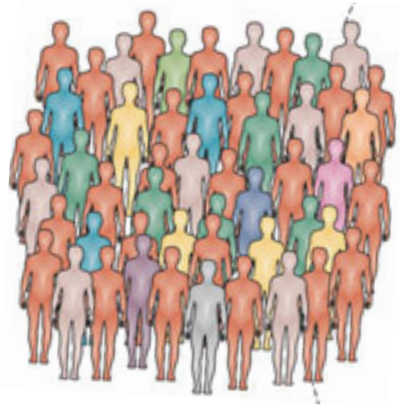
1. cannot speak to a number of important empirical facts, e.g.
    - unequally distributed growth
    - poorest hit hardest in recessions

2. cannot think coherently about welfare – "who gains, who loses?"

Second Generation Macro Theories: 1990 to 2008

(a) First Generation Theories

(b) Second Generation Theories

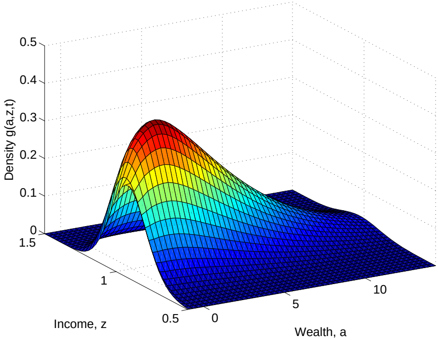


Second generation theories incorporate heterogeneity from micro data, particularly in income and wealth

Second Generation Macro Theories: 1990 to 2008

(a) First Generation Theories

(b) Second Generation Theories



Second generation theories represent economy with a distribution...

Second Generation Macro Theories: 1990 to 2008

(a) First Generation Models

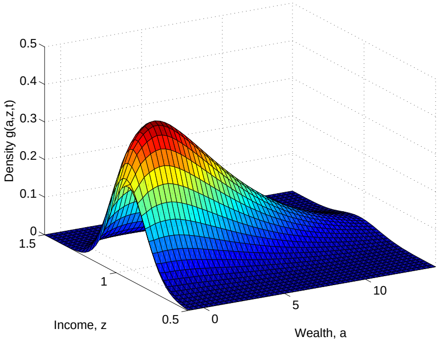
(b) Second Generation Models

Second generation theories represent economy with a distribution...
that moves over time, responding to macroeconomic shocks, policies

Second Generation Macro Theories: 1990 to 2008

(a) First Generation Models

(b) Second Generation Models



To contrast these theories with representative agent models, they are often referred to as "heterogeneous agent models"

- important early contributions in the 1990s by Aiyagari, Bewley, Huggett, Krusell-Smith, Den Haan,...

Second Generation Macro Theories: 1990 to 2008

(a) First Generation Models

(b) Second Generation Models

Second generation theories can potentially speak to

- unequally distributed growth
- poorest hit hardest in recessions

and are useful for welfare analysis

Second Generation Theories: Inequality $\not\Rightarrow$ Macro

- Typical finding: heterogeneity doesn't matter much for macro agg's
- Reason: in these theories, rich and poor differ in wealth but not consumption and saving behavior – rich = scaled version of poor
- Hence "inequality $\not\Rightarrow$ macro", but also a knife-edge result
- Problem: in data, rich $\neq$ scaled version of poor, e.g. rich have
  - lower MPCs out of transitory income changes
  - higher saving rates out of permanent income, wealth
- Note: some important contributions from same time period don't fit my narrative
  - Banerjee-Newman, Benabou, Galor-Zeira, Persson-Tabellini, ...
  - also related: 1950s "capitalist-worker theories" of Kaldor, Pasinetti, ...

