

This is the second half of ECON2149 “Computational Economics” at Harvard, co-taught with Jesus Fernández-Villaverde.

My part of the course will focus partly on substance and partly on tools. In terms of substance, we will be concerned with “distributional macroeconomics” by which I mean the study of macroeconomic questions in terms of distributions of microeconomic variables rather than just aggregates. Equivalently, we will be concerned with macroeconomic theories in which the relevant state variable is a distribution. We will first cover some theories aiming to explain why income and wealth are so unequally distributed and why inequality has increased over time in many developed countries, and then explore the implications of income and wealth heterogeneity for macroeconomic dynamics and macroeconomic policy. Finally (time permitting) we will think about the determinants of firm size distribution and the implications of firm heterogeneity for the macroeconomy. The models we cover have implications for both cross-sectional data and for time series behavior of an economy as a whole.

At the beginning of the course we will cover a number of analytical and computational tools, namely continuous time methods that are useful for macroeconomics, and in particular for the study of theories in which the relevant state variable is a distribution. These include Hamiltonians, Stochastic Calculus, Hamilton-Jacobi-Bellman equations, and Kolmogorov Forward Equations. Rather than presenting an in-depth technical derivation of the methods from first principles, my aim is to provide you with a “cookbook” for you to use these methods in your own research and then to cover a few applications. All lecture slides will be available from my website <http://www.princeton.edu/~moll/notes.htm>. Codes for solving many of the models we will consider are available at <http://www.princeton.edu/~moll/HACTproject.htm>.

Organization: The class meets on Mondays and Tuesdays 10:00 to 11:30am. A syllabus is attached.

Grading: Homeworks will be voluntary/self-graded. Your final grade will be based on either a written research proposal or a take-home final. You can choose which you prefer:

- The **take-home final** will consist of a computational project that you will have to complete.
- The **10-page research proposal** will consist of: a clearly defined question; motivation of why the question is economically interesting; a synthesis of relevant background literature focusing on the points most germane to the question at hand; a discussion of an appropriate model; preliminary results; and a discussion of next steps.

1. Deterministic Continuous Time Methods

1.A Methods:

Lecture Notes: Hamiltonians and Phase Diagrams

Acemoglu (2010), Chapter 7 “Review of the Theory of Optimal Control” in “Introduction to Modern Economic Growth”

1.B Applications:

Lecture Notes: Neoclassical Growth Model in Continuous Time

- http://www.princeton.edu/~moll/ECO503Web/Lecture3_ECO503.pdf
- http://www.princeton.edu/~moll/ECO503Web/Lecture4_ECO503.pdf

Acemoglu (2010), Chapter 8 “The Neoclassical Growth Model” in “Introduction to Modern Economic Growth”

Lecture Notes: “New Keynesian Model in Continuous Time” (useful for HANK papers in Section 3.D)

Werning (2012), “Managing a Liquidity Trap”, Working Paper

2. Stochastic Continuous Time Methods, Modeling Distributions

2.A Methods:

Lecture Notes: Stochastic Calculus, Hamilton-Jacobi-Bellman Equations, Kolmogorov Forward Equations, Finite Difference Methods

Codes for finite difference solution to HJB and KF equations:

<http://www.princeton.edu/~moll/HACTproject.htm>

Candler (1999), “Finite-Difference Methods for Dynamic Programming Problems.” In Computational Methods for the Study of Dynamic Economies., ed. Ramon Marimon and Andrew Scott. Cambridge, England: Cambridge University Press.

Kushner and Dupuis (1992), “Numerical Methods for Stochastic Control Problems in Continuous Time”, Springer New York

2.B Applications:

Gabaix (2009), “Power Laws in Economics and Finance”, Annual Review of Economics

Gabaix, Lasry, Lions and Moll (2016), “The Dynamics of Inequality,” Econometrica

Merton (1975), “An Asymptotic Theory of Growth Under Uncertainty,” Review of Economic Studies

3. Income and Wealth Distribution in Macroeconomics

3.A. Prerequisites and Useful Background (Not Taught)

Aiyagari (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving" *The Quarterly Journal of Economics*

Krusell and Smith (1998). "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*

Den Haan (2010), "Assessing the Accuracy of the Aggregate Law of Motion in Models with Heterogeneous Agents", *Journal of Economic Dynamics and Control*

Heathcote, Storesletten, and Violante (2008), "Quantitative Macroeconomics with Heterogeneous Households", *Annual Review of Economics*

Krueger, Mitman and Perri (2016), "Macroeconomics and Household Heterogeneity," *Handbook of Macroeconomics*

Stoker (2008), "Aggregation", *The New Palgrave Dictionary of Economics* (particularly the last section)

Nakamura and Steinsson (2018), "Identification in Macroeconomics" (particularly Section 3 on "Aggregate Versus Cross-Sectional Identification")

3.B. Resources for numerical solution of discrete-time heterogeneous agent models (not taught)

Carroll et al, "ARK," <https://github.com/econ-ark>, particularly the heterogeneous agent part <https://github.com/econ-ark/HARK>

Stachurski and Sargent, "QuantEcon, <http://quant-econ.net/>, particularly codes for solving Aiyagari model: Python: <http://quant-econ.net/py/ayagari.html>, Julia: <http://quant-econ.net/jl/ayagari.html>

Gianluca Violante's lecture notes and codes:

Macroeconomic Theory I: <https://sites.google.com/a/nyu.edu/glviolante/teaching/macrotheory>

Quantitative Macroeconomics: <https://sites.google.com/a/nyu.edu/glviolante/teaching/quantmacro15>

