Heterogeneous-Agent Macro as a Gateway to Behavioral Macro

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Slides at `https://benjaminmoll.com/SCE_behavioral_macro/`

Society for Computational Economics, Nice

Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy

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Peter Maxted

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Plan

1. Heterogeneous-agent macro as a gateway to behavioral macro: some general considerations

2. Finite-difference methods for solving heterogeneous-agent models

3. “Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy” with Laibson and Maxted

4. Solution methods for HA models with aggregate risk: what we’re doing makes no sense and the problem is rational expectations!

HA Macro as a Gateway to Behavioral Macro

Philosophy of heterogeneous-agent macro:

- build things from ground up, take individual behavior seriously
- flesh out implications for macro policy, fluctuations

Enormously successful research program...

Household finance & behavioral econ literatures:

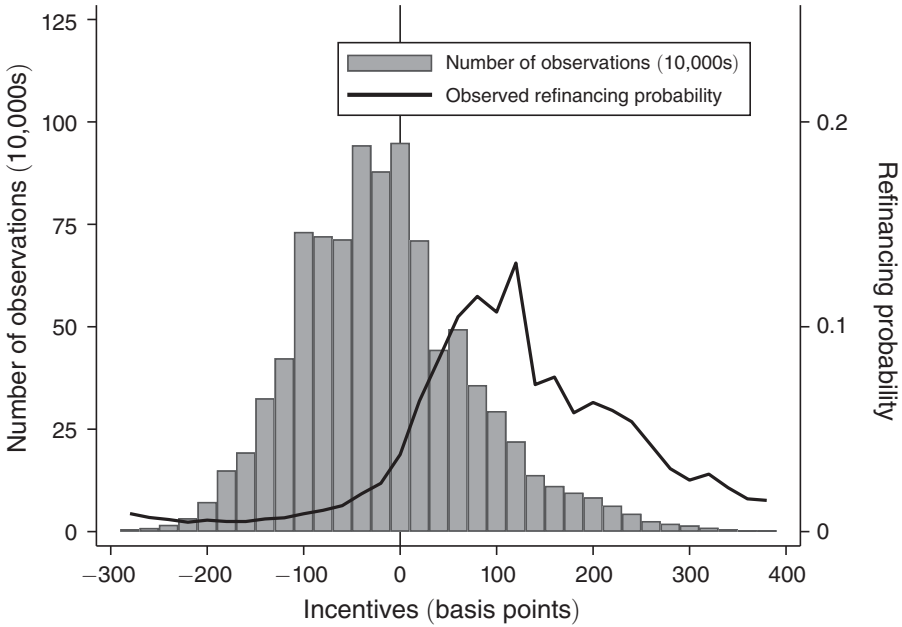
- Empirical findings that are hard to rationalize w optimizing behavior
  1. pension saving
  2. credit card borrowing
  3. mortgage refinancing
  4. ...
- Propose alternative models that do rationalize empirical findings

Logical question: Does incorporating such behavior into our (HA) macro models change our thinking about macro policy, fluctuations?

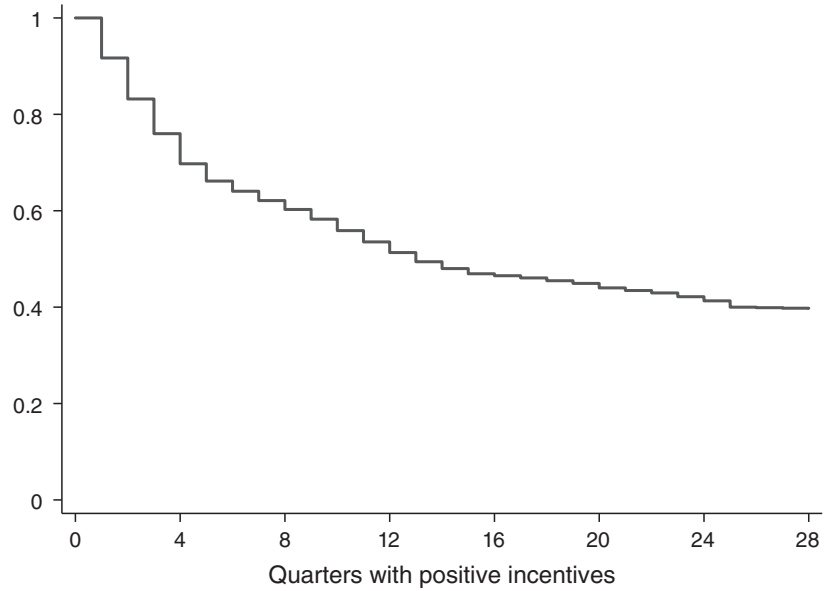
Mortgage refinancing: large delays, sums left on table

Andersen et al (AER 2020) on refinancing of Danish fixed-rate mortgages

(a) Interest savings left on table



(b) Refinancing delays



Note: Prediction of (S, s) model = refinance whenever incentive > 0 where incentive $\approx$ potential savings = $r_{\text{old}} - r_{\text{new}}$ − fixed cost (ADL threshold)

- Also: inconsistencies that violate optimal inaction, instead Calvo

Questions:

1. Where does this inertia come from?
2. Does incorporating it change our thinking about macro policy?

A Bottom-Up Approach to Behavioral Macro

Behavioral macro is well-established field, many important contributions

Most theoretical work uses RA rather than HA models

- RA models hard to connect to micro data
- often top-down approach: pick behavioral biases to fit macro data
- sometimes feels a bit reverse-engineered

Usefulness of heterogeneous-agent modeling? Bottom-up approach

- starting point: empirical findings about individual behavior
- easier to link HA models to huge body of micro work in household finance, behavioral econ, psychology,...

This talk: (baby) attempt at doing this = paper with Laibson and Maxted

A number of other recent HA macro papers move in same direction

Auclert-Rognlie-Straub, Boutros, Maxted, Laibson-Maxted-Moll, Lian, Kueng, ...

Finite-difference methods for solving heterogeneous agent models

Background readings

1. Achdou-Han-Lasry-Lions-Moll (2022) "Income and Wealth Distribution in Macro: A Continuous-Time Approach"

   `https://benjaminmoll.com/HACT/`

2. Website with codes `https://benjaminmoll.com/codes/`

- Key idea: solve HA models as systems of PDEs

Examples of impressive advances by others building on this idea: aggregate risk

- FernandezVillaverde-Hurtado-Nuno “Financial Frictions and the Wealth Distribution”
  https://www.sas.upenn.edu/~jesusfv/Financial_Frictions_Wealth_Distribution.pdf
- Schaab “Micro and Macro Uncertainty”
  https://andreasschaab.com/wp-content/uploads/2020/11/JMP.pdf
- Gu-Lauriere-Merkel-Payne “Deep Learning Solutions to Master Equations for Continuous Time Heterogeneous Agent Macroeconomic Models”
  https://drive.google.com/file/d/10xz4moTUIPwgw7Rp8g7XqbiahDmC81KD/view
- Bilal “Solving Heterogeneous Agent Models with the Master Equation” https://drive.google.com/file/d/19g2RmDK-J7dSmi7YXE0SIfosZpJ_dx5H/view
- Lee “The Macroeconomic Effects of Debt Relief Policies during Recessions ” https://github.com/soyoung-lee-n/files/blob/master/jmp_soyoung.pdf

Textbook Heterogeneous-Agent Model

Households are heterogeneous in their wealth a and income y, solve

$$\max_{\{c_t\}_{t\geq 0}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_t) dt \qquad \text{s.t.}$$

