Lecture 13: Review Session, and New Keynesian Model

ECO 503: Macroeconomic Theory I

Benjamin Moll

Princeton University

Fall 2014

Review of Material

- Course had two components

  1 tools

  2 substance

- Let's go over these, emphasizing what I want you to take away from this course

What I want you to take away: **Tools**

- how to solve deterministic dynamic optimization problems in **discrete time**...

- ... and **continuous time**...

- how to **analyze dynamics** of resulting trajectories (find steady states, analyze their stability)

- how to **numerically solve** dynamic optimization problems (shooting algorithm, linearization)

- how to **calibrate** a model

- how to set up a **Ramsey taxation problem**

- how to construct a **balanced growth path** (BGP)

- how to solve rep. agent models with **externalities**

- how to introduce overlapping generations (**perpetual youth**)

- how to think about a **stationary equilibrium** in a heterogeneous agent model

What I want you to take away:
Substance

- Basic tradeoffs in growth model

- Income vs. substitution effects

- Effects of response to unexpected productivity increase in growth model (building block of business cycle model)

- Quick convergence of growth model to steady state (or BGP)

- Kaldor facts (and caveats)

- Growth model = decent model of growth experience of U.S. (and other developed countries)...

- ... but bad model of "development dynamics"

What I want you to take away:
Substance

- Ramsey taxation of capital:
  - robust prediction: if possible, want to tax more today than tomorrow
  - not robust: $\Rightarrow$ zero capital taxes in long-run

- Endogenous growth: all theories make some linearity assumption
  - if it doesn't jump out at you, it's hidden somewhere

- Upper tails of income and wealth distribution follow Pareto distributions

- Q: where do Pareto distributions come from?
  A: simplest mechanism: combination of
  1. exponential growth
  2. random death (exponentially distributed)

New Keynesian Model

- New Keynesian model $=$ RBC/growth model with sticky prices

- References:

    - Gali (2008): most accessible intro

    - Woodford (2003): New Keynesian bible

    - Clarida, Gali and Gertler (1999): most influential article

    - Gali and Monacelli (2005): small open economy version

Why Should You Care?

- Simple framework to think about relationship between monetary policy, inflation and the business cycle.
- RBC model: cannot even think about these issues! Real variables are completely separate from nominal variables ("monetary neutrality", "classical dichotomy").
- Corollary: monetary policy has **no effect** on any real variables.
- Sticky prices break "monetary neutrality"
- Workhorse model at central banks (see Fed presentation `/DB_EC0521_2012_2013/LectureNotes/MacroModelsAtTheFed.pdf`)
- Makes some sense of newspaper statements like: "a boom leads the economy to overheat and creates inflationary pressure"
- Some reason to believe that "demand shocks" (e.g. consumer confidence, animal spirits) may drive business cycle. Sticky prices = one way to get this story off the ground.

(1) Model with flexible prices

(2) Model with sticky prices

Setup: Flexible Prices

- Households maximize

$$\int_0^\infty e^{-\rho t} \left\{ \log C(t) - \frac{N(t)^{1+\varphi}}{1+\varphi} \right\} dt$$

subject to

$$PC + \dot{B} = iB + WN$$

- C: consumption
- N: labor
- P: price level
- B: bonds
- i: nominal interest rate
- W: nominal wage
- Note: no capital

Households

- Hamiltonian

$$\mathcal{H}(B, C, N, \lambda) = \log C - \frac{N^{1+\varphi}}{1 + \varphi} + \lambda[iB + WN - PC]$$

- Conditions for optimum

$$\dot{\lambda} = \rho\lambda - \lambda i$$

$$\frac{1}{C} = \lambda P \quad \Rightarrow \quad \frac{\dot{C}}{C} = -\frac{\dot{\lambda}}{\lambda} - \frac{\dot{P}}{P}$$

$$N^{\varphi} = \lambda W$$

- Defining the inflation rate $\pi = \dot{P}/P$

$$\frac{\dot{C}}{C} = i - \pi - \rho$$

$$CN^{\varphi} = \frac{W}{P}$$

Firms – Final Goods Producer

- A competitive final goods producer aggregates a continuum of intermediate inputs

$$Y = \left(\int_0^1 y_j^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Cost minimization $\Rightarrow$ demand for intermediate good j

$$y_j(p_j) = \left(\frac{p_j}{P} \right)^{-\varepsilon} Y$$

where

$$P = \left(\int_0^1 p_j^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$$

- For a derivation see the Technical Appendix of

  http://www.crei.cat/people/gali/pdf_files/monograph/slides-ch3.pdf

Firms – Intermediate Goods Producers

- Continuum of monopolistically competitive intermediate goods producers $j \in [0, 1]$.