Boppart, Krusell and Mitman (2017), "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative", Working Paper

3.C. Key Facts on Income and Wealth Distribution

Atkinson, Piketty, and Saez (2011), "Top Incomes in the Long Run of History," *Journal of Economic Literature*

Acemoglu (2002), "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*

Kopczuk (2015), "What Do We Know About Evolution of Top Wealth Shares in the United States?", *Journal of Economic Perspectives*

Papers in RED special issue "Cross-Sectional Facts for Macroeconomists" (Krueger, Perri, Pistaferri and Violante, 2010): <https://www.economicdynamics.org/si-cross-facts/>

Kuhn and Rios-Rull (2016), "2013 Update on the U.S. Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomics," *Federal Reserve Bank of Minneapolis Quarterly Review*

Thewissen, Nolan and Roser (2016), <https://ourworldindata.org/incomes-across-the-distribution/>

3.D Theories of Income and Wealth Inequality

Benhabib and Bisin (2016), "Skewed Wealth Distribution: Theory and Empirics", *Annual Review of Economics*

Cagetti and DeNardi (2006), "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*

Fagereng, Guiso, Malacrino and Pistaferri (2016), "Heterogeneity and Persistence in Returns to Wealth"

Gabaix and Landier (2008), "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics*

Sattinger (1979), "Differential Rents and the Distribution of Earnings," *Oxford Economic Papers*

Sattinger (1993), "Assignment Models of the Distribution of Earnings," *Journal of Economic Literature*

Jones and Kim (2016), "A Schumpeterian Model of Top Income Inequality"

Jones (2015), "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality", *Journal of Economic Perspectives*

3.E Macroeconomic Implications of Income and Wealth Heterogeneity

Achdou, Han, Lasry, Lions and Moll (2017), "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach", Working Paper

Auclert (2016), "Monetary Policy and the Redistribution Channel", Working Paper

Kaplan, Moll and Violante (2018), "Monetary Policy According to HANK", *American Economic Review*

Kaplan and Violante (2018), "Microeconomic Heterogeneity and Macroeconomic Shocks", *Journal of Economic Perspectives*

Werning (2016), "Incomplete Markets and Aggregate Demand", Working Paper

McKay, Nakamura and Steinsson (2016), "The Power of Forward Guidance Revisited", *American Economic Review*

McKay and Reis (2016), "The Role of Automatic Stabilizers in the U.S. Business Cycle", *Econometrica*

Auclert and Rognlie (2016), "Inequality and Aggregate Demand", Working Paper

Straub (2018), "Consumption, Savings, and the Distribution of Permanent Income", Working Paper

Cloyne, Ferreira and Surico (2018) "Monetary Policy when Households have Debt: New Evidence on the Transmission Mechanism"

Ahn, Kaplan, Moll and Winberry (2018), "When Inequality Matters for Macro and Macro Matters for Inequality," *NBER Macroeconomics Annual*

Reiter (2009a), "Solving heterogeneous-agent models by projection and perturbation", *Journal of Economic Dynamics and Control*

Reiter (2009b), "Approximate and Almost-Exact Aggregation in Dynamic Stochastic Heterogeneous-Agent Models"

Mankiw (1986), "The Equity Premium and the Concentration of Aggregate Shocks", *Journal of Financial Economics*

3.F Inequality and Growth/Development

Benabou (2002), "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?", *Econometrica*

Lucas and Moll (2014), "Knowledge Growth and the Allocation of Time", *Journal of Political Economy*

Staley (2011), "Growth and the diffusion of ideas", *Journal of Mathematical Economics*

Luttmer (2012), "Eventually, Noise and Imitation Implies Balanced Growth"

Galor and Zeira (1993), "Income Distribution and Macroeconomics," *Review of Economic Studies*

Piketty, Thomas (1997) "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies*

4. Firm Heterogeneity

4.A Methods: Stopping time problems

Stokey (2008), Chapter 6 "Exercising an Option" from "The Economics of Inaction: Stochastic Control Models with Fixed Costs"

Dixit (1993), "The Art of Smooth Pasting" *Fundamentals of Pure and Applied Economics* 55, The Routledge.

Dixit and Pindyck (1994), "Investment Under Uncertainty," Princeton University Press.

4.B Theories of Firm Size Distribution

Hopenhayn (1992), "Entry, Exit, and firm Dynamics in Long Run Equilibrium", *Econometrica*

Luttmer (2007), "Selection, Growth, and the Size Distribution of Firms," *Quarterly Journal of Economics*

Bertola and Caballero (1994), "Irreversibility and Aggregate Investment"

Caballero and Engel (1999), "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach", *Econometrica*

Khan and Thomas (2008), "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics" *Econometrica*

Winberry (2016), "Lumpy Investment, Business Cycles, and Stimulus Policy"

Zwick and Mahon (2017), "Tax Policy and Heterogeneous Investment Behavior," *American Econ Review*

5. Useful Advice on how to do Research

Dixit (1998), "My System of Work (Not!)", <https://www.princeton.edu/~dixitak/home/dixitwrk.pdf>

Duflo (2004), "Finding the Right Questions," available at <https://www.aeaweb.org/content/file?id=585>

Varian (1997), "How to Build an Economic Model in Your Spare Time," available at <http://people.ischool.berkeley.edu/~hal/Papers/how.pdf>

Also see other articles in CSWEP newsletters <https://www.aeaweb.org/about-aea/committees/cswep/newsletters/archives> and other CSWEP mentoring material <https://www.aeaweb.org/about-aea/committees/cswep/mentoring/reading>