Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains*

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Abstract

Do wealthier households save a larger share of their incomes than poorer ones? We use Norwegian administrative panel data on income and wealth to answer this question and interpret our findings through the lens of economic theory. We find that saving rates net of capital gains are approximately flat across the wealth distribution, i.e., the rich do *not* actively save a larger share of their incomes than the poor. At the same time, saving rates including capital gains rise sharply with wealth because wealthier households hold more appreciating assets and tend to retain these capital gains. Qualitatively, these patterns are consistent with consumption-saving models with standard isoelastic preferences if asset prices appreciate while asset cashflows and labor incomes grow at the same rates. Quantitatively, a housing model designed to match asset-to-income ratios across the wealth distribution can account for the empirical findings.

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1 Introduction

A large and growing literature in macroeconomics studies the determinants of secular trends in income and wealth inequality and how such distributional shifts feed back to macroeconomic aggregates like the economy's saving rate and equilibrium interest rates, or how they affect the transmission of macroeconomic policy. One key ingredient in many of the theories in this literature is how individuals' saving behavior varies across the wealth distribution, in particular whether richer people save a larger share of their incomes than poorer ones.[1] Empirically disciplining the proposed theoretical mechanisms requires evidence on how saving rates vary with wealth. Unfortunately, high-quality empirical evidence on this question is largely lacking.[2]

We fill this gap by using Norwegian administrative panel data on income and wealth to examine how saving rates out of income vary across the wealth distribution, and by interpreting our findings through the lens of economic theory.

Because Norway levies both income and wealth taxes on households, the tax registry data provide a complete account of household income and balance sheets down to the single asset category. We combine the tax registry data with shareholder and housing transaction registries. Together, these data contain detailed third-party-reported information on household-level wealth and income, covering the universe of Norwegians from the very bottom to the very top of the wealth distribution.

Our study highlights that the relation between wealth and saving rates crucially depends on whether saving includes capital gains. We distinguish between two saving concepts which differ by how capital gains are treated when writing the household budget constraint. *Net saving*, or *active saving*, is the change in a household's net worth from one year to the next *holding asset prices constant* – the difference between a household's income (excluding capital gains) and its consumption. *Gross saving* is simply the total change in a household's net worth, including any revaluation effects due to changing asset prices.[3]

[1]For theories of secular inequality trends, see for example Greenwald et al. (2021); Gomez and Gouin-Bonenfant (2024); Kaymak, Leung and Poschke (2020); Benhabib and Bisin (2018); Hubmer, Krusell and Smith (2020); De Nardi and Fella (2017); Gabaix et al. (2016); Kaymak and Poschke (2016); De Nardi (2004); Carroll (1998); Boerma and Karabarbounis (2023). For theories of macro aggregates and policy, see for example Mian, Straub and Sufi (2021); Melcangi and Sterk (2020); Rachel and Summers (2019); Straub (2018); Auclert and Rognlie (2016); Krueger, Mitman and Perri (2016); Kumhof, Rancière and Winant (2015); Krusell and Smith (1998).

[2]A few existing studies do provide related evidence, e.g., on "synthetic saving rates." See the related literature section at the end of this introduction for a more detailed discussion, particularly footnote 5.

[3]The literature has used a number of other names for the same concepts, for example "change-in-wealth saving" in place of "gross saving" and "passive saving" in place of capital gains. Whereas we say that gross saving is the sum of net saving and capital gains, these studies would say that change-in-wealth saving is

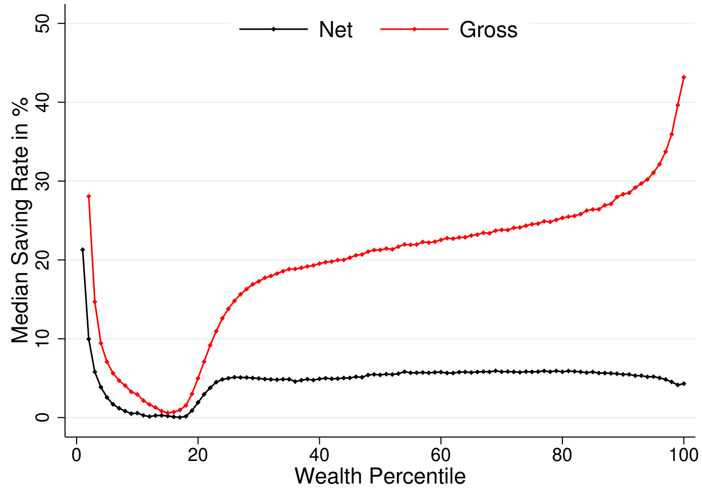


Figure 1: Saving rates across the wealth distribution.

Our main finding is that, among households with positive net worth, net or active saving rates are remarkably flat across the wealth distribution. Gross saving rates instead increase sharply with wealth. Hence, the answer to how saving rates vary with wealth, depends on whether capital gains are included in the definition of saving. Our second contribution is to provide a theoretical interpretation. We show that the empirical finding is consistent with standard models of household wealth accumulation if asset prices appreciate while asset cashflows and labor incomes grow at the same rates.

The empirical relationships between wealth and saving rates are easiest to communicate graphically. Figure 1 plots saving rates against percentiles of net worth. To the left are households with negative net worth, while zero-wealth households are located around the 15th percentile. Among households with negative net worth, net and gross saving rates both decline with wealth. Among the majority of households who have positive wealth, matters are very different. While the gross saving rate (including capital gains) increases sharply up to fifty percent for the top one percent of the wealth distribution, the net saving rate (excluding capital gains) is remarkably stable around seven percent. Moreover, observed saving rates retain these characteristics when we control for the main determinants of heterogeneous saving behavior in economic models, namely age, income, and individual fixed effects in saving propensities. In a few of our alternative specifications the net saving rate is weakly upward-sloping but in none of them do we see a steep increase (or decrease) outside of the top 1 percent of the wealth distribution, where saving rates statistics are sensitive to methodological choice. Our data thus provide a nuanced answer to how saving rates vary with wealth: The wealthier households do not have particularly high net saving rates compared to the poorer ones, but they still accumulate

the sum of active and passive saving. The two statements are equivalent.

more wealth than others through capital gains.

The remarkable fact in Figure 1 is how flat the net saving rate is across the wealth distribution for households with positive wealth. Given the flat net saving rate, the increasing gross saving rate follows mechanically from households' asset-to-income ratios and capital gains: relative to their income, wealthier households hold more assets like housing and stocks that appreciate over time in our sample. A flat net saving rate means that, even though wealthier households hold more assets experiencing capital gains, they do not spend these gains and instead hold on to them. Gross saving rates therefore increase with wealth. We term this phenomenon "saving by holding."

A simple back-of-the-envelope example clarifies this point. Assume the net saving rate is 10% at all points of the wealth distribution and capital gains on all assets are 2%. Now compare two people with different wealth: the first has an income excluding capital gains of \$100,000 and no assets, the second has the same income but owns assets worth \$1,000,000. If neither person consumes out of capital gains, their gross savings are \$10,000 and $\$10,000 + 2\% \times \$1,000,000 = \$30,000$ respectively. The gross saving rate of the first person is then 10% whereas that of the second is $\frac{30,000}{100,000+20,000} = 25\%$. Even quite small capital gains can induce sizable divergence between net and gross saving.

Our empirical findings beg the questions: Why are net saving rates flat? And why do they remain flat in the face of changing asset prices? Our second contribution is to interpret our findings through the lens of economic theory. To this end, we consider a series of theoretical benchmark models and show their predictions for net and gross saving rates by wealth. These models are purposely stylized to highlight the basic economic mechanisms governing saving rates across the wealth distribution with and without capital gains.

We start with a particularly simple consumption-saving model that can be solved analytically and thereafter enrich the framework to address prevalent features of the data. The simple benchmark features a household with standard isoelastic preferences (i.e. CRRA utility) who receives a constant stream of labor income and saves in an asset with an exogenous price. When this asset price is constant, the model predicts that the household's saving rate out of income is independent of wealth.

When the asset price varies over time, net saving (excluding capital gains) differs from gross saving (including capital gains). Capital gains may be due to either rising asset cashflows or other factors ("discount rates"). We show that, with isoelastic preferences, net saving rates do not vary with wealth (as in the data) whenever asset cashflows and labor incomes grow at the same rates, regardless of the source of capital gains. Intuitively, asset cashflows matter more for income at the top of the wealth distribution than at the bottom. Whenever asset cashflows and labor income grow at the same rate, the corresponding consumption and saving incentives do not vary across the wealth distribution. Hence net saving rates do not vary with wealth. The gross saving rate then mechanically rises with wealth because wealthier households hold more assets that experience capital gains. An important special case is when capital gains are exclusively due to declining discount rates while cashflow and labor income growth is zero. Thus, our empirical findings are qualitatively consistent with standard models of household wealth accumulation if asset prices appreciate while asset cashflows and labor income grow at the same rates.

The simple models just discussed deliver a flat net saving rate, but they are not quantitative models and miss on a number of empirical patterns, most notably portfolio shares and asset-to-income ratios across the wealth distribution. We therefore introduce a more fully-fledged model that we can take to the data. The model features housing as this dominates Norwegian households' balance sheets. Preferences for housing are non-homothetic so as to match how the ratio of housing to income varies along the wealth distribution.[4] Different income sources grew at similar rates over our sample period, so the calibrated model implies a relatively flat net saving rate by the same logic as in the simple model. Adjustment frictions to housing and fixed amortization schedules reinforce this tendency, as they induce households to stay in their homes and repay their mortgages. Because the model matches the increasing housing-to-income ratio across the wealth distribution, when house prices increase as dramatically (more than rents) in our model as they did during our sample period, an increasing gross saving rate follows. Overall, the model accounts well for the saving rates in Figure 1 between the 25th and 98th percentiles of the wealth distribution.

We have thus shown that standard models of household wealth accumulation can deliver our main empirical patterns, specifically the remarkably flat net saving rate. These models omit several mechanisms emphasized in the literature, for example, a luxury bequest motive, wealth in the utility function, or behavioral biases. While we demonstrate that our findings can be explained without including such mechanisms, we do not claim that ours is the unique theory that can account for the observed patterns. Such mechanisms may still be relevant. For example, the combination of asset cashflows growing faster than labor income (which rotates net saving rates down) and a luxury bequest motive (which rotates net saving rates up) could balance each other so as to deliver a flat net saving rate. The same is true for other (combinations of) theoretical mechanisms that may interact with and complement the ones we highlight. Still, we find it noteworthy that completely standard economic theory can account for our empirical findings, with potentially far-

[4]Note that this non-homotheticity concerns the *intra*temporal allocation between housing and non-housing consumption (housing is a luxury good). This is different from the non-homothetic preferences over wealth or bequests sometimes used in the literature.

reaching implications.

Our data is from Norway – what lessons generalize? We do not contend that cross-sectional saving patterns are identical across countries, and several institutional features distinguish Norway from other countries. The most important aspect is Norway's welfare state, which provides public pensions, generous insurance against income shocks, and free services such as education, healthcare, and elderly care, all backed by a large sovereign wealth fund. Norwegian households also hold a large share of housing in their portfolios. Our results should, therefore, be read as applying to an economy in which households face low risk and primarily hold housing in their portfolios, as in our quantitative model. The main general lessons then are that our empirical saving behavior across the wealth distribution fits remarkably well with predictions from standard economic theory, and that it is key to treat asset-price movements carefully when analyzing wealth accumulation.

We hope our findings will be useful building blocks for the large and growing literature on macroeconomic implications of micro-level saving behavior cited in the first paragraph. Many of the studies in this literature assume standard isoelastic preferences, and we find that this feature may suffice for explaining observed saving behavior when coupled with the additional assumptions that there are capital gains and that asset cashflows and labor incomes grow at the same rates (as on a balanced growth path). Notably, the existing literature typically abstracts from explicitly modeling asset prices and is therefore silent on the source of capital gains and how these affect saving behavior. Important exceptions are Greenwald et al. (2021) and Gomez and Gouin-Bonenfant (2024). Besides this theoretical literature, our findings are relevant for a nascent empirical literature in macroeconomics and inequality that emphasizes portfolio choice and asset-price changes (Feiveson and Sabelhaus, 2019; Kuhn, Schularick and Steins, 2020; Martínez-Toledano, 2023).

Our main empirical contribution is to provide systematic evidence on individual saving rates out of income over the entire wealth distribution.[5] A contemporaneous paper by

[5] Krueger, Mitman and Perri (2016), Saez and Zucman (2016), and Smith, Zidar and Zwick (2020) contain related evidence. However, none of these papers provide evidence on *individual*-specific saving rates like we do. Krueger, Mitman and Perri document consumption rates out of income (i.e., one minus saving rates) computed as total consumption expenditures for a specific wealth quintile divided by total income in that wealth quintile (they work with quintiles rather than percentiles due to the small sample size of their dataset, the U.S. Panel Study of Income Dynamics). Saez and Zucman and Smith, Zidar and Zwick provide evidence on "synthetic saving rates" that are computed by following percentile groups, rather than individuals, over time. Interestingly and in line with our results, Smith, Zidar and Zwick find that using their preferred capital gains estimates considerably attenuates the saving rate disparities of Saez and Zucman and increases the importance of asset-price growth for understanding wealth growth.

Straub (2018), Alan, Atalay and Crossley (2015), and Dynan, Skinner and Zeldes (2004) document how consumption and saving behavior vary with "permanent income" defined as the permanent component in labor income. Permanent income is not directly observable and must be estimated, typically by means of an instrumental variable strategy. We instead focus on wealth which is readily observable.

Bach, Calvet and Sodini (2018) examines saving behavior across the wealth distribution using Swedish administrative data, but they focus on the saving rate out of *wealth*, i.e., the saving-to-wealth ratio or wealth growth rate, whereas we focus on the saving rate out of *income*.[6] Given our goal of learning about theories of consumption-saving behavior, the saving rate out of income is the more informative object to study.[7]

Our paper is also related to the literature on the consumption effects of asset-price changes, in particular studies estimating how capital gains affect consumption.[8] Broadly consistent with our findings, Di Maggio, Kermani and Majlesi (2020) and Baker, Nagel and Wurgler (2007) argue that marginal propensities to consume (MPCs) out of capital gains are significantly smaller than MPCs out of dividend income.[9] Our evidence differs substantively though as we study households' *average* propensity to save out of income and capital gains rather than their *marginal* responses. As pinpointed by Aguiar, Bils and Boar (2025), theory has very different predictions for the two objects.

Section 2 presents what a series of theoretical benchmarks predict for saving rates across the wealth distribution. Section 3 describes our data and how we measure saving rates. Section 4 presents our empirical results. Section 5 explores how well a quantitative model with housing can account for our findings. Section 6 concludes.

2 Theoretical Benchmarks

While our main contribution is empirical, we first consider a series of simple theoretical benchmarks to fix ideas about saving rates across the wealth distribution and to guide our empirical analysis. We analyze a richer quantitative model in Section 5. Importantly, our theoretical benchmarks motivate "net" and "gross" saving concepts that we explore in the data. We show that simple models with isoelastic preferences predict that net saving rates, i.e., saving rates excluding capital gains, are approximately constant across the wealth distribution if asset prices appreciate while asset cashflows and labor incomes

[6] Bach, Calvet and Sodini (2018) also briefly discuss saving rates out of income in their Swedish data (Table II). Their findings for gross saving rates ("total saving to income") are similar to ours but their measure of the net saving rate ("active saving to income") turns negative for the top 40% of the population.

[7] In contrast, as we explain in the main body of the paper, standard consumption-saving models have no clear prediction for the saving-to-wealth ratio except that it should be mechanically decreasing with wealth.

[8] As opposed to the impact of the *level* of asset prices on the *level* of consumption or, equivalently, *changes* in asset prices on *changes* in consumption. Poterba (2000) reviews the literature on the consumption effects of changes in stock market wealth and Chodorow-Reich, Nenov and Simsek (2021), Paiella and Pistaferri (2017), and Christelis, Georgarakos and Jappelli (2015) are examples of more recent studies. For studies examining the effect of house price changes on consumer spending, both theoretically and empirically, see Berger et al. (2018) and Guren et al. (2021) among others.

[9] Our findings are also consistent with a household finance literature that finds substantial inertia in households' financial decisions (e.g., Calvet, Campbell and Sodini, 2009; Brunnermeier and Nagel, 2008).

grow at the same rates.[10]

2.1 Saving Decisions with Constant Asset Price

Households are infinitely lived and have standard isoelastic preferences

$$\int_0^\infty e^{-\rho t} u(c_t) dt, \quad u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{1}$$

where c_t is consumption. They receive a constant labor income w and can save in an asset a_t paying a constant interest rate r. Their budget constraint is $\dot{a}_t = w + ra_t - c_t$ and they face a natural borrowing constraint $a_t \geq -w/r$. Utility maximization yields a simple analytic solution for the optimal saving policy function $\dot{a}_t = s(a_t)$ (see Appendix A.1 for the proof):

$$s(a_t) = \frac{r-\rho}{\gamma}\left(a_t + \frac{w}{r}\right). \tag{2}$$

That is, households save (and consume) a constant fraction of their effective wealth $a_t + w/r$. It follows that the saving rate out of total income $y_t = w + ra_t$ is:

$$\frac{s_t}{y_t} = \frac{r-\rho}{\gamma r}. \tag{3}$$

Hence, the saving rate out of income is independent of wealth.[11] We show below that many other models inherit this property, at least approximately.

2.2 Saving Decisions with Changing Asset Prices and Labor Incomes

We next extend the simple benchmark model to feature a time-varying asset price and labor income. Households still have isoelastic preferences (1), and they receive labor income w_t. Different from above, they can buy and sell an asset k_t at a price p_t. This asset pays a dividend D_t and is the only saving vehicle available. The wage w_t, the price p_t, and

[10]Naturally, these models are too simple to account for all the facts about saving and inequality that one would want to match to fully address broad macro questions like how inequality affects prices, and how the two interact. We consider them as useful starting points for exploring how saving rates vary with wealth.

[11]Households save a constant fraction of their effective wealth and their income is the constant return to that effective wealth. Hence saving is also a constant fraction of income. Note that the saving rate is also constant over discrete time intervals (not just infinitesimal ones): using (2), we have $a_{t+1} - a_t = \int_0^1 \dot{a}_{t+s} ds = \frac{r-\rho}{\gamma} \int_0^1 \left(a_{t+s} + \frac{w}{r}\right) ds$ and hence saving $a_{t+1} - a_t$ is a constant fraction $\frac{r-\rho}{\gamma r}$ of income $\int_0^1 (w + ra_{t+s})\, ds$.

the dividend D_t follow exogenous and deterministic time paths. The budget constraint is

$$c_t + p_t \dot{k}_t = w_t + D_t k_t. \tag{4}$$

Households maximize (1) subject to (4). Note that we assume away any form of uncertainty because this complication is inessential for the points we want to make. We briefly discuss the role of uncertainty in Section 2.4.

The asset's return, which governs households' investment decisions, is the sum of dividend yield and capital gains:

$$r_t := \frac{D_t + \dot{p}_t}{p_t}. \tag{5}$$

The budget constraint (4) can be written in terms of the market value of wealth $a_t := p_t k_t$ as $\dot{a}_t = w_t + r_t a_t - c_t$. Unlike in the model in Section 2.1, the wage w_t and the return r_t are now potentially time-varying.

The asset-pricing literature distinguishes between different sources of asset-price changes, in particular between asset discount rates and cashflows. In this dichotomy, "discount rates" simply means any source of asset-price changes other than current and expected cashflows. To this end, consider the equation for the asset return (5), but adopt the perspective of the asset-pricing literature to treat required asset returns or discount rates $\{r_t\}_{t \geq 0}$ as a primitive and prices $\{p_t\}_{t \geq 0}$ as an outcome. One way of thinking about this is that, in equilibrium models, it is typically the asset returns that are pinned down, and in turn determine asset prices. Integrating (5) forward in time and assuming a no-bubble condition, the asset price equals the present discounted value of future dividend streams:[12]

$$p_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} D_s ds. \tag{6}$$

There are thus two sources of rising asset prices: rising dividends $\{D_s\}_{s \geq t}$ and declining discount rates $\{r_s\}_{s \geq t}$. Campbell and Shiller (1988) provide a log-linear accounting decomposition of (6) that makes this point precise.

Key Concepts: Net and Gross Saving. Before characterizing households' saving decisions, we define key concepts that we will use in our empirical analysis. These are "net" and "gross" saving and the corresponding net and gross saving rates. Their definitions follow from two different ways of treating capital gains when writing the budget

[12]The no-bubble condition is $\lim_{T \to \infty} e^{-\int_0^T r_s ds} p_T = 0$.

constraint "consumption plus saving equals income":

$$c + \overbrace{p\dot{k}}^{\text{net saving}} = \overbrace{w + Dk}^{\text{disposable income}}, \tag{7}$$

$$c + \underbrace{p\dot{k} + \dot{p}k}_{\text{gross saving}} = \underbrace{w + (D + \dot{p})k}_{\text{Haig-Simons income}}. \tag{8}$$

The difference between these two accounting identities is that the latter adds capital gains $\dot{p}k$ on both sides. Intuitively, since consumption in the two equations is the same, a difference in the saving definition necessarily implies a difference in the income definition. Formulation (7) features disposable income, whereas formulation (8) features "Haig-Simons" income which includes unrealized capital gains (von Schanz, 1896; Haig, 1921; Simons, 1938).[13] We define the "net saving rate" as the ratio of net saving to disposable income and the "gross saving rate" as the ratio of gross saving to Haig-Simons income.

Optimal Consumption and Saving with Changing Asset Prices and Labor Incomes. With these definitions in hand, Proposition 1 characterizes households' optimal choices in the face of changing asset prices.

Proposition 1. *Consider a household with current asset holdings k_t who maximizes the isoelastic utility function (1) subject to the budget constraint (4) with perfect foresight over $\{p_s, D_s\}_{s\geq t}$ and the resulting asset return $\{r_s\}_{s\geq t}$ defined in (5). Its optimal consumption and net saving are*

$$c_t = \xi_t(\omega_t + p_t k_t), \tag{9}$$

$$p_t\dot{k}_t = w_t + D_t k_t - \xi_t(\omega_t + p_t k_t), \tag{10}$$

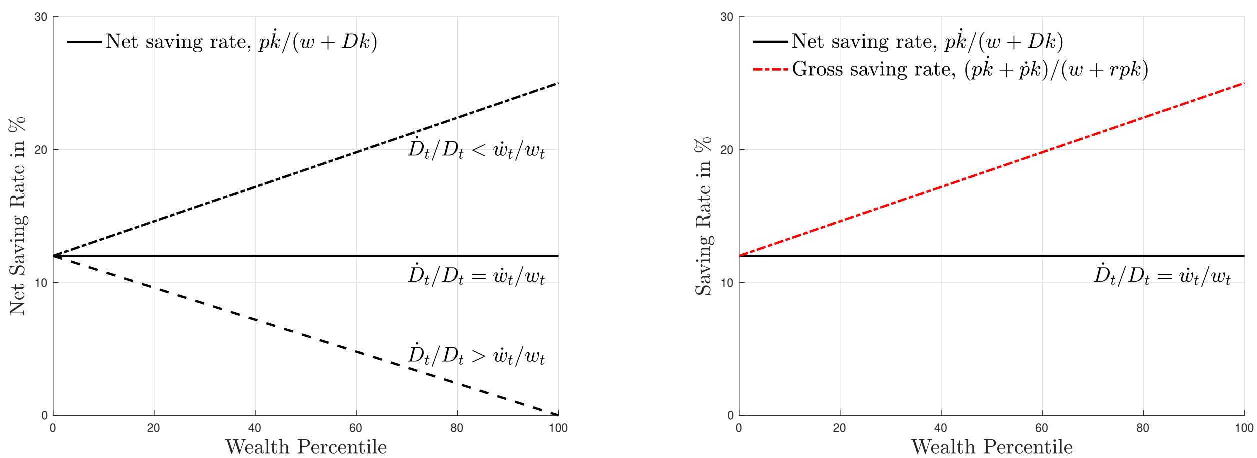
where $\omega_t \equiv \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds$ is "human wealth" (the present-discounted value of labor income) and $\xi_t = 1/\int_t^\infty e^{-\int_t^s (r_\tau - \frac{1}{\gamma}(r_\tau - \rho))d\tau} ds$.

If asset cashflows and labor income grow at the same (time-varying) rate, $\dot{D}_t/D_t = \dot{w}_t/w_t$ for all t, then the yield on financial wealth equals that on human wealth, $D_t/p_t = w_t/\omega_t$ for all t. From (9) and (10) this implies that consumption c_t and net saving $p_t\dot{k}_t$ are proportional to disposable income $w_t + D_t k_t$, and hence the net saving rate is constant across the wealth distribution. For example, this happens on a balanced growth path where labor income, dividends, and the asset price grow at the same constant rate, $\dot{w}_t/w_t = \dot{D}_t/D_t = \dot{p}_t/p_t = g$, and the asset return is constant r.

[13] This income definition forms the basis for the argument in the public finance literature that capital gains should be taxed on accrual rather than realization.

Positive capital gains, either due to rising cashflows $\{D_s\}_{s\geq t}$ or declining discount rates $\{r_s\}_{s\geq t}$, then imply a gross saving rate that rises with wealth so that wealthier households "save by holding." In the more general case where labor income and dividends do not grow at the same rate, higher dividend growth rotates the net saving rate down, while higher labor income growth rotates the net saving rate up across the wealth distribution.

Figure 2 illustrates Proposition 1. Figure 2(a) shows that the key determinant of how the net saving rate varies with wealth is how the growth rate of asset cashflows compares to that of labor income. The intuition is that asset cashflows are a more important income source at the top of the wealth distribution than at the bottom. When cashflows and labor incomes grow at identical rates, the net saving rate is constant and does not vary with wealth. Whenever this is the case and there are capital gains (either due to rising cashflows or declining discount rates), the gross saving rate mechanically rises with wealth simply because wealthier households experience higher capital gains, as in Figure 2(b).