Third Generation Theories: after the Crisis

- 3rd generation theories take micro data more seriously
- Leads them to emphasize things like
  - household balance sheets
  - credit constraints
  - MPCs that are high on average but heterogeneous
  - non-homotheticicities, non-convexities
- Typical finding: distribution matters for macro
- Will see a number of examples throughout the course

Inequality in Macro: Summary

- Before modern macro: 1930 to 1970
  - it's complicated
- 1st generation: 1970 to 1990
  - representative agent models (RBC, New Keynesian etc)
  - no role for inequality by design
- 2nd generation: 1990 to financial crisis
  - early "distributional macro" models
  - "macro $\Rightarrow$ inequality" but "macro $\not\Leftarrow$ inequality"
- 3rd generation: after the financial crisis
  - current "distributional macro" models
  - rich interaction: "inequality $\Longleftrightarrow$ macro"

Inequality in Modern Macro: Summary

Recent Janet Yellen speech "Macroeconomic Research After the Crisis": `http://www.federalreserve.gov/newsevents/speech/yellen20161014a.htm`

- "Prior to the financial crisis, representative-agent models were the dominant paradigm for analyzing many macroeconomic questions [= 1st generation]."
- "However, a disaggregated approach seems needed to understand some key aspects of the Great Recession..."
- "While the economics profession has long been aware that these issues matter, their effects had been incorporated into macro models only to a very limited extent prior to the financial crisis [= 2nd generation]."
- "I am glad to now see a greater emphasis on the possible macroeconomic consequences of heterogeneity [= 3rd generation]."

So: this course is about “3rd generation” models

- Methods for solving them and some fun applications
- “Distributional macro” is hard
  - closed-form solutions are rare
  - computations are challenging
  - large micro datasets that may be hard to think through
- (Note: even though models harder to solve, they are often easier to understand – you have good intuition about micro behavior!)
- Why should you be interested in this?
  - fertile area of research, excellent dissertation topics!
  - many open questions
  - economics is becoming more empirical, macro no exception
  - pays off to be a bit strategic in your choice of topic

What’s the deal with these continuous time methods?

- ... nothing in particular
- I’ve found them useful in my own work
    - analytical results
    - fast computations
- ... so I thought I’d try to pass on some of that knowledge
- But “macroeconomy = distribution” idea is more important to me

Plan for Rest of Lecture

(1) Hamiltonians

(2) Phase diagrams

(3) Finite difference methods and shooting algorithm

Hamiltonians

- Pretty much all deterministic optimal control problems in continuous time can be written as

$$v(x_0) = \max_{\{\alpha(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} r(x(t), \alpha(t))\, dt$$

  subject to the law of motion for the state

$$\dot{x}(t) = f(x(t), \alpha(t)) \quad \text{and} \quad \alpha(t) \in A$$

  for $t \geq 0$, $x(0) = x_0$ given.
- $\rho \geq 0$: discount rate
- $x \in X \subseteq \mathbb{R}^N$: state vector
- $\alpha \in A \subseteq \mathbb{R}^M$: control vector
- $r : X \times A \to \mathbb{R}$: instantaneous return function

Example: Neoclassical Growth Model

$$v(k_0) = \max_{\{c(t)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(c(t))dt$$

subject to

$$\dot{k}(t) = F(k(t)) - \delta k(t) - c(t)$$

for $t \geq 0$, $k(0) = k_0$ given.

- Here the state is $x = k$ and the control $\alpha = c$
- $r(x, \alpha) = u(\alpha)$
- $f(x, \alpha) = F(x) - \delta x - \alpha$

Hamiltonian: General Formulation

- Consider the general optimal control problem two slides back
- Can obtain necessary and sufficient conditions for an optimum using the following procedure ("cookbook")
- Current-value Hamiltonian

$$\mathcal{H}(x, \alpha, \lambda) = r(x, \alpha) + \lambda f(x, \alpha)$$

- $\lambda \in \mathbb{R}^N$: "co-state" vector

Hamiltonian: General Formulation

- Necessary and sufficient conditions:

$$\mathcal{H}_\alpha \left(x\left(t\right), \alpha\left(t\right), \lambda\left(t\right)\right) = 0$$

$$\dot{\lambda}\left(t\right) = \rho\lambda\left(t\right) - \mathcal{H}_x\left(x\left(t\right), \alpha\left(t\right), \lambda\left(t\right)\right)$$

$$\dot{x}\left(t\right) = f\left(x\left(t\right), \alpha\left(t\right)\right)$$

  for all $t \geq 0$

- Initial value for state variable(s): $x(0) = x_0$
- Boundary condition for co-state variable(s) $\lambda\left(t\right)$, called "transversality condition"

$$\lim_{T\to\infty} e^{-\rho T}\lambda\left(T\right)x\left(T\right) = 0$$

- `http://www.princeton.edu/~moll/ECO503Web/Lecture2_ECO503.pdf` (Slide 26 ff)
- Note: initial value of the co-state variable $\lambda\left(0\right)$ not predetermined