- Production uses labor only

$$y_j(t) = A(t)n_j(t).$$

- Solve (drop j subscripts for simplicity)

$$\max_p \; p \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{W(t)}{A(t)} \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t)$$

- Solution

$$p(t) = P(t) = \frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{A(t)}$$

where $P = p_j$ follows because all producers are identical.

Equilibrium with Flexible Prices

- Market clearing:

$$C = AN$$

- Combining with household FOC $CN^\varphi = W/P$ and
$P = \frac{\varepsilon}{\varepsilon-1} W/A$

$$C = Y = A \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{-1}{1+\varphi}}$$

- Note: distortion from monopolistic competition

- Back out real interest rate from

$$r = i - \pi = \rho - \frac{\dot{C}}{C} = \rho + \frac{\dot{A}}{A} = \rho + g$$

Some Notable Features

- Like an RBC model, this model features "monetary neutrality"

  `http://lmgtfy.com/?q=monetary+neutrality`

- Equivalently: there is a "classical dichotomy"

  `http://lmgtfy.com/?q=classical+dichotomy`

- Real variables $(C(t), Y(t), N(t), W(t)/P(t), r(t))$ are determined completely separately from nominal variables $(P(t), W(t), \pi(t), i(t))$.

- In fact, $P(t)$ and $\pi(t)$ are not even determined in the absence of a description of a determination of the economy's money stock (e.g. through monetary policy). But this doesn't matter for real variables.

- As a corollary, monetary policy has **no effect** on real variables

Sticky Prices

- Everything same except intermediate goods producers.

- Per period profits are still

$$\Pi_t(p) = p \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t) - \frac{W(t)}{A(t)} \left(\frac{p}{P(t)} \right)^{-\varepsilon} Y(t)$$

- But now have to pay quadratic price adjustment cost

$$\Theta_t \left(\frac{\dot{p}}{p} \right) = \frac{\theta}{2} \left(\frac{\dot{p}}{p} \right)^2 P(t) Y(t)$$

- Optimal control problem:

$$V_0(p_0) = \max_{p(t), t \geq 0} \int_0^\infty e^{-\int_0^t i(s)ds} \left\{ \Pi_t(p(t)) - \Theta_t \left(\frac{\dot{p}(t)}{p(t)} \right) \right\} dt$$

- θ: degree of price stickiness

Comparison to Literature

- Note: my formulation uses quadratic price adjustment costs as in Rotemberg (1982).

- Different from standard Calvo (1983) pricing formulation: allowed to change price at Poisson rate α

- I like Rotemberg better because pricing is state dependent as opposed time dependent ("Calvo fairy").

- Closer to "menu cost" models.

- Schmitt-Grohe and Uribe (2004), Fernandez-Villaverde et al. (2011) also use Rotemberg

- I also assume that adjustment costs are paid as a transfer to consumers, $T = \Theta_t(\pi) = (\theta/2)\pi^2 PY$. Just a trick to eliminate real resource costs of inflation ($\Theta_t(\pi) \approx 0$ anyway).

Optimal Price Setting

- Hamiltonian (state: p, control: $\dot{p}$, co-state: η):

$$\mathcal{H}(p, \dot{p}, \eta) = p \left(\frac{p}{P}\right)^{-\varepsilon} Y - \frac{W}{A} \left(\frac{p}{P}\right)^{-\varepsilon} Y - \frac{\theta}{2} \left(\frac{\dot{p}}{p}\right)^2 PY + \eta \dot{p}$$

- Conditions for optimum

$$\theta \frac{\dot{p}}{p} \frac{P}{p} Y = \eta$$

$$\dot{\eta} = i\eta - \left[(1-\varepsilon)\left(\frac{p}{P}\right)^{-\varepsilon} Y + \varepsilon \frac{W}{p} \frac{1}{A} \left(\frac{p}{P}\right)^{-\varepsilon} Y + \theta \left(\frac{\dot{p}}{p}\right)^2 \frac{P}{p} Y\right].$$

- Symmetric equilibrium: $p = P$

$$\theta \pi Y = \eta$$

$$\dot{\eta} = i\eta - \left[(1-\varepsilon)Y + \varepsilon \frac{W}{P} \frac{1}{A} Y + \theta \pi^2 Y\right].$$