$$\dot{a}_t = y_t + r a_t - c_t$$

$$y_t \in \{y_1, y_2\} \text{ Poisson with intensities } \lambda_1, \lambda_2$$

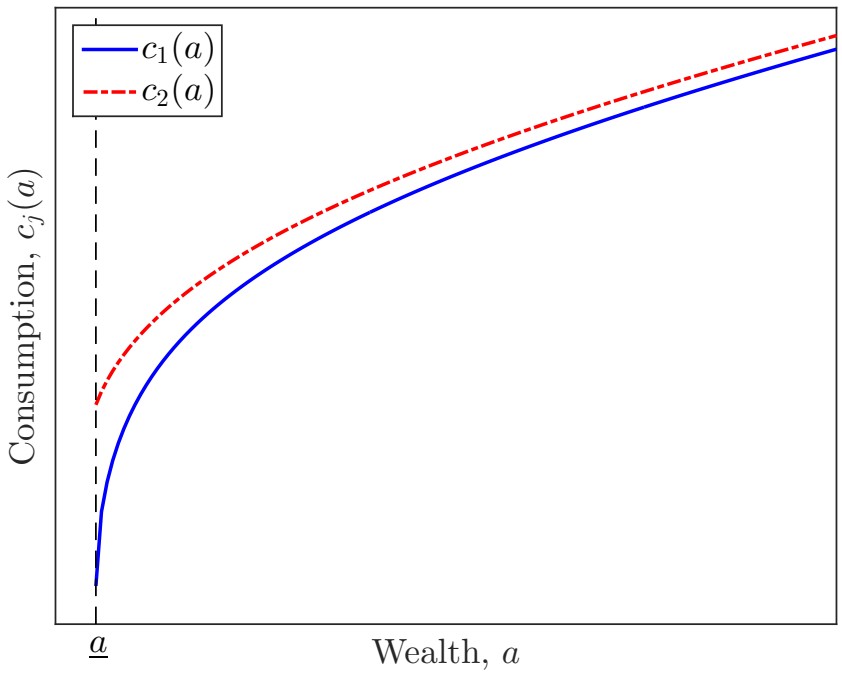
$$a_t \geq \underline{a}$$

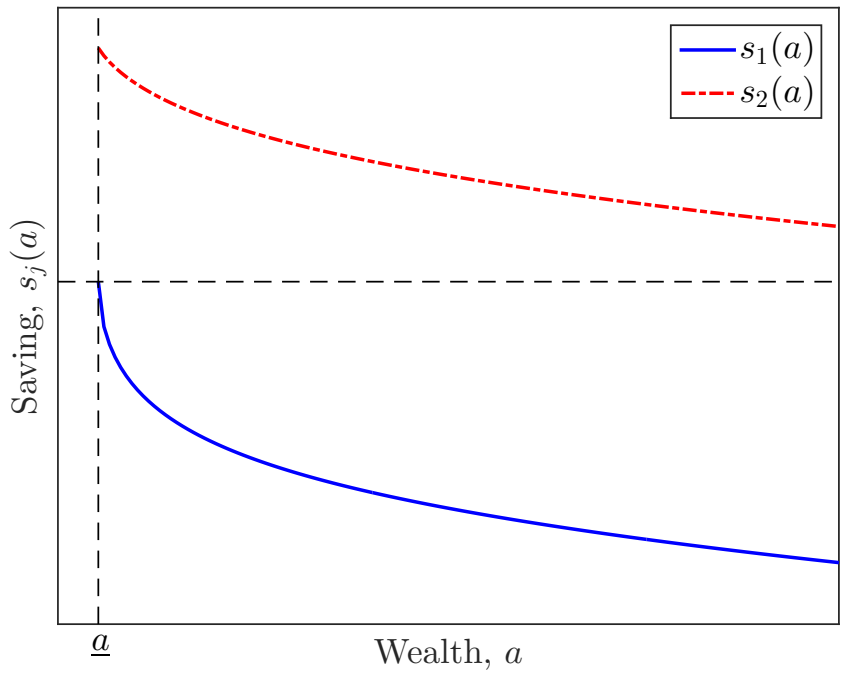
- c_t: consumption
- u: utility function, $u' > 0$, $u'' < 0$
- ρ: discount rate
- r : interest rate
- $\underline{a} \geq -y_1/r$ if $r > 0$: borrowing limit e.g. if $\underline{a} = 0$, can only save

Carries over to y_t = more general processes, e.g. diffusion

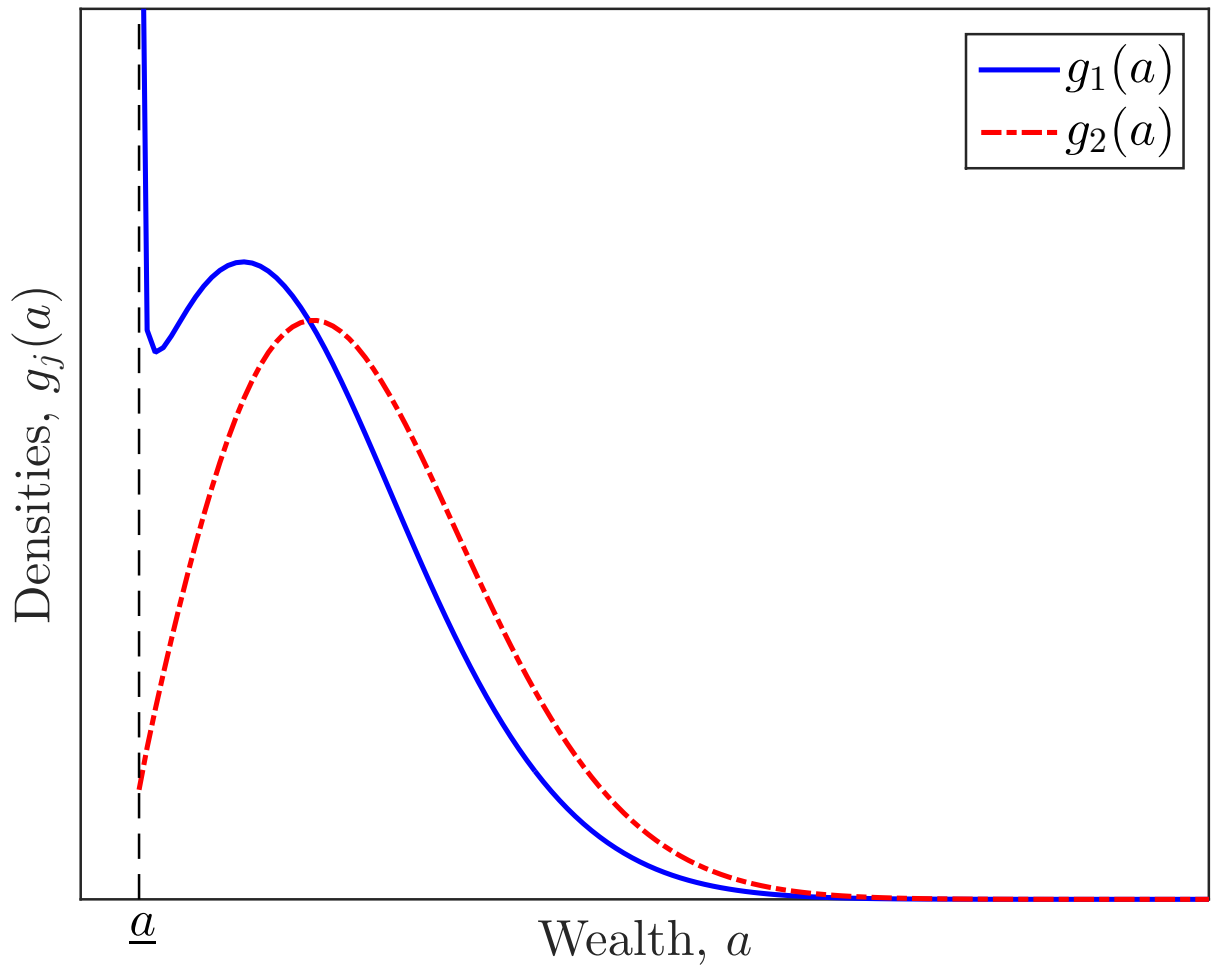
Equilibrium (Huggett): bonds in fixed supply, i.e. aggregate a_t = fixed

Typical Consumption and Saving Policy Functions





Typical Stationary Distribution



Equations for Stationary Equilibrium

$$\rho v_j(a) = \max_c \; u(c) + v_j'(a)(y_j + ra - c) + \lambda_j(v_{-j}(a) - v_j(a)) \qquad \text{(HJB)}$$

$$0 = -\frac{d}{da}[s_j(a)g_j(a)] - \lambda_j g_j(a) + \lambda_{-j} g_{-j}(a), \qquad \text{(KF)}$$

$s_j(a) = y_j + ra - c_j(a) =$ saving policy function from (HJB),

$$\int_{\underline{a}}^{\infty} (g_1(a) + g_2(a))da = 1, \quad g_1, g_2 \geq 0$$

$$S(r) := \int_{\underline{a}}^{\infty} a g_1(a) da + \int_{\underline{a}}^{\infty} a g_2(a) da = B, \qquad B \geq 0 \qquad \text{(EQ)}$$

- The two PDEs (HJB) and (KF) together with (EQ) fully characterize stationary equilibrium

Computational Advantages relative to Discrete Time

1. Borrowing constraints only show up in boundary conditions
   - FOCs always hold with "$=$"
2. "Tomorrow is today"
   - FOCs are "static", compute by hand: $c^{-\gamma} = v_a(a, y)$ (EGM)
3. Sparsity
   - solving Bellman, distribution = inverting matrix
   - but matrices very sparse ("tridiagonal")
   - reason: continuous time $\Rightarrow$ one step left or one step right
4. Two birds with one stone
   - tight link between solving (HJB) and (KF) for distribution
   - matrix in discrete (KF) is transpose of matrix in discrete (HJB)
   - reason: diff. operator in (KF) is adjoint of operator in (HJB)

Real Payoff: extends to more general setups

- non-convexities
- stopping time problems – see Laibson-Maxted-Moll paper
- multiple assets
- transition dynamics
- aggregate shocks