(a) Labor income vs dividend growth

(b) Gross saving rate vs net saving rate

Notes: Panel (a) plots the net saving rate for different assumptions on the growth rates of asset cashflows $\dot{D}_t/D_t$ and labor income $\dot{w}_t/w_t$. Panel (b) plots the net and gross saving rates under the assumption that cashflows and labor income grow at equal rates $\dot{D}_t/D_t = \dot{w}_t/w_t$.

Figure 2: Saving rates with growing asset prices and labor incomes (Proposition 1)

To build intuition for this general result, consider the special case of a balanced growth path (BGP) on which all of labor income, dividend income and the asset price grow at the same constant rate, $\dot{w}_t/w_t = \dot{D}_t/D_t = \dot{p}_t/p_t = g$, rather than following general time paths as in the Proposition. On such a BGP the asset return r is constant, consisting of both a constant capital gains component $\dot{p}_t/p_t = g$ and a constant dividend yield $D_t/p_t = r - g$.

Human wealth equals $\omega_t = w_t/(r-g)$. Equation (9) then reads

$$c_t = \left(r - \frac{r-\rho}{\gamma}\right)(\omega_t + p_t k_t) = \left(r - \frac{r-\rho}{\gamma}\right)\frac{w_t + D_t k_t}{r-g}. \tag{11}$$

Therefore, consumption c_t is proportional to disposable income $w_t + D_t k_t$, and the net saving rate is constant and does not vary with wealth.

If, additionally, $r = \rho + \gamma g$ as on a BGP in the neoclassical growth model, then from (11) households simply hold on to their assets and consume their income flow $c_t = w_t + D_t k_t$. Hence the net saving rate is mechanically constant and equal to zero. What about the gross saving rate in this special case? Observe that the asset price grows at rate g, $\dot{p}_t/p_t = g$, and hence each household's wealth $a_t = p_t k_t$ also increases at rate g. The gross saving rate is then $g a_t/(w_t + r_t a_t)$ which increases with wealth a_t simply because wealthier households hold more assets that experience capital gains.

Examining (11), the key implication of equal cashflow and labor income growth rates that results in consumption c_t being proportional to disposable income $w_t + D_t k_t$ (and hence a constant net saving rate), is that financial and human wealth generate proportional income flows, $D_t = (r-g)p_t$ and $w_t = (r-g)\omega_t$. Whenever this property holds, the net saving rate is constant and does not vary with wealth.

In contrast, suppose that cashflows and the asset price grow at rate g but that labor income grows at a different rate $g_w \neq g$ so that human wealth equals $\omega_t = w_t/(r - g_w)$. The dividend yield equals $D_t/p_t = r-g$, while the yield on human wealth is $w_t/\omega_t = r-g_w \neq r-g$. Then the net saving rate is no longer constant, but varies with wealth. For example, if $g > g_w$, we have $p_t/D_t > \omega_t/w_t$ meaning that financial wealth is worth more than human wealth relative to the corresponding income flows. This generates a greater income effect for dividend income than for labor income, inducing households with more dividend income (those at the top of the wealth distribution) to consume more and save less. The net saving rate thus decreases with wealth if $g > g_w$, and it increases with wealth if $g < g_w$.

Gross saving rates add capital gains to the numerator and denominator of net saving rates. When capital gains are positive, households holding assets have a gross saving rate above the net saving rate. The higher the household's asset-to-income ratio, the higher the gap between gross and net saving, as in Figure 2(b).

The situation is similar outside a BGP when discount rates $\{r_s\}_{s\geq t}$ vary over time. In this case the general formulae (9) and (10) apply. When asset cashflows and labor income grow at the same rate, $\dot{D}_t/D_t = \dot{w}_t/w_t$, the net saving rate is constant across the wealth distribution. An important special case of Proposition 1 is when capital gains are exclusively due to declining discount rates while cashflow and labor income growth are zero, $\dot{D}_t/D_t = \dot{w}_t/w_t = 0$. As stated in the Proposition, capital gains due to declining discount rates leave the relationship between net saving rates and wealth flat. Gross saving rates then slope up, as wealthier households save more by holding on to appreciating assets as in Figure 2(b). Extensive work dating back to Shiller (1981) and Campbell and Shiller (1988) has documented the importance of discount rates movements for understanding asset-price movements. This implication of Proposition 1 may therefore be particularly important.[14]

2.3 Saving Decisions with Housing

Housing is a main asset for households, which provides direct utility and thus does not fit directly with our basic model.[15] We thererforе expand our analysis with housing.

We study a model with two assets, housing and bonds, based on Berger et al. (2018). Households maximize the same type of utility function as in (1), but now over an aggregator C of non-housing consumption c and housing services s:

$$\int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt, \qquad C_t = C(c_t, s_t), \tag{12}$$

where we assume $C(c_t, s_t)$ is homothetic. The budget constraint is

$$c_t + \dot{b}_t + p_t \dot{h}_t = w_t + r_t b_t + R_t(h_t - s_t), \tag{13}$$

where b are bonds, h is housing owned by the household, r is the return on bonds, p is the house price, and R is the rental price per unit of housing. The presence of a perfectly competitive rental market implies a separation between housing consumption and investment decisions. In combination with a no-arbitrage condition, it requires that the housing and bond returns are equal, $(R_t + \dot{p}_t)/p_t = r_t$, and implies that the portfolio allocation between b_t and h_t is indeterminate.

[14] Also see Campbell and Cochrane (1999) and Bansal and Yaron (2004), for example. While most of this research focuses on price variation at higher frequencies than in our setting, some studies consider longer-run price growth. For instance, van Binsbergen (2020) argues that international stock price growth over the past decades can be accounted for by declining interest rates.

[15] As Glaeser (2000) puts it: "A house is both an asset and a necessary outlay. [...] When my house rises in value, that may make me feel wealthier, but since I still need to consume housing there in the future, there is no sense in which I am actually any richer. And because house prices are themselves a major component of the cost of living, one cannot think of changes in housing costs in the same way as changes in the value of a stock market portfolio."

The house price is the net present value of future housing rents, R:

$$p_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} R_s ds. \tag{14}$$

We define a price index P_t so that total expenditure $R_t s_t + c_t = P_t C_t$. Consistent with homotheticity of C, the price index is a function of the house price p only: $P_t = P(p_t)$.

Optimal Consumption and Saving Decisions in a Housing Model. Our interest lies in how saving rates vary across the wealth distribution when households save in and obtain utility from housing assets. Proposition 2 summarizes the main results.

Proposition 2. *Consider a household with current asset holdings* (h_t, b_t) *who maximizes the homothetic utility function* (12) *subject to the budget constraint* (13) *with perfect foresight over* $\{p_s, R_s, r_s\}_{s\geq t}$ *and price index* $P_t = P(p_t)$. *Its optimal consumption and net saving are*

$$P_t C_t = \xi_t \left(\omega_t + b_t + p_t h_t\right), \tag{15}$$

$$\dot{b}_t + p_t \dot{h}_t = w_t + r_t b_t + R_t h_t - \xi_t \left(\omega_t + b_t + p_t h_t\right), \tag{16}$$

where $\omega_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds$ *is "human wealth" (the present-discounted value of labor income) and* $\xi_t = 1/\int_t^\infty \left(\frac{P_s}{P_t}\right)^{1-1/\gamma} e^{-\int_t^s (r_\tau - \frac{1}{\gamma}(r_\tau - \rho))d\tau} ds$.

If housing rents and labor income grow at the same (time-varying) rate, $\dot{R}_t/R_t = \dot{w}_t/w_t$ *for all t and* $b_t = 0$, *then consumption* $P_t C_t$ *and net saving* $\dot{b}_t + p_t \dot{h}_t$ *are proportional to disposable income* $w_t + r_t b_t + R_t h_t$ *and hence the net saving rate is constant across the wealth distribution. For example, this happens if labor income and housing rents grow at the same constant rate,* $\dot{R}_t/R_t = \dot{w}_t/w_t = g$.

Positive capital gains on housing, either due to rising housing rents $\{R_s\}_{s\geq t}$ *or declining discount rates* $\{r_s\}_{s\geq t}$, *then imply a gross saving rate that rises with wealth.*

Equation (16) in Proposition 2 is similar to equation (10) in Proposition 1, except that bond holdings and housing enter instead of capital. Absent bonds, this is a one-asset model with the same implications as in Proposition 1, only with h replacing k. If housing rents and labor income grow at identical rates, the net saving rate is constant and does not vary with wealth. If housing rents grow more (less) than labor income, then households with more housing wealth relative to income will save less (more). This would rotate the net saving rate down (up) across the wealth distribution to the extent that richer households hold more housing wealth relative to their income. Hence, introducing housing as a consumption good does not alter the main insight from Proposition 1.

Housing rent, R, plays the same role in Proposition 2 as dividends in Proposition 1. If house prices rise due to news of higher rents in the future, our proposition implies that households should dissave to consume out of these higher rents immediately. This implication might sound counter-intuitive, as higher rents imply higher living costs as well as higher income. However, households adapt to a higher rent by cutting housing in favor of non-housing consumption. This intratemporal substitution allows them to enjoy higher lifetime utility following capital gains due to higher rent growth. In contrast, if house prices increase due to lower interest rates r alone, then households cannot elevate their current consumption without sacrificing future utility – R is unchanged, so at no time will households gain from altering their consumption baskets.[16]

A further remark to Proposition 2 regards intertemporal substitution effects. These are contained in ξ_t. They depend not only on r, but also on p as house prices influence the cost of living. These intertemporal substitution effects are unimportant for saving across the wealth distribution as they affect all households equally due to the assumption of homothetic preferences.

Finally, note that the discussion above assumes $b = 0$ to isolate the role of housing. If instead households have non-zero bond holdings, then interest rate changes induce income effects, with a similar impact on saving across the wealth distribution as labor income, dividends, and housing rents. If all households hold the same ratio of bonds to their total income, then these income effects are irrelevant for the slope of net saving rates across the wealth distribution. If instead wealthier households hold more bonds relative to income and interest rates are expected to fall (rise), then these income effects rotate net saving rates up (down). The reason is that wealthier households in this scenario lose relatively more income from lower interest rates than poorer households do, and they therefore increase their net saving rate relative to poorer households.

2.4 Extensions

The frameworks above were deliberately simplified to introduce our two saving concepts and highlight our main theoretical predictions in Proposition 1. We next discuss how these predictions extend to more comprehensive models of household wealth accumulation.

The main extensions we consider are asset-price risk, income risk, correlation between age and earnings, and heterogeneity in type-dependent saving propensity (patience). Of these extensions, only the first involves changing asset prices. For the other extensions,

[16]Thus, the underlying logic of Proposition 2 resembles the citation from Glaeser (2000) in footnote 15 when capital gains are due to falling returns, but when house prices increase due to rental growth there is an important difference due to intratemporal substitution between housing and non-housing consumption.

our statements therefore refer to the net saving rate only, and we do not assess how these extensions might interact with each other or the drivers of asset prices. The models corresponding to these extensions are described in Appendix B.

Asset-Price Risk. In models with a stochastic asset price, the net saving rate is approximately flat across the wealth distribution when the average growth rate of asset cashflows equals that of labor income, regardless of the source of capital gains (as in Proposition 1). This statement is based on an extension of the model in Section 2.2 presented in Appendix B.1 in which the asset price is stochastic and driven by dividend and discount-rate shocks.

Income Risk and Borrowing Constraints. In models with income risk, cross-sectional correlation between labor income and wealth can induce an upward-sloping relation between saving rates and wealth. Conditional on households' income, the saving rate is approximately flat. Borrowing constraints increase saving rates for households near constraints in models with labor-income risk and borrowing constraints, as in Aiyagari (1994) and Huggett (1993). We analyze such a model in Appendix B.2 and show that it generates a policy function for saving which, conditional on income, is declining in wealth. The slope is distinct for households close to the borrowing constraint and almost flat for richer households. For households with high labor income realizations, the saving policy function shifts upward. If we instead consider the cross-sectional relationship between saving rates and wealth in the model's stationary distribution, without conditioning on income, the relationship is first decreasing and then increasing, due to two opposing forces. First, conditional on income, saving rates decrease with wealth. Second, saving rates increase with labor income, which is in turn positively correlated with wealth. In our empirical exercises, we will condition on labor income to account for these mechanisms.

Life-Cycle Earnings Profile. If income and wealth vary systematically over the life cycle, this correlation with age could mechanically induce an upward-sloping relationship between wealth and saving rates. For example, in a standard life-cycle model, households save little (or borrow, if possible) when they are young and have low income and wealth. As age and income rise, they begin to save and accumulate wealth. Consequently, life-cycle considerations introduce a cross-sectional correlation between saving rates and wealth because both are correlated with age and income. Conditional on age and income, however, the saving rate is approximately flat under the same conditions as in Proposition 1, or slightly decreasing for the reasons explained in the paragraph above, as noted by De Nardi and Fella (2017). Hence, in our empirical exercises, we will condition on age and labor income.

Heterogeneous Type-Dependent Saving Propensity. In models where households differ in their saving propensity due to heterogeneous patience or returns, cross-sectional correlation between saving inclination and wealth can induce an upward-sloping relation between saving rates and wealth. Such heterogeneity is a popular device for generating wealth dispersion in economic models (e.g., Krusell and Smith, 1998). Patient individuals save a significant amount and consequently accumulate high wealth, resulting in a positive cross-sectional correlation between wealth and saving rates. For our empirical purposes, saving rates in this case are "type-dependent" (Gabaix et al., 2016), and the cross-sectional wealth-saving correlation is explained by individual-fixed saving propensities. Conditional on households' saving propensities, the saving rate is approximately flat under the same conditions as in Proposition 1. Hence, in our empirical exercises, we will condition on past saving behavior.

Other Extensions. Several other mechanisms explored in the macroeconomic literature can shape how saving rates vary by wealth. One example is portfolio adjustment frictions that prevent asset holders from re-balancing their portfolios in response to price changes (see e.g. Grossman and Laroque, 1990; Kaplan and Violante, 2014; Kaplan, Moll and Violante, 2018; Guren et al., 2021; Fagereng, Gottlieb and Guiso, 2017). We introduce this mechanism in our quantitative housing model in Section 5.

Another mechanism is non-homotheticity, for example a warm-glow bequest motive with a different curvature than period utility or preferences depending directly on wealth (see e.g. Atkinson, 1971; Carroll, 1998; De Nardi, 2004; Straub, 2018; Saez and Stantcheva, 2018; Mian, Straub and Sufi, 2021). Non-homothetic preferences rotate the saving rates in Figure 2. For example, a luxury bequest motive makes the net saving rate more upward-sloping in wealth.

In addition, there are behavioral mechanisms, like limits to households' attention to asset prices or income growth (Sims, 2003; Gabaix, 2019), or "mental accounting" where different income sources are treated differently (Shefrin and Thaler, 1988; Baker, Nagel and Wurgler, 2007; Di Maggio, Kermani and Majlesi, 2020). These mechanisms do not generally rotate net saving rates in one unique direction.

Finally, there could be bounds to households' horizons. Our models assume an infinite horizon, which means households do not derive benefits from capital gains per se, but only from the income growth that accompanies these gains. If, instead, households plan to sell the assets they hold before they die (more generally, sell more than they buy), a lasting price hike would make them richer (Fagereng et al., 2025) and allow them to consume more. This mechanism would also rotate the saving rates in Figure 2, depending on how wealth correlates with horizons.

3 Institutional Setting, Data and Definitions

We use Norwegian administrative data. Norway levies a wealth tax on households, on top of income taxes.[17] For this reason, the government compiles a complete account of each household's income and balance sheet components down to the single asset category every year. Below, we describe these data in more detail and explain the saving rate measures we derive from them.

3.1 Institutional Setting

Like other Northern European countries, Norway has a generous welfare state that provides relatively rich insurance against income loss, illness, and other life events. Services such as child care, education, and health care are subsidized and provided at low or no cost. The welfare state is financed by taxes (the ratio of tax income to GDP was around 40% in our sample period) and returns from a sovereign wealth fund, the Norwegian Pension Fund Global, designed to smooth public spending and support future governments' ability to meet public pension entitlements.[18] In principle, all households in our sample thus hold a claim to a share of Norway's oil and gas wealth through their pension entitlements, which start from a minimum guaranteed level and accumulate with individual earnings. We pay specific attention to households' public pension claims in Section 4.5.

3.2 Data Sources

We link a set of Norwegian administrative registries, maintained by Statistics Norway and made available for research, with pseudonymized identifiers at the individual, household,

[17] Taxable wealth above an exemption threshold is taxed at a flat rate which has been around 1% during our sample period. The exemption threshold has increased over time. Toward the end of our sample, the threshold is approximately USD 260,000 (NOK 1.5 million) for a married couple (and half of that for a single person). Importantly for our purposes, assets are reported and recorded even if the household's total wealth falls short of the threshold. For several asset classes, values are discounted when measuring taxable wealth. We revert these discounted values to market values when computing household wealth.

[18] See https://www.wider.unu.edu/project/government-revenue-dataset for how the tax-to-GDP ratio has evolved over time. The Norwegian Pension Fund Global is part of Norway's fiscal mechanism for smoothing the use of national oil revenues. The mechanism postulates that the flow government income from oil activity is invested abroad, while an estimated normal return (3% currently, 4% previously) on the existing fund may be spent each year. Currently, the fund is worth more than three times Norwegian GDP.

and firm levels. We combine a rich longitudinal database covering every resident (containing socioeconomic information including sex, age, marital status, family links, educational attainment, and geographical identifiers), the individual tax registry, the Norwegian shareholder registry on listed and unlisted stock holdings, balance sheet data for listed and unlisted companies, and registries of housing transactions and ownership. Income flows are yearly, and assets are valued at the end of the year (December 31). Appendix C provides details on the data sets.

These data have several advantages. First, we observe wealth together with income at the household level for the entire population. Neither income nor wealth is top- or bottom-coded. The only sources of attrition are mortality and emigration out of Norway. Most of the data are reported electronically by third parties, which limits the scope for tax evasion and other sources of measurement error associated with self reporting.

In the data, households are defined as either single individuals, married couples, or cohabiting couples. Arguably, saving and consumption decisions are made at the household level, and we construct our variables accordingly. When individuals are single, they constitute their own household, and variables such as income, wealth, and saving are attributed directly. When individuals are married or cohabiting, we assume that economic resources are shared equally between the two adults; in these cases, we attribute half of the relevant household-level variables to each individual and retain both individuals in the dataset. This approach is consistent with the Norwegian practice of levying the wealth tax at the household level, as well as with legal norms requiring equal division of assets between spouses in the event of separation. It also allows us to follow individuals as they transition across different household structures over time. Thus, while we often refer to our observations as "households" throughout the paper, the unit of observation in the tables and figures is the individual.

3.3 Variable Definitions

Wealth. We define wealth as net worth, equal to the sum of safe assets, housing wealth, private businesses, public equity, and vehicles, minus debt.

"Safe assets" consists of the separate entries of bank deposits, cash holdings, informal loans, and bond holdings in the tax registry. The bond holdings primarily consist of bond mutual funds and money market funds, but also include direct holdings of government and corporate bonds.

"Housing wealth" includes the value of a household's principal residence, secondary homes and recreational estates. Traditionally, housing values in the tax records have been linked to original purchase prices which often deviate substantially from current market values. To improve on this, we utilize transaction-level data from the Norwegian Mapping Authority, combined with data from the land registration office and the population census, to regress the price per square meter on house characteristics. The predicted values from this procedure are then used as housing wealth over our sample period.[19]

"Private business" wealth is ownership of firms that are not publicly traded and hence do not have an observed market price. We apply the "assessed value" which private businesses (by law) must report to the tax authority. The tax authority distributes this value to the shareholders of the firm in proportion to ownership shares. The assessed value is derived from the book value, but omits intangibles (goodwill). We extract listed stocks and debt and place them, respectively, in the public equity and debt categories on the owners' balance sheets (see Appendix C.5). Medium- and large-sized firms (turnover above about USD 500,000) are required to have their balance sheet audited by an approved auditing entity. In addition, the tax authority has control routines to scrutinize reporting.

"Public equity" consists of publicly-listed stocks owned privately, stock funds, and publicly-listed stocks owned via private businesses. "Vehicles" are valued according to a fixed valuation schedule used by the tax authorities, which depends on the vehicle's brand, model, and age. "Debt" consists of mortgages, student loans, consumer debt, private loans, and debt in private businesses owned by the individual.

Assets directly held abroad are self reported, but the reporting categories are not consistent throughout our sample period and we therefore exclude directly held foreign assets from our analysis.[20] Households may also hold assets indirectly abroad through internationally diversified funds. Fagereng, Gottlieb and Guiso (2017) find that 28% of Norwegian stock funds' wealth under management was invested abroad. These investments are included in the stock fund category of our data.

Households also have pension claims that we do not observe and do not include in wealth. The Norwegian government provides a relatively generous pension scheme. Some workers additionally have private retirement accounts held by their employers, so as to top up the public pension plan which is capped. In extensions of our baseline analysis, we impute each household's public pension entitlement and include pensions

[19] The price per square meter is estimated from house characteristics such as the number of rooms and bathrooms, location, time periods, and their interactions using machine learning techniques. We employ an ensemble method that combines a random forest algorithm, a regression tree, and LASSO, as described in Mullainathan and Spiess (2017). Fagereng, Holm and Torstensen (2020) evaluate the predictive power of the method and find that 85% of the predicted housing values are within ±10% of the realized transaction value.

[20] In the raw data around 4% of households hold positive foreign wealth. Recently, Alstadsæter, Johannesen and Zucman (2019) have investigated the prevalence of hidden offshore wealth, finding that 10% of Scandinavian offshore wealth was declared to tax authorities in 2006, and that 77% of the offshore wealth belonged to the wealthiest 0.1%.

in measures of income, saving, and wealth – see Section 4.5 and Appendix C.7.

Income. Net (disposable) income is defined as the sum of labor income, business income, capital income, transfers, housing service flows, and retained earnings in corporate firms owned by the household, minus taxes paid by the household. Labor income, business income, capital income, transfers, and taxes are observed directly in the tax registries. We compute housing service flows using a rental equivalence approach that aims to value owner-occupied housing services as the rental income the homeowner could have received if the house had been let out.[21] Retained earnings are computed by linking the ownership registry with firm-level income statements, where retained earnings are defined as profits net of dividends paid. This measure includes retained earnings from directly held publicly listed firms, private businesses, as well as publicly listed firms that are owned indirectly through private business ownership structures.

Saving. As explained in Section 2.2, we distinguish between two saving concepts: *net* and *gross saving*. While we introduced these concepts in a simple model with only one asset, all our definitions extend to the case of multiple assets (and liabilities) – see Appendix C.2. We now describe how we measure our two saving concepts in the data.

We directly observe the year-to-year change in each household's net worth, and thus gross saving. The next step is to separate gross saving into net saving and capital gains. Our approach differs by asset class (for details see Appendix C.3).

To compute net saving in housing, we use the housing transactions data. Each year, we observe whether the household engaged in any housing transactions, and if it did, the transaction value. For households without housing transactions, we attribute changes in the value of housing to capital gains. For households with housing transactions, we compute net saving as the sum of all housing purchases minus sales throughout the year, and capital gains as the difference between gross and net saving. We do not include housing maintenance and improvements in net saving.[22]

For private businesses, our data provide assessed tax values that are related to book values, as explained above. If book values are not marked to market, changes in them should in principle be added to net saving. Recall that we take the private businesses'

[21] We follow Eika, Mogstad and Vestad (2020) and distribute the aggregate value of owner-occupied housing services from the national accounts across households in proportion to the value of their house, which implies a rent-to-value ratio that decreases from about 1.98% in 2006 to 1.78% in 2015.

[22] An alternative approach is to include maintenance and home improvements in net saving. In Appendix D, we impute maintenance and home improvements using data from the Norwegian Consumer Expenditure Survey and allocate these expenditures to net saving in proportion to home sizes, illustrating that this adjustment does not affect our main results.

holdings of traded stocks out of their balance sheet and attribute them to their owners. Year-to-year changes in the remaining private business values we add to net saving. This addition includes retained earnings in private businesses and any changes due to stock transactions by the owners. Since private businesses are most important at the top of the wealth distribution, this may lead to an underestimation of capital gains and, consequently, an underestimate of the gap between the gross and net saving rates among the very wealthiest.

Public equity consists of directly held stocks, indirectly held stocks via private businesses, and stock funds. For directly and indirectly held stocks, we use the Norwegian shareholder registry, which contains year-end information on holdings and prices of individual stocks at the security level. We thus compute a measure of capital gains for each individual stock. Net saving in a particular stock becomes the change in the value of holdings of that stock minus capital gains, adjusted for time aggregation.[23] Also, publicly listed firms save themselves in the form of retained earnings, and part of the observed capital gains are due to this saving. We allocate these retained earnings to the firms' owners in proportion to ownership shares and include them in net saving.[24] For stock funds, we observe the end-of-year value of stock fund holdings, and we use capital gains from the financial accounts to impute individual capital gains for stock funds.[25]

Households' holdings in bond and money market funds are concentrated in government bonds with maturities below one year and medium-term bonds (two to four years), according to data from the Norwegian Fund and Asset Management Association. Among these, short-term bonds make up approximately 70 percent. Since such bonds are unlikely to experience substantial price changes—and bond holdings account for only 0.4% of households' total portfolios over the sample period—we make the simplifying assumption that bonds do not experience capital gains.

The remaining assets and liabilities listed in Section 3.2 do not experience asset-price changes.[26] Lastly, we define gross income as the sum of net income and capital gains.

[23]Stock holdings are observed at the end of the year. We assume net purchases in any given stock happen in the middle of the year when computing net saving. Alternative assumptions (such as all purchases happen at the first or last day of the year, or throughout the year) yield similar results.

[24]Hence, like in Mian, Straub and Sufi (2020) we allocate corporate saving to households. Our approach differs from theirs as we use individual firm data on retained earnings and directly observed ownership shares to allocate this saving to the owners, whereas Mian, Straub and Sufi (2020) pursue a more aggregate approach using national accounts data. In contrast to them, we only allocate observed corporate saving by Norwegian firms and do not attempt to impute unobserved corporate saving by foreign firms.