Optimal Price Setting

- Recall the FOC: $\theta \pi Y = \eta$. Differentiate with respect to time

$$\theta \dot{\pi} Y + \theta \pi \dot{Y} = \dot{\eta}$$

- Substitute into equation for co-state and rearrange

Lemma

The price setting of firms implies that the inflation rate $\pi = \dot{P}/P$ is determined by

$$\left(i - \pi - \frac{\dot{Y}}{Y} \right) \pi = \frac{\varepsilon - 1}{\theta} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W}{P} \frac{1}{A} - 1 \right) + \dot{\pi}.$$

Optimal Price Setting in Equilibrium

- In equilibrium $C = Y$ and Euler equation

$$\frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = i - \pi - \rho$$

- Substitute into expression on previous slide $\Rightarrow$ Inflation determined by

$$\rho\pi = \frac{\varepsilon - 1}{\theta} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W}{P} \frac{1}{A} - 1 \right) + \dot{\pi}. \qquad (*)$$

- In integral form (check that differentiating gives back above)

$$\pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W(s)}{P(s)} \frac{1}{A(s)} - 1 \right) ds$$

- Compare with equation (16) in Chapter 3.3. of Gali's book and expression just below.

Optimal Price Setting in Equilibrium

- Inflation determined by

$$\pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W(s)}{P(s)} \frac{1}{A(s)} - 1 \right) ds$$

- Intuition: term in brackets = marginal payoff to a firm from increasing its price

$$\Pi'_t(P(t)) = (\varepsilon - 1) Y(t) \left(\frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{P(t)} \frac{1}{A(t)} - 1 \right).$$

- Positive whenever P less than optimal markup $\frac{\varepsilon}{\varepsilon - 1}$ over marginal cost W/A.

- With flexible prices, $\theta = 0$: $\Pi'_t(P(t)) = 0$ for all t, $P = \frac{\varepsilon}{\varepsilon - 1} \frac{W}{A}$.

- With sticky prices, $\theta > 0$: π = PDV of all future $\Pi'_t(P(t))$.

- Adjustment cost is convex. So if expect reason to adjust in the future – e.g. $W(t)/A(t) \uparrow$ – already adjust now.

IS Curve and Phillips Curve

- Call outcomes under flexible prices, $\theta = 0$, "natural" output Y^n and "natural" real interest rate. Recall

$$Y^n = A \left(\frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{-1}{1+\varphi}}, \quad \frac{\dot{Y}^n}{Y^n} = r - \rho, \quad r = \rho + \frac{\dot{A}}{A}$$

- Define output gap: $X = Y/Y^n$. Recall Euler equation under sticky prices

$$\frac{\dot{Y}}{Y} = i - \pi - \rho$$

- Euler equation in terms of output gap $\dot{X}/X = \dot{Y}/Y - \dot{Y}^n/Y^n$

$$\frac{\dot{X}}{X} = i - \pi - r$$

- This is basically an IS curve.

IS Curve and Phillips Curve

- Can obtain "Phillips Curve" in similar way. Recall

$$P^n = \frac{\varepsilon}{\varepsilon - 1} \frac{W^n}{A} \quad \Rightarrow \quad \frac{W}{P} \frac{1}{A} = \frac{W/P}{W^n/P^n}$$

- Equation for inflation $(*)$ becomes

$$\rho \pi = \frac{\varepsilon - 1}{\theta} \frac{W/P - W^n/P^n}{W^n/P^n} + \dot{\pi}.$$

- From FOC $CN^\varphi = \frac{W}{P}$, and mkt clearing $C = Y, N = Y/A$

$$\frac{W/P}{W^n/P^n} = \left(\frac{Y}{Y^n} \right)^{1+\varphi} = X^{1+\varphi}.$$

IS Curve and Phillips Curve

- Relation between inflation and output gap: "New Keynesian Phillips Curve"

$$\rho\pi = \frac{\varepsilon - 1}{\theta} \left(X^{1+\varphi} - 1 \right) + \dot{\pi}.$$

- In integral form

$$\pi(t) = \frac{\varepsilon - 1}{\theta} \int_t^\infty e^{-\rho(s-t)} \left(X(s)^{1+\varphi} - 1 \right) ds.$$

- Inflation high when future output gaps are high, i.e. when economy "overheats"

Three Equation Model

- Recall: IS curve and Phillips curve

$$\frac{\dot{X}}{X} = i - \pi - r \qquad \text{(IS)}$$

$$\rho\pi = \frac{\varepsilon - 1}{\theta}\left(X^{1+\varphi} - 1\right) + \dot{\pi} \qquad \text{(PC)}$$

- To close model: Taylor rule

$$i = i^* + \phi\pi + \phi_x \log X \qquad \text{(TR)}$$

- "Three equation model," see modern undergraduate textbooks (e.g. Carlin and Soskice)

- Substitute (TR) into (IS) $\Rightarrow$ system of two ODEs in (π, X), analyze with phase diagram.