Finite-difference methods for solving HJB equation

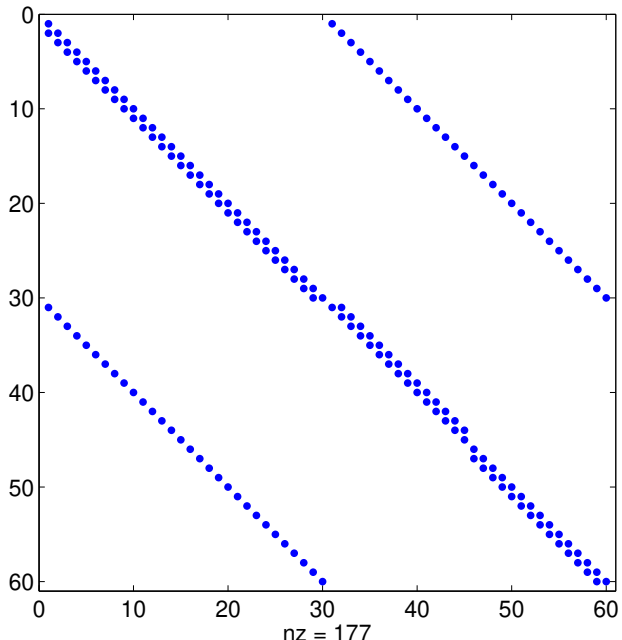
- HJB equation in HA model

$$\rho v_j(a) = \max_c \; u(c) + v_j'(a)(y_j + ra - c) + \lambda_j(v_{-j}(a) - v_j(a)), \quad j = 1, 2$$

- Will discretize and solve using finite difference method
- Discretization $\Rightarrow$ system of non-linear equations

$$\rho \mathbf{v} = \mathbf{r}(\mathbf{v}) + \mathbf{A}(\mathbf{v})\mathbf{v}$$

where **A** is a sparse (tri-diagonal) transition matrix



Finite-difference methods for solving HJB equation

- Generic HJB equation (operator $\mathcal{A}$ = infinitesimal generator)

$$\rho v(x) = \max_{\alpha} \; r(x,\alpha) + (\mathcal{A}_\alpha v)(x)$$

- Will discretize and solve using finite difference method
- Discretization $\Rightarrow$ system of non-linear equations

$$\rho \mathbf{v} = \mathbf{r}(\mathbf{v}) + \mathbf{A}(\mathbf{v})\mathbf{v}$$

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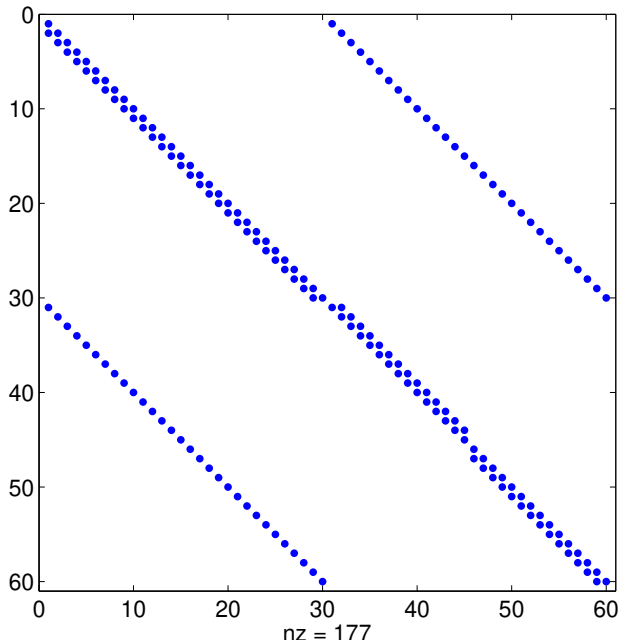

Finite-difference methods for solving HA models

- Use finite difference method: `https://benjaminmoll.com/codes/`
- Discretize state space a_i, $i = 1, ..., I$ with step size Δa

$$v_j'(a_i) \approx \frac{v_{i+1,j} - v_{i,j}}{\Delta a} \quad \text{or} \quad \frac{v_{i,j} - v_{i-1,j}}{\Delta a}$$

$$\text{Denote} \quad \mathbf{v} = \begin{bmatrix} v_1(a_1) \\ \vdots \\ v_2(a_I) \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} g_1(a_1) \\ \vdots \\ g_2(a_I) \end{bmatrix}, \quad \text{dimension} = 2I \times 1$$

- End product of FD method: system of sparse matrix equations

$$\rho \mathbf{v} = \mathbf{u}(\mathbf{v}) + \mathbf{A}(\mathbf{v}; r)\mathbf{v}$$
$$\mathbf{0} = \mathbf{A}(\mathbf{v}; r)^\top \mathbf{g}$$
$$B = S(\mathbf{g}; r)$$

which is easy to solve on computer

Present Bias Amplifies the Household Balance-Sheet Channels of Macroeconomic Policy

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Question

Idea with long tradition (Strotz 1956, ...)

- dynamically inconsistent preferences alter dynamic choices
- particular form with strong empirical support: present bias
  (e.g. Ashraf-Karlan-Yin, Augenblick-Niederle-Sprenger, Laibson-Maxted-Repetto-Tobacman, ...)

Monetary and fiscal policy $\Rightarrow$ household consumption and saving

- = leading examples of dynamic choices affected by present bias

To what extent does present bias alter impact of these policy tools?

(To be clear: present bias = β-δ preferences = quasi-hyperbolic discounting)

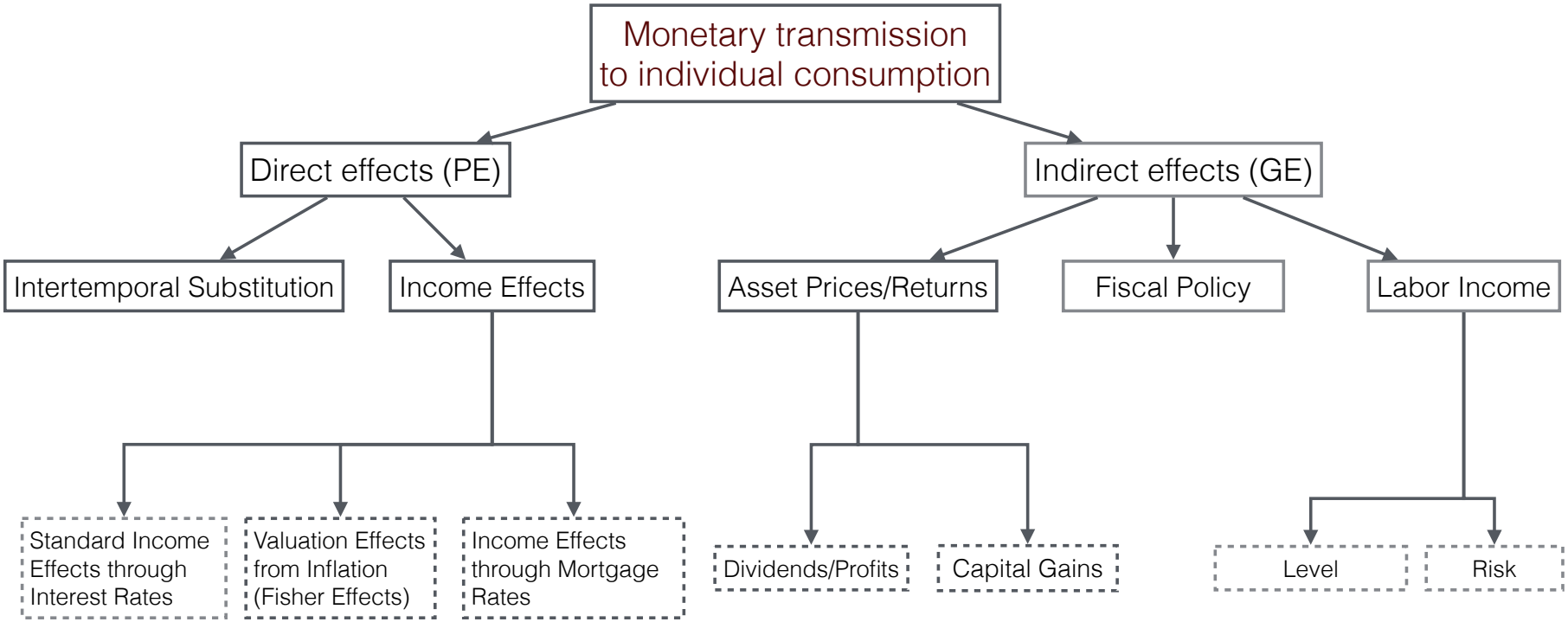
What We Do

Develop partial-equilibrium heterogeneous-household model with

1. rich household balance sheets ("Aiyagari w mortgages & housing")
   (e.g. Guerrieri-Lorenzoni-Prato, Wong, Eichenbaum-Rebelo-Wong, Kaplan-Mitman-Violante,...)
   - assets: liquid wealth and illiquid housing
   - liabilities: credit card debt and fixed-rate mortgages
   - liquidity constraints
2. present biased preferences
   - naïve present bias with procrastination