[25]We use the capital appreciation of category "F520 Investment fund shares or units" in the national accounts to measure capital gains for stock funds. This category corresponds to stock mutual funds in the tax accounts (4.1.4 Investment funds stocks).

[26]Vehicles depreciate over time and we use the tax office's valuation schedule to infer depreciation rates (see Section 2.1). However, we count depreciation of an asset as a decline in the physical units of that asset,

With the above measures of net saving and capital gains within asset classes, we have all the necessary components to compute the net and gross saving rates in the data.

	Mean	SD	P10	Median	P90	Participation Rate
Age	50.00	17.62	27	48	75	
Male	0.49	0.50	0	0	1	
Years of education	12.18	3.20	8	12	16	
Less than high school	0.30	0.46	0	0	1	
High school	0.39	0.49	0	0	1	
College education	0.31	0.46	0	0	1	
Safe assets	46,594	193,899	894	15,195	111,020	0.99
Housing	462,303	573,979	0	355,981	978,749	0.80
Debt	142,722	1,818,209	0	63,811	296,877	0.85
Public equity	13,891	1,113,091	0	0	14,088	0.38
Private businesses	76,835	3,309,814	0	0	6,496	0.14
Vehicles	7,187	121,556	0	1,059	19,588	0.57
Net wealth	464,087	2,436,925	-6,793	298,823	990,334	
Net (disposable) income	67,743	582,495	31,028	56,133	96,437	
Gross (Haig-Simons) income	86,424	877,637	5,560	59,729	202,425	
Net saving	6,231	1,361,871	-30,954	2,237	33,234	
Gross saving	24,711	1,021,692	-78,676	6,487	138,857	

Notes: The table summarizes demographic characteristics and asset holdings of households pooling data for our sample period 2006-15 with a total of 33,422,884 individual-year observations. Values are in USD, 2011 prices - using the 2011-average exchange rate between USD and NOK in 2011 (NOK/USD = 5.607), throughout the paper. Safe assets are defined as deposits, bonds, and private loans. Debt is defined as the sum of debt held privately and debt held in private businesses. Public equity is the sum of directly held listed stocks, stock funds, and listed stocks held in private businesses. Private businesses is total assets in private businesses excluding listed stocks.

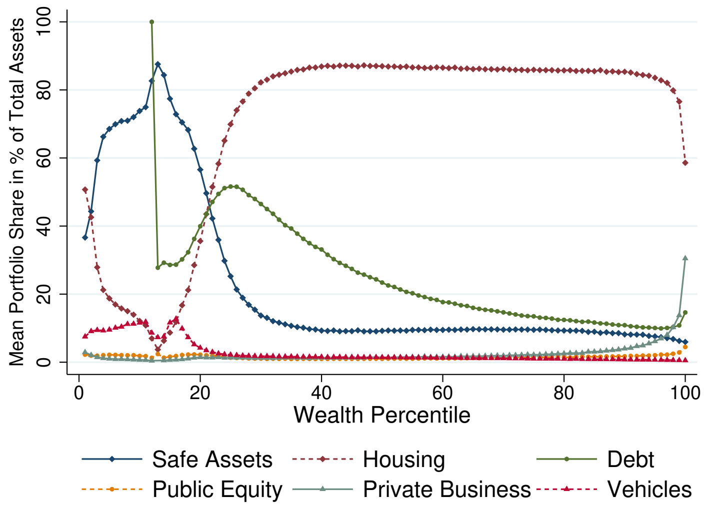
Table 1: Descriptive statistics.

3.4 Sample Restrictions and Descriptive Statistics

Sample Restrictions. For the majority of our analysis, we use data from 2006 to 2015.[27] The sample is restricted to households residing in Norway in which at least one adult

as opposed to merely a revaluation effect. That is, depreciation leads to negative net saving as opposed to a capital loss. Therefore, also for vehicles, net saving equals gross saving.

[27]The same combination of tax and transaction registry data is also available for 2005. However, we exclude 2005 from our main sample because a shareholder income tax reform (Alstadsæter and Fjærli, 2009; Holm et al., 2024) makes that year atypical along several dimensions, including a substantial increase in observed firm dividend payouts.



Notes: The figure displays the mean portfolio share in percent of total assets across the wealth distribution, by percentile group.

Figure 3: Asset class shares in household portfolios.

is above the age of 20, yielding nearly 38 million individual-year observations over the 10 years. We then exclude years in which household disposable income falls below the base amount defined in the Norwegian Social Insurance Scheme (equal to NOK 79,216, or about USD 13,700 in 2011), or where the household was newly formed or dissolved in that year. Applying these restrictions reduces the sample by approximately 12 percent, resulting in a baseline sample of 33.4 million individual-year observations.

Descriptive Statistics. Table 1 shows descriptive statistics from our sample. The table is split in three; demographics, wealth components, income and saving concepts.

Figure 3 plots the average holdings of different asset classes and debt relative to total assets across the wealth distribution. To the left are households with negative net worth and a debt-to-asset ratio above 100 percent. We note that the least wealthy households hold a high ratio of housing wealth relative to total assets. As we move rightward, the housing share declines until we reach households with approximately zero net worth around the 15th wealth percentile. These households hold almost no assets and no debt. From the 15th to approximately the 25th wealth percentile, the housing portfolio share grows rapidly, after which it flattens out around 80 percent. The portfolio share of safe assets is relatively high across the entire wealth distribution, while the portfolio shares of public equity and vehicles are small. The wealthiest households hold private businesses.

Appendix C.6 compares the data on household portfolios in Norway with the Wealth and Asset Survey for the U.K. and the Survey of Consumer Finances for the U.S. As in

Norway, a significant share of households are reported with negative wealth. Among households with positive wealth, the main difference is that housing constitutes a larger fraction of household portfolios in Norway, especially among the richest. In the interquartile range (from the 25th to 75th percentile) of the wealth distribution, housing constitutes a similar portfolio share in Norway compared to the U.S. and somewhat higher than in the U.K. As we move towards the top of the wealth distributions, however, the housing portfolio share remains high in Norway, whereas the portfolios in the top deciles of the U.K. and U.S. are shifted more towards other assets. For instance, among the top 1%, housing is 23% of net worth in the U.S. and 45% in Norway. The bigger share of housing in the portfolios is mainly due to the lower holding of stocks among Norwegian households and the lower values of vehicles (primarily cars). Cash holdings (including bank deposits and cash) and the share of the value of private businesses are relatively similar across the wealth distribution.

Year	**Labor Income Growth**	**Capital Income Growth**	**House Price Growth**	**Housing Rent Growth**	**Real Mortgage Rate**	**Real Deposit Rate**
2006	-0.36	-12.39	12.96	0.02	1.95	-0.29
2007	2.25	7.17	10.55	1.19	4.95	2.76
2008	0.87	1.03	-8.24	-0.79	3.52	1.21
2009	4.43	0.97	1.11	1.22	2.75	0.27
2010	1.88	0.92	6.15	0.47	2.07	-0.36
2011	3.38	2.08	8.56	0.96	3.45	1.06
2012	3.11	3.26	7.45	1.24	4.20	1.76
2013	0.68	3.36	1.03	0.92	2.62	0.13
2014	1.95	0.28	0.75	0.66	2.52	0.04
2015	2.47	4.85	8.11	0.63	1.78	-0.74
Average	2.07	1.15	4.84	0.65	2.98	0.58
Standard Deviation	1.41	5.21	6.22	0.64	1.03	1.09

Notes: The growth rates of labor and capital income are the real annual growth rates of per capita labor income (salary and transfer income net of taxes) and capital income (disposable income minus labor income) from Table 10799 at Statistics Norway, deflated by the consumer price index. Real house prices are computed using the nominal house prices in Oslo from Statistics Norway deflated by the consumer price index. Real rental prices are computed using data on rental prices of imputed housing services in the consumer price index, deflated by the consumer price index. Mortgage and deposit rates are the average interest rates on outstanding loans and deposits from Table 08175 at Statistics Norway, adjusted for inflation in the consumer price index. The standard deviation is computed as the standard deviation of the annual growth rates.

Table 2: Income growth, house prices, housing rent and interest rates from 2006 to 2015.

Income Growth and Price Changes. The theoretical benchmark models in Section 2 attribute a key role to the expected growth rates of labor and capital income (asset cash-flows). We therefore display their annual growth rates from the national accounts in our sample period in Table 2. During our sample period, average labor income growth was somewhat higher than capital income growth, suggesting, if it was expected, that the net saving rate should increase with wealth. However, the growth rate of capital income varied significantly, making it difficult to place much weight on this difference in averages.

	Wealth Percentile										
	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-99	Top 1%
Average	1.42	1.53	2.32	2.53	2.57	2.56	2.57	2.63	2.73	2.43	-0.25
Standard Deviation	2.36	1.57	1.52	1.12	1.04	1.24	1.29	1.46	2.01	4.12	21.49

Notes: The table displays the average growth rate of net income across wealth deciles for the years 2006-2015. The standard deviation is computed as the standard deviation of the annual growth rates in each decile.

Table 3: Net income growth by wealth deciles, 2006 to 2015.

Another set of descriptive statistics that the theory points to, is net (disposable) income growth rates across the wealth distribution, shown in Table 3. Average net income growth was almost identical across most of the wealth distribution, apart from at the very top where it was extremely volatile and in the bottom two deciles where it was lower. Hence, to the extent that realized income growth is informative about households' income expectations, the similarity in income growth rates would in our theoretical benchmark models imply a fairly flat net saving rate across most of the wealth distribution.

Table 2 also shows that house prices increased in all but one year, the financial crisis year of 2008, and on average, they increased significantly more than housing rent. Real interest rates on mortgages and deposits fluctuated and declined toward the end of our sample period. These data are key inputs to our quantitative model exercise in Section 5.

4 Results

This section provides our empirical findings. We first plot the median net and gross saving rates within wealth percentiles. The advantage of medians is that they are robust to outliers and therefore represent the typical behavior of individuals in a wealth percentile. Average saving rates are sensitive to outliers, in particular to households with income close to zero in any given period, so we study them separately.

We proceed by accounting for the key factors identified in Section 2.4: income, age, education, and heterogeneous saving propensities. We then study two alternative average measures, and decompose saving by asset class. Thereafter we consider the role of

Norway's public pensions, extending our definitions of saving and wealth to encompass them. Finally we provide additional evidence on saving excluding housing and saving over the life cycle.

4.1 Saving Rates across the Wealth Distribution

Figure 4(a) plots net and gross saving rates against percentiles of wealth. For every year we have computed the median saving rate within each year-specific wealth percentile. The two lines plot the average of these within-year medians over our sample period.[28] The shaded areas are 95% confidence intervals from quantile regressions.[29] To unveil the importance of capital gains for the pattern in panel (a), panel (b) shows how capital gains and households' asset holdings relative to income vary across the wealth distribution. Panel (b) uses the same time-aggregation approach as panel (a).

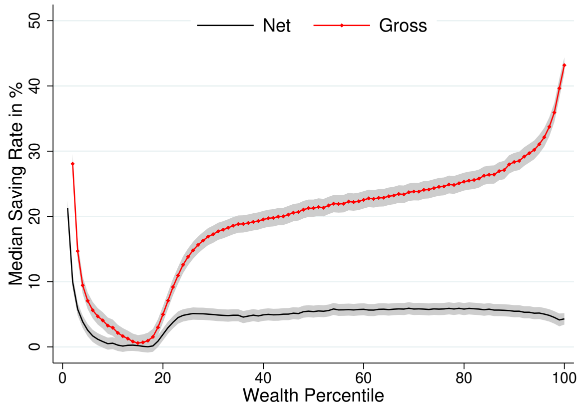
Consider first the *net* saving rate in panel (a). To the left, we see that it is decreasing with wealth among the poorest households. Then, after reaching a trough around the 15th wealth percentile, it rises up to the 25th wealth percentile. The trough lies where net worth is approximately zero (see Figure 3). The net saving rate flattens out at $6-8\%$ from the 25th wealth percentile and up.

Consider next the gross saving rate. As capital gains have been positive on average over the sample period, the gross saving rate necessarily lies above the net saving rate. Our interest lies in its shape. To the left, we see that it decreases just like the net saving rate. The net and gross saving rates follow each other down to the trough around zero net worth. Thereafter, the gross saving rate diverges by rising almost monotonically up to the very wealthiest households. The increase is steepest among households immediately above zero wealth, tapers off somewhat, and then picks up again among the $10-15\%$ wealthiest households. To simplify exposition, we coin this distinct pattern a "swoosh" (in analogy with the logo of a well-known footwear brand).

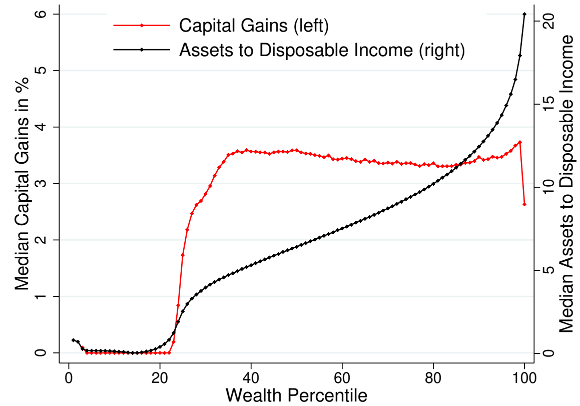
Instead of averaging across time, Figures 4(c) and (d) plot saving against wealth for each year separately. The lighter gray lines are constructed for each specific year. The thicker black lines are our baseline time averages. Notice that the scale on the vertical axes differs between the two panels.

[28] We present results from an alternative time-aggregation approach in Section 4.6.

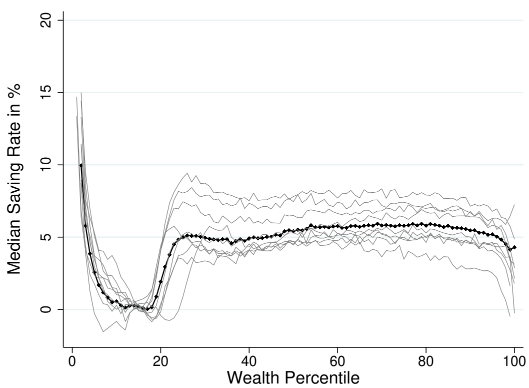
[29] The confidence bands in Figure 4(a) are computed as follows. For every year t we run a quantile regression of saving rates on indicators of wealth percentiles. Based on the resulting variance-covariance matrices, we compute the following time-averaged standard error for each percentile: $\widehat{\sigma}_p = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\widehat{\sigma}_{pt}^2}$, where T is the number of years and $\widehat{\sigma}_{pt}$ is the standard error of the regression coefficient on the indicator for percentile p in year t.



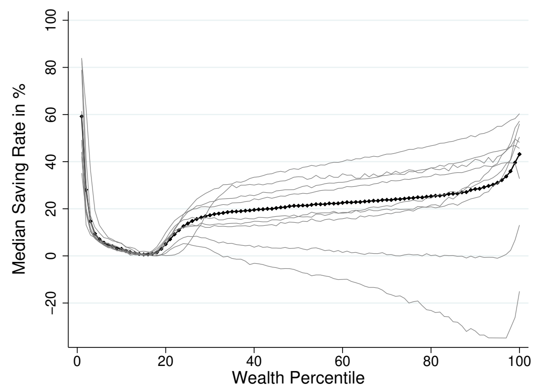
(a) Saving rates by wealth percentiles



(b) Average capital gains and asset-to-income ratio



(c) Net saving rates in different years



(d) Gross saving rates in different years

Notes: (a) displays the median saving rates within wealth percentile and year, averaged across all years. The grey areas show 95% confidence intervals computed from the variance-covariance matrix from quantile regressions of saving rates on indicators of wealth percentiles each year, averaged across all years (see footnote 29). (b) displays households' capital gains as a fraction of their assets, the persistent component of these capital gains, and median total assets as a share of disposable income. All variables in (a) and (b) are computed as the median within-wealth percentile and year, averaged across all years. (c) and (d) display median saving rates for each year of the sample (grey), together with the average across all years (black).

Figure 4: Saving rates across the wealth distribution.

Figures 4(c) and (d) illustrate the importance of capital gains for saving rates across the wealth distribution. In all years, including the two where asset prices fell for large parts of the wealth distribution (2008 and 2009), the relationship between wealth and net saving rates has the same shape. In particular, it is approximately flat from the 25th wealth percentile and out. In contrast, the shape of the gross saving rate varies considerably from year to year. It increases most distinctly with wealth in good years with high capital gains, and then *de*creases with wealth in bad years where large parts of the distribution experienced capital losses.

We began this paper by asking whether wealthier households have higher saving rates. Figure 4 suggests a nuanced answer. On the one hand, net saving rates are almost flat across the wealth distribution, especially among households with some positive net worth – which is to say that, no, wealthier households do *not* have higher saving rates in the traditional sense.[30] On the other hand, gross saving rates increase sharply with wealth – that is, even though the net saving rate is flat, wealthier households still accumulate more wealth through capital gains.

The proximate explanation for our observed patterns of saving rates has two logical components. First, wealthier households hold more assets relative to their income. Hence, their total asset holdings automatically appreciate by a higher dollar amount relative to their income (unless they systematically invest in assets with lower capital gains than poorer households do). In our data we can observe this component of the mechanism: Figure 4(b) visualizes how wealthier households hold more assets relative to their income, and capital gains rates do not systematically decline with wealth. Second, households do not systematically adjust their saving in response to asset-price changes. In the face of rising asset prices, households do not sell off assets to consume part of the capital gains. Conversely, in the face of falling asset prices, households do not purchase more assets to compensate for the paper loss in wealth. Instead, households tend to hold on to assets as their price fluctuates, leading to flat net saving rates every year and gross saving rates that increase or decrease depending on the sign of capital gains. Because capital gains are positive on average, we term this phenomenon "saving by holding."

Having documented our main empirical finding in Figure 4, we can now link it back to the theoretical discussion in Section 2. What explains the observed patterns for net and gross saving rates? In particular, what explains the remarkably flat net saving rate for households with positive net worth?

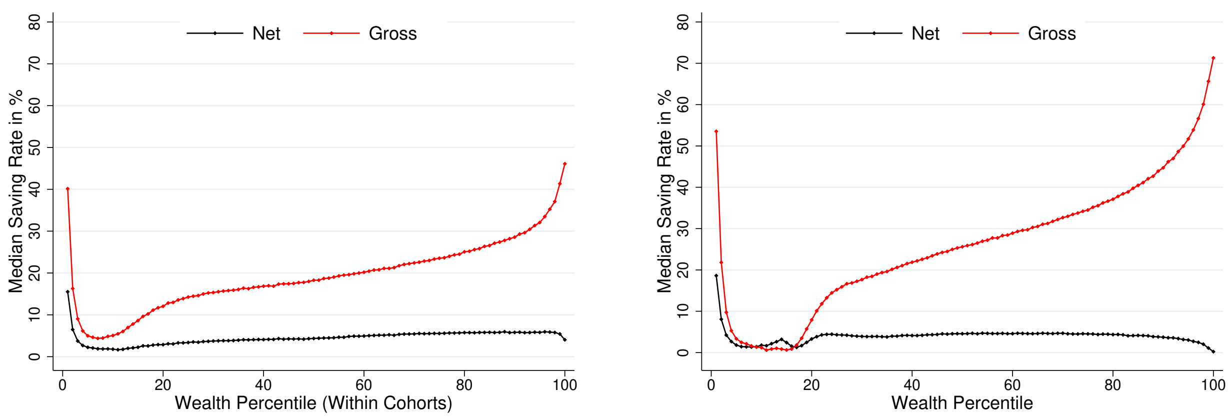
The theoretical predictions in Figure 2 suggest a simple explanation for the flat net saving rate: households behave according to a completely standard theory of household wealth accumulation with isoelastic preferences but one additional twist, namely that asset cashflows and labor incomes grow at the same rates. Wealthier households then optimally save by holding on to most of their assets experiencing capital gains.

[30]While we confidently claim that the net saving rate is approximately flat from the 25th to the 99th percentile of the wealth distribution, we caution about concluding regarding the wealthiest households (top 1%) because their net saving rates are extremely dispersed, as illustrated in Figure D.2(a)).

4.2 Controlling for Age, Earnings, and Education

As summarized in Section 2.4, common extensions of consumption-saving models imply that, in the cross-section, saving rates might correlate with wealth because both saving and wealth are related to labor income and age. Earnings risk may motivate households with high income realizations to save more, thereby inducing a positive correlation between saving rates and wealth. Life-cycle considerations may lead to a positive relationship between saving rates and wealth because both are correlated with age.[31] In this section, we therefore explore to what extent these factors can account for the patterns in Figure 4.

We utilize three approaches: (1) we compute median saving rates by *within-cohort* wealth percentiles, thereby eliminating the cross-sectional correlation between age and wealth, (2) we control parametrically for covariates in a quantile regression, and (3) we plot saving rates by wealth within age, earnings, and education groups.



(a) Saving rates by within-cohort wealth percentiles (b) Controlling for age, earnings and education

Notes: (a) displays the median saving rates by within-cohort wealth percentile (with cohorts defined as birth-years) and by year, averaged across all years. The net saving rate is defined in equation (7) as net saving divided by disposable income. The gross saving rate is defined in equation (8) as gross saving divided by gross income. (b) displays the coefficients on wealth percentiles (ϕ_p) from estimating equation (17) as a quantile regression for net and gross saving rates, respectively.

Figure 5: Saving rates adjusted for age, cohort, earnings and education.

Figure 5(a) displays the results when using within-cohort wealth percentiles. It breaks the mechanical link between wealth and age: because we rank households by the wealth percentiles for their respective cohort, every percentile now consists of an equal number of households from all age groups. As before, we compute the median saving rate within

[31] An alternative mechanism with similar implications is that personal experience with macroeconomic events shapes expectations. Malmendier and Nagel (2011) find evidence in this direction, documenting that individuals' experienced returns shape their portfolio choice and optimism. If wealth correlates with age, this could lead to a systematic relationship between expected capital gains and wealth.

each percentile each year, and then average these medians over time. The results are very similar to those in Figure 4. The main difference is that the net saving rate now increases slightly with wealth.

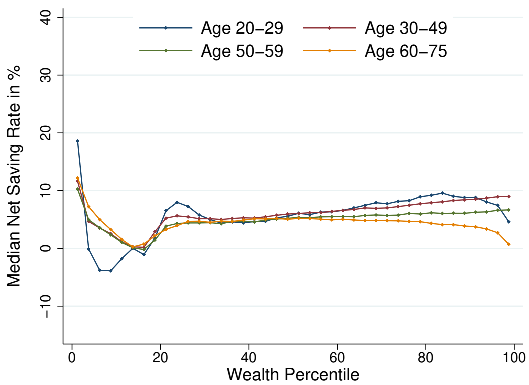
In approach (2), we estimate the quantile regression

$$\frac{s_{it}}{y_{it}} = \sum_{p=1}^{100} \phi_p D_{it,p} + f(x_{it}) + \tau_t + \varepsilon_{it}, \tag{17}$$

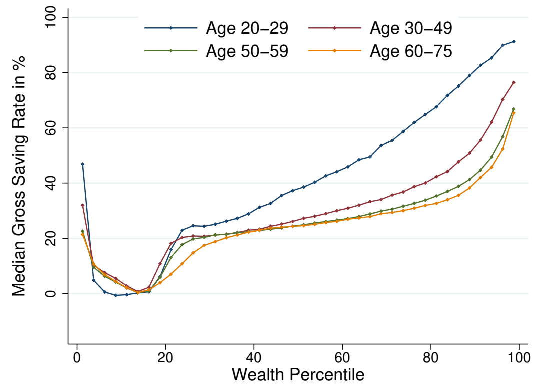
where s_{it}/y_{it} is the net or gross saving rate, $D_{it,p}$ is a dummy for being in wealth percentile p at the beginning of year t, ϕ_p is the corresponding regression coefficient for percentile p, x_{it} is a vector of control variables, τ_t are time-fixed effects, and ε_{it} is an error term. We specify $f(\cdot)$ to include a set of age, education (no high school, high school, college), and household labor income (in quintiles) indicator variables. We have considered several alternatives and the results are insensitive to the exact specification. Our interest lies in the ϕ_p coefficients displayed graphically in Figure 5(b): the median saving rates across the wealth distribution after controlling for age, earnings, and education.

Figure 5(b) shows that conditional on age, earnings and education, the relation between wealth and saving rates is qualitatively similar to its unconditional counterpart in Figure 4. Consistent with the discussion in Section 2.4, the net saving rate is slightly flatter after controlling for the three variables. This indicates that some of the correlation between saving rates and wealth is due to age, earnings, and education. The main takeaway is that also conditional on these observables, the saving graphs maintain their main characteristics: a "swoosh-shaped" gross saving rate, and a remarkably flat net saving rate among households with positive net worth.

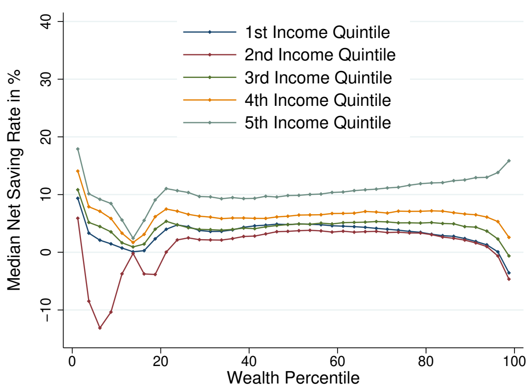
In approach (3), we produce our main graph within groups defined by age, earnings and education. For age we divide households into four strata (20-29, 30-49, 50-59, and 60-75 years), for earnings we stratify by earnings decile, and for education we stratify by no high school, high school, and college. Within each group, we then compute the median saving rate for different wealth percentiles, just like in Figure 4. Figure 6 shows the median net and gross saving rates within groups (the results for education are in Appendix Figure D.1). Overall, we observe decreasing and then relatively flat net saving rates, and "swooshes" for gross saving rates. In more detail, we observe some differences in saving slopes across age groups. Net saving rates slope upwards with wealth among younger households, and slope downward in the oldest age group. Gross saving is steeper among the youngest, reflecting that in this group asset-to-income ratios increase particularly steeply with wealth. Conditioning on earnings and education affects the level of saving rates, but only modestly affects how these covary with wealth.