Example: Neoclassical Growth Model

- Recall: $r(x, \alpha) = u(\alpha)$ and $f(x, \alpha) = F(x) - \delta x - \alpha$
- Using the "cookbook"

$$\mathcal{H}(k, c, \lambda) = u(c) + \lambda[F(k) - \delta k - c]$$

- We have

$$\mathcal{H}_c(k, c, \lambda) = u'(c) - \lambda$$

$$\mathcal{H}_k(k, c, \lambda) = \lambda(F'(k) - \delta)$$

- Therefore conditions for optimum are:

$$\dot{\lambda} = \lambda(\rho + \delta - F'(k))$$

$$\dot{k} = F(k) - \delta k - c \qquad \text{(ODE)}$$

$$u'(c) = \lambda$$

with $k(0) = k_0$ and $\lim_{T\to\infty} e^{-\rho T}\lambda(T)k(T) = 0$.

Example: Neoclassical Growth Model

- Interpretation: continuous time Euler equation
- In discrete time

$$\lambda_t = \beta\lambda_{t+1}(F'(k_{t+1}) + 1 - \delta)$$

$$k_{t+1} = F(k_t) + (1-\delta)k_t - c_t$$

$$u'(c_t) = \lambda_t$$

- (ODE) is continous-time analogue

Phase Diagrams

- How analyze (ODE)? In one-dimensional case (scalar x): use phase-diagram
- Two possible phase-diagrams:
  (i) in (λ, k)-space: more general strategy
  (ii) in (c, k)-space: nicer in terms of the economics
- For (i), use $u'(c) = \lambda$ or $c = (u')^{-1}(\lambda)$ to write (ODE) as

$$\begin{aligned} \dot{\lambda} &= \lambda(\rho + \delta - F'(k)) \\ \dot{k} &= F(k) - \delta k - (u')^{-1}(\lambda) \end{aligned} \tag{ODE'}$$

  with $k(0) = k_0$ and $\lim_{T\to\infty} e^{-\rho T}\lambda(T)k(T) = 0$.
- Homework 1: draw phase-diagram in (λ, k)-space.

Phase Diagrams

- For (ii), note that
$$\dot{\lambda} = u''(c)\dot{c}$$
and substitute into equation for $\dot{\lambda}$:
$$u''(c)\dot{c} = u'(c)(\rho + \delta - F'(k))$$
- Or define the "coefficient of relative risk aversion"
$$\sigma(c) := -\frac{u''(c)c}{u'(c)} > 0$$
and write (ODE) as
$$\frac{\dot{c}}{c} = \frac{1}{\sigma(c)}(F'(k) - \rho - \delta)$$
$$\dot{k} = F(k) - \delta k - c \qquad \text{(ODE")}$$
with $k(0) = k_0$ and $\lim_{T\to\infty} e^{-\rho T} u'(c(T))k(T) = 0$.
- Note: $\frac{1}{\sigma(c)}$ = "intertemporal elasticity of substitution" (IES)

Steady State

- In steady state $\dot{k} = \dot{c} = 0$. Therefore

$$F'(k^*) = \rho + \delta$$

$$c^* = F(k^*) - \delta k^*$$

- Same as in discrete time with $\beta = 1/(1+\rho)$.

- For example, if $F(k) = Ak^{\alpha}, \alpha < 1$. Then

$$k^* = \left(\frac{\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}}$$

Phase Diagram

- See graph that I drew in lecture by hand or Figure 8.1 in Acemoglu's textbook
- Obtain saddle path
- Prove stability of steady state
- Important: saddle path is not a "knife edge" case in the sense that the system only converges to steady state if $(c(0), k(0))$ happens to lie on the saddle path and diverges for all other initial conditions
- In contrast to the state variable $k(t)$, $c(t)$ is a "jump variable." That is, $c(0)$ is free and always adjusts so as to lie on the saddle path

Violations of Transversality Condition

- Question: how do you know that trajectories with $c(0)$ off the saddle path violate the transversality condition?