Goal: understand how interaction of (1)+(2) affects policy transmission

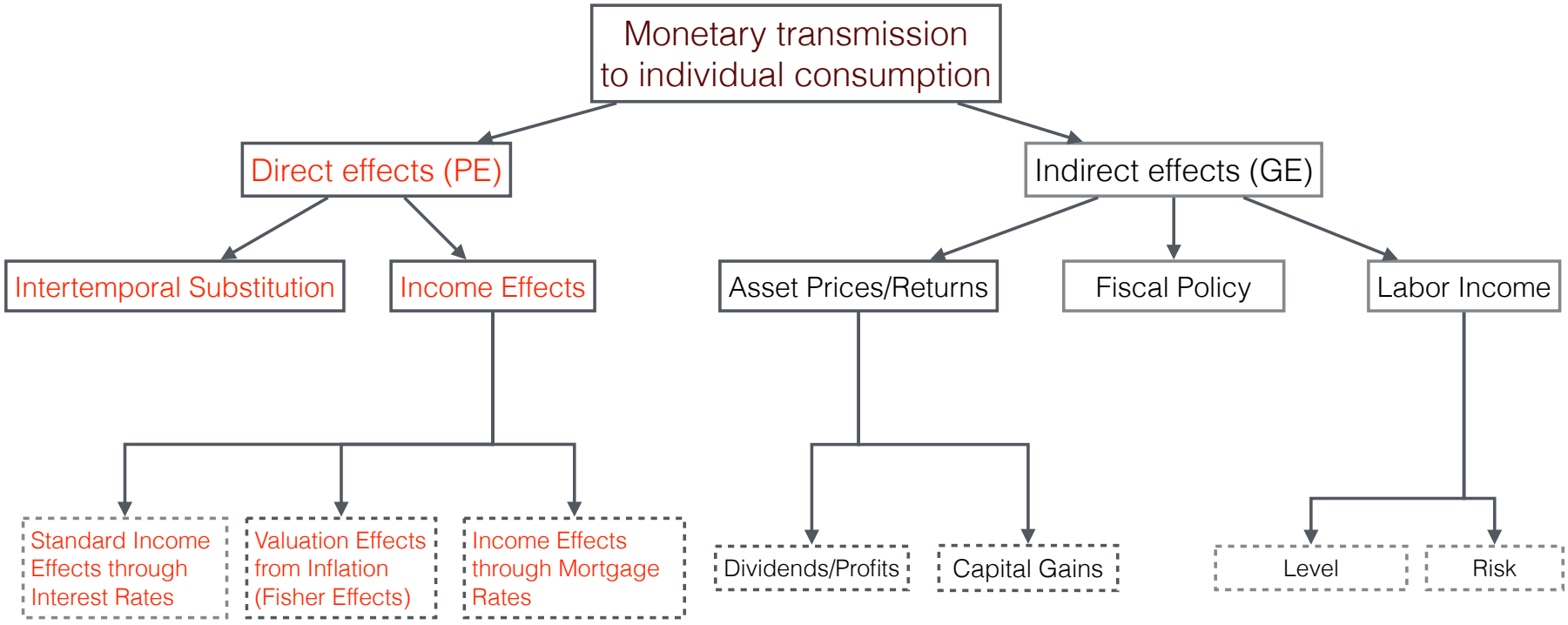
Our Scope: Monetary Policy Transmission



Important: today $\neq$ GE analysis, want to first understand PE

Paper: speculative discussion through lens of HANK literature

Our Scope: Monetary Policy Transmission



Important: today ≠ GE analysis, want to first understand PE

Paper: speculative discussion through lens of HANK literature

What We Find

1. Fiscal policy
   - present bias amplifies potency
   - generically increases economy's average MPC
2. Monetary policy
   - present bias amplifies potency...
     - cash-out refis = liquidity injections to high-MPC households
   - ... but at same time slows down transmission speed
     - refinancing inertia due to procrastination

Both effects of present bias move model toward data

Monetary policy is struggling to tame inflation – help explain this?

3. Methods
   - continuous-time present bias, option value problem via HJBQVI

Model

Plan for model exposition

1. Household balance sheets: “Aiyagari with mortgages & housing”
2. Time preferences: naïve present bias
3. Refinancing procrastination

Household Balance Sheets

- Continuum of households
- Stochastic income y_t, liquid wealth b_t, housing h, mortgage m_t
- Can refinance mortgage at cost (both \$ and effort – details later)
- When not refinancing:

$$\dot{b}_t = y_t + r_t b_t + \omega^{cc} b_t^- - (r_t^m + \xi) m_t - c_t$$

$$\dot{m}_t = -\xi m_t$$

  - credit card limit: $b_t \geq \underline{b}$
  - LTV constraint: $m_t \leq \theta h$

- Note shortcut: housing h is fixed and cannot be adjusted
  $\Rightarrow$ when taking to data, restrict to home-owners who do not move
- "Monetary policy": exogenous process for liquid rate r_t
- Mortgage interest rate r_t^m fixed until refinance, then $r_t^m = r_t + \omega^m$

Why refinance?

1. Rate refinancing motive
    - Lower mortgage interest payments if market rate falls
2. Cash-out refinancing motive
    - Access home equity during low-income spells (c smoothing)
    - Replace expensive credit card debt w cheaper mortgage debt

- Model: refinancing is costly
    - fixed cost κ^{refi}, effort cost $\bar{\varepsilon} \approx 0$

Time preferences: naïve present bias

Key behavioral element: present bias = β-δ discounting

Additional assumption: households are naïve about their present bias

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Key behavioral element: present bias = β-δ discounting

Additional assumption: households are naïve about their present bias

Discrete-time warmup:

- Current self discounts all future selves by $\beta < 1$

$$u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t)$$

- Naïveté: current self believes future selves time-consistent ($\beta = 1$)
  $\Rightarrow$ no game between current and future selves

Time preferences: naïve present bias

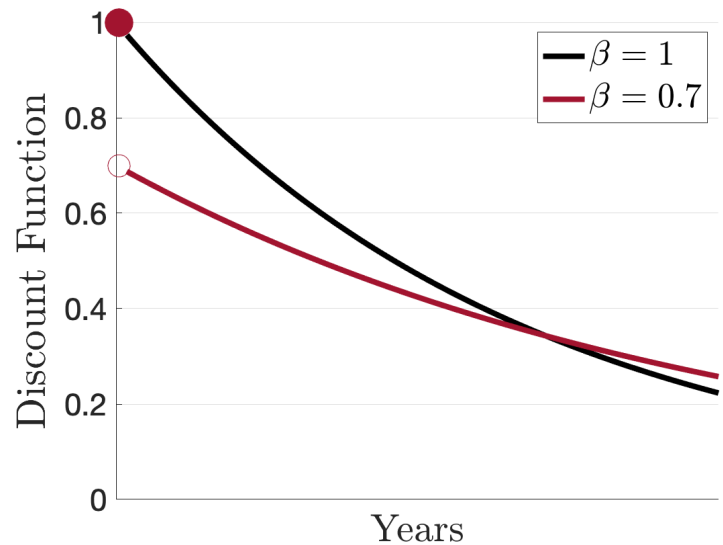
Key behavioral element: present bias = β-δ discounting

Additional assumption: households are naïve about their present bias

Continuous time:

- Current self discounts all future selves by $\beta < 1$
- Take period length $\to 0$

Discount function $D(s) = \begin{cases} 1 & \text{if } s = 0 \\ \beta e^{-\rho s} & \text{if } s > 0 \end{cases}$



Why continuous time? Tractable approx. of daily/weekly time-steps

(Laibson-Maxted, Augenblick, Augenblick-Rabin, McClure et al.)

Refinancing Procrastination

Large empirical literature: households slow to refinance – think Calvo
(e.g. Andersen-Campbell-Nielsen-Ramadorai, Keys-Pope-Pope,...)

Naïve $\beta < 1$ naturally generates such refinancing procrastination

- Key ingredient: effort cost $\bar{\varepsilon} \approx 0$
- Application of result from theory literature (O'Donoghue-Rabin): naïfs procrastinate on immediate-cost delayed-benefit tasks
- Take $\bar{\varepsilon} \to 0$: no effect when $\beta = 1$ but procrastination when $\beta < 1$
- Monetary cost not enough. See discussion in paper.

How get Calvo? Stochastic $\varepsilon_t \in \{\underline{\varepsilon}, \bar{\varepsilon}\}$, flicks from $\bar{\varepsilon}$ to $\underline{\varepsilon}$ at rate ϕ

- $\underline{\varepsilon} < \beta\bar{\varepsilon} \Rightarrow$ procrastinate whenever $\varepsilon_t = \bar{\varepsilon}$, refi whenever $\varepsilon_t = \underline{\varepsilon}$
- True even though we take limit as $\underline{\varepsilon}, \bar{\varepsilon} \to 0$

Methods
Effect of $\beta < 1$ on Policy Functions

Methods: option-value problems (HJBQVI)

Without mortgage adjustment: standard HJB equation

$$\rho v(x) = \max_c \; u(c) + (\mathcal{A}v)(x)$$

With mortgage adjustment: "HJB quasi-variational inequality"

$$\rho v(x) = \max\left\{ \max_c u(c) + (\mathcal{A}v)(x) \;,\; \rho(v^*(x) - \varepsilon) \right\}$$

- $x = (b, m, y, r, r^m)$ = household state variables
- operator $\mathcal{A}$ = infinitesimal generator for x (no adjustment)
- $v^*(x) - \varepsilon$ = value of mortgage adjustment
- ε = effort cost

How solve this? Linear complementarity problem (LCP)

- In contrast to "smooth pasting", works beautifully even w 5D state
- **`http://benjaminmoll.com/Lecture2_Rochester/`**, section "Stopping Time Problems"
- Codes labelled "Stopping Time Problems" at **`http://benjaminmoll.com/codes/`**