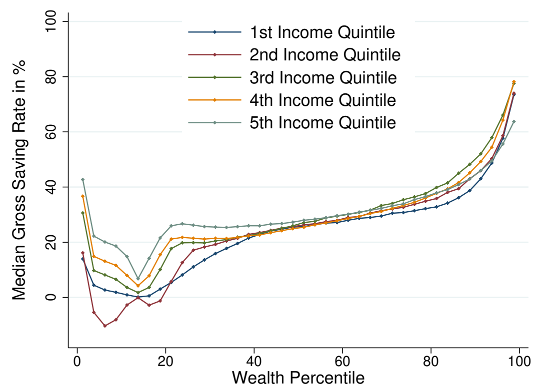
(a) Net saving rate by age group



(b) Gross saving rate by age group



(c) Net saving rate by earnings group



(d) Gross saving rate by earnings group

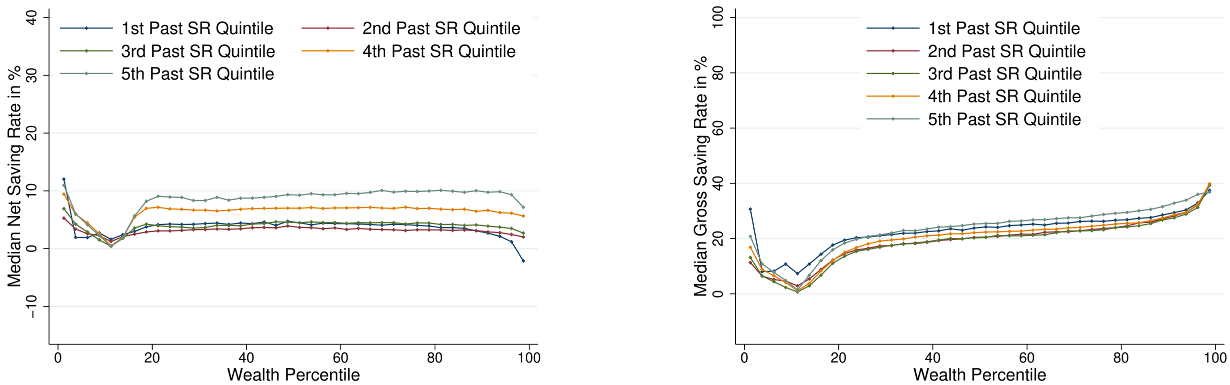
Notes: The figures display the median net saving rates (left column) and median gross saving rates (right column) within age, labor earnings, and education groups. All saving rates are computed as the median saving rate within wealth percentile and year, averaged across all years.

Figure 6: Saving rates across the wealth distribution by age and earnings.

4.3 Controlling for Heterogeneous Saving Propensities

As explained in Section 2.4, persistent differences in individual saving rates might result in a positive correlation between (gross) saving rates and wealth. For example, models with discount rate heterogeneity predict that relatively patient households have persistently higher saving rates and, over time, become wealthier than impatient ones. Are the saving patterns we observe rotated upward by a causal effect of saving propensity on wealth?

We exploit the panel dimension of our data to gauge the role of such "type dependence" (Gabaix et al., 2016) in driving our results. From year five of our sample period and on, we compute each household's average saving rate in the past (at least four years). Then, for each year we stratify households by their quintile of past saving rates. Within each past saving rate decile, we compute the median saving rate by wealth percentile. Finally,



(a) Net saving rate by past saving rate group (b) Gross saving rate by past saving rate group

Notes: The figures display the median net saving rates (left) and median gross saving rates (right) within percentiles of past saving rates. Conditional upon observing a household for at least 4 prior years, we compute each household's past gross saving rate for every year, and thereafter stratify each individual-year observation by average past gross saving rate. All variables are computed as the median within wealth percentile and year, averaged across all years.

Figure 7: Saving rates across the wealth distribution within deciles of past saving rates.

we construct our main graph by averaging the group-specific medians over the years.

Figure 7 displays the results. In short, our main findings remain the same also within groups defined by their saving history. It does not seem that our findings are driven by persistent heterogeneity in saving behavior.[32]

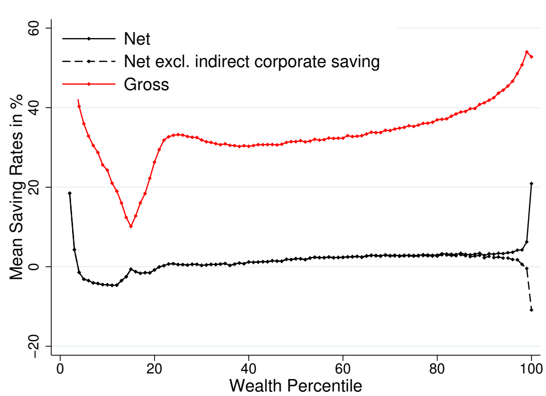
4.4 Mean Saving Rates and the Role of Infrequent Transactions

Our main graph depicts *median* saving rates within wealth percentiles. Medians have a straightforward interpretation and are robust to outliers, but they ignore infrequent transactions that might be large and important for saving when they take place. For example, if households buy a new home every five years, the median household would tend to be a non-transactor. This could be problematic for interpreting saving behavior if households let housing capital gains influence their saving only when these gains are realized. Studying median saving rates might then be misleading. We therefore explore mean saving rates instead of medians, and decompose saving by asset class to zoom in on the role of infrequent housing transactions.

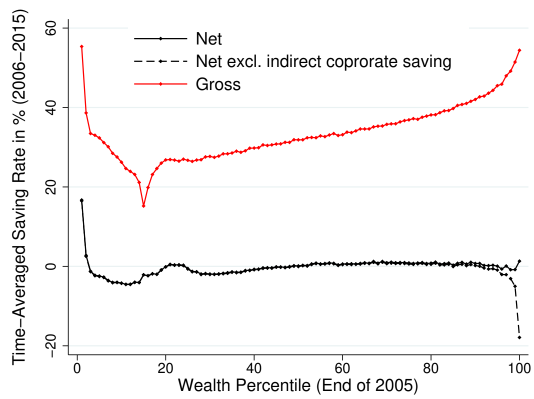
The simple arithmetic mean is problematic because it is sensitive to outliers and heavily influenced by observations with low income. For example, if a household consumes

[32]Moreover, the different saving curves lie close to each other and are not monotonically ranked by past saving rates. Hence, while propensities to save might well be heterogeneous, this force is not pervasive enough to alone drive persistent differences in saving rates in our data.

$100,000 in two consecutive periods with incomes \$50,000 and \$150,000 respectively, the unweighted mean saving rate would be -33.3% ($\frac{1}{2}\frac{50-100}{50} + \frac{1}{2}\frac{150-100}{150} = -33.3\%$) even though total net saving is zero. We therefore consider two alternative averages, mean saving divided by mean income and time-averaged saving rates.



(a) Mean saving divided by mean income



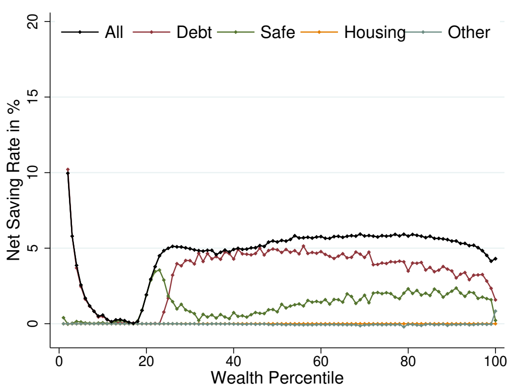
(b) Time-averaged saving rates

Notes: Panel (a) displays the ratios of mean saving to mean income within each wealth percentile and averaged across years. Panel (b) displays the mean time-averaged net and gross saving rate, computed as the average saving rate for each household across all years, with households sorted according to their position in the wealth distribution at the beginning of 2005. Before computing mean saving rates in panel (a), we trim the bottom and top 2.5% of saving and income within each wealth percentile. To compute time-averaged saving rates (b), we first calculate averages of individual saving rates across the years, trim these for outliers (bottom and top 2.5% within percentiles), and then compute averages of these net saving rates within percentiles.

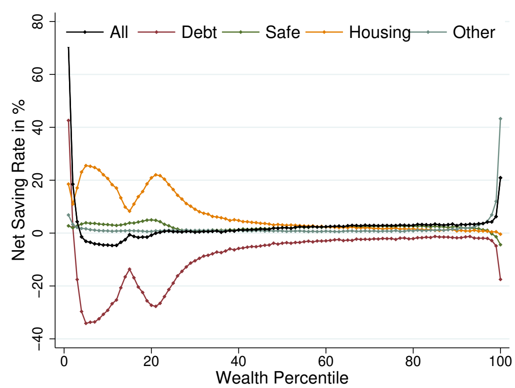
Figure 8: Mean saving rates.

We first compute the ratio of mean saving to mean income (equivalently, the income-weighted mean) within each percentile every year, and then compute the average of these means over time. The results are displayed in Figure 8(a). Both saving rates experience a level shift, the gross rate shifts up and the net rate shifts down (except for the top percentile). Below the very wealthiest, the mean net saving rate is slightly more upward-sloping than the median was, increasing from around half a percent in the 20-30th percentiles to around three percent in the 70-95th percentiles. At the very top of the wealth distribution, indirect saving via corporations owned (either directly or indirectly) by the top 1 percent becomes influential. Our baseline includes all retained earnings by these corporations as both net saving and net income for the owner, even if they are in listed or indirectly held corporations. In this case the mean net saving rate spikes up to 20 percent at the top. An alternative is to include only the retained earnings accrued by non-listed companies owned directly, and thus to a greater extent controlled, by the household. This gives the figure's dashed curve labelled "Net excluding indirect corporate saving" which drops down to minus 10 percent at the top. The main take-away is that the net saving rate still is relatively flat over most of the wealth distribution, while the gross saving rate increases distinctly, similar to our main findings.[33] The exception might be among the very wealthiest, where the measurement of mean net saving rates is sensitive to how one includes retained earnings.

Our second approach to averaging is to compute total saving over total income for each household in the entire sample period (2006-2015). This way, each household's saving rate includes the years in our sample when they transact. Moreover, by summing over all years rather than considering each year at a time, we avoid that single years with low income distort our results. Thereafter, we stratify households by their percentile in the wealth distribution at the end of 2005 and compute the income-weighted mean of households' time-averaged saving rate in each wealth percentile. Figure 8(b) presents the mean net and gross saving rates plotted against the 2005 wealth distribution on the horizontal axis. Again the net saving rate in the top percentile is sensitive to how we deal with retained earnings, but otherwise the pattern is the familiar one. With this approach too, the mean net saving rate is relatively flat while the mean gross saving rate increases distinctly with wealth.



(a) Median net saving rates decomposed



(b) Mean net saving rates decomposed

Notes: In panel (a), the line 'All' is computed as the averages within percentiles after taking medians within percentiles and years, as in Figure 4. For 'Debt', 'Safe', 'Housing', and 'Other', we use the saving behavior of the median household in the net saving distribution within percentiles and years, and then take averages across all years. In panel (b), before computing (income-weighted) mean saving rates, we trim the bottom and top 2.5% of saving and income within each wealth percentile.

Figure 9: Decomposed net saving rates.

[33]Using PSID data, Krueger, Mitman and Perri (2016, Table 2) document a sizable decline in expenditure as a share of disposable across wealth quintiles for the U.S., implying a mean net saving rate that is increasing with wealth. The net saving rate we find for Norway is considerably flatter.

Figure 9 decomposes saving into repayment of debt and accumulation of housing, safe and other assets (public equity, private business, vehicles). The figure plots net saving in each category relative to total net income. A positive value for debt means repayment, a positive number for assets means accumulation. The left plot considers medians as in our main graphs, whereas the right panel displays means computed as in Figure 8(a).

Figure 9(a) reflects the infrequency of housing transactions. Median savers do not buy or sell real estate, so the median net saving in housing is zero across the wealth distribution. We also see that median households' main form of net saving is debt repayment – the total net saving rate lies very close to the debt repayment curve. But a gap between net saving and repayment opens as we move rightward, accompanied by saving in safe assets.[34]

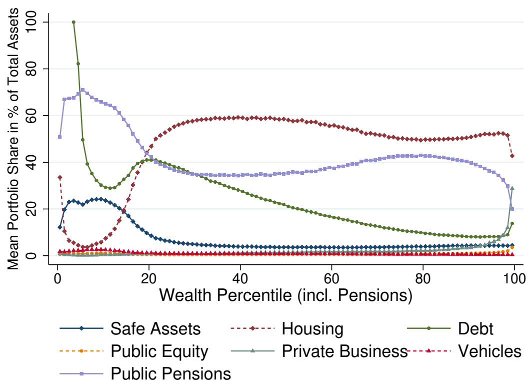
Figure 9(b) shows the decomposition when we switch to income-weighted means, which include transactors. Poorer households are net buyers of housing, richer ones are net sellers. This pattern feeds into debt accumulation, which is sizable for poorer households. In contrast to median debt repayment, means are negative in all wealth percentiles reflecting that total household debt increased in this time period. Compared to housing and debt, mean safe asset accumulation is negligible. Other assets are unimportant too, except at the top where public equity and private businesses are held and traded.

To sum up, medians ignore infrequent housing transactions, but this omission does not drive our main findings. When observations are averaged, we again find that the net saving rate is flat or moderately increasing, while the gross saving rate is distinctly upward-sloping.

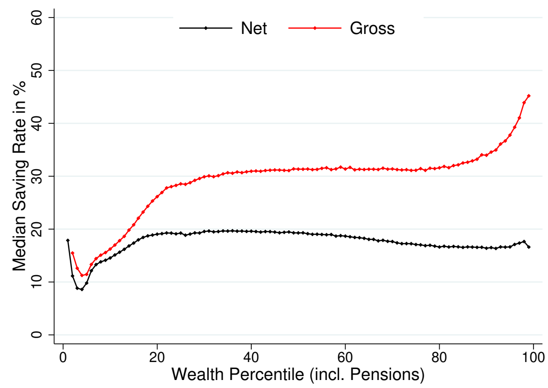
4.5 Including Public Pensions in Wealth and Saving

Norway has a public pension system with full coverage of all citizens, backed by a large sovereign wealth fund. Through their pension entitlements, each Norwegian thus holds a claim to the country's oil and gas wealth. Moreover, this claim grows with lifetime earnings. We therefore include the stock of accumulated pension entitlements in our wealth measure and the flow of additional entitlements accumulating over the year in our saving measures. We compute public pension saving as the change in the discounted value of future pension benefits, taking into account expected wage growth and expected life time of the household. We describe the details of the system and how we compute pension saving and pension wealth in Appendix C.7.

[34] Note that home equity lines of credit were widely offered by banks and common in Norway in our sample period, as they were rolled out from 2006 and onwards. In 2009 the use of HELOCs peaked and they accounted for 25 percent of all household debt, before gradually falling to about 15 percent in 2015. Hence, apart from among the highly leveraged households who by regulation cannot have HELOCs and must amortize, households are not forced to save by repaying their mortgage.



(a) Portfolio shares incl. pensions



(b) Saving incl. pensions by wealth incl. pensions

Notes: (a) displays portfolio shares including pension wealth, (b) displays the median saving rates when we include pension saving and income from the public pension system in the saving rate, and wealth includes public pension wealth. Appendix C.7 describes the public pension system and how we compute pension wealth, income, and saving. All variables are computed as the median within wealth percentile and year, averaged across all years.

Figure 10: Saving rates and wealth including public pensions.

Figure 10 presents the portfolio shares and saving rates across the wealth distribution when we include pensions.[35] Note that because the public pension system has complete coverage of the Norwegian population, the inclusion of public pensions increases wealth for everyone. Furthermore, the public pension system is relatively generous. Everyone is entitled to a minimum pension equal to approximately $20,000 per year after retirement. Since the discounting approximately cancels out the real wage growth, the value of this pension claim is approximately equal to $20,000 per year multiplied by 20 years, net of taxes on pension benefits (approximately 17%) and the probability of living until age 67. This implies that every 20-year old has a pension wealth approximately equal to $300,000. Almost no Norwegian has negative net wealth when we count their claims to public pensions. Figure 10 shows that pensions dominate other wealth components at the bottom of the wealth distribution. Furthermore, public pensions are a substantial share of net worth across the wealth distribution, ranging from around 90% in the bottom two deciles to about 20% in the top wealth percentile.[36]

Figure 10(b) shows saving rates across the wealth distribution when we include pen-

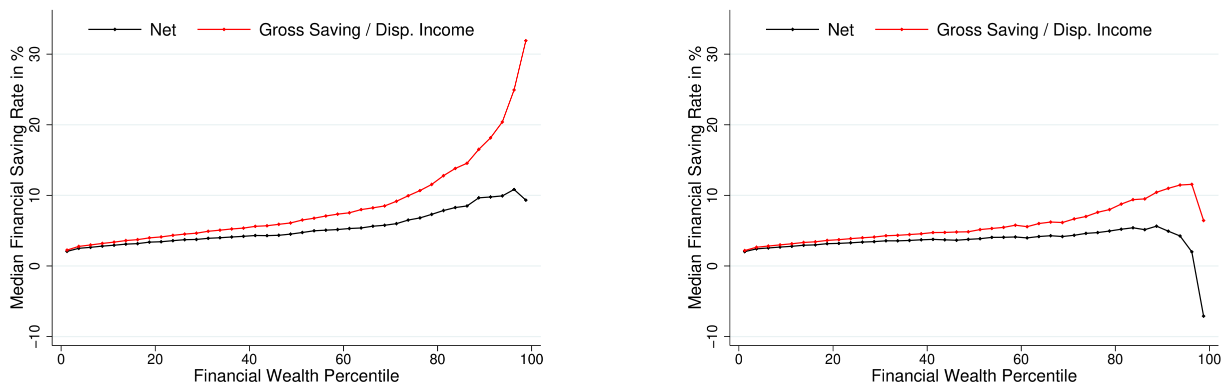
[35] We exclude retirees from the sample when we look at portfolios and saving rates including pensions. This is because retirees typically have approximately zero income when we include pension income (their income = pension benefits - reduction in pension wealth ≈ 0), making saving rates explosive.

[36] The 20% share at the top may seem large. However, most individuals in the top 1% wealth group have the maximum public pension, equal to about $55,000 per year and worth about $900,000. Taking into account that the threshold for entering the top 1% wealth is a little less than $2,000,000 (excluding pensions), 20% is a reasonable average.

sions. Saving rates are shifted up, reflecting that households' pension wealth tends to increase from one year to the other. There are two reasons why. People work that year and add to their pension wealth, and people live another year such that the probability of living until age 67 increases. Still, the main findings are similar to our baseline results: the net saving rate is relatively flat while gross saving increases with wealth.

4.6 Saving Rates Excluding Housing and over the Life Cycle

We have explored several extensions of our empirical investigation. We here present two particularly interesting ones, saving excluding housing and saving over the life cycle.[37]



(a) Financial saving rates, excluding 2008 and 2009

(b) Financial saving rates, all years

Notes: The figures display the median financial saving rate for the sample of households who hold at least USD 1,000 in public equity and at least 25% of their financial wealth in public equity (10.4% of benchmark sample). All the reported variables are medians within wealth percentile and year, averaged across years.

Figure 11: Relation between financial saving rates and financial wealth.

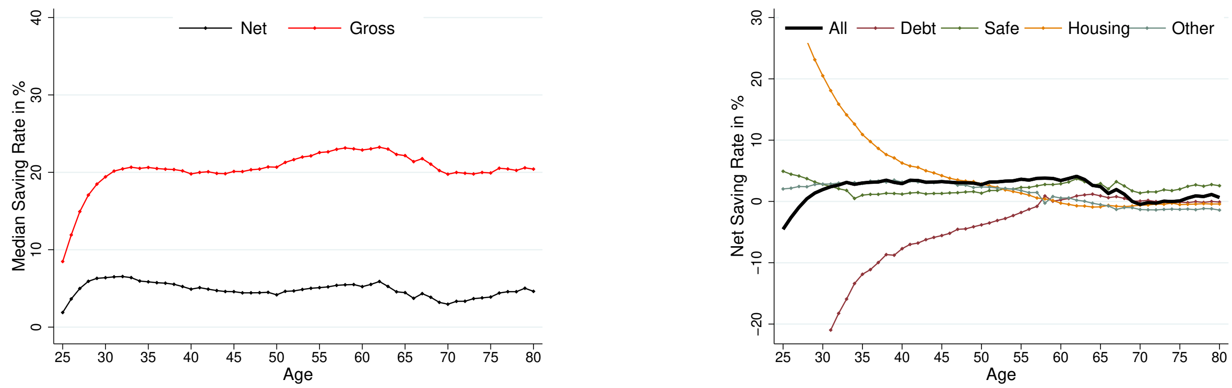
Financial Saving Rates with Housing Taken Out. Housing dominates Norwegians' portfolios, so our main graphs reflect that while house prices rise, households largely hold on to their residences and repay mortgages. Do we see similar patterns if housing is "taken out" of the picture? This question is hard to answer because the vast majority of

[37] Among the other extensions we have explored, are saving rates relative to wealth, i.e., the growth rate of wealth, as in Bach, Calvet and Sodini (2018). Like they find for Sweden, saving over wealth falls with wealth in Norway. We have also explored saving rates over the income distribution, which are increasing. For details, see Appendix D. There we also display saving rates when imputed home maintenance and improvements from the Norwegian Consumer Expenditure Survey are included in net saving. This exercise elevates net saving rates from around 7 to around 12 percent and thus closes some of the gap between the two saving concepts, but otherwise it does not affect results materially. We also zoom in on the top part of the wealth distribution in Appendix D.

Norwegians hold relatively few assets other than housing (see Figure 3). We address it by restricting attention to households with at least twenty-five percent of their non-housing assets held in stocks, directly or via funds. We sort households by financial wealth defined as net worth excluding housing and debt, and plot measures of their net and gross *financial* saving rates. The net financial saving rate is the change in financial wealth minus capital gains, divided by disposable income excluding housing income. Gross financial saving is the change in financial wealth. Because our sample includes the 2008-09 financial crisis, financial capital gains are strongly negative for a substantial part of this restricted sample. This leads to problems with computing gross saving *rates* (which divide by Haig-Simons income including capital gains), and we therefore instead report gross saving divided by non-housing income excluding capital gains which is always well-defined. Finally, recall that for stock-fund holdings, as opposed to directly held stocks, we must use an aggregate index to identify capital gains, as explained in Section 3.3. The split of gross saving into net saving and capital gains becomes considerably more sensitive to this source of measurement error now that we do an exercise where funds play a bigger role.[38]

Figure 11 plots these financial saving rates against financial wealth for this restricted sample. The sample is relatively small (10.4% of the baseline sample), so we report medians in 2.5% rather than 1% bins. Panel (a) reports results when we exclude the financial crisis years 2008-09 while panel (b) reports results for all years. In both panels the net financial saving rate is relatively flat. However, in contrast to Figure 4, the net saving rate is weakly upward-sloping with financial wealth until around the 90th percentile of financial wealth: from 2 percent at the bottom to 8 percent just below the top in panel (a), and from 2 percent to 5 percent in panel (b). Thereafter, the net saving rate falls sharply, and even turns negative at the very top, but here our measurement of capital gains is particularly problematic as explained above. We therefore conclude that, unlike in our baseline results that include all assets and liabilities, the net saving rate is weakly upward-sloping when we "take out" housing.

[38]The problems with measuring net saving in stock funds call for caution when interpreting the quantitative relationship between net financial saving and financial wealth. In particular, a non-zero correlation between individual-level stock fund capital gains and financial wealth would bias the relation between the net saving rate and financial wealth. For instance, suppose that true stock fund capital gains are positively correlated with financial wealth, i.e., wealthier individuals experience larger capital gains on stocks held in stock funds. Then, by using an aggregate index, we underestimate stock fund capital gains for wealthy individuals and hence attribute too large a share of their gross saving in stock funds to net saving rather than capital gains. This would then result in an upward bias of the relationship between the net saving rates and financial wealth. Conversely, a negative correlation between wealth and stock fund capital gains would result in a downward bias.



(a) Saving rates by age

(b) Mean net saving rates by age, decomposed

Notes: The figures show the median net and gross saving rates by age (a) and the decomposed income-weighted mean net saving rate by age (b). In panel (b), before computing mean saving rates, we trim the bottom and top 2.5% of saving and income within each wealth percentile.

Figure 12: Net and gross saving rates over the life cycle.

Saving Rates over the Life Cycle. How do saving rates vary over the life cycle? Section 4.2 provided a breakdown of saving rates plotted against wealth by age groups, but did not plot saving rates against age itself. Since saving behavior over the life cycle is key for many macroeconomic and distributional questions, we now provide such evidence. Figure 12(a) plots the median net and gross saving rates against age, whereas Figure 12(b) plots the (income-weighted) mean net saving rate against age and breaks it down by asset categories. Except for the youngest households, the median net saving rate in panel (a) is remarkably flat around 5%. There is more life-cycle variation in mean net saving rates, with some dissaving among the young and the old. Although the overall net saving rate is relatively constant across the life cycle, panel (b) reveals that net saving rates for individual asset classes vary systematically with age. Specifically, young households are net buyers of housing, but this positive net saving is offset by the young's uptake of debt; in contrast, old households are net sellers of housing. To complement Figure 12, Appendix Figures D.5 and D.6 plot saving rates by age within wealth percentiles (D.5) and life-cycle wealth profiles (D.6). The latter breaks these down by asset category, and also shows an important role for cohort effects in shaping cross-sectional wealth profiles by age.

5 Saving Behavior in a Quantitative Housing Model

Our main empirical finding of a flat net saving rate is qualitatively consistent with the theoretical benchmark models in Section 2. However, as we discuss below, these simple models miss other key empirical patterns. They also omit a number of features that are important in reality like housing adjustment costs. This leaves open the question: How well can a richer quantitative model calibrated to the Norwegian data account for the observed net and gross saving rates across the wealth distribution?