Three Equation Model in Literature

- Literature uses log-linearization all over the place.
- Obtain exact analogues by defining

$$x \equiv \log X = \log Y - \log Y^n$$

- Using that for small x (Taylor-series)

$$X^{1+\varphi} - 1 = e^{(1+\varphi)x} - 1 \approx (1+\varphi)x$$

- and defining $\kappa \equiv (\varepsilon - 1)(1 + \varphi)/\theta$

$$\dot{x} = i - \pi - r \tag{IS'}$$

$$\rho\pi = \kappa x + \dot{\pi} \tag{PC'}$$

$$i = i^* + \phi\pi + \phi_x x \tag{TR'}$$

- Exact continuous time analogues of (21), (22), (25) in Chapter 3 of Gali's book, same as in Werning (2012)
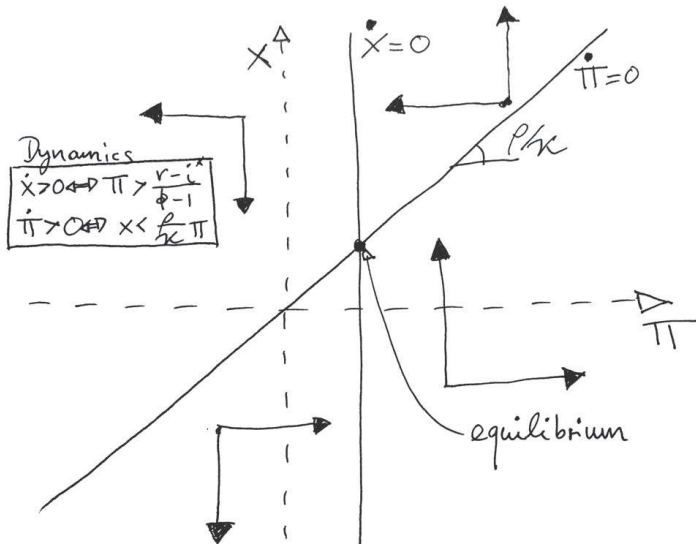
Phase Diagrams

- For simplicity, assume $\phi_x = 0$. Makes some math easier.
- Also ignore ZLB, $i \geq 0$ (next time).
- Substitute (TR') into (IS')

$$\dot{x} = i^* - r + (\phi - 1)\pi$$
$$\dot{\pi} = \rho\pi - \kappa x$$
(ODE)

- See phase diagrams on next slide
- Important: both π and x are jump-variables. No state variables.
- Two cases:
    - $\phi > 1$: unique equilibrium. "Taylor principle": i increases more than one-for-one with π so that also real rates increase.
    - $\phi < 1$: equilibrium indeterminacy
- From now assume $\phi > 1$

Dynamics
$$\dot{x} > 0 \Leftrightarrow \pi > \frac{r - i^*}{\phi - 1}$$
$$\dot{\pi} > 0 \Leftrightarrow x < \frac{\rho}{\kappa} \pi$$

$\dot{x} = 0$

$\dot{\pi} = 0$

ρ/κ

equilibrium

x

π

Monetary Policy

- Can achieve $\pi = 0$ and $x = 0$ by setting $i^* = r$ (and $\phi > 1$).

- Scenario 1: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, r increases, e.g. because TFP growth increases (recall $r = \rho + \dot{A}/A$).

- Scenario 2: suppose economy is in $(x, \pi) = (0, 0)$ equilibrium. But at $t = T$, someone at the Fed goes crazy and increases i^* (e.g. because mistakenly think that TFP growth goes up).

- Draw time paths for $(x(t), \pi(t))$ for both scenarios.

Optimal Monetary Policy

- Huge literature

- See in particular Clarida, Gali and Gertler (1999) "The science of monetary policy: a new Keynesian perspective"

Zero Lower Bound

- So far: New Keynesian 3 equation model, derived from micro foundations

- Ignored ZLB (or "liquidity trap"), $i(t) \geq 0$.

- Some references

  - Eggertsson and Woodford (2003), "The zero interest-rate bound and optimal monetary policy"

  - Christiano, Eichenbaum, Rebelo (2011) "When is the Government Spending Multiplier Large?"

  - Werning (2012), "Managing a Liquidity Trap: Monetary and Fiscal Policy"