Effect of present bias on consumption

Warmup: continuous-time FOC and Euler equation with $\beta = 1$

1. **FOC for today vs future:**

$$u'(c) = \frac{\partial v(x)}{\partial b}$$

   where $x = (b, m, y, r, r^m)$ = household state variables

2. **Euler equation:**

$$\frac{\mathbb{E}_t[du'(c_t)]/dt}{u'(c_t)} = \rho - r_t(b_t)$$

Note: no discounting in FOC, unlike discrete-time $u'(c) = \delta \mathbb{E}\left[\frac{\partial}{\partial b} v(x')\right]$

(Comes from HJB equation $\rho v(x) = \max_c u(c) + \frac{\partial v(x)}{\partial b}(y + rb + ... - c)$)

Effect of present bias on consumption

Continuous-time FOC and Euler equation with present bias, $\beta < 1$

1. **FOC for today vs future:**

$$u'(c) = \beta \frac{\partial v(x)}{\partial b}$$

and naïveté $\Rightarrow v(x)$ = time-consistent value function ($\beta = 1$)

2. **Euler equation:** (Maxted, 2022)

$$\frac{\mathbb{E}_t[du'(c_t)]/dt}{u'(c_t)} = \left[\rho + \gamma\left(1 - \beta^{\frac{1}{\gamma}}\right)\frac{\partial c(x_t)}{\partial b}\right] - r_t(b_t)$$

3. When unconstrained, households overconsume by $\beta^{-1/\gamma} > 1$

$$c(x) = \beta^{-1/\gamma}\widehat{c}(x) \quad \text{where} \quad \widehat{c}(x) = \text{time-consistent policy fn} \quad (*)$$

Observation: interaction of $\beta < 1$ with liquidity constraint is critical. Otherwise $(*) \Rightarrow \beta < 1$ and $\beta = 1$ observationally equivalent

Calibration and Results

Calibration and results

Always show results for 3 cases

1. Rational Benchmark: $\beta = 1$, ~~Procrastination~~
2. Intermediate Case: $\beta < 1$, ~~Procrastination~~
3. Behavioral Benchmark: $\beta < 1$, Procrastination

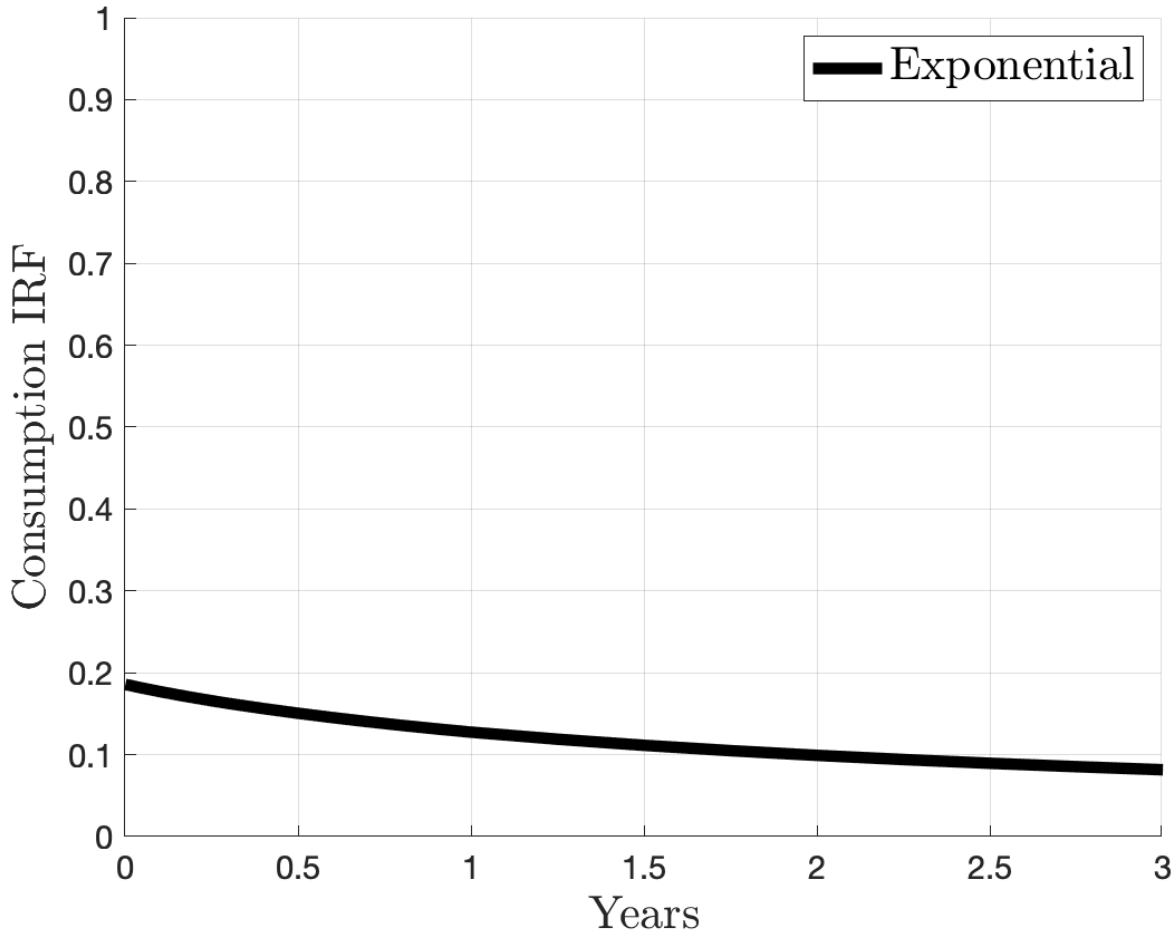
Discount Function

- Calibrate discount function to match empirical wealth moments
- 2016 SCF wave of home owners who don't move:
  - Average LTV = 0.54
  - Average credit card debt to income ratio = 0.09

	Data	Exponential Benchmark	Intermediate Case	Present-Bias Benchmark
Discount Function				
β	-	1	0.7	0.83
ρ	-	1.65%	0.66%	1.08%
Calibration Targets				
LTV	0.54	**0.54**	**0.54**	**0.54**
Avg. CC Debt	0.09	0.04	**0.09**	**0.09**
Share CC Debt >0	60%	27%	51%	46%