5.1 What the Simple Housing Model Misses and How to Fix it

As discussed in Section 2.3, a simple housing model with homothetic utility, a perfectly competitive rental market, and no adjustment costs can produce a flat net saving rate. However, it turns out that this model misses other key empirical patterns. Specifically, as we discuss in Appendix E.1, homothetic housing utility (which implies a constant expenditure share of housing services) creates a tension with the empirical housing-wealth-to-income ratio, which is strongly increasing with wealth. For example, if we take the housing-wealth-to-income ratio from the data,[39] this would imply that wealthy households rent *out* much of their housing wealth. This stands in contrast to only 4.2% of Norwegian households declaring rental income above USD 1,000. Alternatively, eliminating the perfectly competitive rental market while retaining homotheticity between housing and consumption would yield a flat housing-wealth-to-income ratio and, by implication, a flat gross saving rate, which is also inconsistent with the data.

How can we change the model to fit those salient features of the data and still generate a flat net saving rate? As we show in Appendix E.1, this can be achieved by considering a more general preference specification for housing: instead of the homothetic aggregator $C(c, s)$ in Section 2.3, non-homothetic preferences allows housing expenditure as a share of income to increase with wealth. The key theoretical insight is thus that this extended model improves over our simple baseline models by delivering not only our main finding of a flat net saving rate but also that richer households hold proportionately more housing than poorer ones as in the data. The remaining question is whether these findings survive once our model takes on board various other important features of reality.

5.2 The Quantitative Housing Model

We extend the housing model from Section 2.3 in four dimensions. First, as explained above, we allow for non-homothetic preferences for housing. Second, we explicitly include the three main assets held by Norwegian households: housing, mortgages, and liquid wealth. Third, the mortgage contract includes an amortization requirement and a

[39] Recall from Section 2.3 that, with a perfect rental market, the housing portfolio share is indeterminate.

down-payment constraint, in line with the lending regulation in Norway. Fourth, households can adjust their mortgage and housing only infrequently and subject to transaction costs. To simplify, we also eliminate the perfectly competitive housing rental market, but this is not essential.

A continuum of households differ in their holdings of liquid wealth b, mortgages m, housing h, and permanent non-capital income y. Each period they maximize the net present value of utility from consumption c and housing h

$$\int_0^\infty e^{-\rho t} \log C_t dt, \quad C_t = C(c_t, h_t), \tag{18}$$

where ρ is the discount rate. We specify a utility function that generates portfolio shares that may vary with wealth as in Wachter and Yogo (2010) and Cioffi (2021):

$$C(c,h) = \left(\alpha^{\frac{1}{\varepsilon_c}} c^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + (1-\alpha)^{\frac{1}{\varepsilon_c}} \frac{1-\varepsilon_c^{-1}}{1-\varepsilon_h^{-1}} h^{\frac{\varepsilon_h - 1}{\varepsilon_h}} \right)^{\frac{\varepsilon_c}{\varepsilon_c - 1}}, \tag{19}$$

where α is the weight on consumption, and ε_c and ε_h govern the degree of non-homotheticity.[40] If $\varepsilon_h > \varepsilon_c$, housing expenditure as a share of income increases with wealth, as in the data.

When the household cannot adjust, its constraints are

$$\dot{b}_t = y + r^b b_t - r^m m_t - \zeta m_t - c_t, \tag{20}$$

$$b_t \geq 0, \tag{21}$$

where r^b and r^m are interest rates on liquid wealth and mortgages, respectively, and ζ is the amortization requirement.

At Poisson rate λ households can adjust housing and mortgages. When they transact housing, they pay a fixed cost κ. Households who are allowed to adjust solve

$$\{b^*, m^*, h^*\} = \arg\max_{\{b', m', h'\} \in \Gamma(b,m,h)} V(b', m', h'), \tag{22}$$

where $\Gamma(b, m, h)$ is the set of (b', m', h') that satisfy the budget constraint, the downpayment constraint, and the housing choice. Formally, $\Gamma(b, m, h)$ has to satisfy

$$\begin{aligned} b' &\in [0, b + (m' - m) - p(h' - h) - \kappa \mathbb{1}_{h' \neq h}], \\ m' &\in [0, \theta p h'], \\ h' &\in \{h_0, h_1, ..., h_N\} \equiv \mathcal{H}. \end{aligned} \tag{23}$$

[40]This specification nests CES preferences ($\varepsilon_h = \varepsilon_c$) and Cobb-Douglas preferences ($\varepsilon_h = \varepsilon_c = 1$).

5.3 Quantification

Our goal is to compare the empirical net and gross saving rates across the wealth distribution with the corresponding saving rates implied by our model's policy functions.[41] We here explain how we map the model to the data, calibrate the model and conduct our experiment in the model.

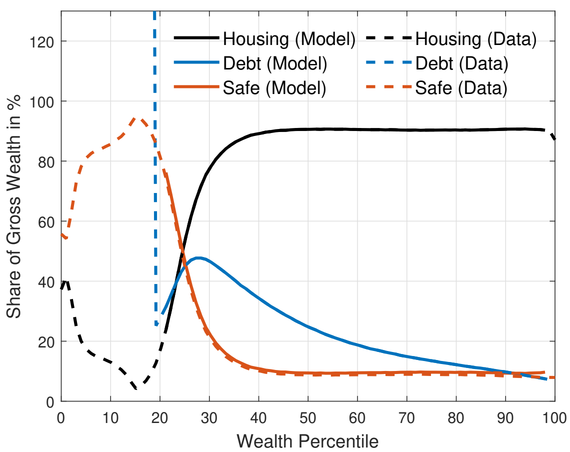
Mapping Model to Data. We map the wealth and portfolio composition in the data to the housing, mortgage, and liquid wealth states in the model by normalizing the model so that one unit equals the average annual income in the bottom decile of the income distribution. We define non-financial income y in the model as the sum of labor income and transfers, net of taxes.[42] Because our model does not feature all asset categories from the data (no public equity or private businesses), we map our model to empirical counterparts that are consistent with those in the model. Wealth is the sum of housing, deposits, bonds, and outstanding claims, minus mortgages; liquid wealth is the sum of deposits, bonds, and outstanding claims; net income is the sum of labor income, transfers, and interest income, minus interest costs and taxes; net saving is the sum of net housing transactions, debt repayment, and saving in deposits, bonds, and outstanding claims. These data adjustments imply that the moments in Figures 13 and 14 differ slightly from those in Section 4.

We compute model-implied saving behavior across the wealth distribution as follows. The model's saving policy function is defined for each point in the state space, i.e., for different combinations of housing, mortgages, and liquid wealth, and for different levels of permanent income. We compute the average level of housing, mortgages, and liquid wealth within every wealth percentile in the data. We then use the saving policy function to compute model-implied net saving at each of these points and for each permanent income level. Similarly, we compute net income and capital gains at the same points as well as the net and gross saving rates by dividing saving by income. Finally, we compute the weighted average of these saving rates across permanent income deciles within every wealth percentile.

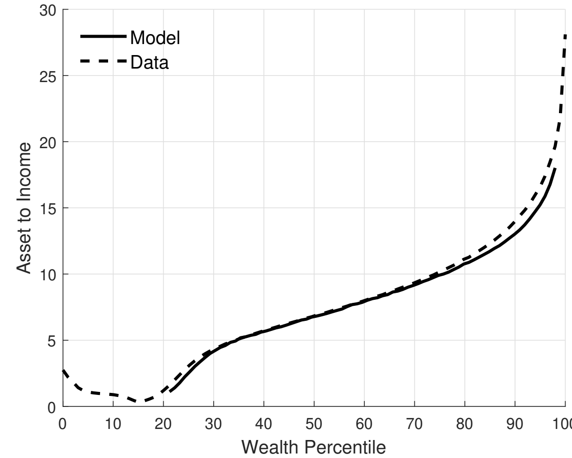
Figure 13 illustrates that our model, by design, matches the empirical portfolio shares (panel (a)) and asset-to-income ratios (panel (b)) across the wealth distribution.

[41] We do not match the wealth distribution implied by our model to that in the data and instead focus on the model's policy functions. Building a model that generates the same wealth distribution and portfolio choices as in the data is beyond the scope of this paper.

[42] We solve the model with a grid of ten income (y) states, one per decile of the empirical income distribution. The latter are average income within each decile every year, averaged over our sample period.



(a) Portfolio shares.



(b) Asset to income.

Notes: The housing and debt portfolio shares are housing/(housing + liquid wealth) and debt/(housing + liquid wealth), respectively. Asset-to-income is (housing + liquid wealth)/net income.

Figure 13: Mapping the model to data

Calibration. Table 4 summarizes the calibration. We externally calibrate a set of parameters to institutional features. The amortization rate ζ and the down-payment constraint θ are set in line with the Norwegian lending regulation. The transaction cost in housing is set equal to the average price of transacting in housing from NOU 2021:7 (2021, p. 18).[43] The probability of being allowed to transact λ is set to match the 9.2 percent share of households transacting in the housing market per year in our sample. The real interest rates on mortgages r^m and liquid wealth r^b are set to their sample averages in Table 2.

We calibrate the four remaining parameters (α, ε_c, ε_h, and ρ) to match the net saving rate across the wealth distribution in an initial scenario with constant prices, interest rates and income. We adjust ρ to ensure that the *level* of the net saving rate for non-adjusters matches the level of median net saving rates in the data.[44] As discussed in Section 5.1, non-homothetic preferences over housing ($\varepsilon_h > \varepsilon_c$) are key to simultaneously match flat net saving rates and increasing asset-to-income ratios across the wealth distribution. With CES preferences ($\varepsilon_h = \varepsilon_c$), households desire the same housing expenditure share and thus the same asset-to-income ratio irrespective of their wealth. Hence, if we assume CES preferences and study states such that asset-to-income ratios increase in wealth, the net saving rate will decline with wealth because richer households hold more housing than

[43]In NOU 2021:7 (2021), the average transaction price to the realtor is NOK 60,000 excl. value-added taxes in 2021, approximately equal to 60% of the average income in the lowest decile in the income distribution. Additionally, there are costs associated with relocating. We add NOK 40,000 excl. value-added taxes, such that the cost is set to 100% of the average income in the lowest decline in the income distribution.

[44]The discount rate ρ governs the substitution effect which affects the level of the net saving rate equally across the wealth distribution (the ξ_t in Proposition 1 and 2).

Parameter	Description	Value	Target
Externally set			
ζ	amortization rate	0.025	approx. amortization requirement in the Norwegian lending regulation
θ	down-payment constraint	0.850	15% down-payment requirement in the Norwegian lending regulation
κ	fixed adjustment cost	1.000	average price of housing sale (NOU 2021:7, 2021, p. 18)
λ	probability of transacting	0.092	share of households transacting in the housing market
r^m	real mortgage rate	0.0292	average real mortgage rate in Table 2
r^b	real rate on liquid wealth	0.0051	average real deposit rate in Table 2
Calibrated to match net saving rate under constant prices, interest rates and income.			
α	the consumption share	0.9864	
ε_c	preference parameter	1.1	
ε_h	preference parameter	19	
ρ	discount rate	0.0260	

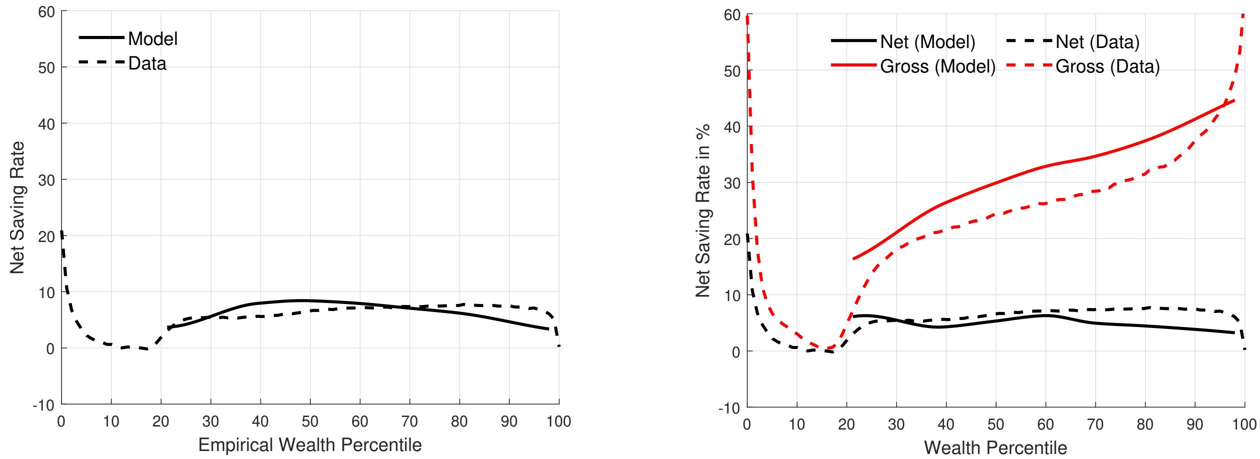
Table 4: Model calibration.

they want and therefore dissave relative to poorer households.[45]

Experiment. Initially, there are no capital gains, so the net saving rate equals the gross saving rate in the model. We construct a model experiment with capital gains by simulating the model responses to a surprise one-year shock (a so-called 'MIT'-shock) of the same magnitude as the empirically observed average growth rates for labor income, house prices, housing rent, and mortgage rates in Table 2. Our experiment thus provides implied behavior under the assumption that households are surprised by learning that labor income, house prices, and interest rates will change from this year to the next, expect these changes to be permanent, and do not expect any further changes.[46]

[45]Figure E.2 in Appendix E presents the net and gross saving rates across the wealth distribution with the calibration in Table 4 except that we assume Cobb-Douglas preferences ($\varepsilon_h = \varepsilon_c = 1$).

[46]We have also experimented with scenarios in which households learn in 2006 (the beginning of our sample) that house prices will gradually grow and mortgage rates will gradually fall until 2015 (the end of our sample) as in Table 2. These perfect-foresight scenarios generate similar saving patterns as those in Figures 14 (a) and (b). However, the unrealistic assumption that households know with certainty that house prices will grow by almost 5 percent annually over the next 10 years generates too much portfolio rebalancing from liquid wealth toward housing relative to the data. We therefore prefer the one-year shock experiment.



(a) Initial net saving rate.

(b) Saving rates in transition.

Notes: The figures display the model-implied saving rates compared with their data counterparts. The net and gross saving rates are net saving/net income and (net saving + capital gains in housing)/(net income + capital gains in housing), respectively. Panel (a) refers to the initial scenario with constant prices, interest rates, and income. Panel (b) compares the model's average saving rates with the empirical medians when prices, interest rates, and income grow in the model as in the data.

Figure 14: Saving behavior across the wealth distribution in the model.

5.4 Saving Behavior in the Model versus the Data

Figure 14 shows the saving rates across the wealth distribution in the initial scenario without growth, and the saving rates in our experiment with growth in prices, interest rates and income. The model is calibrated so that the net saving rate in our initial scenario, with zero growth in prices, interest rates, and income, matches the net saving rate in the data, as shown in panel (a). Figure 14(b) then displays the average net and gross saving rates from our experiment and compares these with the median saving rates in the data. The net saving rate remains approximately flat across the wealth distribution, consistent with our empirical findings, even when prices, interest rates, and income move as observed. In contrast, the gross saving rate grows across the wealth distribution as in the data. In sum, the model accurately accounts for the variation in the empirical saving rates between the 25th and 98th percentiles of the wealth distribution.

6 Conclusion

Little is known about the distribution of saving rates and how these vary across the wealth distribution. Using Norwegian administrative panel data on income and wealth, we document that how saving rates vary with wealth crucially depends on whether saving includes capital gains. Net or active saving rates are approximately constant or moderately increasing across the wealth distribution. In contrast, gross saving rates increase sharply with wealth so that wealthier households nevertheless accumulate more wealth through capital gains. These distinct relationships are present because richer households tend to hold on to assets experiencing capital gains – they "save by holding".

We show that these findings are broadly consistent with standard models of household wealth accumulation with isoelastic preferences if asset prices appreciate while asset cashflows and labor incomes grow at the same rates. A quantitative model with housing as a durable good, calibrated to the Norwegian data, can account for how net and gross saving rates vary across the wealth distribution under the observed trends in house prices, interest rates and non-capital income.

Capital gains are an important contributor to changes in aggregate saving and the wealth distribution. From our discussion, it might be tempting to conclude that capital gains induced by falling asset returns and the accompanying changes in wealth distribution are welfare-irrelevant valuation effects, or "paper gains." This conclusion would be incorrect: while the welfare implications of such asset-price changes are subtle, they are certainly not zero (e.g. Paish, 1940; Whalley, 1979; Auclert, 2019; Moll, 2020; Greenwald et al., 2021). In Fagereng et al. (2025) we conduct a thorough exploration of exactly these welfare effects, based on the insight that investment plans matter: whether investors benefit from rising asset prices depends not only on the assets they hold but whether they intend to buy, sell or keep their portfolios unchanged.

Norway has particularly high social insurance and public pensions backed by a large sovereign wealth fund. Our evidence should therefore be read as stemming from an economy with relatively low individual risk compared to for instance the U.S. or the U.K., and the exact slopes of saving profiles might well look different elsewhere. Still we believe that our findings have some broader implications. The main general insight is that both the macroeconomics and wealth inequality literature need to consider changing asset prices with care. Asset-price changes are not merely pesky "valuation effects" but a prevalent feature of the data: the majority of year-to-year movements in household wealth are due to asset-price changes (i.e., capital gains or losses) rather than asset purchases or sales (net saving or dissaving). Fortunately, a nascent literature in macroeconomics and the study of inequality is starting to emphasize portfolio choice and asset-price changes (Greenwald et al., 2021; Gomez and Gouin-Bonenfant, 2024; Feiveson and Sabelhaus, 2019; Kuhn, Schularick and Steins, 2020; Martínez-Toledano, 2023; Hubmer, Krusell and Smith, 2020; Gomez, 2025). We hope that our empirical findings and their theoretical interpretation will be useful building blocks for future contributions to this literature.

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Online Appendix for "Saving Behavior Across the Wealth Distributon: The Importance of Capital Gains"

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A Details and Proofs for the Theoretical Benchmarks

A.1 Constant Asset Prices (Section 2.1)

Derivation of Saving Policy Function (2). Households maximize (1) subject to $\dot{a}_t = w + ra_t - c_t$. The corresponding HJB equation is

$$\rho v(a) = \max_c \; u(c) + v'(a)(w + ra - c) \tag{A.1}$$

We solve this equation by using a guess-and-verify strategy: guess

$$v(a) = B\frac{(a + w/r)^{1-\gamma}}{1-\gamma}$$

which implies $v'(a) = B(a + w/r)^{-\gamma}$ and

$$c(a) = v'(a)^{-1/\gamma} = B^{-1/\gamma}(a + w/r) \tag{A.2}$$

Substituting into (A.1) and dividing by $(a + w/r)^{1-\gamma}$

$$\rho B\frac{1}{1-\gamma} = \frac{1}{1-\gamma}B^{-(1-\gamma)/\gamma} + Br - BB^{-1/\gamma}$$

Dividing by B and collecting terms we have $B^{-1/\gamma} = r - \frac{r-\rho}{\gamma}$ and hence from (A.2) we have

$$c(a) = \left(r - \frac{r-\rho}{\gamma}\right)\left(a + \frac{w}{r}\right) \tag{A.3}$$

Since the saving policy function is given by $s(a) = w + ra - c$, this yields (2).

A.2 Changing Asset Prices (Section 2.2)

Lemma A.1. *Consumption satisfies the usual Euler equation*

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\gamma}(r_t - \rho) \tag{A.4}$$

Proof of Lemma A.1: Consider the formulation in terms of net worth $a_t := p_t k_t$ with the budget constraint $\dot{a}_t = w_t + r_t a_t - c_t$ with r_t defined in (5). The current-value Hamiltonian is

$$\mathcal{H} = u(c_t) + \lambda_t[w_t + r_t a_t - c_t].$$

The first-order condition and law of motion of the co-state are

$$c_t^{-\gamma} = \lambda_t, \qquad \dot{\lambda}_t = (\rho - r_t)\lambda_t.$$

□

Proof of Proposition 1: The present-value budget constraint is

$$\int_t^\infty e^{-\int_t^s r_\tau d\tau} c_s ds = \omega_t + p_t k_t \quad \text{with} \quad \omega_t \equiv \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds \tag{A.5}$$

Combining (A.4) and (A.5), we get

$$c_t = \xi_t\left(\omega_t + p_t k_t\right) \quad \text{with} \quad \xi_t = \frac{1}{\int_t^\infty e^{-\int_t^s (r_\tau - \frac{1}{\gamma}(r_\tau - \rho))d\tau} ds} \tag{A.6}$$

which is (9) in the Proposition. The expression for net saving (10) follows from the budget constraint (4). This proves the first part of the Proposition.

Turning to the second part of the Proposition, the main step consists of proving the following Lemma:

Lemma A.2. *If asset cashflows and labor income grow at the same rate, $\dot{D}_t/D_t = \dot{w}_t/w_t$ for all t, then the yield on financial wealth equals that on human wealth $D_t/p_t = w_t/\omega_t$ for all t.*

Proof of Lemma A.2 Define the yields on financial and human wealth (the dividend yield and the wage-to-human-wealth ratio):

$$\theta_t^k \equiv \frac{D_t}{p_t} \quad \text{and} \quad \theta_t^w \equiv \frac{w_t}{\omega_t}. \tag{A.7}$$

The goal is to find a condition under which the two yields are equalized, $\theta_t^k = \theta_t^w$ for all t.

Human wealth $\omega_t \equiv \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds$ satisfies the differential equation

$$\dot{\omega}_t = r_t \omega_t - w_t \tag{A.8}$$

with terminal condition $\lim_{T\to\infty} e^{-\int_0^T r_s ds}\omega_T = 0$. Similarly, rearranging the definition for the asset return (5) we have

$$\dot{p}_t = r_t p_t - D_t \tag{A.9}$$

with terminal condition $\lim_{T\to\infty} e^{-\int_0^T r_s ds} p_T = 0$.

Using that $\dot{\theta}_t^k/\theta_t^k = \dot{D}_t/D_t - \dot{p}_t/p_t$ and $\dot{\theta}_t^w/\theta_t^w = \dot{w}_t/w_t - \dot{\omega}_t/\omega_t$, the two yields satisfy

$$\frac{\dot{\theta}_t^k}{\theta_t^k} = \frac{\dot{D}_t}{D_t} - r_t + \theta_t^k \tag{A.10}$$

$$\frac{\dot{\theta}_t^w}{\theta_t^w} = \frac{\dot{w}_t}{w_t} - r_t + \theta_t^w \tag{A.11}$$

with terminal conditions $\lim_{T\to\infty} e^{-\int_0^T r_s ds}(D_T/\theta_T^k) = 0$ and $\lim_{T\to\infty} e^{-\int_0^T r_s ds}(w_T/\theta_T^w) = 0$, where we have used (A.8) and (A.9).

If $\dot{D}_t/D_t = \dot{w}_t/w_t$, then θ_t^k and θ_t^w satisfy the same differential equations for all t. Similarly, $\dot{D}_t/D_t = \dot{w}_t/w_t$ for all t implies $D_T/D_0 = w_T/w_0 \equiv \kappa_T$ for some scaling factor $\kappa_T \geq 0$ so that the terminal conditions become

$$\lim_{T\to\infty} e^{-\int_0^T r_s ds}\left(\frac{\kappa_T}{\theta_T^k}\right) = 0 \quad \text{and} \quad \lim_{T\to\infty} e^{-\int_0^T r_s ds}\left(\frac{\kappa_T}{\theta_T^w}\right) = 0. \tag{A.12}$$

Hence, if $\dot{D}_t/D_t = \dot{w}_t/w_t$ for all t, then θ_t^k and θ_t^w satisfy the same differential equations (A.10) and (A.11) for all t as well as the same terminal conditions (A.12). Hence the solutions coincide so that $\theta_t^k = \theta_t^w$ for all t. □

The final step is to show that the net saving rate is constant across the wealth distribution whenever Lemma A.2 holds so that $D_t/p_t = w_t/\omega_t$ for all t. To see this note that $p_t = D_t/\theta_t$ and $\omega_t = w_t/\theta_t$ for the same factor θ_t. Plugging into (10) we have

$$p_t \dot{k}_t = \left(1 - \frac{\xi_t}{\theta_t}\right)(w_t + D_t k_t).$$

Therefore the net saving rate $p_t \dot{k}_t/(w_t + D_t k_t)$ is independent of k_t, i.e. constant across the wealth distribution. This concludes the proof of Proposition 1. □

A.3 Housing (Section 2.3)

Lemma A.3. *Consumption satisfies the usual Euler equation*

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\gamma}(r_t - \rho) - \frac{1}{\gamma}\frac{\dot{P}_t}{P_t} \tag{A.13}$$

Proof of Lemma A.3: Consider the formulation using the final consumption good C with the budget constraint $\dot{b}_t + p_t\dot{h}_t = w_t + r_t b_t + R_t h_t - P_t C_t$. The current-value Hamiltonian is

$$\mathcal{H} = u(C_t) + \lambda_t[w_t + r_t b_t + R_t h_t - P_t C_t].$$

The first-order condition and law of motion of the co-state are

$$\lambda_t = C_t^{-\gamma}/P_t, \qquad \dot{\lambda}_t = (\rho - r_t)\lambda_t. \qquad \square$$

Proof of Proposition 2: The present value budget constraint is

$$\int_t^\infty e^{-\int_t^s r_\tau d\tau} P_s C_s ds = \int_t^\infty e^{-\int_t^s r_\tau d\tau}(w_s + r_s b_t + R_s h_t) ds. \tag{A.14}$$

Combining (A.13) and (A.14), we get

$$P_t C_t = \xi_t \left(\int_t^\infty e^{-\int_t^s r_\tau d\tau}(w_s + r_s b_t + R_s h_t) ds \right) = \xi_t \left(\omega_t + b_t + p_t h_t\right) \tag{A.15}$$

where $\omega_t = \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds$, the last equality holds because $\int_t^\infty e^{-\int_t^s r_\tau d\tau} r_s ds = 1$, and

$$\xi_t = \frac{1}{\int_t^\infty \left(\frac{P_s}{P_t}\right)^{1-1/\gamma} e^{-\int_t^s (r_\tau - \frac{1}{\gamma}(r_\tau - \rho))d\tau} ds}. \tag{A.16}$$

Net saving is then

$$\dot{b}_t + p_t \dot{h}_t = w_t + r_t b_t + R_t h_t - \xi_t\left(\omega_t + b_t + p_t h_t\right). \tag{A.17}$$

This proves the first part of the proposition. Turning to the second part of the Proposition, the main step consists of proving the following Lemma:

Lemma A.4. *If housing income and labor income grow at the same rate, $\dot{R}_t/R_t = \dot{r}_t/r_t = \dot{w}_t/w_t$ for all t and $b_t = 0$, then the yield on wealth and human wealth are the same for all t.*

Proof of Lemma A.4 Define the yields on housing, bonds, and human wealth:

$$\theta_t^h \equiv \frac{R_t}{p_t} \quad \text{and} \quad \theta_t^w \equiv \frac{w_t}{\omega_t}. \tag{A.18}$$

The goal is to find a condition under which the yields are equalized, $\theta_t^h = \theta_t^w$ for all t. Rearranging the no-arbitrage condition for bond and housing returns $(R_t + \dot{p}_t)/p_t = r_t$ we have

$$\dot{p}_t = r_t p_t - R_t \tag{A.19}$$

with terminal condition $\lim_{T\to\infty} e^{-\int_0^T r_s ds} p_T = 0$. Similarly, human wealth $\omega_t \equiv \int_t^\infty e^{-\int_t^s r_\tau d\tau} w_s ds$ satisfies the differential equation

$$\dot{\omega}_t = r_t \omega_t - w_t \tag{A.20}$$

with terminal condition $\lim_{T\to\infty} e^{-\int_0^T r_s ds} \omega_T = 0$. Using that $\dot{\theta}_t^h/\theta_t^h = \dot{R}_t/R_t - \dot{p}_t/p_t$ and $\dot{\theta}_t^w/\theta_t^w = \dot{w}_t/w_t - \dot{\omega}_t/\omega_t$, the yields satisfy

$$\frac{\dot{\theta}_t^h}{\theta_t^h} = \frac{\dot{R}_t}{R_t} - r_t + \theta_t^h \quad \text{and} \quad \frac{\dot{\theta}_t^w}{\theta_t^w} = \frac{\dot{w}_t}{w_t} - r_t + \theta_t^w \tag{A.21}$$

with terminal conditions $\lim_{T\to\infty} e^{-\int_0^T r_s ds}(R_T/\theta_T^h) = 0$ and $\lim_{T\to\infty} e^{-\int_0^T r_s ds}(w_T/\theta_T^w) = 0$.