- See Acemoglu, chapter 8 "The Neoclassical Growth Model" section 5 "Transitional Dynamics"
  - if $c(0)$ below saddle path, $k(t) \to k_{\max}$ and $c(t) \to 0$
  - if $c(0)$ above saddle path, $k(t) \to 0$ in finite time while $c(t) > 0$. Violates feasibility.
  - local analysis/linearization gives same answer
    `http://www.princeton.edu/~moll/ECO503Web/Lecture4_ECO503.pdf`
  - notes that most rigorous and straightforward way is to use that concave problems have unique solution (his Theorem 7.14)

Numerical Solution: Finite-Difference Method

- By far the simplest and most transparent method for numerically solving differential equations.
- Approximate $k(t)$ and $c(t)$ at N discrete points in the time dimension, t^n, $n = 1, ..., N$. Denote distance between grid points by Δt.
- Use short-hand notation $k^n = k(t^n)$.
- Approximate derivatives

$$\dot{k}(t^n) \approx \frac{k^{n+1} - k^n}{\Delta t}$$

- Approximate (ODE") as

$$\frac{c^{n+1} - c^n}{\Delta t}\frac{1}{c^n} = \frac{1}{\sigma(c^n)}(F'(k^n) - \rho - \delta)$$

$$\frac{k^{n+1} - k^n}{\Delta t} = F(k^n) - \delta k^n - c^n$$

Finite-Difference Method/Shooting Algorithm

- Or

$$
\begin{aligned}
c^{n+1} &= \Delta t c^n \frac{1}{\sigma(c^n)}(F'(k^n) - \rho - \delta) + c^n \\
k^{n+1} &= \Delta t(F(k^n) - \delta k^n - c^n) + k^n
\end{aligned} \tag{FD}
$$

  with $k^0 = k_0$ given.

- Homework 2: draw phase diagram/saddle path in MATLAB.
- Assume $F(k) = Ak^\alpha$, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $A = 1$, $\alpha = 0.3$, $\sigma = 2$, $\rho = \delta = 0.05$, $k_0 = \frac{1}{2}k^*$, $\Delta t = 0.1$, $N = 700$.
- Algorithm:
  (i) guess c^0
  (ii) obtain (c^n, k^n), $n = 1, ..., N$ by running (FD) forward in time.
  (iii) If the sequence converges to (c^*, k^*), then you have obtained the correct saddle path. If not, back to (i) and try different c^0.
- This is called a "shooting algorithm"

References: Some "Third Generation" Papers

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- Auclert (2016) "Monetary Policy and the Redistribution Channel"
- Auclert & Rognlie (2016) "Inequality and Aggregate Demand"
- Bayer, Pham, Luetticke & Tjaden (2015) "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk"
- Carroll, Slacalek & Tokuoka (2016) "The Distribution of Wealth and the Marginal Propensity to Consume"
- Den Haan, Rendahl & Riegler (2017) "Unemployment (fears) and Deflationary Spirals,"
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- Guerrieri & Lorenzoni (2017) "Credit Crises, Precautionary Savings, and the Liquidity Trap"
- Kaplan, Moll & Violante (2017) "Monetary Policy According to HANK"

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- McKay & Reis (2016), “The Role of Automatic Stabilizers in the U.S. Business Cycle”
- McKay, Nakamura & Steinsson (2016) “The Power of Forward Guidance Revisited”
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- Hagedorn, Manovskii & Mitman (2017) “The Fiscal Multiplier”
- Oh & Reis (2012), “Targeted Transfers and the Fiscal Response to the Great Recession”
- Ravn & Sterk (2016), “Job Uncertainty and Deep Recessions”
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- Heathcote, Perri & Violante (2010), “An Empirical Analysis of Economic Inequality in the United States, 1967-2006”
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- Persson & Tabellini (1994) “Is Inequality Harmful for Growth?”

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- Blinder (2014) "The Supply-Side Case for Government Redistribution" http://economistsview.typepad.com/economistsview/2014/08/the-supply-side-case-for-government-redistribution.html
- Coeure (2013) "The Relevance of Household-Level Data for Monetary Policy and Financial Stability Analysis" https://www.ecb.europa.eu/press/key/date/2013/html/sp131017.en.html
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