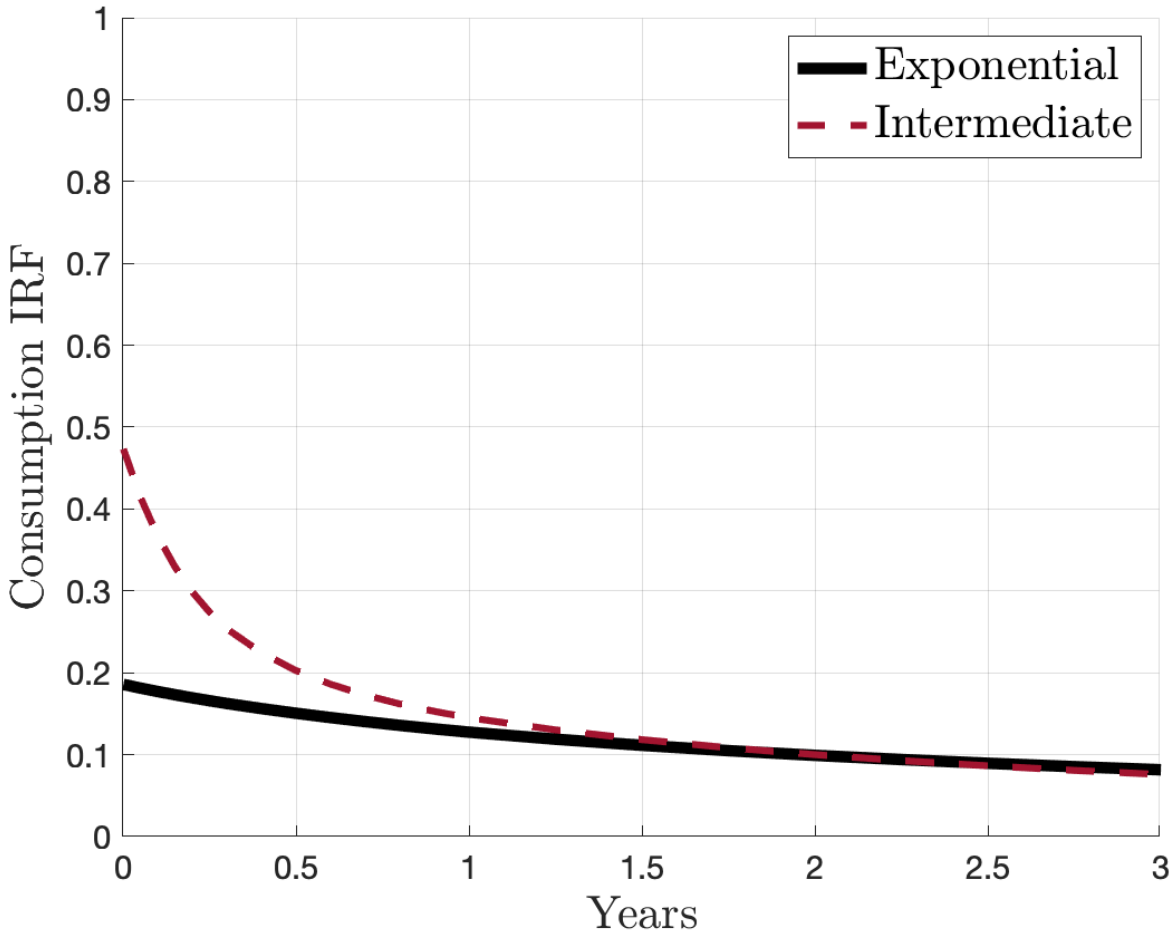
▸ SCF Details ▸ Calibration Details

Fiscal Policy: $1000 Helicopter Drop



- Present bias $\beta < 1$ robustly amplifies potency of fiscal policy

Fiscal Policy: $1000 Helicopter Drop



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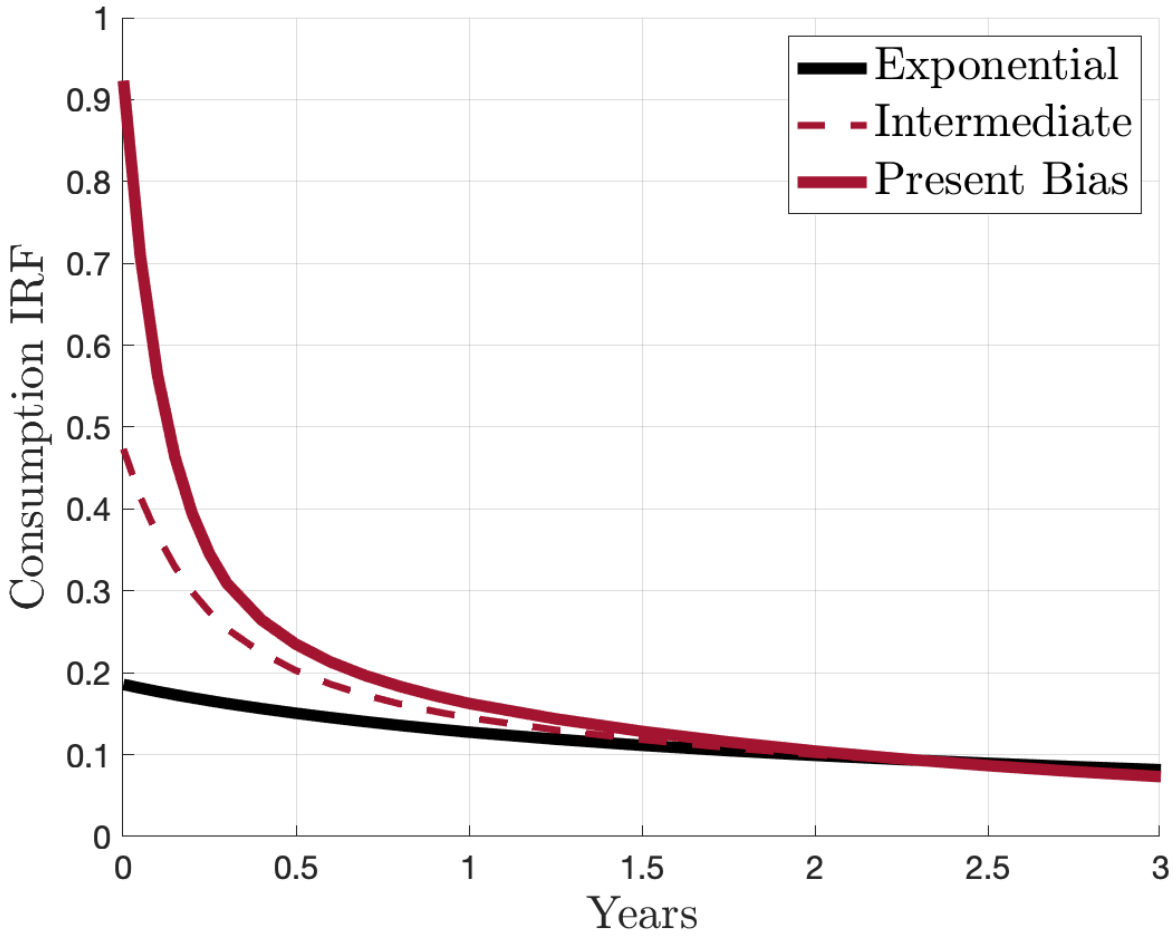
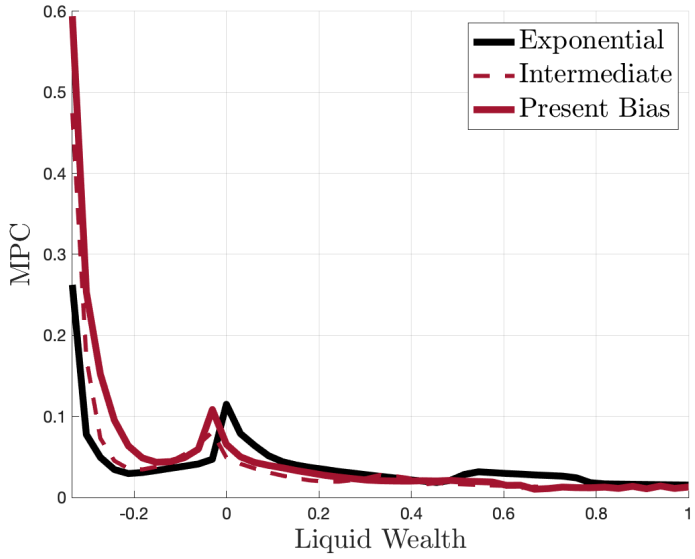
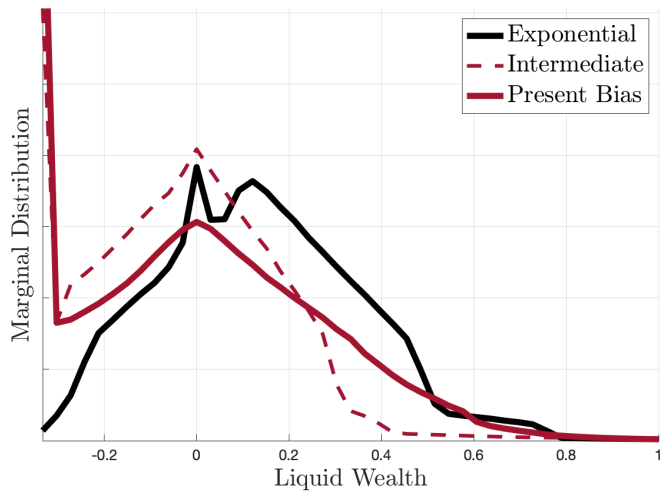


- Present bias $\beta < 1$ robustly amplifies potency of fiscal policy

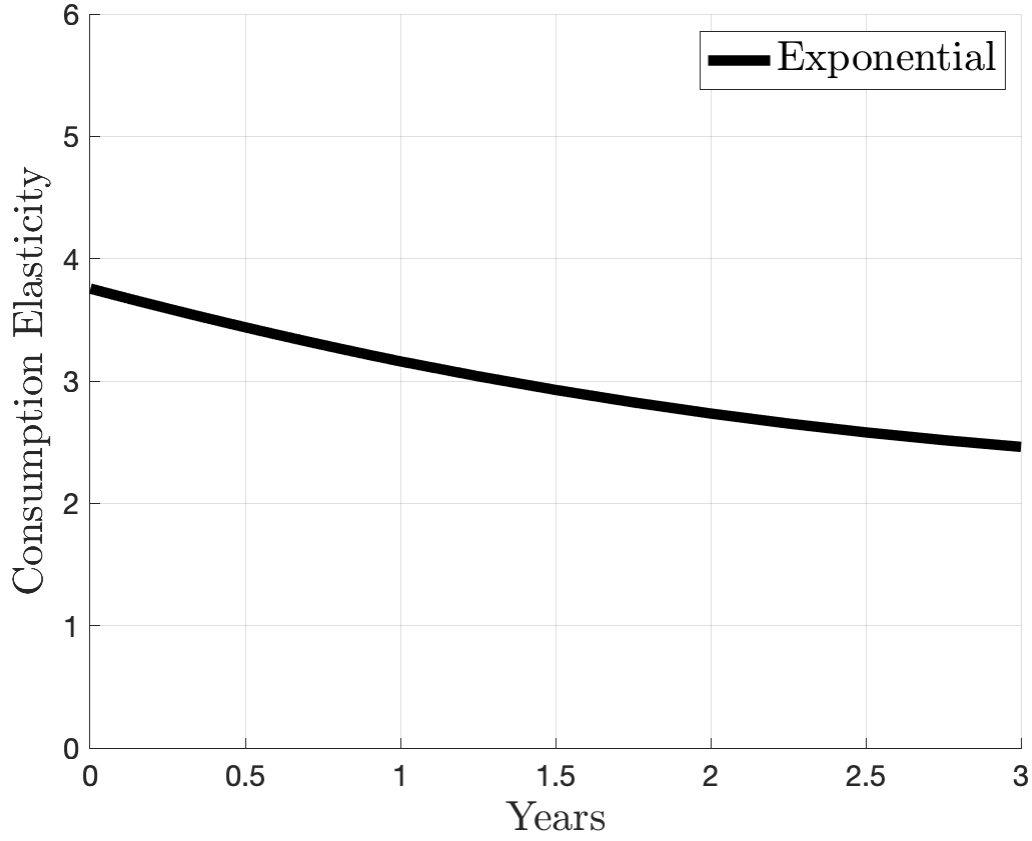
Present bias amplifies potency of fiscal policy: intuition