If $\dot{R}_t/R_t = \dot{w}_t/w_t$, then θ_t^h and θ_t^w satisfy the same differential equations for all t. Similarly, $\dot{R}_t/R_t = \dot{w}_t/w_t$ for all t implies $R_T/R_0 = w_T/w_0 \equiv \kappa_T$ for some scaling factor $\kappa_T \geq 0$ so that the terminal conditions become

$$\lim_{T\to\infty} e^{-\int_0^T r_s ds}\left(\frac{\kappa_T}{\theta_T^h}\right) = 0 \quad \text{and} \quad \lim_{T\to\infty} e^{-\int_0^T r_s ds}\left(\frac{\kappa_T}{\theta_T^w}\right) = 0. \tag{A.22}$$

Hence, if $\dot{R}_t/R_t = \dot{w}_t/w_t$ for all t, then θ_t^h and θ_t^w satisfy the same differential equations (A.21) for all t as well as the same terminal conditions (A.22). Hence, the solutions coincide so that $\theta_t^h = \theta_t^w$ for all t. □

The final step is to show that the net saving rate is constant across the wealth distribution whenever Lemma A.4 holds so that $R_t/p_t = w_t/\omega_t$ for all t. To see this, note that $p_t = R_t/\theta_t$ and $\omega_t = w_t/\theta_t$ for the same factor θ_t. Plugging into (16) we have

$$\dot{b}_t + p_t\dot{h}_t = \left(1 - \frac{\xi_t}{\theta_t}\right)(w_t + R_t h_t).$$

Therefore, the net saving rate $(\dot{b}_t + p_t\dot{h}_t)/(w_t + R_t h_t)$ is independent of wealth, i.e., constant across the wealth distribution. This concludes the proof of Proposition 2. □

B Extensions (Section 2.4)

B.1 Asset-Price Risk

We extend the model in Section 2.2 to feature a risky asset price and a constant growth rate of labor income. Individuals maximize

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt, \tag{B.1}$$

subject to the budget constraint (4). Writing the budget constraint (4) in terms of market wealth $a_t := p_t k_t$ we have

$$da_t = (w_t - c_t)\, dt + a_t dR_t, \tag{B.2}$$

$$dw_t = g w_t dt \tag{B.3}$$

where $dR_t := \frac{D_t dt + dp_t}{p_t}$ is the asset's instantaneous return over a time interval of length dt, and g is the constant growth rate of labor income. We adopt the perspective of the asset-pricing literature, treating the required asset return as a primitive and the price as an outcome. To this end, define the expected return

$$r_t dt := \mathbb{E}_t[dR_t] = \frac{D_t dt + \mathbb{E}[dp_t]}{p_t}. \tag{B.4}$$

Note that, in general, the expected return r_t itself follows a stochastic process. Rearranging (B.4) as $r_t p_t = D_t + \mathbb{E}_t[dp_t]/dt$ and integrating forward in time and assuming a no-bubble condition, the asset price is given by the analogue of (6):

$$p_t = \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} D_s \right] ds, \tag{B.5}$$

where the expectation $\mathbb{E}_t$ is taken over future realizations of returns and dividends $\{r_s, D_s\}_{s \geq t}$, conditional on information available at time t. Because the return and dividend are stochastic, so is the asset price p_t.

From (B.5), there are two reasons why asset prices may fluctuate: due to fluctuations in cashflows (dividends) or returns. We analyze households' optimal saving decisions in two polar cases. We first consider the case in which all asset-price movements are due to changing cashflows, and then turn to the opposite case of asset-price fluctuations that are due to discount rate changes.

B.1.1 Asset-price changes accounted for by changing dividends

In this case we assume that the expected return is constant over time, $r_t = r$ for all t, and that asset-price fluctuations are driven by dividend fluctuations. In particular, assume that dividends follow a geometric Brownian motion with drift μ and variance σ^2:

$$dD_t = \mu D_t dt + \sigma D_t dW_t. \tag{B.6}$$

With a constant discount rate, $r_t = r$ for all t, (B.5) simplifies to

$$p_t = \mathbb{E}_t \int_t^\infty e^{-r(s-t)} D_s ds. \tag{B.7}$$

Because dividend follows a geometric Brownian motion, $\mathbb{E}_t[D_s] = D_t e^{\mu(s-t)}$ for $s \geq t$, and

$$p_t = \int_t^\infty e^{-r(s-t)} \mathbb{E}_t[D_s] ds = D_t \int_t^\infty e^{-(r-\mu)(s-t)} ds = \frac{D_t}{r-\mu}. \tag{B.8}$$

Hence, the asset price p_t also evolves according to a geometric Brownian motion

$$dp_t = \mu p_t dt + \sigma p_t dW_t \tag{B.9}$$

with the same drift and variance as dividends. Furthermore, the dividend yield $D_t/p_t = r - \mu$ is constant over time and the instantaneous return dR_t is given by

$$dR_t = \frac{dp_t + D_t dt}{p_t} = r dt + \sigma dW_t.$$

Substituting into (B.2) the budget constraint becomes

$$da_t = (w_t + r a_t - c_t)\, dt + \sigma a_t dW_t. \tag{B.10}$$

Households therefore maximize (B.1) subject to (B.10) and (B.3). When returns feature a stochastic component $\sigma > 0$ and labor income is positive $w_t > 0$, it is no longer possible to solve the consumption-saving problem analytically. However, Proposition B.3 (which is proved in Appendix B.1.3 below) derives a useful approximation to the value and policy function under the assumption that labor income is small and constant, $w_t = w \approx 0$:

Proposition B.3. *The value function, consumption policy function, and net saving function that* *maximize* (B.1) *subject to* (B.10) *and* (B.3) *satisfy*

$$\begin{aligned} v(a) &= \bar{c}^{-\gamma}\left(\frac{a^{1-\gamma}}{1-\gamma} + \frac{wa^{-\gamma}}{r-g-\gamma\sigma^2}\right) + O(w^2) \\ c(a) &= \bar{c}\left(a + \frac{w}{r-g-\gamma\sigma^2}\right) + O(w^2) \\ p_t\dot{k}_t &\approx D_t k_t + w_t - \bar{c}\left(\frac{D_t k_t}{r-\mu} + \frac{w_t}{r-g-\gamma\sigma^2}\right) \\ \bar{c} &:= \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \end{aligned} \tag{B.11}$$

The approximation is exact if either $\sigma^2 = 0$ *– so that we are back in the case with a deterministic return – or* $w = 0$ *– in which case the problem is a simplified version of Merton (1969) without portfolio choice.*

From (B.11), the net saving rate is approximately flat across the wealth distribution if dividends grow on average at the same rate as wages ($\mu \approx g$) and σ is small, similar to the result in Proposition 1.

B.1.2 Asset-price changes accounted for by discount rate shocks

In this case, we first assume that dividends and wage income are constant over time, $D_t = D$ and $w_t = w$ for all t, and that asset-price fluctuations are entirely driven by fluctuations in expected returns. In particular, assume that expected returns evolve according to a diffusion process

$$dr_t = \mu_r(r_t)dt + \sigma_r(r_t)dW_t. \tag{B.12}$$

With a constant dividend, $D_t = D$ for all t, (B.5) simplifies to

$$p_t = D\mathbb{E}_t\left[\int_t^\infty e^{-\int_t^s r_\tau d\tau}\right]ds. \tag{B.13}$$

and a particular choice of the diffusion process (B.12) implies a stochastic process for the asset price p_t. Given this price process, we then compute the instantaneous return dR_t defined in (B.2) and solve the household's problem. For any expected-return process, we already know from (B.4) that $\mathbb{E}_t[dR_t] = r_t dt$ or, equivalently,

$$dR_t = r_t dt + \tilde{\sigma}(r_t)dW_t$$

for some function $\tilde{\sigma}$. Substituting into (B.2) the budget constraint becomes

$$da_t = (w_t + r_t a_t - c_t)\, dt + \tilde{\sigma}(r_t) a_t dW_t. \tag{B.14}$$

Households therefore maximize (B.1) subject to (B.14).

Following the same steps as in the proof of Proposition 1, one can show that optimal consumption satisfies the analogue of (A.6):

$$c_t = \xi_t \left(a_t + w\mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \right] \right) \quad \text{with} \quad \xi_t = \frac{1}{\int_t^\infty e^{-\int_t^s (r_\tau - \frac{1}{\gamma}(r_\tau - \rho))d\tau} ds}, \tag{B.15}$$

i.e., households optimally consume a fraction ξ_t of their lifetime income. Using the expression for the asset price (B.13) to replace for $a_t = p_t k_t$

$$c_t = \xi_t \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \right] (w + Dk_t)$$

$$p_t \dot{k}_t = \phi(w + Dk_t), \qquad \phi_t := 1 - \xi_t \mathbb{E}_t \left[\int_t^\infty e^{-\int_t^s r_\tau d\tau} ds \right]$$

Therefore, when asset-price movements are entirely accounted for by expected-return movements, the net saving rate is flat across the wealth distribution.

B.1.3 Proof of Proposition B.3

Households maximize (B.1) subject to (B.10) and (B.3), yielding the HJB equation

$$\rho v(a, w) = \max_c \; u(c) + \partial_a v(a, w)(w + ra - c) + \frac{\sigma^2 a^2}{2} \partial_{aa} v(a, w) + gw\partial_w v(a, w). \tag{B.16}$$

We provide an approximate solution relying on a perturbation method around small w. We first solve (B.16) analytically when $w = 0$ and then perturb v that solves (B.16) for $w > 0$ around the solution for the case $w = 0$. Our argument makes use of a standard perturbation method as in Fleming (1971), Judd (1996), Anderson, Hansen and Sargent (2012) and Kasa and Lei (2018).

Step 1: Closed form with $w = 0$. For the first step, consider the value function for $w = 0$ which we denote by $v_0(a)$. It solves:

$$\rho v_0(a) = \max_c \; u(c) + v_0'(a)(ra - c) + \frac{\sigma^2 a^2}{2} v_0''(a) \tag{B.17}$$

This is the HJB equation for a simplified version without portfolio choice of the problem analyzed by Merton (1969) and it is well-known to have a closed-form solution.

Lemma B.5. *The value function and consumption policy function with* $w = 0$ *are*

$$v_0(a) = B_0^{-\gamma}\frac{a^{1-\gamma}}{1-\gamma}, \quad c_0(a) = B_0 a, \quad B_0 := \frac{\rho - r}{\gamma} + r + (1-\gamma)\frac{\sigma^2}{2} \tag{B.18}$$

Proof of Lemma B.5: the proof uses a guess-and-verify strategy. Start by guessing that $v_0(a) = B_0^{-\gamma}\frac{a^{1-\gamma}}{1-\gamma}$ for a constant B_0 to be determined. Then $v'(a) = (B_0 a)^{-\gamma}, c(a) = B_0 a$ and $v''(a) = -\gamma B_0^{-\gamma} a^{-\gamma-1}$. Substituting into (B.17), we have

$$\rho B_0^{-\gamma}\frac{1}{1-\gamma} = B_0^{1-\gamma}/(1-\gamma) + B_0^{-\gamma} r - B_0^{1-\gamma} - \frac{\sigma^2}{2}\gamma B_0^{-\gamma}$$

Rearranging

$$\rho\frac{1}{1-\gamma} = \gamma B_0/(1-\gamma) + r - \frac{\sigma^2}{2}\gamma.$$

Rearranging again we obtain the expression for B_0 in (B.18). □

Step 2: Perturbation around $w = 0$. As already mentioned, it is no longer possible to solve (B.16) in closed form. However, we can look for approximate solutions of the form

$$v(a, w) = v_0(a) + w v_1(a) + O(w^2), \quad c(a, w) = c_0(a) + w c_1(a) + O(w^2) \tag{B.19}$$

where v_0 and c_0 are the value and consumption policy functions from Lemma B.5 and where v_1 and c_1 are to be determined. Further, we restrict our attention to solutions such that $v_1'(a) > 0$ for all a which ensures that $\partial_a v(a, w) \approx v_0'(a) + w v_1'(a) > 0$ for all $w > 0$ and therefore consumption $c(a, w) = (\partial_a v(a, w))^{-1/\gamma}$ is positive. However, we do not make any assumptions about the sign of $v_1(a)$. Substituting (B.19) into (B.16)

$$\rho(v_0(a) + w v_1(a)) = \max_c \; u(c) + (v_0'(a) + w v_1'(a))(w + ra - c) + (v_0''(a) + w v_1''(a))\frac{\sigma^2}{2}a^2 + g w v_1(a)$$

and the first-order condition is $(c_0(a) + w c_1(a))^{-\gamma} = v_0'(a) + w v_1'(a)$. Differentiating both the HJB equation and the first-order condition with respect to w and evaluating at $w = 0$:

$$\rho v_1(a) = v_1'(a)(ra - c_0(a)) + v_1''(a)\frac{\sigma^2}{2}a^2 + v_0'(a) + g v_1(a), \tag{B.20}$$

$$-\gamma c_0(a)^{-\gamma-1} c_1(a) = v_1'(a). \tag{B.21}$$

Substituting the expressions for v_0 and c_0 from Lemma B.5 into (B.20)

$$\rho v_1(a) = v_1'(a)\left(\frac{r-\rho}{\gamma} + (\gamma-1)\frac{\sigma^2}{2}\right)a + v_1''(a)\frac{\sigma^2}{2}a^2 + B_0^{-\gamma}a^{-\gamma} + g v_1(a) \tag{B.22}$$

It remains to find a solution $v_1(a)$ that solves (B.22). We solve it using a guess-and-verify strategy. Guess that $v_1(a) = B_1 a^{-\gamma}$, such that $v_1'(a) = -\gamma B_1 a^{-\gamma-1}$ and $v_1''(a) = \gamma(1+\gamma)B_1 a^{-\gamma-2}$. Substituting into (B.22)

$$\rho B_1 a^{-\gamma} = -\gamma B_1 a^{-\gamma}\left(\frac{r-\rho}{\gamma} + (\gamma-1)\frac{\sigma^2}{2}\right) + \gamma(1+\gamma)B_1 a^{-\gamma}\frac{\sigma^2}{2} + B_0^{-\gamma}a^{-\gamma} + gB_1 a^{-\gamma}$$

Rearranging we find that $B_1 = \frac{1}{r-g-\gamma\sigma^2}B_0^{-\gamma}$. Therefore

$$v_1(a) = \frac{1}{r-g-\gamma\sigma^2}B_0^{-\gamma}a^{-\gamma}$$

Similarly, substituting the expressions for $v_1'(a)$ and $c_0(a) = B_0 a$ into (B.21) we find

$$c_1(a) = \frac{1}{r-g-\gamma\sigma^2}B_0$$

Substituting v_1 and c_1 as well as v_0 and c_0 from Lemma B.5 into (B.19), we obtain the value function and the consumption policy function in (B.11) with $\bar{c} = B_0$. To obtain the net saving rate, start from the gross saving rate (B.10)

$$da_t = (w_t + ra_t - c_t)dt + \sigma a_t dW_t$$

and insert for the consumption policy function and rewrite it as the net saving rate

$$p_t dk_t = (w_t + rp_t k_t - B_0\left(p_t k_t + \frac{w_t}{r-g-\gamma\sigma^2}\right)dt - dp_t k_t + \sigma p_t k_t dW_t.$$

Use (B.8) and (B.9) to replace for p_t and dp_t

$$p_t dk_t = \left(D_t k_t + w_t - B_0\left(\frac{D_t k_t}{r-\mu} + \frac{w_t}{r-g-\gamma\sigma^2}\right)\right)dt,$$

which is net saving in Proposition B.3. □

B.2 Labor Income Risk

Another extension is to allow for labor income risk and borrowing constraints, as in Aiyagari (1994) and Huggett (1993). A continuum of ex-ante identical and infinitely-lived households maximize the discounted utility flow from consumption,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt,$$

subject to a budget constraint

$$c_t + \dot{a}_t = w_t + r a_t,$$

where w_t is labor income that evolves stochastically according to an Ornstein-Uhlenbeck process (continuous-time analogue of an AR(1) process) in logs

$$d \log w_t = -\nu \log w_t + \sigma_w dW_t.$$

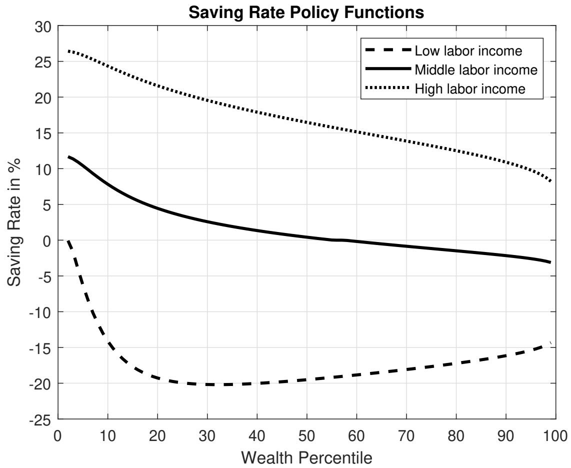
We impose a no-borrowing constraint, $a_t \geq 0$. Markets are incomplete and households self-insure by accumulating wealth a. Conditional on their earnings history, households differ in their level of wealth and income. Table B.1 presents our calibration.

	Value	
γ	2.000	Relative risk aversion / inverse IES
ρ	0.050	Discount rate
r	0.045	Dividend yield
ν	0.030	Persistence of income innovations (annual autocorrelation = 0.97)
σ_w	0.140	Standard deviation of income innovations

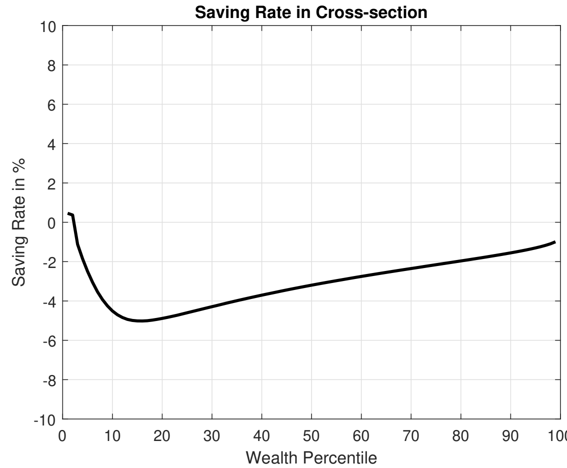
Table B.1: Calibration of Huggett model

The model generates a saving policy function $\dot{a} = s(a, w)$ where w is labor income. Figure B.1 plots the resultant saving rate against wealth in this environment. The left panel displays the saving rate $s(a, w)/(w + ra)$ for three different levels of labor income w. Conditional on labor income, the saving rate tends to decline with wealth, and this decline is more pronounced the closer a household is to the borrowing constraint. As we change income, the saving rate shifts up or down.[1]

[1] It is straightforward to show that the flat-saving-rate result from Section 2.1 now applies as wealth becomes large. More precisely, for all w, $s(a, w)/(w + ra) \to (r - \rho)/(\gamma r)$ as $a \to \infty$ meaning that the saving rate policy function even converges to the same value as in Section 2.1. The steep decline close to the borrowing constraint reflects two familiar effects. First, precautionary saving of high-income, low-wealth households.



(a) Policy function



(b) Saving across the wealth distribution

Figure B.1: Saving rates with income risk and borrowing constraints.

The right panel displays saving rates without conditioning on income, displaying the cross-sectional relationship between saving rates and wealth in the model's stationary distribution. The saving rate initially decreases and then increases with wealth. This reflects two opposing forces. On the one hand, conditional on income, saving rates tend to decrease with wealth. On the other hand, saving rates increase with labor income, and labor income and wealth are positively correlated in the stationary distribution.

Second, low-income households rapidly decumulate wealth and then hit the constraint (i.e., their saving rate is zero at the constraint but strongly negative above the constraint).

C Appendix for Section 3

C.1 Data Sources and Variables

Source:	Variables:
Income and wealth from tax returns	Labor income
Annual, 1993 -	Business income
	Capital income
	Transfers received
	Taxes paid
	Asset holdings (e.g deposits, mutual funds, bonds, real estate)
	Debt (total debt)
	Pensionable income (since 1967)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/inntekt	
Housing wealth database	Value of housing (including cabins and secondary homes)
Annual, 1993 -	(as in Fagereng, Holm and Torstensen, 2019)
Norwegian educational database	Highest completed education (length and type)
Annual, 1964 -	
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/utdanning	
Stockholder registry	ISIN / firm ID
Annual, 2004 -	Owner ID
	Quantity owned of stock
https://www.ssb.no/383782/utlan-av-data-om-aksjonaerer-aksjeselskaper-og-allmennaksjeselskaper	
Firm balance sheet and tax return data data	Balance sheet information (e.g., book value of equity, retained earnings)
Annual, 1995 -	Assessed value of private companies
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/regnskap	
The central population register	Region of residence at the end of the year
Annual, 1964 -	Date (month) of birth
	Gender, indicator variable for gender
	Marital status indicator variable
	Spousal id (unique identifier of spouse)
https://www.ssb.no/en/omssb/tjenester-og-verktoy/data-til-forskning/befolkning	
Norwegian mapping authority	Buyer/seller ID
Event data, 1993 -	Price
https://www.ssb.no/bygg-bolig-og-eiendom/eiendom/statistikk/eiendomsomsetning	
Other public data sources:	
Consumer price index, Statistics Norway	
https://www.ssb.no/en/priser-og-prisindekser/statistikker/kpi	
Flow of funds, Statistics Norway	
https://www.ssb.no/en/nasjonalregnskap-og-konjunkturer/statistikker/finsekv	
Stock price index and general stock prices, Oslo Børs	
https://www.oslobors.no/ob_eng/markedsaktivitet/#/details/OBX.OSE	
House price indices, Eitrheim and Erlandsen (2004)	
https://www.norges-bank.no/en/topics/Statistics/Historical-monetary-statistics/	
Exchange rate data, Norges Bank	
https://www.norges-bank.no/en/topics/Statistics/exchange_rates/	
MSCI stock index	
https://finance.yahoo.com/quote/MSCI/history/	

C.2 Net and Gross Saving with Multiple Assets

Section 2.2 defined net and gross saving with one asset. The data feature multiple assets and in this section, we generalize the saving definitions.