- $\beta < 1$ creates large MPCs + large mass of households at $\underline{b}$

Monetary Policy: 1% Interest-Rate Cut



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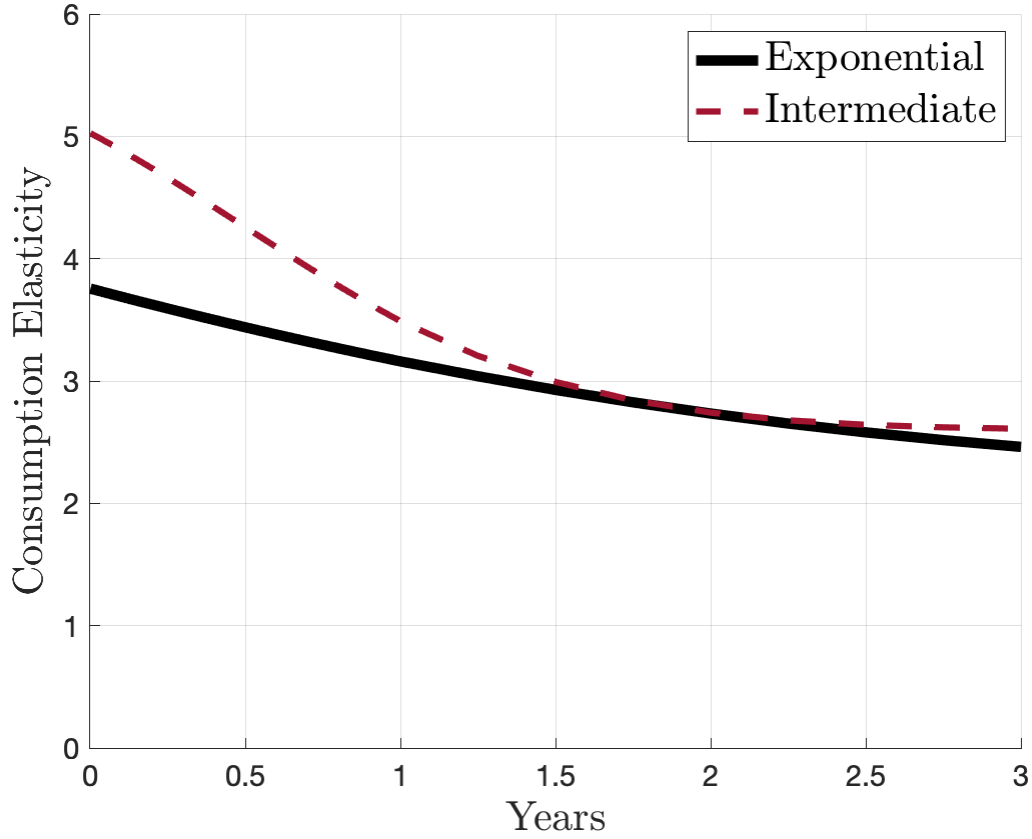
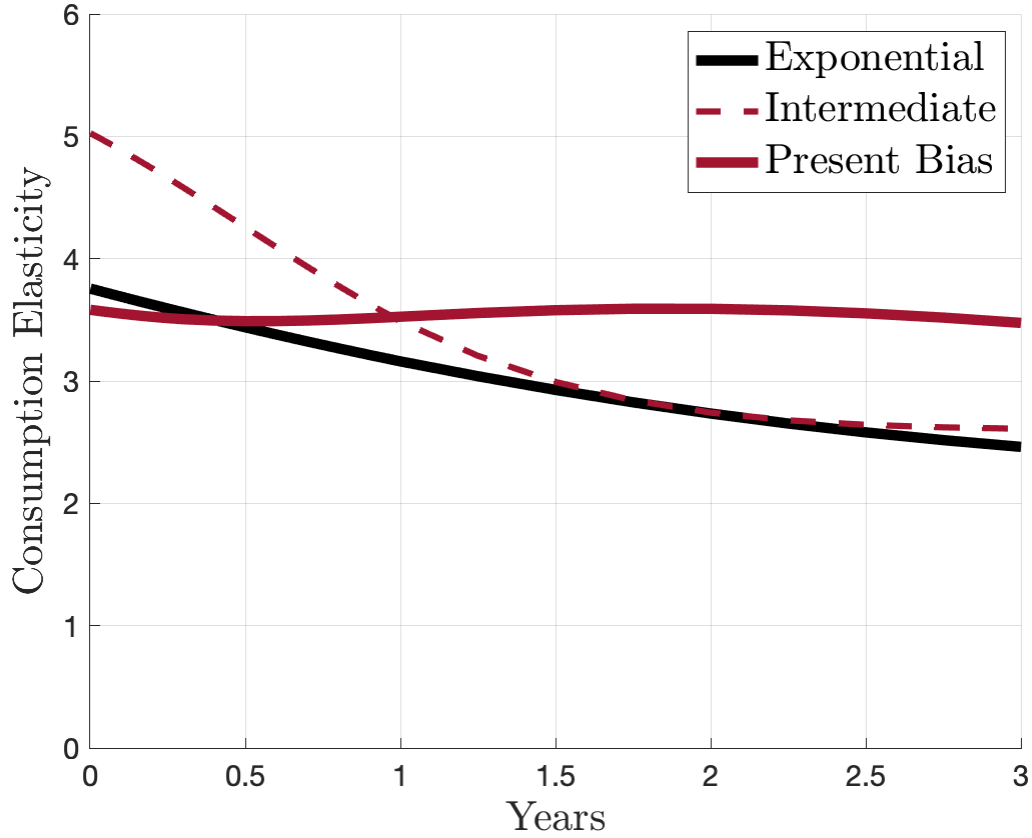


- Present bias $\beta < 1$ amplifies potency of monetary policy ...
  - cash-out refis imitate liquidity-injection of fiscal policy

Monetary Policy: 1% Interest-Rate Cut

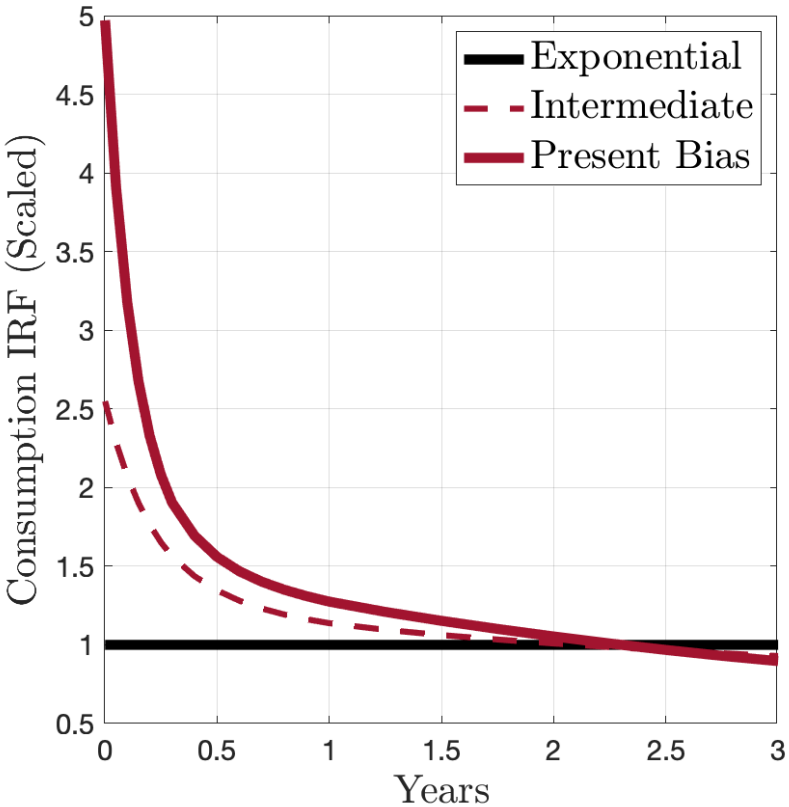


- Present bias $\beta < 1$ amplifies potency of monetary policy ...
- ... but slows transmission speed
  - refi procrastination $\Rightarrow$ "dry powder" ignited more slowly

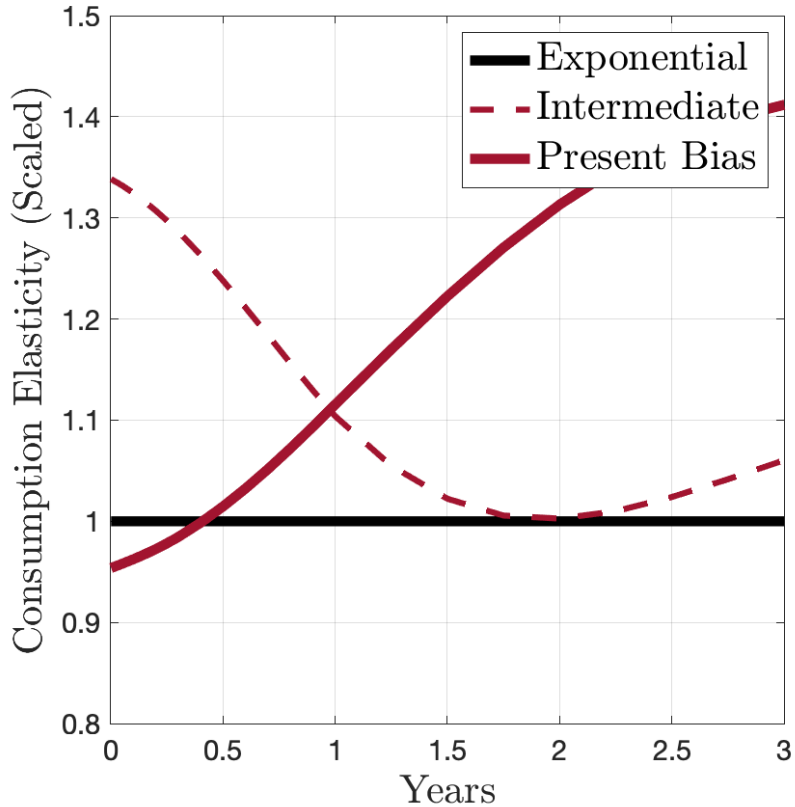
Summary: Effect of $\beta < 1$ on Magnitude and Timing

- Fiscal and Monetary Policy scaled to impact of $\beta = 1$ case

(a) Fiscal policy



(b) Monetary policy



- Fiscal Policy: $\beta < 1$ amplifies potency
- Monetary Policy: $\beta < 1$ amplifies potency but slows transmission

HA models with aggregate risk

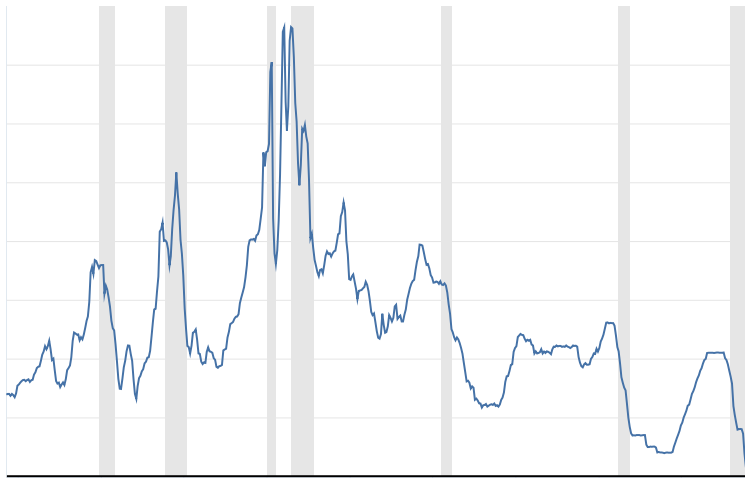
What we’re doing makes no sense

Extremely important model class for macro: heterogeneous-agent models with aggregate risk

- Classic papers by Krusell-Smith and Den Haan from late 90s
- Key challenge: rational expectations + general equilibrium
  $\Rightarrow$ cross-sectional distribution enters household decision problem
  - true even though households/firms do not really care about distribution and only care about prices
- Lots of extremely impressive advances solving such models
  - see beginning of slides for continuous-time methods
  - but also very impressive discrete-time advances
- My argument in next slides: we're spending a lot of intellectual and computational horse power solving a nonsensical problem

The problem with rational expectations in HA models

- Intuition: suppose I live in one of our models, only care about r



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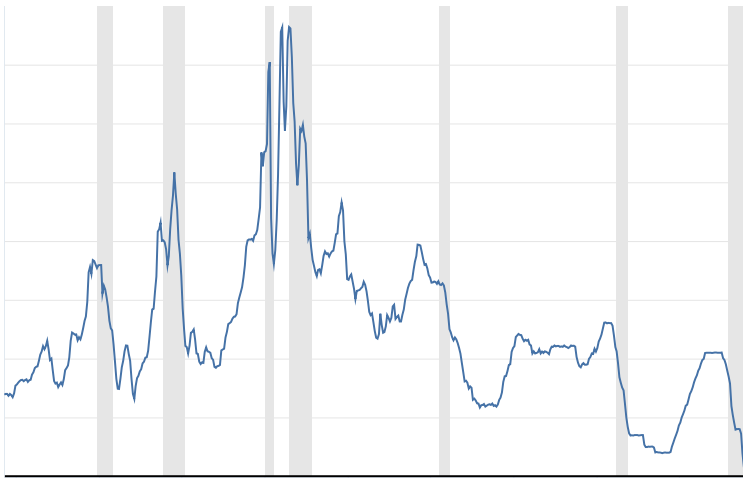

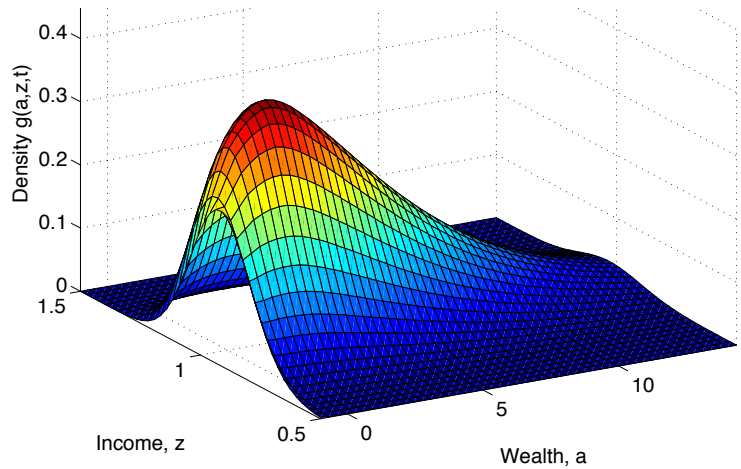

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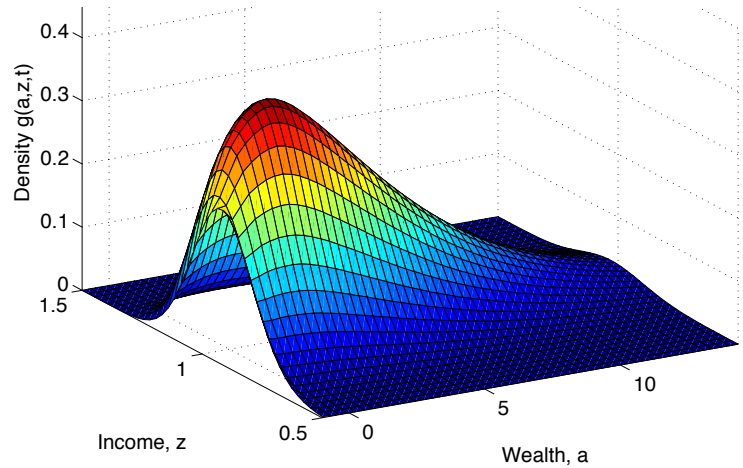

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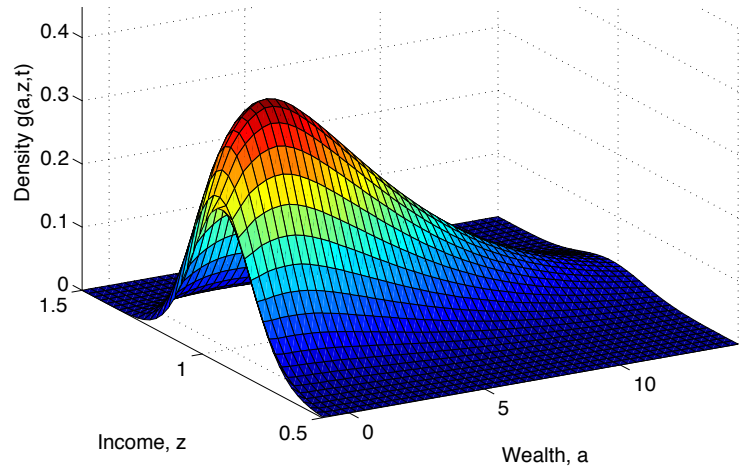

- Makes solution hard/impossible
- But do we really think people do this? I definitely don't

In HA models, rational expectations about equilibrium prices makes no sense. But what should replace it?

- Clear to me: we need to drop RE about equilibrium prices
- Payoff: kill two birds with one stone
  1. make model more realistic
  2. and solution feasible
- But what should replace RE?
  - natural solution: form expectations about prices directly $\neq$ RE
  - note: different from KS = forecast prices using moments of dist, say mean (exception: moment = price, e.g. Favilukis-L-V)
  - but how exactly? I'm not sure either!
- In summary:
  - I only know the problem, not the solution
  - huge payoff for figuring out sensible solution $\Rightarrow$ go for it!

Conclusion

Present bias amplifies household balance-sheet channels of macroeconomic policy

1. Fiscal policy
   - present bias amplifies potency, increases economy's average MPC
2. Monetary policy
   - present bias amplifies potency but...
   - ... at same time slows down speed of monetary transmission

Heterogeneous-agent macro as a gateway to behavioral macro

- bottom-up rather than top-down
- for more see `https://benjaminmoll.com/research_agenda_2020/`

In HA models with aggregate risk, we spend lots of intellectual and computational horsepower solving nonsensical problem

- need to drop rational expectations about equilibrium prices
- open question: what should replace it?

Thanks!