A household receives annual income w_t (labor income and transfers) and pays taxes τ_t. There are J assets indexed by $j = 1, ..., J$. Let $k_{j,t-1}$ denote the household's holdings of asset j at the end of period $t-1$. To simplify notation, assume that the household holds each asset $k_{j,t-1}$ throughout the year and only makes transactions at the end of the year. Throughout the year, asset holdings $k_{j,t-1}$ earn capital income $\theta_{j,t} p_{j,t} k_{j,t-1}$. The general versions of (7) and (8) are then

$$c_t + \underbrace{\sum_{j=1}^{J} p_{j,t}(k_{j,t} - k_{j,t-1})}_{\text{net saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J} \theta_{j,t} p_{j,t} k_{j,t-1}}_{\text{disposable income}} \tag{C.1}$$

$$c_t + \underbrace{\sum_{j=1}^{J}(p_{j,t}k_{j,t} - p_{j,t-1}k_{j,t-1})}_{\text{gross saving}} = \underbrace{w_t - \tau_t + \sum_{j=1}^{J}\left(\theta_{j,t} + \frac{p_{j,t} - p_{j,t-1}}{p_{j,t-1}}\right) p_{j,t-1}k_{j,t-1}}_{\text{Haig-Simons income}}\,. \tag{C.2}$$

C.3 Separating Gross Saving into Net Saving and Capital Gains

C.3.1 Housing: Using Transaction Data

To explain our approach, it is helpful to introduce some notation. Time is continuous and we consider a household that makes housing transactions at discrete time intervals. We denote by $h(t), p(t)$ and $a_h(t) = p(t)h(t)$ the household's physical number of housing units, the price of housing and the value of the house at the beginning of year t. Throughout a year, i.e., between dates t and $t+1$, the household makes $N \geq 0$ transactions at ordered dates τ_n: $t \leq \tau_1 < \tau_2 < ... < \tau_N < t+1$. We decompose gross saving, i.e., the change over the year in housing wealth $a_h(t) = p(t)h(t)$, into net saving and capital gains using the following decomposition

$$\underbrace{a_h(t+1) - a_h(t)}_{\text{gross saving}} = \underbrace{\sum_{n=1}^{N+1}(p(\tau_n) - p(\tau_{n-1}))h(\tau_{n-1})}_{\text{capital gains}} + \underbrace{\sum_{n=1}^{N} p(\tau_n)(h(\tau_n) - h(\tau_{n-1}))}_{\text{net saving}}\,. \tag{C.3}$$

For our purpose, the main implication is that net saving can only be non-zero for households with housing transactions. Hence, for households without housing transactions, net saving in housing is zero and all changes in housing wealth are due to capital gains. In the case with transactions, on the other hand, (C.3) implies that net saving is equal to net transactions at market value during the year. Capital gains in housing is then the change in housing wealth minus net transactions at market value.

C.3.2 Stocks: Using Ownership Data

For most asset classes, we do not know the individual transactions within the year. We therefore approximate capital gains and net saving based only on available information at time t and $t+1$. For example, for stocks we know the number of shares q in each stock and its price p at the beginning and end of the year. We make three simplifying assumptions to compute capital gains and net saving:

1. All transactions are of the same size and direction: $dq_{\tau_n} = \frac{q_{t+1}-q_t}{N}$.
2. All prices move monotonically and with same step size within a year: $p_{\tau_n} = (\tau_n - t)(p_{t+1} - p_t) + p_t = \frac{n}{N+1}p_{t+1} + \frac{N+1-n}{N+1}p_t$.
3. All transactions are distributed uniformly across the year: $\tau_n - \tau_{n-1} = d\tau \; \forall n$.

Under these assumptions, we derive an expression for net saving from observables

$$\begin{aligned}
\sum_{n=1}^{N} p_{\tau_n} dq_{\tau_n} &= \sum_{n=1}^{N} \left(\frac{n}{N+1} p_{t+1} + \frac{N+1-n}{N+1} p_t \right) \frac{q_{t+1}-q_t}{N} \\
&= \frac{q_{t+1}-q_t}{N} \sum_{n=1}^{N} \left(\frac{n p_{t+1} + (N+1-n) p_t}{N+1} \right) \\
&= \frac{q_{t+1}-q_t}{N} \sum_{n=1}^{N} \left(\frac{n(p_{t+1}-p_t)}{N+1} + p_t \right) \\
&= \frac{q_{t+1}-q_t}{N} (N p_t + \frac{1}{2} N (p_{t+1}-p_t)) \\
&= \frac{1}{2}(p_t + p_{t+1})(q_{t+1}-q_t)
\end{aligned}$$

Capital gains is next defined as the change in total value of an asset not accounted for by net saving.

C.4 Private Businesses

The portfolio shares in Figure 3 show that many of the wealthiest households hold a substantial share of their wealth in private businesses. Since these firms are not publicly traded, there is no available market price. In this appendix, we describe how we account for private businesses.

A private business is a company that is not listed on a stock exchange and owned by a small number of shareholders. Control of the firm is therefore limited to a few persons. These firms are typically small to medium sized businesses or holding companies. In 2006, Norway introduced a dividend tax at the individual level. One response to this tax reform was that the number of holding companies grew such that individuals could retain earnings in firms to avoid paying the dividend tax. These holding companies are therefore common, especially at the top of the wealth distribution.

Our aim is to find the ultimate owners of private businesses to be able to allocate retained earnings, public stock ownership, debt, and capital gains onto the ultimate owner's balance sheet. The approach is similar to other papers using Norwegian data (Alstadsæter et al., 2016; Fagereng et al., 2020).

Ultimate owners of private businesses. We use the stock holder registry to find the ultimate owners of private businesses. The stock holder registry contains information of individuals' and firms' ownership of stocks in all companies in Norway. Some companies are held directly. In this case, the ownership share is the fraction of total shares owned by the individual. However, many companies are owned by other firms. To fix ideas, assume an individual owns shares in company A and company A owns shares in company B. In this case, the individual holds an ownership share in company B equal to that individual's ownership share in company A multiplied with company A's ownership share in company B. We compute indirect ownership through up to 7 layers.

Retained earnings. Retained earnings is the profit of the firm that is withheld in the firm by not paying dividends. These are profits that accrue to the company but will not be accounted for on the income statement of individuals. Alstadsæter et al. (2016) show that retained earnings in private businesses have grown sharply after the dividend tax reform in 2006. By not accounting for retained earnings properly, we underestimate earnings of (wealthy) individuals.

To compute retained earnings, we follow the method in Alstadsæter et al. (2016) and exploit the Norwegian accounting concept of earned equity, defined as accumulated retained earnings. Retained earnings in a private business in year t is therefore the difference between earned equity at the end of year t and the beginning of year t. After obtaining retained earnings at the level of the private business, we allocate it to the ultimate owners' income using the ownership register.

Balance sheets. To more precisely measure individuals' portfolio shares and exposure to risky assets and debt, we allocate all publicly-traded stocks and debt onto the ultimate owners' balance sheets. At the end of each year, the private business reports its balance sheet to the tax authorities. Both publicly-traded stocks and debt are directly observed on these firm balance sheets and we allocate these to the ultimate owners' balance sheet using the ownership register.

Capital gains. Private businesses hold publicly traded stocks that accumulate capital gains. From the stockholder registry, we can see which stocks a private business hold. We are therefore able to compute capital gains in a private business in the same way as for publicly traded stocks held by individuals in Appendix C.3.2. Once we obtain a measure of capital gains at the level of the private business, we allocate these capital gains to ultimate owners' capital gains using the ownership register.

C.5 Stock Ownership in Norway

The portfolio shares in Figure 3 show that ownership of publicly-traded stocks, held either directly by the individual or indirectly though stock funds or private businesses, is relatively low in Norway compared with other OECD countries. For example, the mean portfolio share in publicly-traded stocks is about 1.5% for all individuals and less than 5% for the top 1% of the wealth distribution. In contrast, the top 1% in the U.S. hold more than 40% of their assets in public equity (Campbell, 2006). There are two main reasons why the portfolio share of publicly-traded stocks is lower in Norway than in the US. First, Norway has a public pension system that holds a substantial position of Oslo Stock Exchange (OSE) on behalf of the Norwegian population. This indirect ownership of publicly-traded stocks does not enter individuals' balance sheets in the way for example 401k accounts enter the balance sheets of U.S. citizens. Second, Oslo Stock Exchange is smaller as a share of GDP than in other similar countries. For example, the market capitalization of listed domestic companies relative to GDP is about twice as large in Sweden as in Norway.[2] In this appendix we document the ownership structure of Oslo Stock Exchange and to what extent we are able to account for aggregate stock ownership at the individual level.

Ownership structure of Oslo Stock Exchange. Table C.1 presents the ownership structure in aggregate data from the Oslo Stock Exchange and in the ownership registry in 2015. In 2015, 33.6% of the market capitalization of Oslo Stock Exchange was held by the

[2] See https://data.worldbank.org/indicator/CM.MKT.LCAP.GD.ZS?locations=NO-SE.

Owner sector	OSE Annual report	Ownership registry	Potentially controlled by individuals	Allocated to individuals
Government[1]	33.6%	33.0%		
Foreign investors	36.8%	32.4%		
Financial sector[2]	8.8%	8.4%		
Other companies	16.7%	11.2%	9.7%	6.0%
Private investors	3.9%	3.7%	3.7%	3.5%
Others	0.1%	0.0%		
Sum	100.0%	88.6%	13.4%	9.4%

Notes: "OSE Annual report" refers to the annual report of Oslo Stock Exchange and "Ownership registry" refers to the ownership registry that is available with individual owners. "Potentially held by individuals" refers to the share of Oslo Stock Exchange that is potentially controlled directly by individuals. The difference between "ownership registry" and "potentially controlled by individuals" is the stocks that are held by companies that are listed on the stock exchange. "Allocated to individuals" is the share of stocks at Oslo Stock Exchange that we ultimately allocate to individuals, either via indirect ownership through private businesses or directly held stocks.

[1]Government includes the categories "government and municipalities" and "companies with government ownership."
[2]Financial sector includes the categories "banks and mortgage companies," "private pension funds/life insurance," "stock funds," and "general insurance."

Table C.1: Ownership structure of Oslo Stock Exchange (OSE), 2015

government sector. There are two main reasons why the government sector has such a large ownership share. First, the government sector includes pension funds both at the state and municipality level.[3] Norway has a public pension system where all citizens are enrolled. A part of the pension funds is invested in public stocks in Norway. Second, it includes the government's direct ownership of firms. The Norwegian government owns substantial fractions of many publicly-traded companies in Norway, both for historic and strategic reasons. For example, the Norwegian government still holds large positions in many Norwegian banks after the re-capitalization of the banking system in the early 1990s.

In 2015, foreign investors held 36.8% of Oslo Stock Exchange. This ownership share has also been stable between 33% and 41% in our sample period. Next, 8.8% of the stock exchange is held by the financial sector. This is mainly stock funds (6.8%), while the rest (2.0%) is held by banks and mortgage companies, private pension funds or life insurance companies, and general insurance companies.

[3]Note that the Norwegian Sovereign Wealth Fund does not hold stocks in Norway and is therefore not included in this ownership share.

The next two ownership categories, "other companies" and "private investors," are the most interesting for our purpose because a large share of these categories are controlled by Norwegian individuals. Other companies includes all stocks that are held by Norwegian companies. Many individuals hold stocks through private businesses and these are included in this sector. The private investors sector includes all stocks that are directly held by Norwegian individuals. The sum of other companies and private investors is therefore an upper bound the share of the stock exchange that is controlled by Norwegian individuals and that we may be able to find using the available data.[4]

Although the sum of "other companies" and "private investors" is the upper bound for the share of stocks that are directly controlled by Norwegian individuals in the annual report, it is not the upper bound that we can unravel from the registry data for two reasons. First, there is a discrepancy between the ownership registry in the micro data and the official data from Oslo Stock Exchange. This discrepancy brings the ownership share of other companies down from 16.7% to 11.2%. Second, a share of the sector other companies are stocks that are either held by the company itself, held by other publicly traded companies or held by foreign companies. By excluding the share that is owned by other publicly-traded companies, the share of the sector "other companies" declines further from 11.2% to 9.7%. 9.7% is therefore the upper bound on what share of stocks on Oslo Stock Exchange that we potentially can allocate to individuals. At the end of the day, we are able to allocate 6.0% of the stock exchange to the ultimate owners held through private businesses.

[4]Note that the sum of "other company" and "private investors" is not the strict upper bound of ownership that is held by Norwegian individuals since they can hold stocks on Oslo Stock Exchange indirectly through foreign companies. For example, Alstadsæter, Johannesen and Zucman (2019) document that almost 30% of taxes among the top 0.01% are evaded in Norway.

C.6 Comparing Household Portfolios in Norway and Other Countries

	United States				Norway			
	25th-50th	**50th-75th**	**75th-99th**	**Top 1%**	**25th-50th**	**50th-75th**	**75th-99th**	**Top 1%**
Real Estate	152%	103%	62%	23%	153%	109%	95%	45%
Vehicles	30%	9%	4%	1%	3%	1%	1%	1%
Stocks	14%	14%	30%	32%	2%	1%	2%	2%
Cash	14%	7%	6%	2%	14%	10%	8%	4%
Business	2%	4%	9%	41%	2%	2%	6%	52%
Other Assets	2%	3%	4%	5%	0%	0%	1%	2%
Mortgages	-93%	-36%	-14%	-3%	-70%	-24%	-12%	-5%
Credit Card Debt	-4%	-1%	0%	0%	-1%	0%	0%	0%
Education Loans	-12%	-2%	-1%	0%	-2%	-1%	0%	0%
Other Debts	-5%	-1%	0%	-1%				

Notes: The table compares portfolio shares of different groups of assets and debts across the net wealth distribution in the US and Norway. For the US, we use data from the Survey of Consumer Finances (Jones and Neelakantan, 2023). Portfolio shares are defined as the fraction relative to net worth, with debts expressed as negative shares. For Norway, the data and variables are adjusted to match those of the SCF. Following Jones and Neelakantan (2023) (Figure 1) we exclude households below the 25th percentile to avoid negative or small values in the denominator of the portfolio shares. The category "Other Debts" are in the Norwegian data part of the category "Mortgages".

Table C.2: Portfolio shares by net wealth, U.S. vs. Norway.

Decile	United Kingdom Property	Financial	Business	Physical	Norway Property	Financial	Business	Physical
2	4%	17%	1%	78%				
3	5%	25%	3%	66%	56%	36%	3%	5%
4	44%	19%	4%	34%	78%	17%	2%	3%
5	71%	10%	2%	17%	83%	13%	2%	2%
6	75%	12%	2%	11%	84%	12%	2%	2%
7	76%	14%	1%	9%	84%	12%	2%	1%
8	73%	17%	3%	7%	84%	12%	2%	1%
9	70%	21%	3%	6%	84%	11%	3%	1%
10	47%	31%	19%	3%	67%	9%	23%	1%

Notes: The table compares the average share of the total net wealth contributed from different asset classes by family net wealth percentile for the UK and Norway. For the UK we use data from the Wealth and Asset Survey of the Office of National Statistics, as reported by Advani, Bangham and Leslie (2021) (Figure 4). For Norway, the data and variables are adjusted to match those of the UK. Households are allocated to deciles based on wealth measured at the family level. The lowest decile in the UK and the two lowest deciles in Norway are excluded as net wealth is negative. Property wealth is real estate measured net of mortgage debt, and financial wealth (including bank deposits, bonds, stocks and mutual funds) net of other financial liabilities. Business wealth includes ownership in private businesses. Physical wealth includes ownership of vehicles such as cars and boats. For comparison, pensions are excluded from the measures.

Table C.3: Asset shares by net wealth, U.K. vs. Norway.

	United States <25	25-34	35-44	45-54	55-64	65+	Norway <25	25-34	35-44	45-54	55-64	65+
Assets:												
Real Estate	15%	38%	46%	50%	53%	54%	66%	85%	82%	79%	79%	80%
Vehicles	42%	27%	22%	19%	15%	14%	1%	1%	1%	1%	1%	1%
Stocks	6%	11%	13%	14%	16%	15%	1%	1%	1%	1%	2%	2%
Cash	32%	18%	13%	11%	9%	11%	10%	6%	4%	5%	7%	12%
Business	3%	3%	4%	5%	5%	3%	22%	7%	11%	13%	11%	5%
Other Assets	2%	2%	2%	1%	2%	4%	0%	0%	0%	1%	1%	1%
Debts:												
Mortgages	-16%	-44%	-57%	-60%	-62%	-54%	-77%	-89%	-96%	-98%	-98%	-99%
Credit Card Debt	-24%	-18%	-15%	-18%	-20%	-30%	0%	-1%	-1%	-1%	-2%	-1%
Education Loans	-43%	-29%	-19%	-14%	-8%	-4%	-23%	-10%	-3%	-1%	0%	0%
Other Debts	-18%	-9%	-9%	-8%	-10%	-12%						

Notes: The table compares the composition of assets and debts across ages in the US and Norway. For the US, data from the Survey of Consumer Finances (Jones and Neelakantan, 2023) is utilized. Asset shares are expressed as fractions of total assets, and debt shares as fractions of total debt. For Norway, the data and variables are adjusted to match those of the SCF. The category "Other Debts" is in the Norwegian data part of the category "Mortgages".

Table C.4: Composition of Assets and Debts by age, U.S. vs. Norway.

C.7 Public Pension Wealth

This appendix describes how we compute public pension wealth. We define public pension wealth as the net present value of future pension income, discounted at the risk free interest rate and accounting for the probability of living to the retirement age. The main complication is that there are currently four different pension systems depending on birth cohorts. We describe each system in detail before we define public pension wealth, savings, and income. A common feature of all public pension systems is that an individual accumulates claims pen_t^c in units of the basis-amount in the social security system G_t.

C.7.1 The Four Pension Systems

Cohorts born prior to 1944. For these cohorts, we observe pension transfers in our data. Pension wealth is therefore computed as the net present value of these pension earnings.

Cohorts born between 1944 and 1953. For these cohorts, there are two parts of the pension system: the social security and the service pension.

1. **Social Security.** The social security system is based on a point system. Each year, individuals accumulate pension points based on the following formula

$$point_t = \begin{cases} \frac{y_t}{G_t} - 1 & \text{if } G_t \leq y_t \leq 6G_t \\ 5 + \frac{y_t - 6G_t}{3G_t} & \text{if } 6G_t \leq y_t \leq 12G_t \\ 7 & \text{if } y_t \geq 12G_t \end{cases}$$

   where y_t is gross earnings in year t and G_t is the basis-amount in the social security system.

   An individual's pension number P is then defined as the average of the 20 years with the highest pension points, or the average of the n years that person has earned pension points. The payouts from the social security pension is approximately

$$pen^{SC} = \alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}$$

   where $\alpha = 1$ for singles and 0.85 for couples, and κ is a proportionality factor equal to 0.45 if income was accumulated prior to 1992 and 0.42 after 1992.

2. **Service Pension.** All public sector employees and about 50 percent of private sector employees have an additional service pension on top of their social security

pension. This service pension guarantees an individual a fraction ψ of their final income. This fraction depends on how long the individual has worked for the company/government, but the maximum is $\psi = 0.66$.

The service pension pays the difference between the sum implied by the fraction rule and the level received from the social security pension. We can therefore approximate the pensions in units of G as

$$pen_t^c = \max\left\{\alpha + \max\left\{\kappa P \frac{min\{n, 40\}}{40}, 1\right\}, \frac{0.66 y^{final}}{G^{final}}\right\}$$

where n is the number of working years, and y^{final} and G^{final} are income and basis-unit in social security in your final working year, respectively.

Cohorts born between 1954 and 1962. Pensions in units of G is a linear combination of the system for those born prior to 1954 outlined above and those born after 1963 outlined below:

$$pen_t^c = \frac{63 - c}{10} pen_t^{44 \leq c \leq 53} + \frac{c - 53}{10} pen_t^{c \geq 63}.$$

Cohorts born after 1963. In 2010, the government simplified the pension system for all earners born after 1964. The new system implies that every year, 18.1 % of your gross income below 7.1 G is added to your pension holdings ("pensjonsbeholdning," P_t).

$$P_t^c = \left(\sum_{\tau=c+13}^{t} \max\left\{0.181 \frac{y_\tau}{G_\tau}, 0.181 \cdot 7.1\right\}\right)$$

P_t^c is, as before, defined in units of G, the basis-amount in the Norwegian pension system.

One complication is that you can start taking out pensions from age 62. However, to simplify the exposition, we assume that all households value their pension as if they would start taking out pensions from age 67. From age 67, your income from pensions is the value of your pension holdings P_t^c in units of G divided by your expected remaining years (e.g., 16.02 for the cohort born in 1964).

$$pen_t^c = \frac{P_t^c}{d_c}$$

C.7.2 Pension Wealth, Saving, and Income

We define pension wealth as the net present value of pension income from age 67. That income is $pen_t^c G_t$ in each year t when t is greater than 67, where pen_t^c is defined by one of the four systems above depending on your birth cohort. In order to calculate the net present value, we discount the pension contributions net of taxes by the risk free real interest rate and the survival probability[5]

$$V_t^c = (1-\tau) pen_t^c G_t \mathcal{M}_{t,c+67} \left\{ \sum_{\tau=\max\{c+67,t\}}^{\max\{c+67+d_c,t\}} \frac{\prod_{s=t}^{\tau}(1+\pi_{w,s})}{\prod_{s=t}^{\tau}(1+r_s)} \right\} \tag{C.4}$$

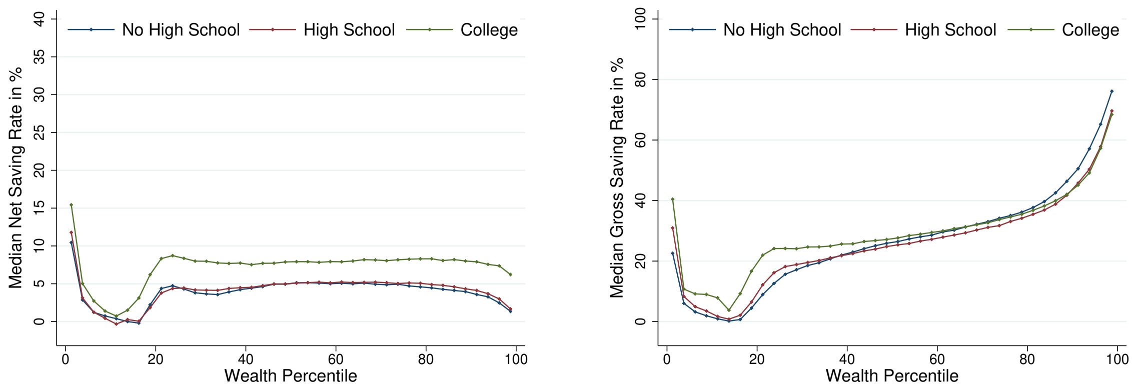
where τ is the median tax rate on pensions (17 %), $\mathcal{M}_{t,c+67}$ is the probability of surviving from year t to year $c+67$ when the household is born in year c, and $\pi_{w,s}$ is the real growth rate in G in year s and r_s is the real interest rate in year s. The max operator is there because an individual start withdrawing from the pension account after age 67. We have to make two assumptions to calculate pension wealth in the data. First, we assume perfect foresight in the years where we observe r_s and $\pi_{w,s}$. Second, we assume that after 2015, the expected real interest rate and growth rate of G are the observed geometric mean in the years from 1993 to 2015. For example, in order to calculate pension wealth in 2006, we discount by the observed real interest rate from 2006 to 2015, and with the mean real interest rate after 2015.

We define pension saving as the change in pension wealth. Arguably, changes in pension wealth may be due one of the following three reasons: (i) net withdrawals or contributions, (ii) revaluation due to discounting by the real interest rate, or (iii) revaluation because the probability of surviving to age 67 increases. We count all changes in pension wealth as net saving. Furthermore, we define pension income in such a way to ensure that the budget constraint adds up. This implies that pension income always equals pension saving.

[5]We calculate the survival probability of living from age t to $c+67$ from the Norwegian mortality tables. It is about 90 % for a 20 year old in our sample and increases toward 1 as the individual ages.

D Appendix for Section 4: Additional Exercises

Saving Rates by Education. These results and the approach behind them are discussed in the main text. Figure D.1 plots the results.



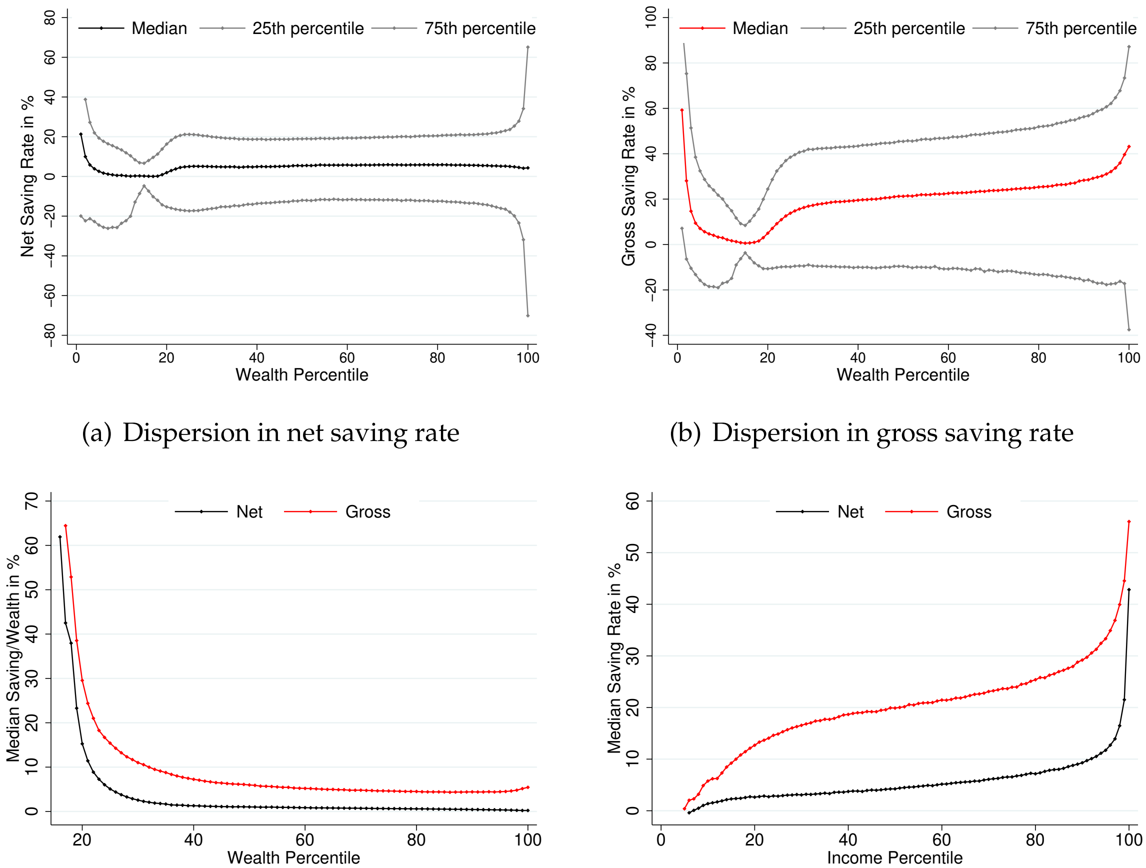
(a) Net saving rate by education group (b) Gross saving rate by education group

Notes: The figures display the median net saving rates (left) and median gross saving rates (right) within education groups. All variables are computed as the median within wealth percentile and year, averaged across all years.

Figure D.1: Saving rates across the wealth distribution by education

Dispersion in Saving Rates. In our main exercise, we compute medians within wealth percentiles. While interesting and informative for models, our approach ignores the dispersion in saving rates that exists within wealth percentiles. Figure D.2(a) and D.2(b) present the net saving rate and the gross saving rate together with their respective 25th and 75th percentile within the wealth percentile. The additional lines are computed in the same way as the main graph. For example, the 25th percentile line is computed by first computing the 25th percentile within the wealth percentile in each year, and then averaging across all years. The most striking feature of the dispersion graphs is that, dispersion is relatively stable across the wealth distribution except for the tails.

Saving Rates as a Fraction of Wealth. Because our objective is to test the saving behavior implied by economic theory, our objects of interest are saving relative to income. However, alternative definitions of saving rates are interesting for other purposes. In particular, Bach, Calvet and Sodini (2018) investigate the role of saving as a fraction of wealth, i.e., the growth rate of wealth, across the wealth distribution, to address how saving behavior affects the dynamics of wealth inequality. To more easily compare our results with theirs,



(a) Dispersion in net saving rate

(b) Dispersion in gross saving rate

(c) Saving as fraction of wealth

(d) Saving rates by income

Figure D.2: Additional exercises.

we present saving as a fraction of wealth across the wealth distribution in Figure D.2(c).[6] The figure reveals a similar pattern as Bach, Calvet and Sodini (2018)'s Figure 4.[7] Part of the downward sloping nature of saving as a fraction of wealth is somewhat mechanical: even low-wealth households have some labor income out of which they save, so the ratio of these quantities to wealth blows up as wealth becomes small.[8]

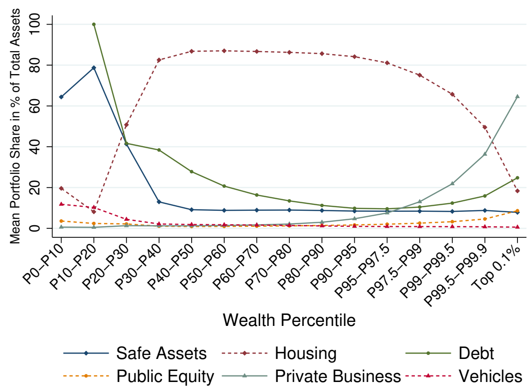
Saving Rates by Income. Figure D.2(d) plots our two saving rates measures across the distribution of *income*, as opposed to *wealth* like in our main Figure 4. The main takeaway

[6]We follow Bach, Calvet and Sodini (2018) and cut the figure at the bottom of the distribution, namely at the percentile below which net worth is zero or negative. The rationale is that the ratio of saving to wealth is ill-defined when wealth is zero or negative.

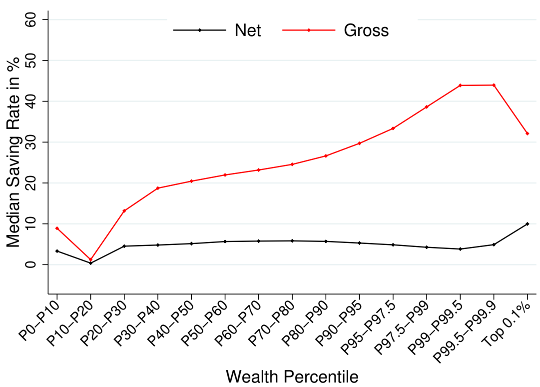
[7]Note that our gross saving over wealth is the same as their total saving over wealth while the other definitions (our net saving and their active saving) are not comparable.

[8]For instance consider the saving and consumption policy functions in our benchmark model, namely (2) and $c(a) = \left(\rho - \frac{r-\rho}{\gamma}\right)\left(a + \frac{w}{r}\right)$. We have $s(a)/a = \frac{r-\rho}{r}\left(1 + \frac{w}{ra}\right)$ and $c(a)/a = \left(\rho - \frac{r-\rho}{\gamma}\right)\left(1 + \frac{w}{ra}\right)$, both of which decrease with wealth and blow up as $a \to 0$, just like in Figure D.2(c).

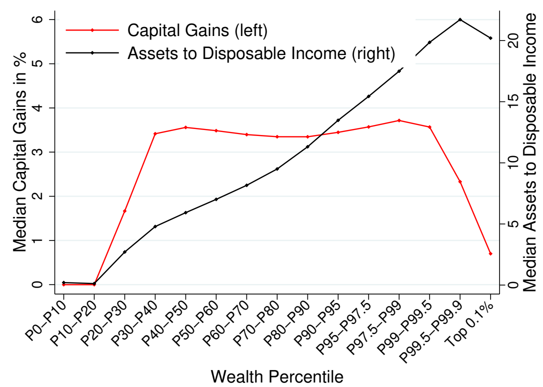
is that not just gross saving rates but also net saving rates are strongly increasing with income. There are a number of theoretical reasons why this is the qualitative relationship to expect, for example the permanent income hypothesis suggests that households will save high transitory income realizations.



(a) Portfolio shares across the wealth distribution



(b) Saving rates across the wealth distribution



(c) Capital gains and asset-to-income ratio

Notes: Figure (a) displays the mean portfolio share in percent of total assets across the wealth distribution, by percentile group. Figures (b) and (c) display the median saving rates, capital gains in percent of assets, and assets to income ratio. All variables in Figure (b) and (c) are computed as the median within wealth percentile and year, averaged across all years.

Figure D.3: Saving behavior in the right tail of the wealth distribution.

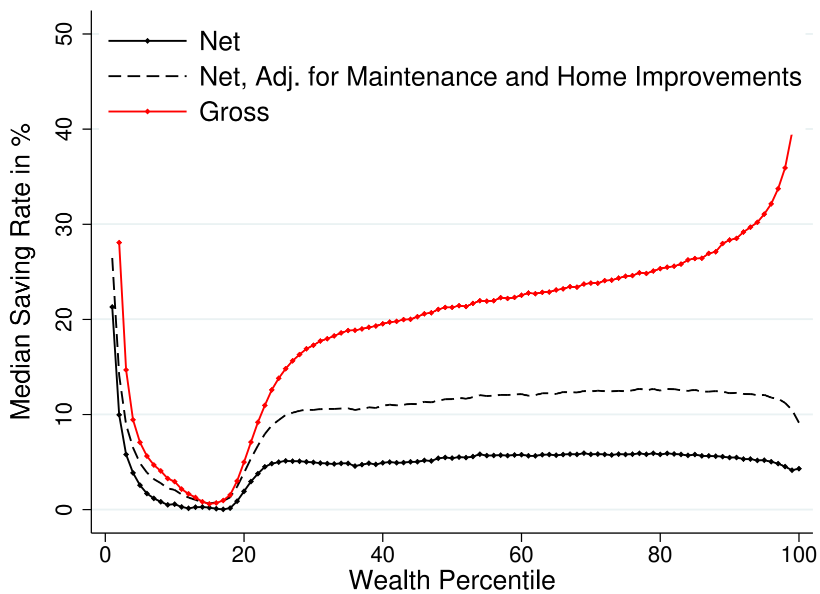
The Right Tail of the Wealth Distribution. The right tail of the wealth distribution is particularly interesting because it contains a disproportionately large share of aggregate wealth and an asset portfolio that is considerably less tilted toward housing, as seen in Figure D.3(a). We zoom in on these households in Figures D.3(b) and D.3(c). Figure D.3(b) has one substantial difference from our main plots. Within the top percentile, the gross saving rate first continues to increase similarly as in the main part of the population, but drops as we move into the top 0.1 percent. Figure D.3(c) plots capital gains relative to asset value. This ratio drops markedly as we step into the top 0.1 percent group. Hence, the saving pattern observed in Figure D.3(b) reflects that households at the very top hold relatively less wealth in assets that experience yearly capital gains and hence their gross saving rate lies closer to the net saving rate.[9]

Maintenance and Home Improvements Costs. Maintenance and home improvement represents a large share of household spending. According to the consumer expenditure survey for Norway in the years 2005, 2006, 2007, 2008, 2009, and 2012, maintenance and home improvements make up 6.3% of household spending on average.[10] In our baseline approach to computing saving rates, we do not include such expenses in net saving and thus implicitly count them as consumption. One might instead argue that maintenance and home improvement mostly are a form of saving, upholding home value and gross wealth, and therefore should be included in the net saving of households. To explore the importance of this assumption for our results, we here make the alternative assumption that all maintenance and home improvement expenses are part of saving. We cannot directly observe these expenses, so we use the average from the consumer expenditure survey (6.3% of aggregate spending) and attribute it to homeowners according to their home sizes. In this imputation procedure, we impose a linear relation between home size and expenses that aggregates to 6.3% share of total spending. Note that the inclusion of maintenance and home improvements only affects net saving. Gross saving is always computed as the change in wealth.

Figure D.4 displays the effect of adjusting for maintenance and home improvements. The adjustment raises the net saving rate by approximately 6 percentage points, as expected, and thus closes part of the gap between net and gross saving. However, including all maintenance and home improvements in net saving does not alter the pattern that net saving rates are approximately flat across the wealth distribution.

[9]The low measured capital gains for the top 0.1% are likely in large part due to the fact that tax values of private businesses are related to book values rather than market values and therefore measured capital gains on private businesses are zero, see the discussion in Sections 3.2. Since private business wealth accounts for a large share of the assets held by the top 0.1% (see panel (a)), the observed capital gains in this group are likely an underestimate of the capital gains in market values.

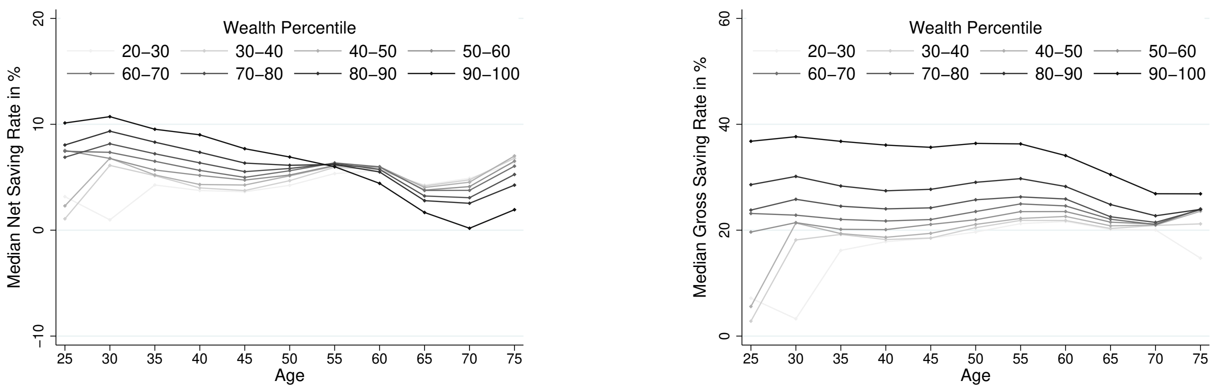
[10]The data are publicly available online: https://www.ssb.no/en/inntekt-og-forbruk/forbruk/statistikk/forbruksundersokelsen.



Notes: The figure displays the median net and gross saving rates by wealth percentile. The dashed line displays the net saving rate when maintenance and home improvements are imputed in proportion to each homeowner's home size and added to their net saving.

Figure D.4: Saving rates across the wealth distribution with maintenance and home improvement added to net saving.

Life-cycle Facts. Figure D.5 displays the median net and gross saving rate by age, broken down by wealth (within-age wealth percentiles). Figure 12(a) in the paper shows that the net and gross saving rates are relatively flat over the lifecycle. Figure D.5 illustrates that this also holds across most wealth percentiles. However, there is a tendency for the saving rates to decline with age among the wealthiest households.



(a) Net saving rates by age within wealth deciles

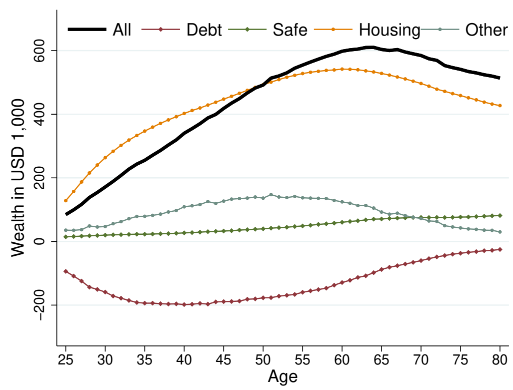
(b) Gross saving rates by age within wealth deciles

Notes: The figures show the median net and gross saving rates across the lifecycle and by (within age-group) wealth percentiles.

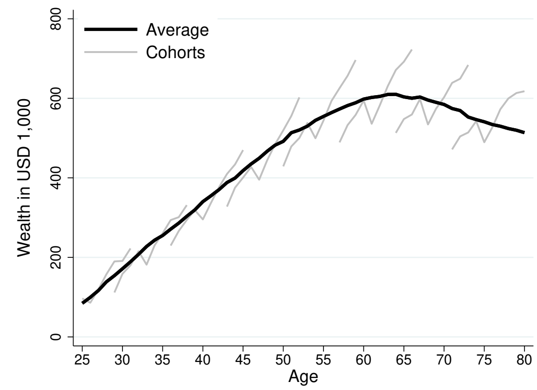
Figure D.5: Net and gross saving rates by age and wealth.

Figure D.6(a) displays wealth decomposed by the main asset classes. Housing is the main asset across all age groups. The holding of houses peaks at the age of 60, after which it falls. Unlike the portfolio shares displayed in Figure 3, where we divide holdings by total assets, we plot holdings in dollar *amount* here. The portfolio shares of households with low asset holdings will therefore not matter much for the graphs in Figure D.6 relative to their impact on the graphs in Figure 3, and this is why the level of safe asset holdings lies below ”other assets” in Figure D.6, while the share of safe assets over total individual asset holdings came out above the other assets in Figure 3.

Finally, we note that there are time and cohort effects in the background, which means that the changes in wealth across age groups in Figure D.6(a) need not mirror the saving rates in Figure 12(b) in the paper. For instance, time and cohort effects explain why debt falls from age 50 to age 70 in D.6(a), even though the net saving rates in debt for these age groups are negative or close to zero in Figure 12(b). Figure D.6(b) illustrates the role of time and cohort effects. The grey lines track different cohorts as they age in our sample. In contrast to the average wealth profile, the cohort wealth profiles tend to increase in almost all years (except years with significant capital losses, also known as time effects). Because the average wealth profile by age differs significantly from the evolution of wealth within a cohort, one should not expect saving rates to align with the average wealth profile by age.



(a) Wealth and assets by age



(b) Wealth by age with cohort means

Notes: The figures show decomposed wealth, and wealth by age with cohort averages. The category "other" contains all remaining asset categories.

Figure D.6: Wealth over the life cycle, across and within cohorts.

E Additional Model Exercises

E.1 Appendix for Section 5.1

This appendix presents a stripped-down version of the model in Section 5.2 to illustrate the importance of non-homothetic preferences (19). Compared with the model in Section 5.2, there are no adjustment frictions, no amortization requirement, no downpayment constraint, and the same interest rate on bonds and mortgages. The model collapses to a one-asset model, similar to the housing model in Section 2.3.

Mapping Model to Data. We adjust the data similarly to Section 5.2. The main difference is that the current model does not separate between liquid wealth and debt, and hence, we combine these two categories into net liquid wealth in this Section.

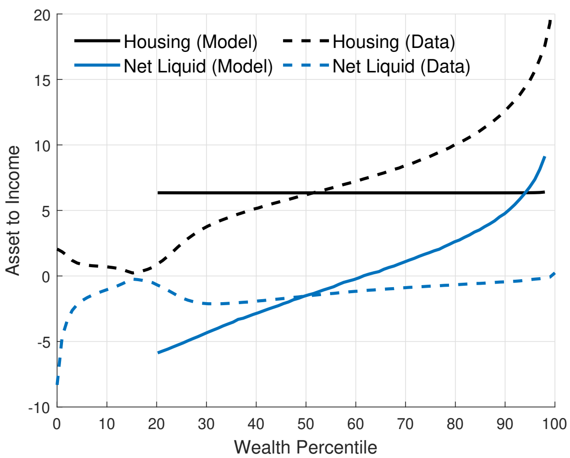
We compute model-implied saving behavior across the wealth distribution as follows. The model's saving policy function is defined for each level of wealth and permanent income. We compute the average level of wealth within every wealth percentile in the data. We then use the saving policy function to compute model-implied net saving at each combination of wealth percentile and permanent income level. Similarly, we compute net income and capital gains at the same points as well as the net and gross saving rates by dividing saving by income. Finally, we compute the weighted average of these saving rates across permanent income deciles within every wealth percentile.

Parameter	Description	Value	Target
Externally set			
r^m	real interest rate	0.0292	average real mortgage rate in Table 2
Calibrated to match the housing share of income and the average level of the net saving rate.			
α	the consumption share	0.9864	
ε_c	preference parameter	1.1	
ε_h	preference parameter	19	
ρ	discount rate	0.025	

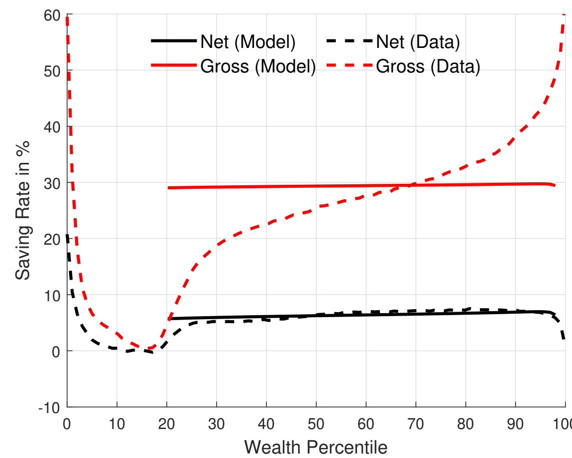
Table E.1: Model calibration.

Calibration. Table E.1 summarizes the calibration. We externally calibrate the interest rate to the average real mortgage rate in the sample period. We calibrate the four remaining parameters (α, ε_c, ε_h, and ρ) to match the share of housing in income across the wealth distribution and the average net saving rate.

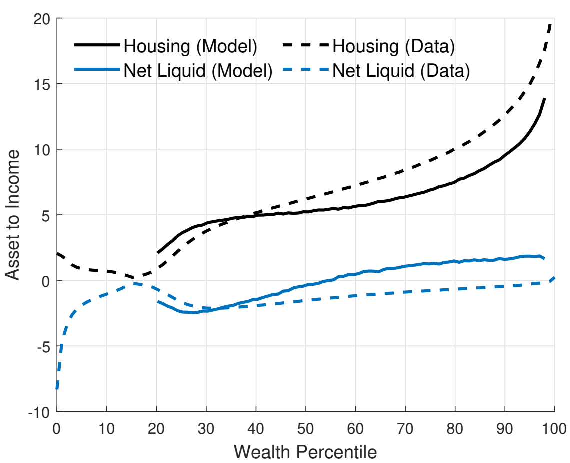
Experiment. Initially, there are no capital gains, so the net saving rate equals the gross saving rate in the model. We construct a model experiment with capital gains by simulating the model responses to a surprise one-year shock (a so-called 'MIT'-shock) of the same magnitude as the empirically observed average growth rates for labor income, house prices, housing rent, and mortgage rates in Table 2. Our experiment thus provides implied behavior under the assumption that households are surprised by the house-price and interest-rate movements, expect them to be permanent, and do not expect any further changes.



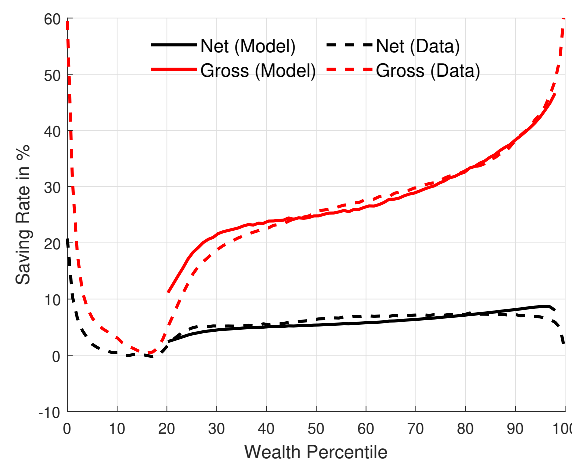
(a) Asset-to-income, Cobb-Douglas.



(b) Saving rates, Cobb-Douglas.



(c) Asset-to-income, non-homothetic.



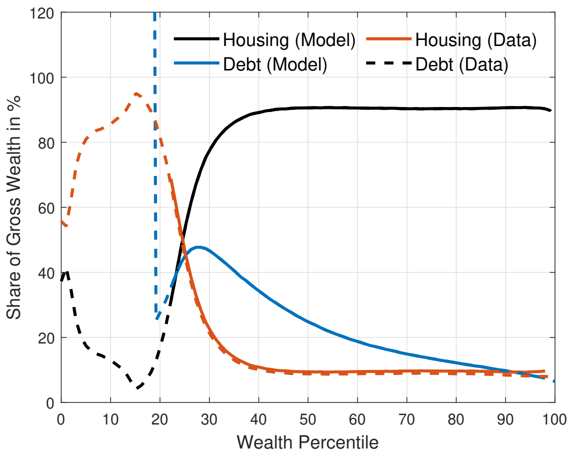
(d) Saving rates, non-homothetic.

Notes: Housing-to-income is defined as housing wealth as a share of net income. The net and gross saving rates are net saving / net income and (net saving + capital gains in housing) / (net income + capital gains in housing), respectively.

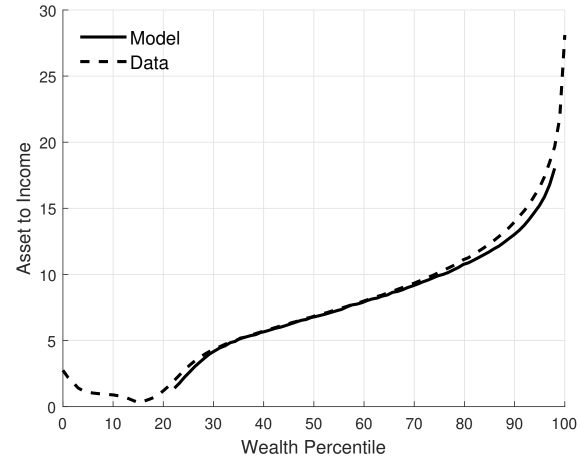
Figure E.1: Asset-to-income shares and saving behavior in a frictionless model.

Saving Behavior in the Model versus the Data. Figure E.1 shows the asset-to-income shares and saving rates in our experiment with growth in prices for two cases: Cobb-Douglas ($\varepsilon_h = \varepsilon_c = 1$) and Wachter and Yogo (2010) preferences ($\varepsilon_h > \varepsilon_c$). With Cobb-Douglas preferences, the share of housing in income remains constant across the wealth distribution. The model generates a flat net saving rate across the wealth distribution. However, because housing as a share of income is constant, the gross saving rate is a parallel shift of the net saving rate. In contrast, with $\varepsilon_h > \varepsilon_c$, the model generates a housing-to-income ratio that increases with wealth, as in the data. Hence, the model can generate a saving rate that is approximately flat across the wealth distribution and a gross saving rate that increases with wealth. Hence, non-homothetic preferences are key to generating housing wealth as a share of income in the data, and thus, how the gross saving rate varies with wealth.

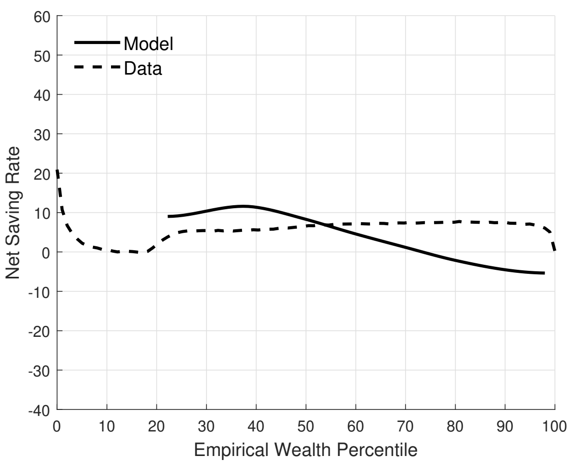
E.2 Model Exercises with Cobb-Douglas Preferences



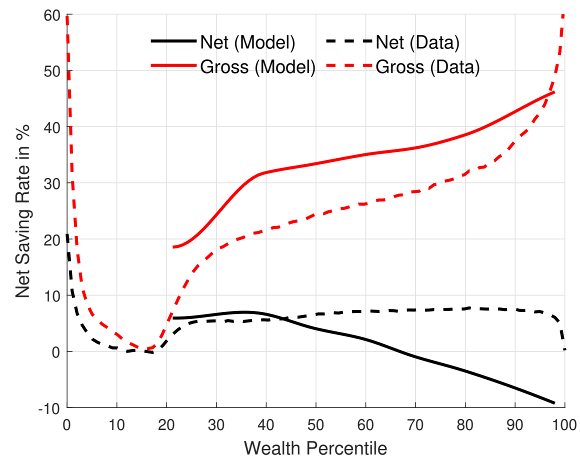
(a) Portfolio shares.



(b) Asset to income.



(c) Initial net saving rate.



(d) Saving rates in transition.

Notes: The figures display the model-implied asset-to-income ratio, portfolio shares, and saving rates, compared with their data counterparts. Asset-to-income equals (housing + liquid wealth)/net income. The housing and debt portfolio shares are housing/(housing + liquid wealth) and debt/(housing + liquid wealth), respectively. The net and gross saving rates are net saving/net income and (net saving + capital gains in housing)/(net income + capital gains in housing), respectively. Panel (c) refers to the initial scenario with constant prices, interest rates, and income. Panel (d) compares the model's average saving rates with the empirical medians when prices, interest rates, and income grow in the model as in the data.

Figure E.2: Saving behavior across the wealth distribution in the model, $\varepsilon_h = \varepsilon_c = 1$.